

Neural RRT*

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Core Idea

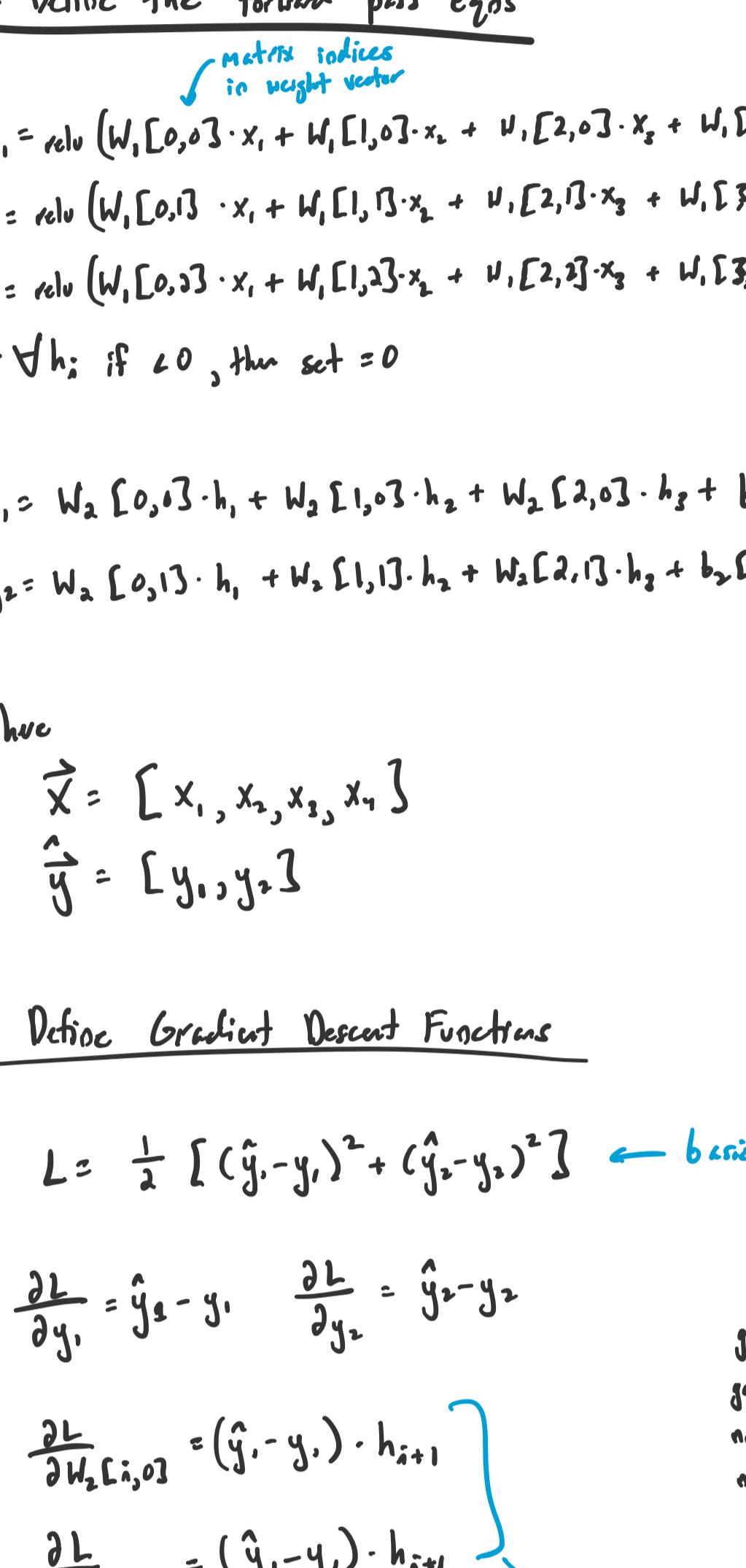
Neural RRT* is Neural Network + RRT*

- RRT* randomly samples points to build its tree
- if we are smart about where we sample, RRT* becomes much better
- if we are smart about which connections to explore, RRT* can converge to solution / optimal solution faster

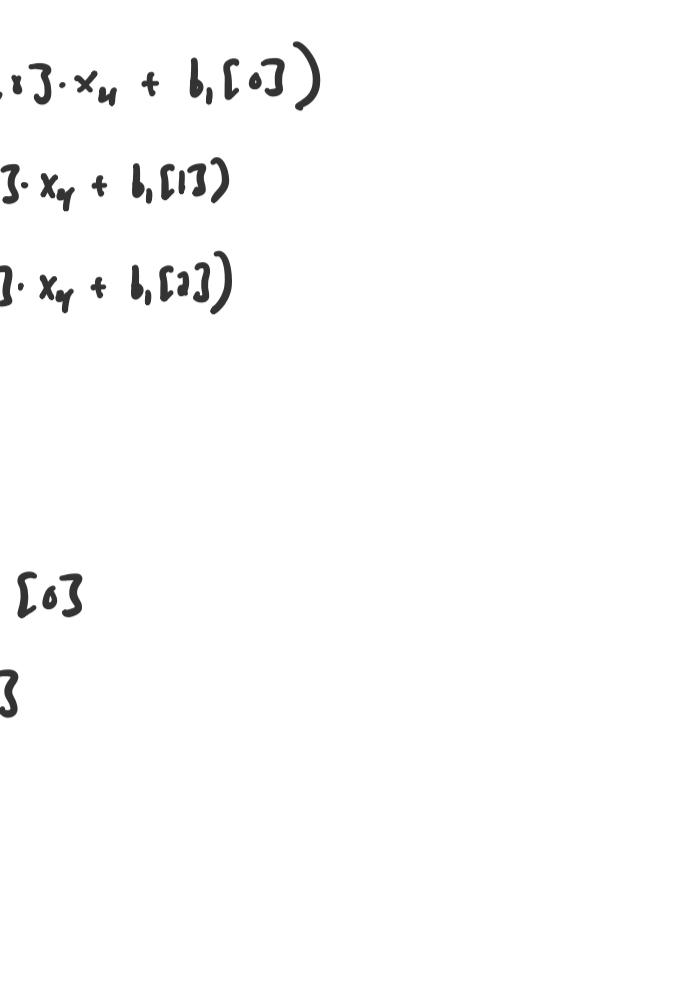
The Neural Net!

Step 1 - Initialization

$$\text{Start} = [1, 1] \quad \text{Goal} = [8, 8]$$



A. Neural Net Definition



We define a set of vectors to store weights

W_1 : input \rightarrow hidden
 W_2 : hidden \rightarrow output
 b_1 : hidden bias
 b_2 : output bias

B. Randomly Initialize weights

$$W_1 = \begin{bmatrix} [0.5, -0.3, 0.2] \\ [1.1, 0.4, -0.1] \\ [-0.2, 0.3, 0.6] \\ [-0.4, -0.2, 0.1] \end{bmatrix} \quad W_2 = \begin{bmatrix} [0.8, 0.2] \\ [-0.1, 0.7] \\ [0.3, -0.4] \end{bmatrix}$$

$$b_1 = [0.1, -0.2, 0.3] \quad b_2 = [0, 0]$$

C. Define the forward pass eqns

$$h_1 = \text{relu}(W_1[0,0] \cdot x_1 + W_1[1,0] \cdot x_2 + W_1[2,0] \cdot x_3 + W_1[3,0] \cdot x_4 + b_1[0])$$

$$h_2 = \text{relu}(W_1[0,1] \cdot x_1 + W_1[1,1] \cdot x_2 + W_1[2,1] \cdot x_3 + W_1[3,1] \cdot x_4 + b_1[1])$$

$$h_3 = \text{relu}(W_1[0,2] \cdot x_1 + W_1[1,2] \cdot x_2 + W_1[2,2] \cdot x_3 + W_1[3,2] \cdot x_4 + b_1[2])$$

* If $h_i < 0$, then set = 0

$$y_1 = W_2[0,0] \cdot h_1 + W_2[1,0] \cdot h_2 + W_2[2,0] \cdot h_3 + b_2[0]$$

$$y_2 = W_2[0,1] \cdot h_1 + W_2[1,1] \cdot h_2 + W_2[2,1] \cdot h_3 + b_2[1]$$

Where

$$\vec{x} = [x_1, x_2, x_3, x_4]$$

$$\vec{y} = [y_1, y_2]$$

D. Define Gradient Descent Functions

$$L = \frac{1}{2} [(\hat{y}_1 - y_1)^2 + (\hat{y}_2 - y_2)^2] \quad \leftarrow \text{basic MSE}$$

$$\frac{\partial L}{\partial y_1} = \hat{y}_2 - y_1 \quad \frac{\partial L}{\partial y_2} = \hat{y}_1 - y_2$$

$$\frac{\partial L}{\partial W_1[0,0]} = (\hat{y}_1 - y_1) \cdot h_{1+1}$$

$$\frac{\partial L}{\partial W_1[1,0]} = (\hat{y}_1 - y_1) \cdot h_{2+1}$$

$$\frac{\partial L}{\partial W_1[2,0]} = (\hat{y}_1 - y_1) \cdot h_{3+1}$$

$$\frac{\partial L}{\partial W_1[3,0]} = (\hat{y}_1 - y_1) \cdot h_{4+1}$$

$$\frac{\partial L}{\partial b_1[0]} = \frac{\partial L}{\partial h_{1+1}}$$

$$\frac{\partial L}{\partial b_1[1]} = \frac{\partial L}{\partial h_{2+1}}$$

$$\frac{\partial L}{\partial b_1[2]} = \frac{\partial L}{\partial h_{3+1}}$$

$$\frac{\partial L}{\partial b_1[3]} = \frac{\partial L}{\partial h_{4+1}}$$

$$\frac{\partial L}{\partial W_2[0,0]} = (\hat{y}_2 - y_2) \cdot W_2[0,0]$$

$$\frac{\partial L}{\partial W_2[1,0]} = (\hat{y}_2 - y_2) \cdot W_2[1,0]$$

$$\frac{\partial L}{\partial W_2[2,0]} = (\hat{y}_2 - y_2) \cdot W_2[2,0]$$

$$\frac{\partial L}{\partial W_2[3,0]} = (\hat{y}_2 - y_2) \cdot W_2[3,0]$$

$$\frac{\partial L}{\partial b_2[0]} = \frac{\partial L}{\partial \hat{y}_1}$$

$$\frac{\partial L}{\partial b_2[1]} = \frac{\partial L}{\partial \hat{y}_2}$$

E. Weight update rules

$$W_1[i,j] = W_1[i,j] - \alpha \cdot \frac{\partial L}{\partial W_1[i,j]}$$

$$W_2[i,j] = W_2[i,j] - \alpha \cdot \frac{\partial L}{\partial W_2[i,j]}$$

$$b_1[i] = b_1[i] - \alpha \cdot \frac{\partial L}{\partial b_1[i]}$$

$$b_2[i] = b_2[i] - \alpha \cdot \frac{\partial L}{\partial b_2[i]}$$

Step 2: Iteration 1

A. Analyze Tree

- Coverage - bottom left of grid
- 1 node (S)
- Distance to Goal: $d = \sqrt{(8-1)^2 + (8-1)^2} = 9.9$

B. Determine NN Input

$$[\text{goal}-x, \text{goal}-y, \text{nn}-x, \text{nn}-y]$$

$$[8, 8, 1, 1]$$

C. NN Forward Pass

$$\hat{y} = [4.487, 0.562]$$

* But check this is a bad guess!

$$A = [4.487, 0.562]$$

$$B = [8, 8]$$

$$S = [1, 1]$$

$$G = [8, 8]$$

$$\text{Distance to goal} = \sqrt{(8-1)^2 + (8-1)^2} = 9.9$$

$$\text{Distance to nearest node} = \sqrt{(4.487-1)^2 + (0.562-1)^2} = 3.56$$

$$S(3.56) \rightarrow S is closest$$

Check for collision \rightarrow NO collision ✓

Step 3 - Iteration 2

A. Analyze Tree

- Nodes: 2 [$A(7.11), S(9.90)$]

B. NN Input

$$[\text{goal}-x, \text{goal}-y, \text{nn}-x, \text{nn}-y]$$

$$[8, 8, 4.487, 0.562]$$

C. NN Forward Pass

$$\hat{y} = [7.23, 0.23]$$

* But check this is a bad guess!

$$A = [7.23, 0.23]$$

$$B = [8, 8]$$

$$S = [1, 1]$$

$$G = [8, 8]$$

$$\text{Distance to goal} = \sqrt{(8-1)^2 + (8-1)^2} = 9.9$$

$$\text{Distance to nearest node} = \sqrt{(7.23-1)^2 + (0.23-1)^2} = 3.46$$

$$A(3.46) \rightarrow A is closer$$

$$d = 0 + 3.46 = 3.46$$

$$\text{Lower! Revise}$$

$$- via (4.487, 0.562) (current)$$

$$d = 3.56 + .26 = 3.82$$

F. Performance Assessment

$$[A(7.11), B(8.25), S(9.9)]$$

Further

NN will not be able to reach goal...

Step 4 - Training

A. Collect Bad suggestions

Training Sample:

$$\text{input state: } [8, 8, 4.487, 0.562]$$

$$\hat{y} = [7.23, 0.23]$$

* we choose this ground truth

$$y = [7, 2]$$

B. Calculate Loss

$$L = \frac{1}{2} [(\hat{y}_1 - y_1)^2 + (\hat{y}_2 - y_2)^2]$$

$$L = 4.318$$

C. Backprop

$$\alpha = .01$$

This isn't a backprop tutorial, so...

after the updates we re-run the same point

$$\text{Initial prediction: } [4.487, 0.562]$$

Epoch 1: [8.665, 2.745] closer

Epoch 2: [6.69, 1.78] VERY close!

Step 5 - Repeat

It's clear now how this works

once the net is trained, it will react

appropriately to the given situation it's to

check for collision \rightarrow NO collision ✓

NN will not be able to reach goal...

Step 6 - Summary

Neural RRT* is Neural Network + RRT*

RRT* randomly samples points to build its tree

if we are smart about where we sample, RRT* becomes much better

if we are smart about which connections to explore, RRT* can converge to solution / optimal solution faster

The Neural Net!

Forward pass: $y = W_1 x + b_1$

Backprop: $\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial W_1}$

$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial b_1}$

Loss function: $L = \frac{1}{2} [(\hat{y}_1 - y_1)^2 + (\hat{y}_2 - y_2)^2]$

Gradient descent: $W_1[i,j] = W_1[i,j] - \alpha \cdot \frac{\partial L}{\partial W_1[i,j]}$

$b_1[i] = b_1[i] - \alpha \cdot \frac{\partial L}{\partial b_1[i]}$

Epoch 1 guess: [8, 8]

Epoch 2 guess: [8.665, 2.745]

Epoch 3 guess: [6.69, 1.78]

Epoch 4 guess: [4.487, 0.562]

Epoch 5 guess: [7.23, 0.23]

Epoch 6 guess: [8, 8]