CS 477/677 Analysis of Algorithms

Fall 2025

Homework 4

Due date: October 9, 2025

1. (U&G-required) [40 points] Assume that you are given a black-box algorithm to find the median, which runs in linear time in the worst case. Write pseudocode for an algorithm that finds the i-th order statistic (i.e., solves the selection problem for an arbitrary order statistic) in linear time.

```
RANDOMIZED-SELECT(A, p, r, i )
     if p = r
         then return A[p]
     x ← Median(A, p, r) // use Median to get a good pivot
     q ← Partition(A, p, r, x) // partition around median
     k ← q - p + 1
     if i = k                 // pivot value is the answer
        then return A[q]
     elseif i < k
             then return RANDOMIZED-SELECT(A, p, q-1, i )
     else return RANDOMIZED-SELECT(A, q + 1, r, i-k)
```

Since the pivot splits the array in half, the recurrence for the algorithm is T(n) = T(n/2) + n, with solution Theta(n).

2. (U&G-required) [30 points]

(a) [15 points] Suppose you have a weighted coin in which heads comes up with probability ¼ and tails with probability ¾. If you flip heads you win \$4, but if you flip tails, you lose \$2. What is the expected value of a coin flip?

$$E = 4 * \frac{1}{4} - 2 * \frac{3}{4} = 1 - \frac{3}{2} = 1 - 1.5 = -0.5$$

(b) [15 points] Supposed that you are drawing a card from a standard 52-card deck (13 cards each for clubs, diamonds, hearts and spades). If you draw a card that is red (diamonds or hearts) you win $10, if the card is clubs you win $4 and if the card is spades you lose $8. What is the expected value of your winnings?

$$E = 10 * \frac{1}{2} + 4 * \frac{1}{4} - 8 * \frac{1}{4} = 5 + 1 - 2 = 4$$

3. (U & G-required) [30 points] Answer the following questions:

(a) [15 points] For the second `PARTITION` algorithm discussed in class (non-randomized, lecture 8, slide 21), what value of `q` does it return when all elements in the array `A[p…r]` have the same value?

The algorithm will return r, the last index of the array. This is due to the following (refer to the algorithm below):

- The algorithm keeps/moves the elements equal to the pivot in the left partition (line 4), which runs from p to i.
- At the end of the for loop, since all the elements are equal to the pivot the left partition will be from index p to i, with i being equal to r-1.
- Line 8 returns the value i+1, which is (r-1) + 1 = r

```
Alg.: PARTITION2(A, p, r)
1. x ← A[r]
2. i ← p - 1
3. for j ← p to r - 1
4.     do if A[ j ] ≤ x
5.              then i ← i + 1
6.                     exchange A[i] ↔ A[j]
7. exchange A[i + 1] ↔ A[r]
8. return i + 1
```

(b) [15 points] Suppose that `RANDOMIZED-SELECT` is used to select the maximum element of the array `A = [2, 3, 0, 5, 7, 9, 1, 8, 6, 4]`. Describe a sequence of partitions that results in a worst-case performance of `RANDOMIZED-SELECT`.

The worst case partitioning happens when the pivot is chosen to be the minimum or the maximum. However, since we are actually seeking the maximum value, choosing the largest element first would be the best case behavior, as the pivot is the maximum, and we will return after the first partition. Therefore, the worst case scenario happens if the pivots are chosen in order from largest to smallest: 0, then 1, then 2, …, then 9.

4. (G-required) [20 points] Write pseudocode for an iterative version of `RANDOMIZED-SELECT`.

Note: alternative solutions are also possible.

```
Alg.: ITERATIVE-RANDOMIZED-SELECT(A, left, right, i )
     while  right > left
     {
          q ← RANDOMIZED-PARTITION(A, left, right)
          k ← q - left + 1
          if i = k                    pivot value is the answer
               then return A[q]
          elseif i < k                // search left partition
                then right ← q - 1
          else left ← q + 1    // search right partition
               i ← i - k
     }
```

Extra credit:

5. [20 points] Suppose you have a bag with 15 slips of paper in it. Some of the slips have a 3 on them and the rest have a 9 on them. If the expected value of the number shown on a slip randomly drawn from the bag is 5, then how many slips have a 3?

$E = 3 * \frac{x}{15} + 9 * \frac{15-x}{15} = 5$ Solve for x and get x =10.