Kalman_Filter

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Core Idea

- Estimate True state of system GIVEN noisy inputs & imperfect model

- Algorithm

1. Predict the next step of the system & how uncertain we are w/ the prediction
2. When the next measurement comes in, combine prediction & measurement, weighing each on their uncertainties.

- The filter both estimates the state AND the state uncertainty (covariance)

biggest advantage

Simple Example

Ball rolling on a line

Setup

- State: [pos, vel]
- We measure position directly (not vel)
- Time step: 1 sec ← We don't have an exact measurement of the system
- Process Noise: Ball might speed up or slow down unpredictably
- Measurement Noise: Position sensor has some error

Initial Conditions

Estimated State: [0, 1]

Uncertainty (P): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Process Noise: (Q) $\begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}$ Measurement Noise: (R) [.5]

State Transition Model (F)

pos-new = [1, 1] [pos-old]
vel-new = [0, 1] [vel-old] $\Rightarrow F = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

t = 1

- Predict

State Prediction = F × prev-state = [1, 1] [0, 1] = [1, 1] Where will the system be next?

Uncertainty Prediction: $P = \underbrace{F \times P \times F^T}_{\text{Prediction}} + \underbrace{Q}_{\text{Environment}}$ How uncertain am I about my State Prediction?

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix} = \begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix}$$

Note: (1) Position uncertainty grew 1 → 2.1 b/c vel affects pos
(2) Correlation appeared in off-diagonal vel & pos uncertainties linked

- Update

position measurement = 1.2

Measurement Matrix: H = [1, 0] what of the state we are measuring

H = [1, 0] = [1 × pos, 0 × vel]

Innovation The diff b/t what we measured vs. expected to measure

1.2 − (1 × 1 + 0 × 1) = .2 measurement was .2 higher than expected

Kalman Gain: Calculate how much to trust the measurement vs. our prediction

$$k = P \times H^T \,/\, (H \times P \times H^T + R)$$

$$\frac{\begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix} \times [1\ 0]^T}{[1\ 0] \begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix} [1\ 0]^T + [.5]} = \frac{[2.1\ 1]^T}{2.1 + .5} = [.81\ .38]$$

Update State Combine prediction & measurement weighted kalman Gain

$$\underbrace{[1\ 1]}_{\text{Predicted State}} + \underbrace{[.81\ .38]}_{\text{kalman Gain}} \times \underbrace{.2}_{\text{Innovation}} = [1.16\ 1.08]$$

Notice how velocity also updated

Updated uncertainty: Reduces uncertainty given the data we recieved

$$P = (I - k \times H) \times P$$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - [.81\ .38] [1\ 0] \right) \times \begin{bmatrix} 2.1 & 1 \\ 1 & 1.1 \end{bmatrix}$$

$$= \begin{bmatrix} .29 & .19 \\ .19 & .69 \end{bmatrix}$$

t = N

measured pos

$\sqrt{\sigma^2}$ if $P = \begin{bmatrix} \sigma^2 & \\ & \sigma^2 \end{bmatrix}$

t	Predicted State	pos	Updated state	P_{pos}	P_{vel}
1	[1.00 1.00]	1.2	[1.16 1.08]	.54	.83
2	[2.24 1.08]	2.1	[2.14 1.02]	.61	.63
3	[3.15 1.02]	3.3	[3.26 1.07]	.60	.53
4	[4.33 1.07]	4.2	[4.24 1.03]	.59	.50
5	[5.27 1.03]	5.1	[5.16 .99]	.58	.50
6	[6.14 .99]	6.0	[6.05 .95]	.57	.50