TrajOpt

Monday, August 4, 2025 4:54 PM

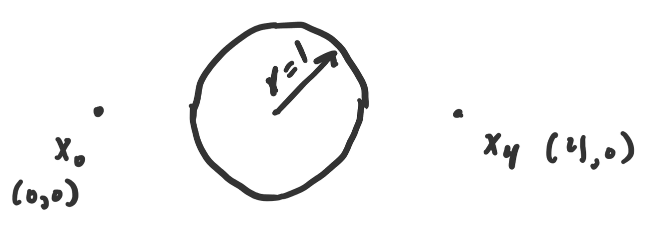
Trajectory Optimization

Core Idea

Trajectory Optimization problems are often not convex

TrajOpt breaks the problem into a set of convex approximations

Example

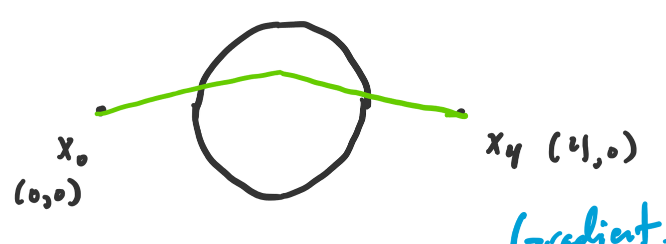


Optimize x_1, x_2, x_3

Objective: $\text{Min } \sum \|x_{i+1} - x_i\|^2$ } smoothness

Constraint: $\forall x_i,\ \|x_i - (2,0)\| \geq 1$ } Do Not intersect with circle

Step 1: Initialization



$x_1 = (1, .2)$
$x_2 = (2, .3)$
$x_3 = (3, .2)$

Gradients

$$\nabla_x d = \frac{(x-2)}{\sqrt{(x-2)^2+y^2}} = \frac{(x-2)}{d} \qquad \nabla_y d = \frac{y}{\sqrt{(x-2)^2+y^2}} = \frac{y}{d}$$

Step 2: Iteration 1

A. Check constraint violations:

- $x_1 = (1, .2)$
  - Distance from (2,0)
    $d_1 = \sqrt{(1-2)^2 + (.2-0)^2} = 1.02$
  - constraint
    $d_1 - 1 = 1.02 - 1 = .02 \geq 0$ ✓

- $x_2 = (2, .3)$
  - Distance from (2,0)
    $d_2 = .3$
  - constraint
    $.3 - 1 = -.7 < 0$ X violated

- $x_3 = (3, .2)$
  - Distance from (2,0)
    $d_3 = 1.02$
  - constraint
    $.02 \geq 0$ ✓

B. Calculate Gradient for Violation

Constraint: $g(x,y) = \sqrt{(x-2)^2+y^2} - 1$

Gradient: $\nabla g = \left(\frac{x-2}{\sqrt{(x-2)^2+y^2}}, \frac{y}{\sqrt{(x-2)^2+y^2}} \right)$

$$\nabla g = \left(\frac{x-2}{d}, \frac{y}{d} \right)$$

- $x_2 = (2, .3)$
  $d_2 = .3$
  $\nabla g(2,.3) = \left(\frac{2-2}{.3}, \frac{.3}{.3} \right)$
  $\nabla g(2,.3) = (0,1)$

C. Create Linear Approximation

The linear approximation of g around $(2, .3)$ is

$g(x,y) \approx g(2,.3) + \nabla g(2,.3) \cdot [x-2, y-.3]$ } 1st order Taylor expansion

Given: (1) $g(x,y) = \sqrt{(x-2)^2+y^2} - 1 \Rightarrow g(2,.3) = -.7$

(2) $\nabla g = \left(\frac{x-2}{\sqrt{(x-2)^2+y^2}}, \frac{y}{\sqrt{(x-2)^2+y^2}} \right) \Rightarrow \nabla g(2,.3) = [0,1]$

Plugging in...

$$g(x,y) \approx -.7 + \cancel{0 \cdot (x-2)}^{0} + 1 \cdot (y - .3)$$
$$\approx -.7 + y - .3$$
$$g(x,y) \approx y - 1$$

From constraint eq

$$g(x,y) = d - 1 \geq 0$$
$$g(x,y) \geq 0$$

Substituting

$$y - 1 \geq 0$$
$$y \geq 1$$

D. Convex Optimization

Objective

Solve for this

$$\text{Min } \|x_1 - x_0\|^2 + \|x_2 - x_1\|^2 + \|x_3 - x_2\|^2 + \|x_4 - x_3\|^2$$
$$= \|(1,.2) - (0,0)\|^2 + \|(x_2) - (1,.2)\|^2 + \|(3,.2) - (x_2)\|^2 + \|(4,0) - (3,.2)\|^2$$
$$= (1^2 + .2^2) + (x_{2x} - 1)^2 + (x_{2y} - .2)^2 + (3 - x_{2x})^2 + (.2 - x_{2y})^2 + (1^2 + .2^2)$$

...

$$f(x_{2x}, x_{2y}) = 2x_{2x}^2 - 8x_{2x} + 2x_{2y}^2 - .8x_{2y} + 10.08$$

$$\frac{\partial f}{\partial x_{2x}} = 4x_{2x} - 8, \quad \frac{\partial f}{\partial x_{2y}} = 4x_{2y} - .8$$

Set partial derivatives $= 0$

$x_{2x} = 2$ $\qquad$ $x_{2y} = .2$

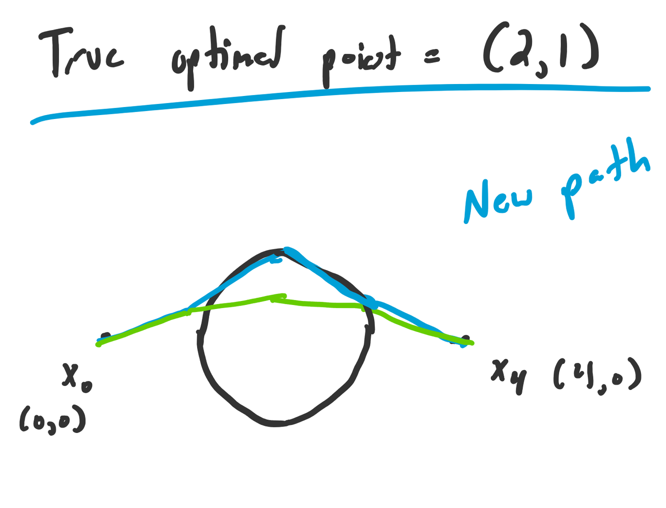
So, the optimal min is $(2, .2)$ **BUT** $.2 \ngtr 1$ X, so the constraint is violated

Accounting for constraints, x_{2x} is the same & x_{2y} is forced to 1

Quick gut check...

$$f(x_{2x}, 1) = 2x_{2x}^2 - 8x_{2x} + 11.28$$
$$\frac{\partial f}{\partial x_{2x}} = 4x_{2x} - 8 \Rightarrow x_{2x} = 2 \checkmark$$

True optimal point = (2,1)



Step 3: Continue until convergence or # iterations reached

Will stop here b/c algorithm converges to (2,1)