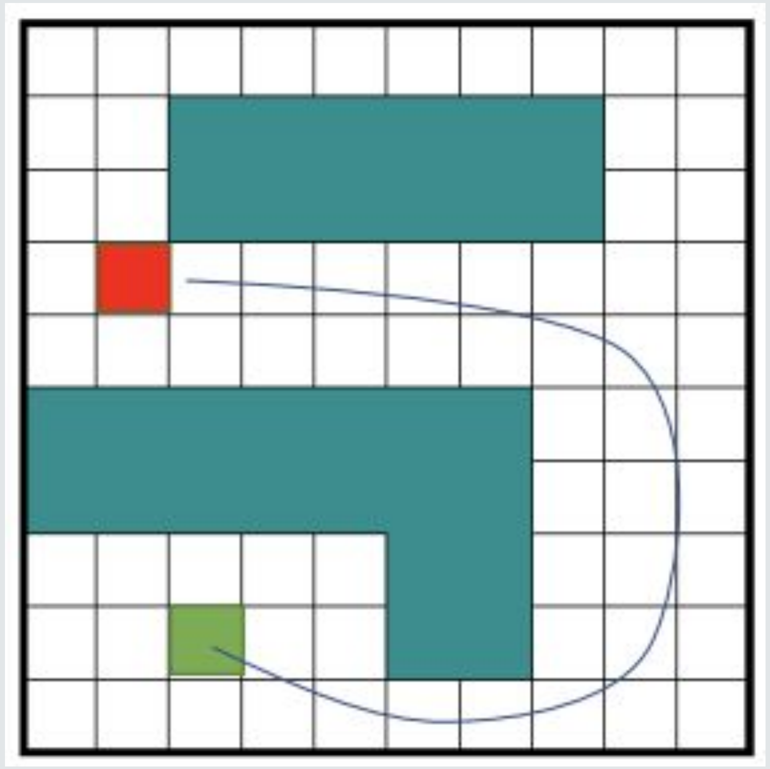
Robot Autonomy

DR. RISMAN ADNAN MATTOTORANG

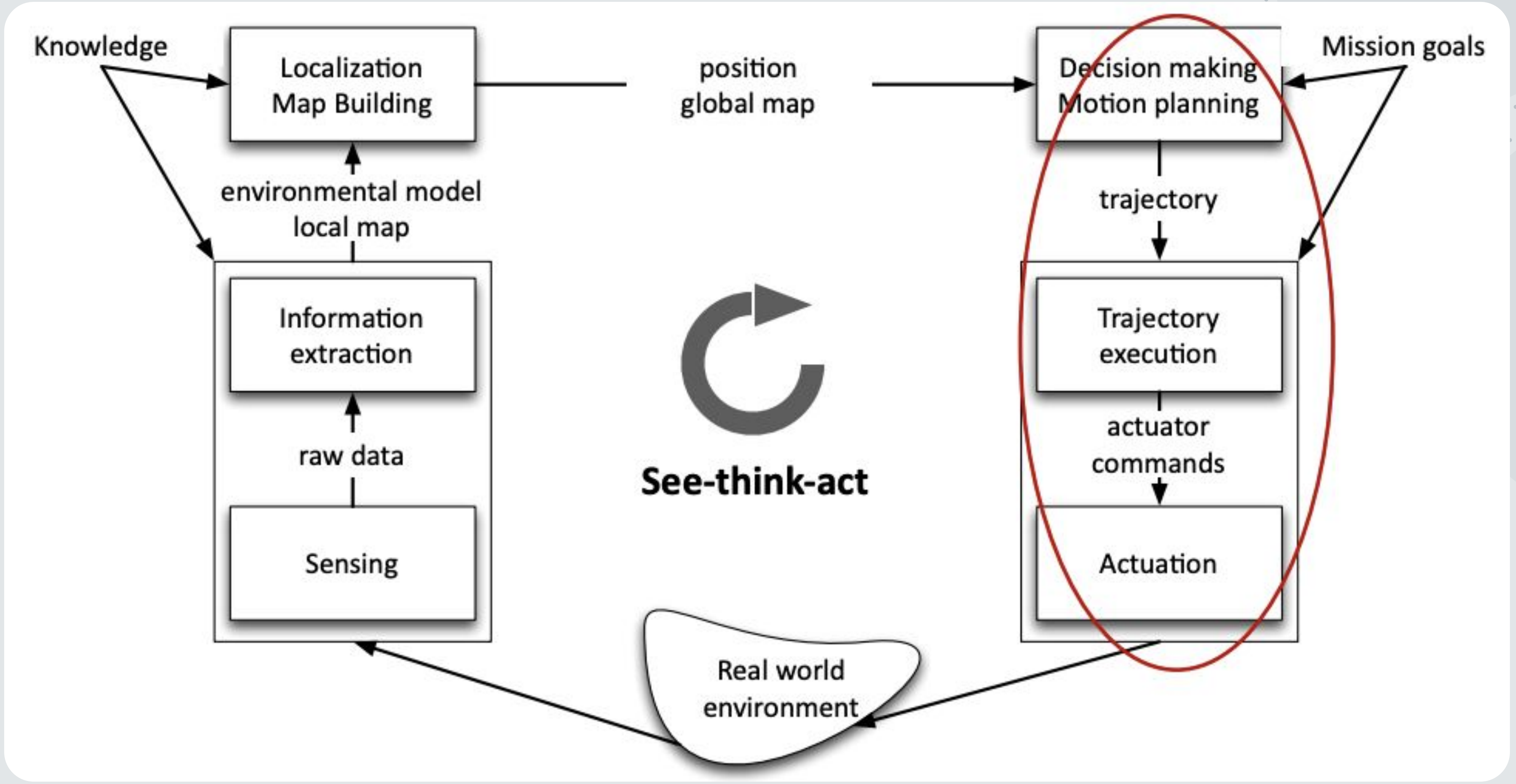
TELKOM UNIVERSITY

Principles of Robot Autonomy I

W5: Motion planning I: Graph Search Methods



The see-think-act cycle

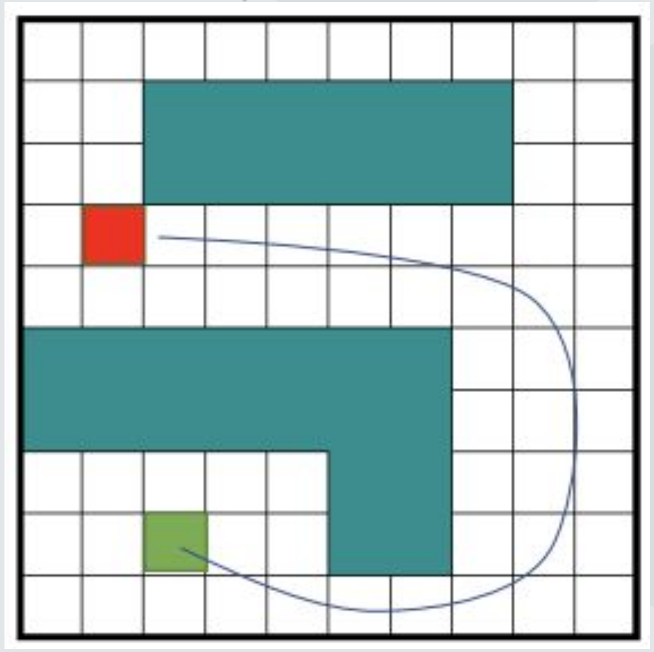


Motion planning

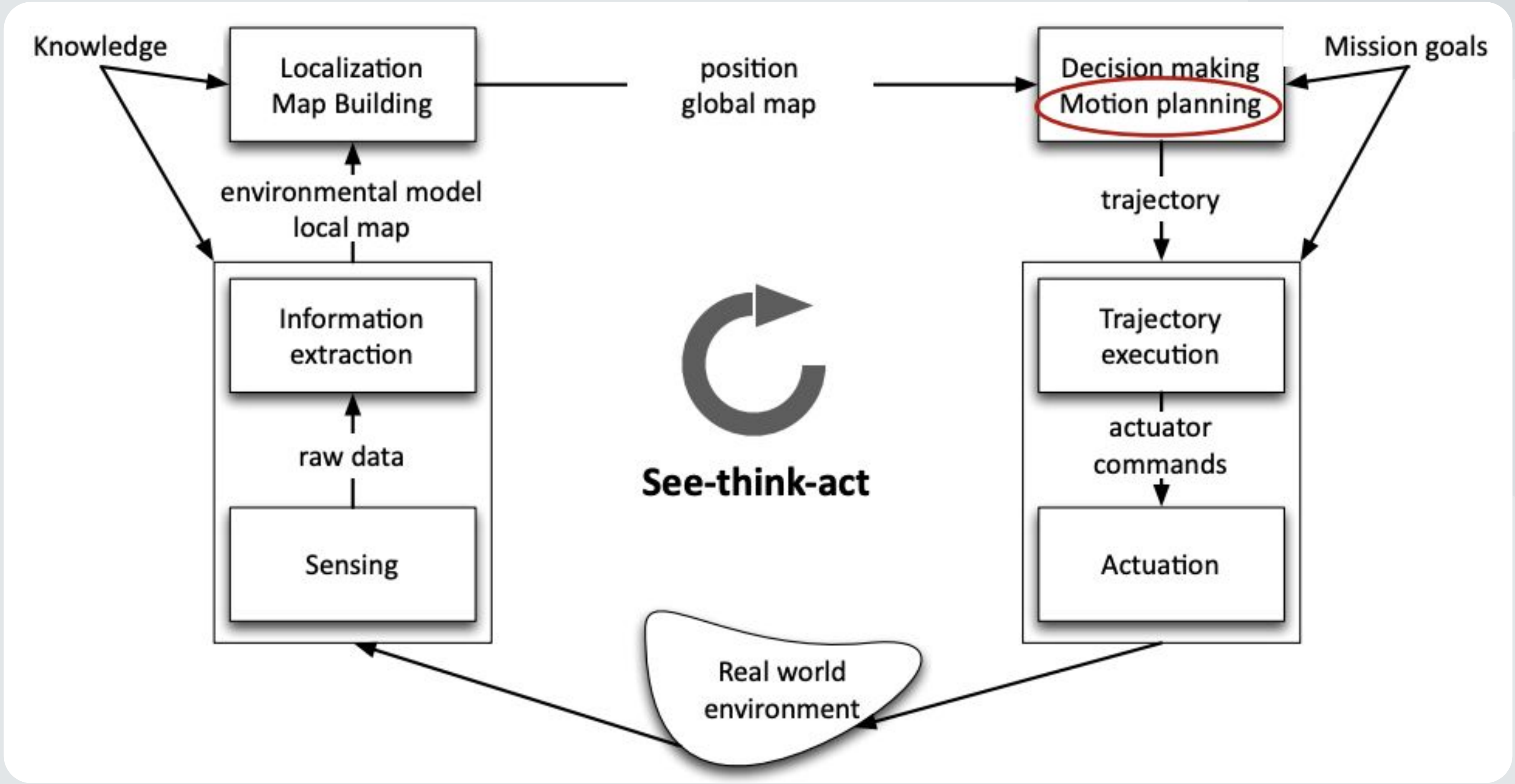
Compute sequence of actions that drives a robot from an initial condition to a terminal condition while avoiding obstacles, respecting motion constraints, and possibly optimizing a cost function

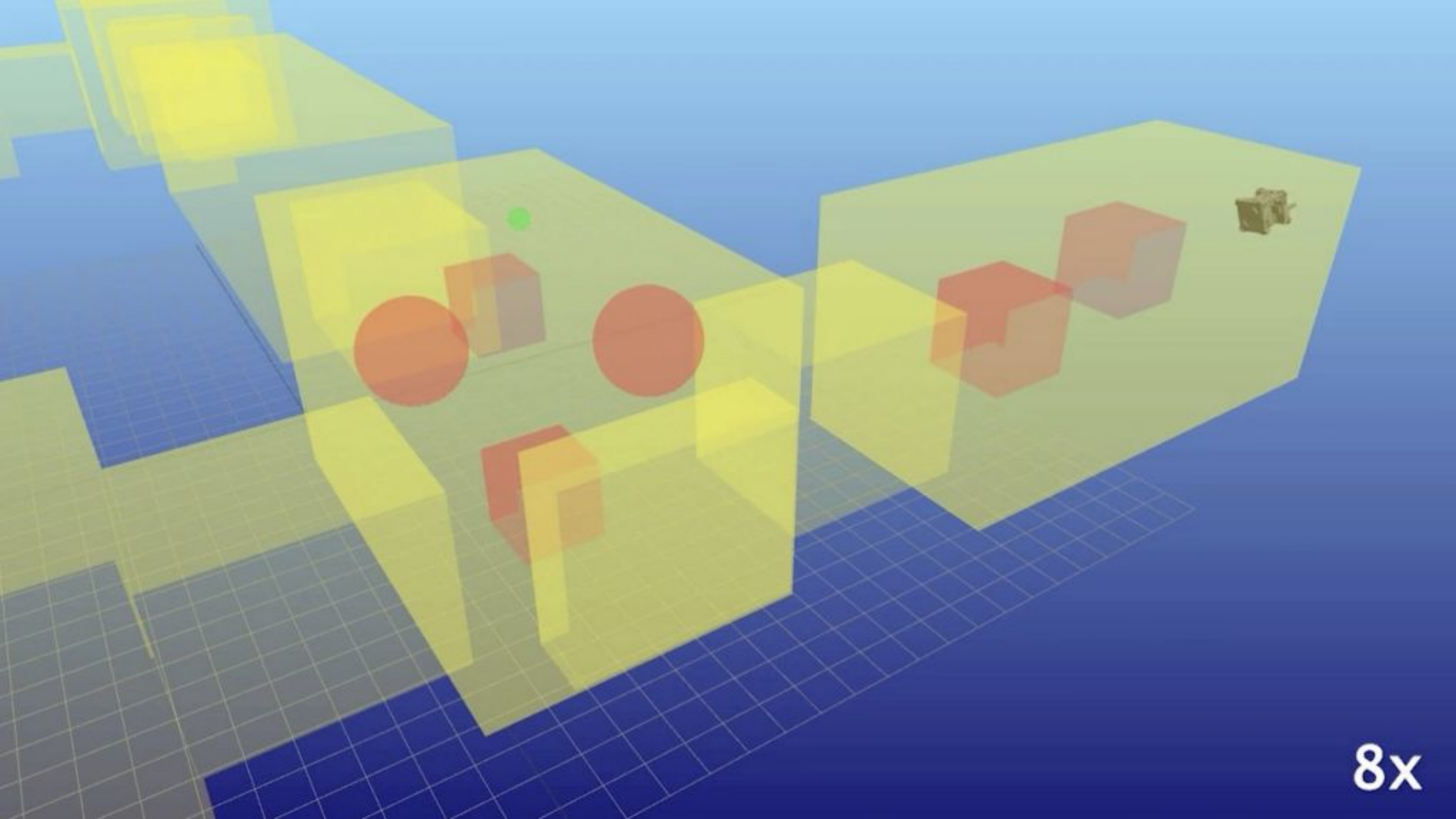
- Aim
  - Introduction to motion planning
  - Learn about search-based methods for motion planning

- Readings:
  - D. Bertsekas. Dynamic Programming and Optimal Control, Vol I. Section 2.3.
  - S. LaValle. Planning Algorithms. Sections 6.1-6.3, 6.5.



The see-think-act cycle

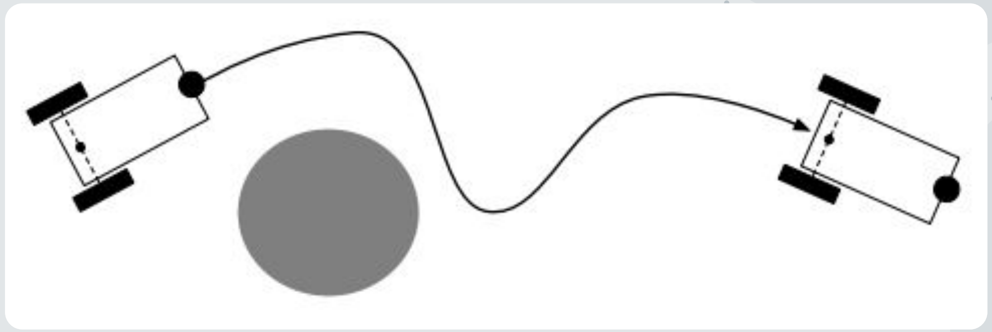


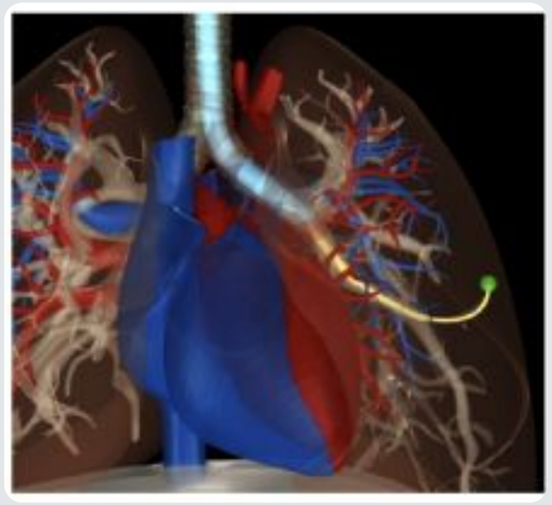

8x

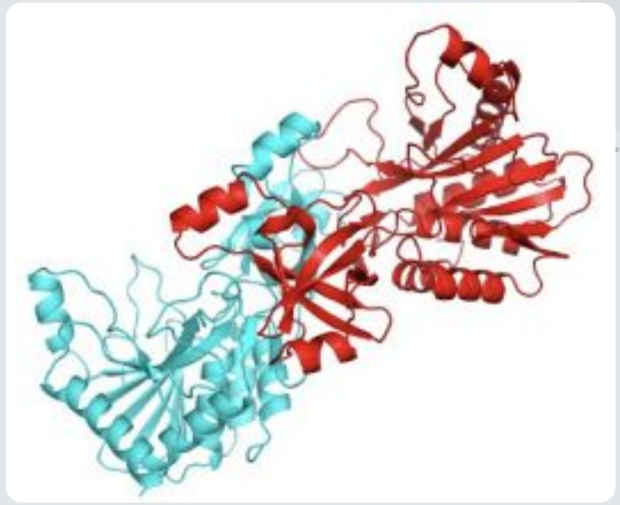
Examples from:
https://ompl.kavrakilab.org/gallery.html

More examples of motion planning

- Steering autonomous vehicles
- Controlling humanoid robot
- Surgery planning
- Protein folding
- …





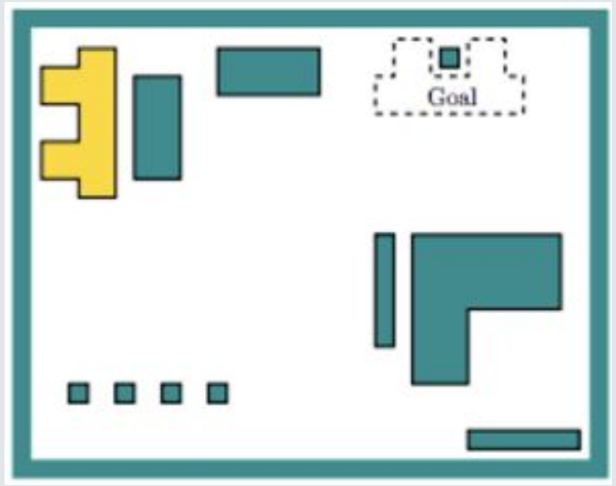


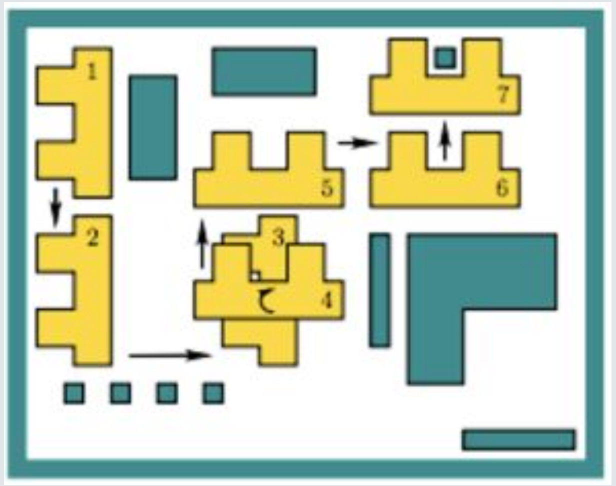
Some History

- Formally defined in the 1970s
- Development of exact, combinatorial solutions in the 1980s
- Development of sampling-based methods in the 1990s
- Deployment on real-time systems in the 2000s
- Current research: inclusion of differential and logical constraints, planning under uncertainty, parallel implementation, feedback plans and more

Simplest Setup

- Assume 2D workspace: $\mathcal{W} \subseteq \mathbb{R}^2$
- $\mathcal{O} \subset \mathcal{W}$ is the obstacle region with polygonal boundary
- Robot is a rigid polygon
- Problem: given initial placement of robot, compute how to gradually move it into a desired goal placement so that it never touches the obstacle region





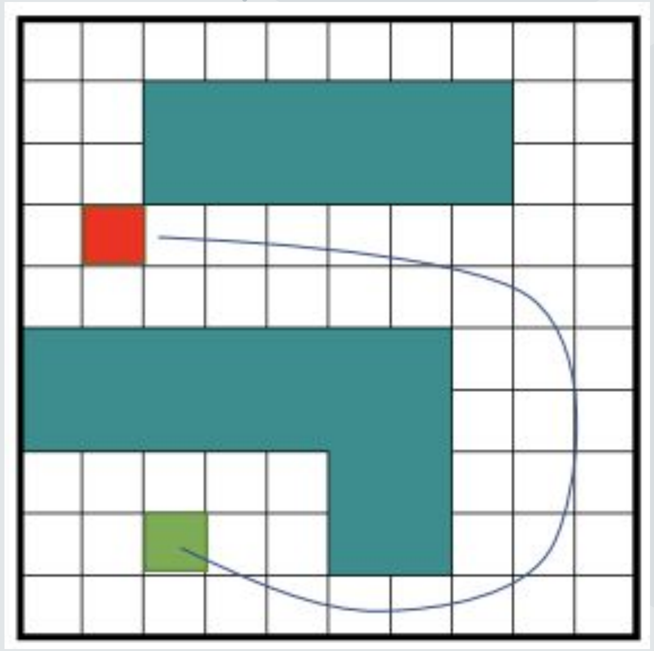
Popular approaches

- *Potential fields* **[Rimon, Koditschek, '92]**: create forces on the robot that pull it toward the goal and push it away from obstacles
- *Grid-based planning* **[Stentz, '94]**: discretizes problem into grid and runs a graph-search algorithm (Dijkstra, A*, …)
- *Combinatorial planning* **[LaValle, '06]**: constructs structures in the configuration (C-) space that completely capture all information needed for planning
- *Sampling-based planning* **[Kavraki et al, '96; LaValle, Kuffner, '06, etc.]**: uses collision detection algorithms to probe and incrementally search the C-space for a solution, rather than completely characterizing all of the C-… structure

Grid-based Approaches

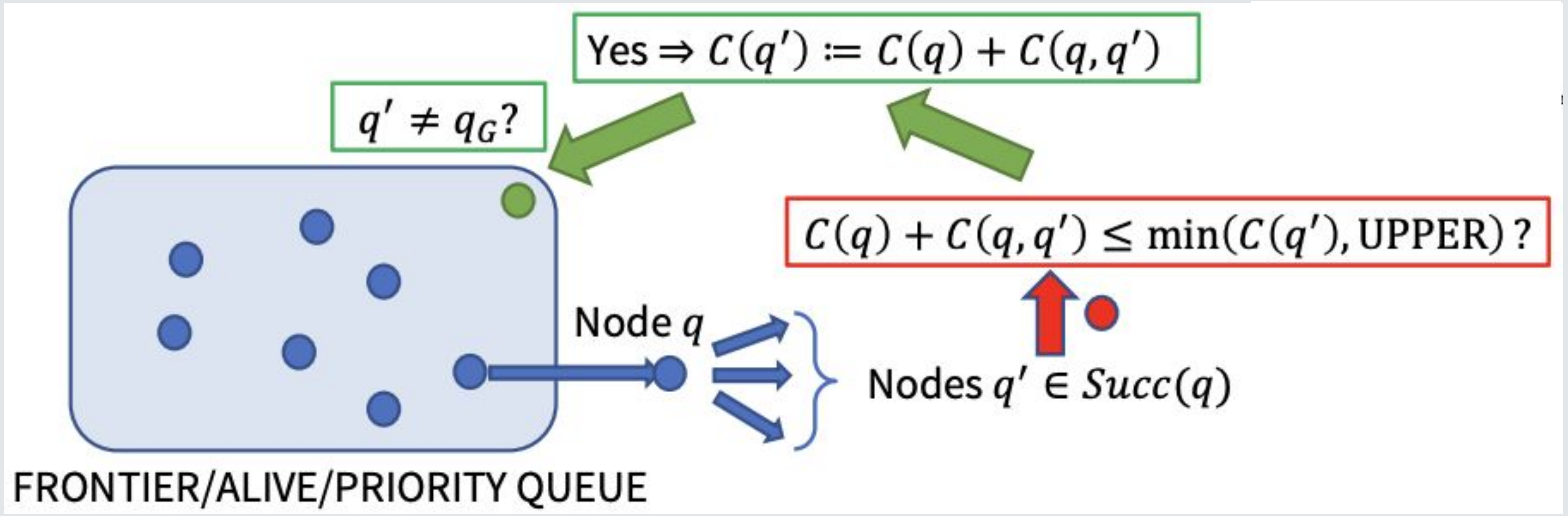
- Discretize the continuous world into a grid
  - Each grid cell is either free or forbidden
  - Robot moves between adjacent free cells
  - Goal: find sequence of free cells from start to goal

- Mathematically, this corresponds to pathfinding in a discrete graph $G = (V, E)$
  - Each vertex $v \in V$ represents a free cell
  - Edges $(v, u) \in E$ connect adjacent grid cells



Graph Search Algorithms

- Having determined decomposition, how to find "best" path?
- **Label-Correcting Algorithms:** $C(q)$ cost-of-arrival from q_1 to q



Yes $\Rightarrow C(q') \coloneqq C(q) + C(q,q')$
$q' \neq q_G$?
$C(q) + C(q,q') \leq \min(C(q'), \text{UPPER})$?
Node q
Nodes $q' \in Succ(q)$
FRONTIER/ALIVE/PRIORITY QUEUE

Illustration of Dijkstra's algorithm finding a path from a start node (lower left, red) to a goal node (upper right, green) in a robot motion planning problem. Open nodes represent the "tentative" set (aka set of "unvisited" nodes). Filled nodes are the visited ones, with color representing the distance: the greener, the closer. Nodes in all the different directions are explored uniformly, appearing more-or-less as a circular wavefront as Dijkstra's algorithm uses a heuristic identically equal to 0.

https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

Label Correcting Algorithm

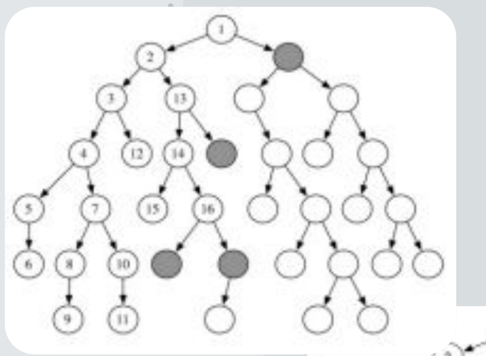
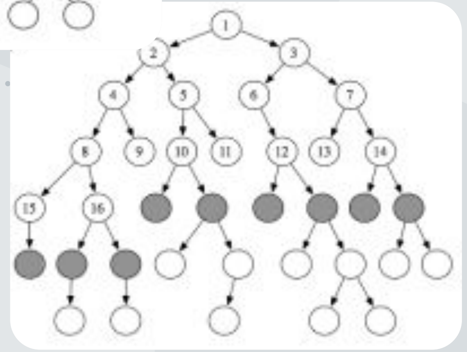
- **Step 1.** Remove a node $\boldsymbol{q}$ from frontier queue and for each child $\boldsymbol{q'}$ of $\boldsymbol{q}$, execute step 2

- **Step 2.** If $c(q) + C(q, q') \leq \min(C(q'), UPPER)$, $set\ C(q') \coloneqq C(q) + C(q, q')$ and set ▯ to be the parent of q' . In addition, if $q' \neq q_G, place\ q'$ in the frontier queue if it is not already there, while if $q' = q_G$, set UPPER to the new value $C(q) + C(q, q_G)$

- **Step 3.** If the frontier queue is empty, terminate, else go to step 1

- **Initialization:** set the labels of all nodes to ∞, except for the label of the origin node, which is set to 0

GetNext() ?

- Depth-First-Search (DFS): Maintain Q as a **stack** – Last in/first out
  - Lower memory requirement (only need to store part of graph)

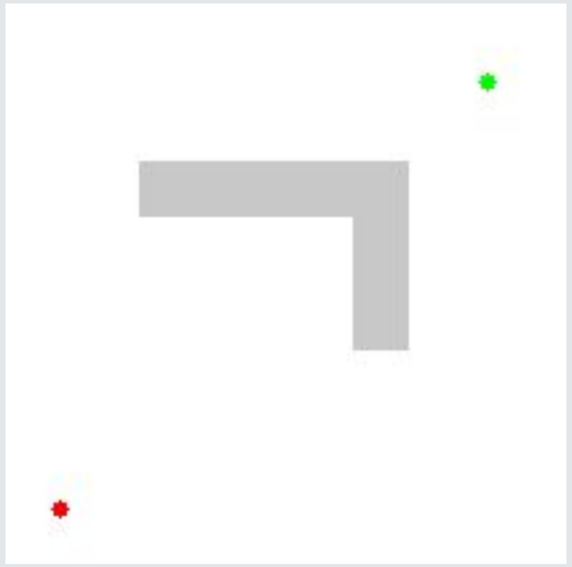
- Breadth-First-Search (BFS, Bellman-Ford): Maintain Q as a **list** – First in/first first out
  - Update cost for all edges up to current depth before proceeding to greater depth
  - Can deal with negative edge (transition) costs

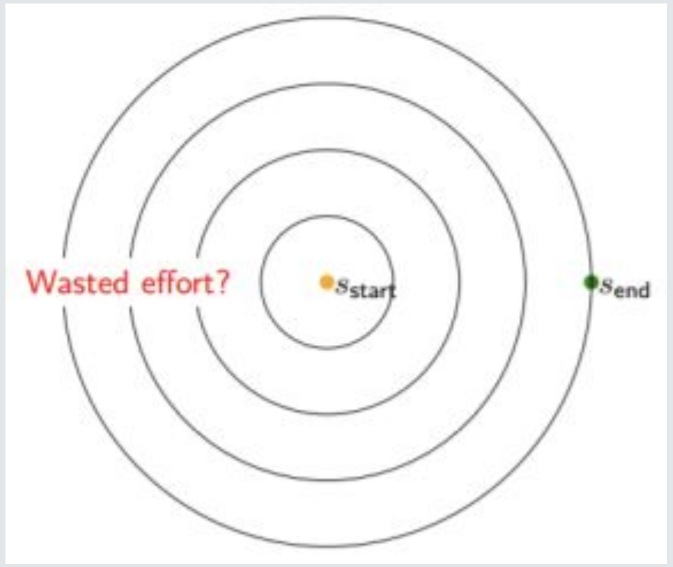
- Best-First (BF, Dijkstra): Greedily select next q: $q = \mathbf{argmin}_{q \in Q} C(q)$
  - Node will enter the frontier queue at most *once*
  - Requires costs to be non-negative

Correctness and Improvements

Theorem

If a feasible path exists from q_1 to q_G , then algorithm terminates in finite time with $\mathtt{C}(q_G)$ equal to the optimal cost of traversal, $\mathtt{C}^*(q_G)$





A*: Improving Dijkstra

- Dijkstra orders by optimal **"cost-to-arrival"**
- Faster results if order by **"cost-to-arrival"+ (approximate) "cost-to-go"**

That is, strengthen test

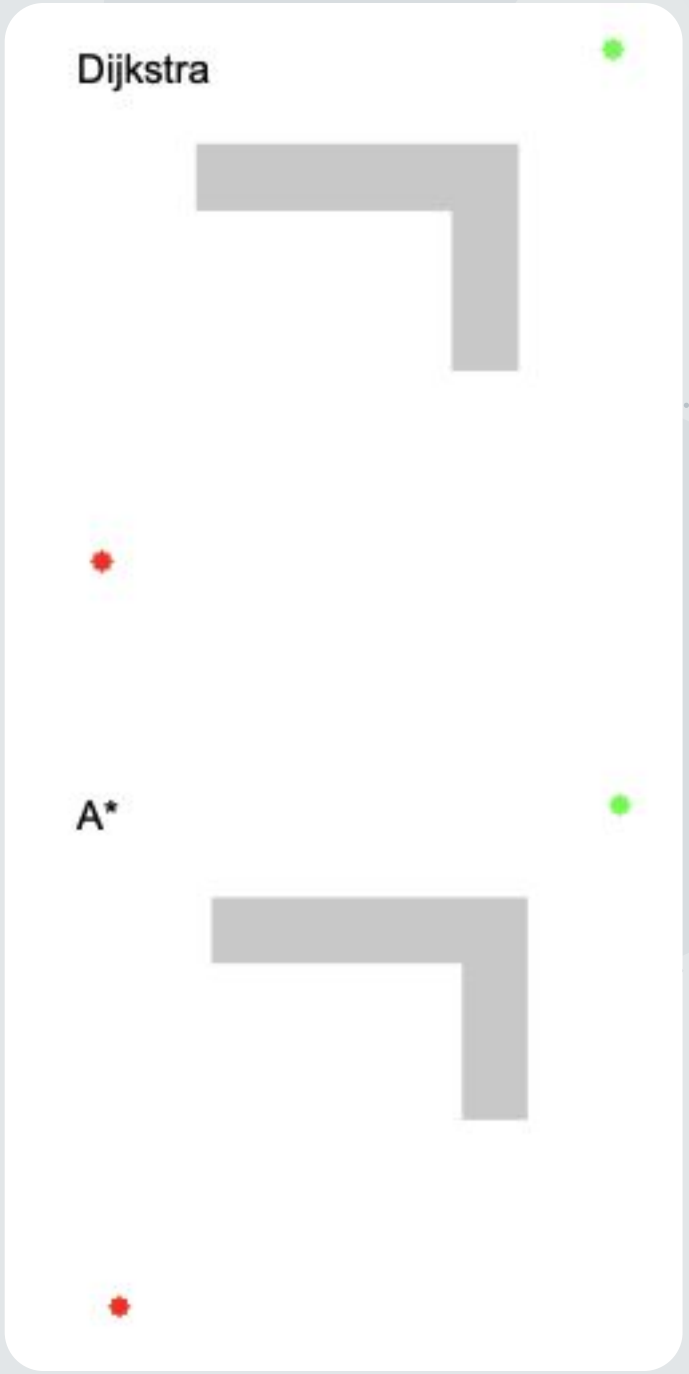
$$C(q) + C(q, q') \leq \text{UPPER}$$

to

$$C(q) + C(q, q') + h(q') \leq \text{UPPER}$$

where $h(q)$ is a heuristic for optimal cost-to-go (specifically, a positive underestimate)

- In this way, fewer nodes will be placed in the frontier queue
- This modification still guarantees that the algorithm will terminate with a shortest path



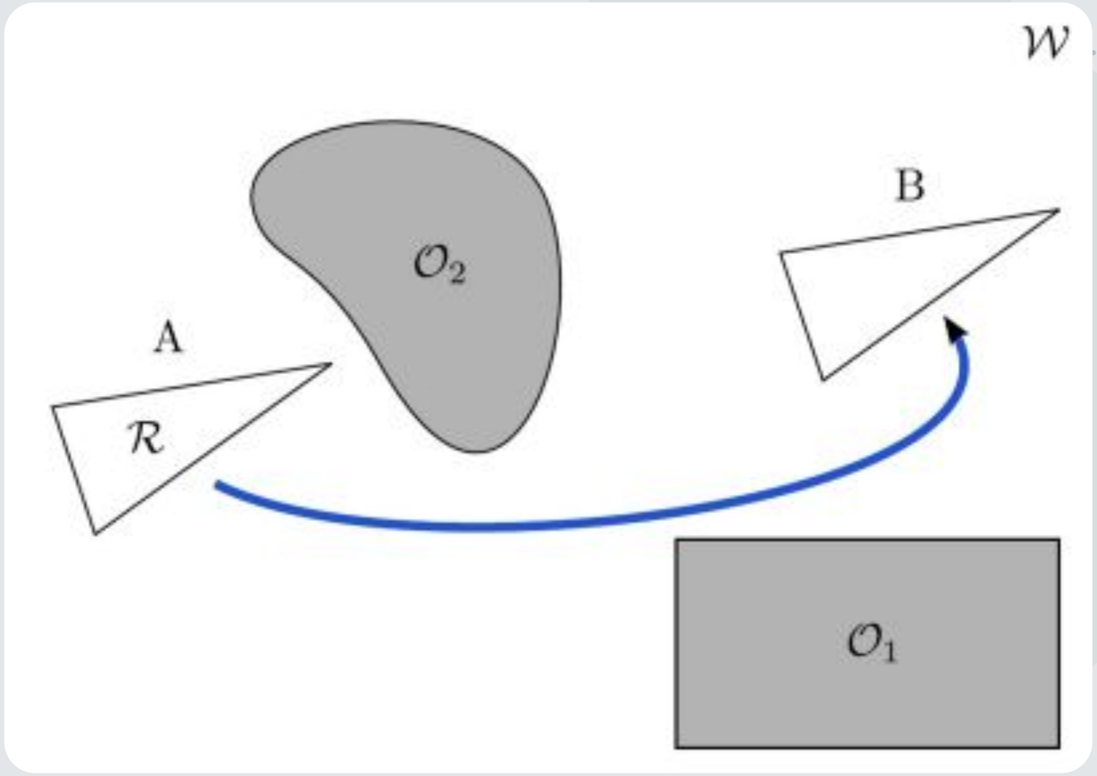
Grid-based approaches: Summary

- **Pros:**
  - Simple and easy to use
  - Fast (for some problems)

- **Cons:**
  - Resolution dependent
  - Not guaranteed to find solution if grid resolution is not small enough
  - Limited to simple robots
  - Grid size is exponential in the number of DOFs

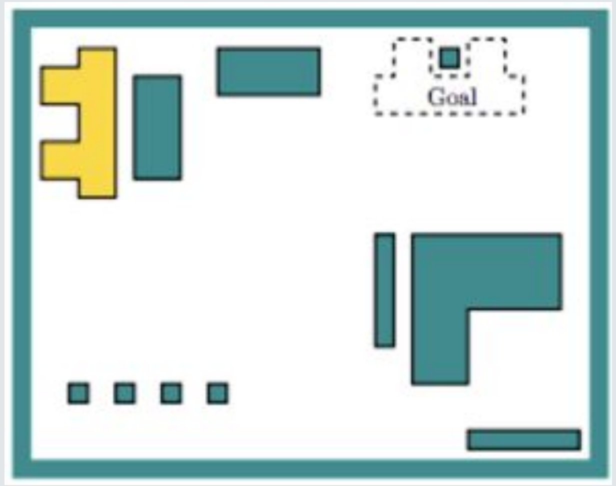
Back to Continuous Motion Planning

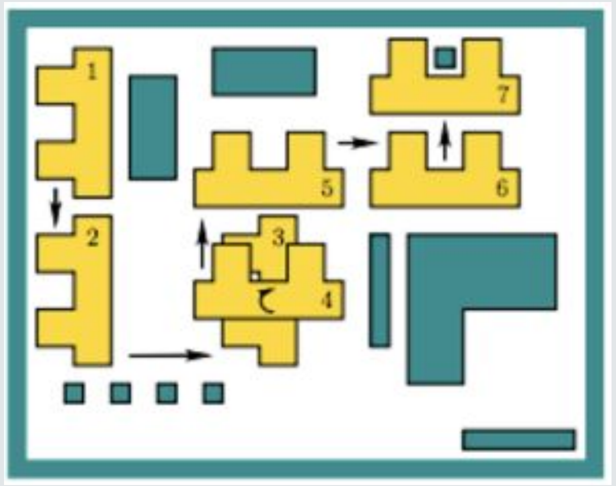
- A robot is a geometric entity operating in continuous space
- ***Combinatorial techniques*** for motion planning capture the structure of this continuous space
  - Particularly, the regions in which the robot is not in collision with obstacles
- Such approaches are typically complete
  - i.e., guaranteed to find a solution;
  - and sometimes even an optimal one



Simplest Setup Revisited

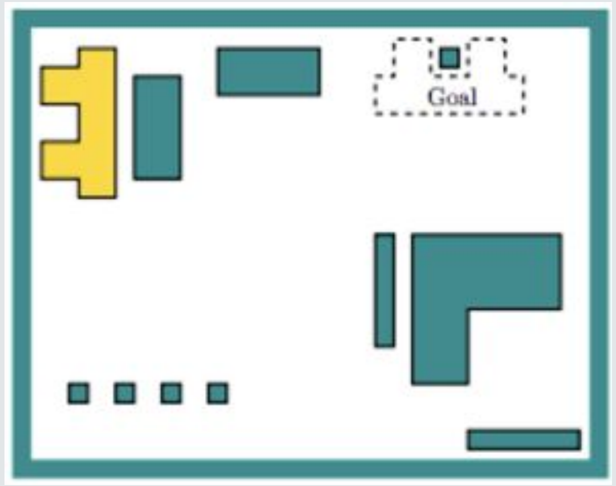
- Assume 2D workspace: $\mathcal{W} \subseteq \mathbb{R}^2$
- $\mathcal{O} \subset \mathcal{W}$ is the obstacle region with polygonal boundary
- Robot is a rigid polygon
- Problem: given initial placement of robot, compute how to gradually move it into a desired goal placement so that it never touches the obstacle region

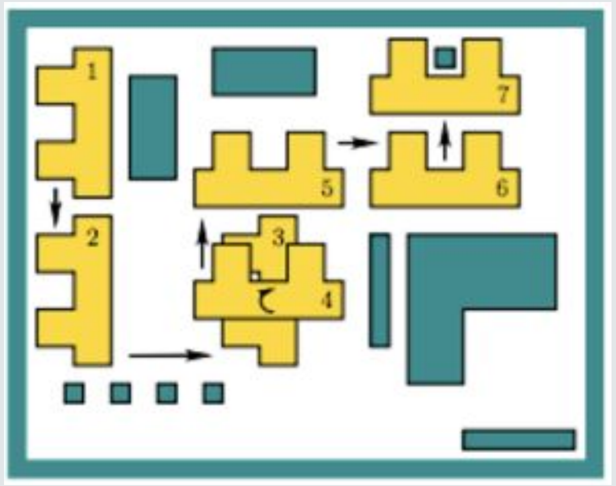




Simplest Setup

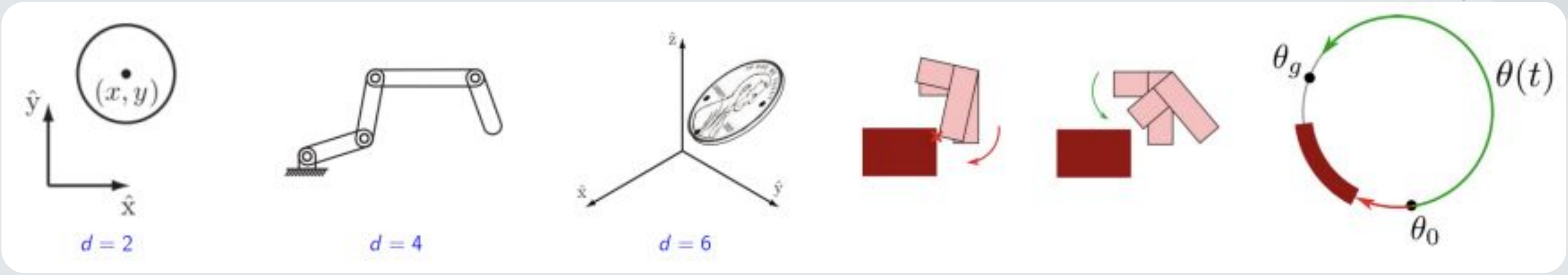
- **Key point:** motion planning problem described in the real-world, but it really lives in another space -- the **configuration** (C-) space!





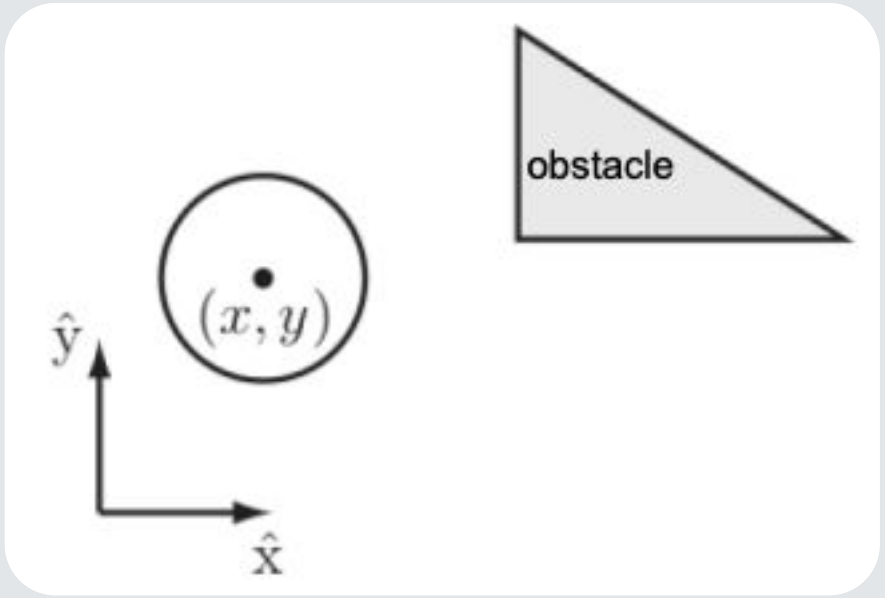
Configuration space

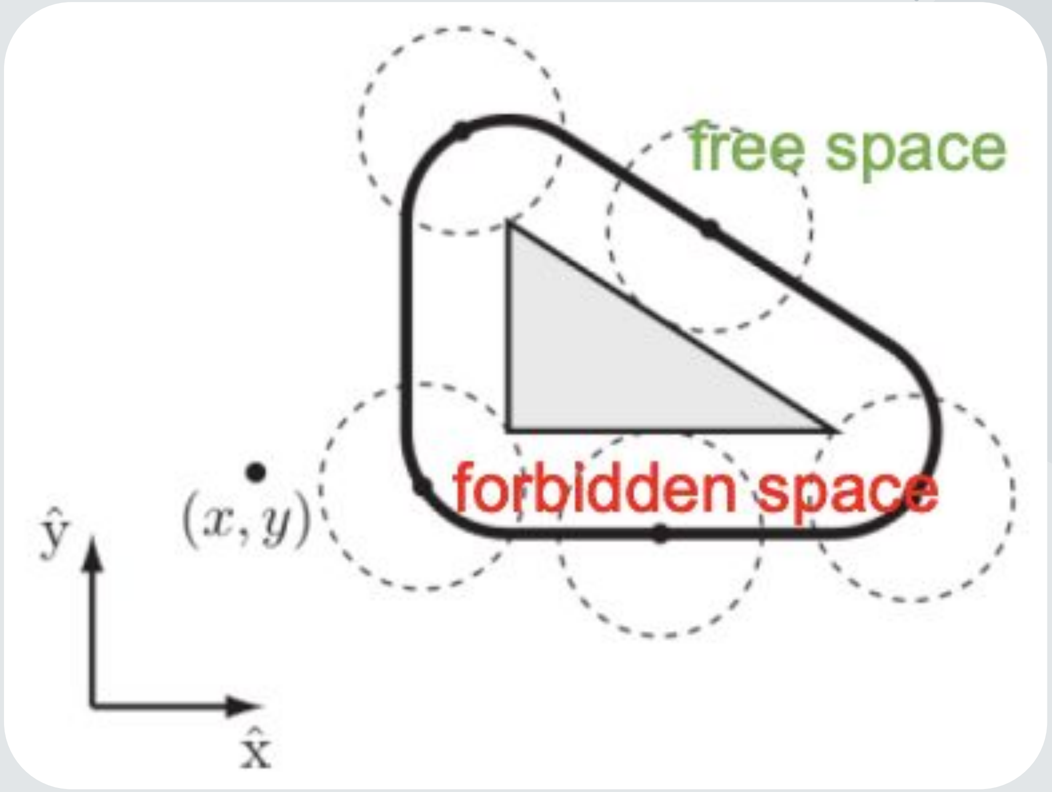
- C- space: captures all degrees of freedom (all rigid body transformations)
- More in detail, let $\mathcal{R} \subset \mathbb{R}^2$ be a polygonal robot (e.g., a triangle)
- The robot can rotate by angle θ or translate $(x_t, yt) \subset \mathbb{R}^2$
- Every combination $q = (x_t, y_t, \theta)$ yields a *unique* robot placement: **configuration**
- So, C- space is a subset of $\mathbb{R}^3$
- Note: $\theta \pm 2\pi$ yields equivalent rotations $\Rightarrow$ C- space is: $\mathbb{R}^2\ x\ \mathcal{S}^1$
- Concept of C- space extends naturally to higher dimensions (e.g., robot linkages)



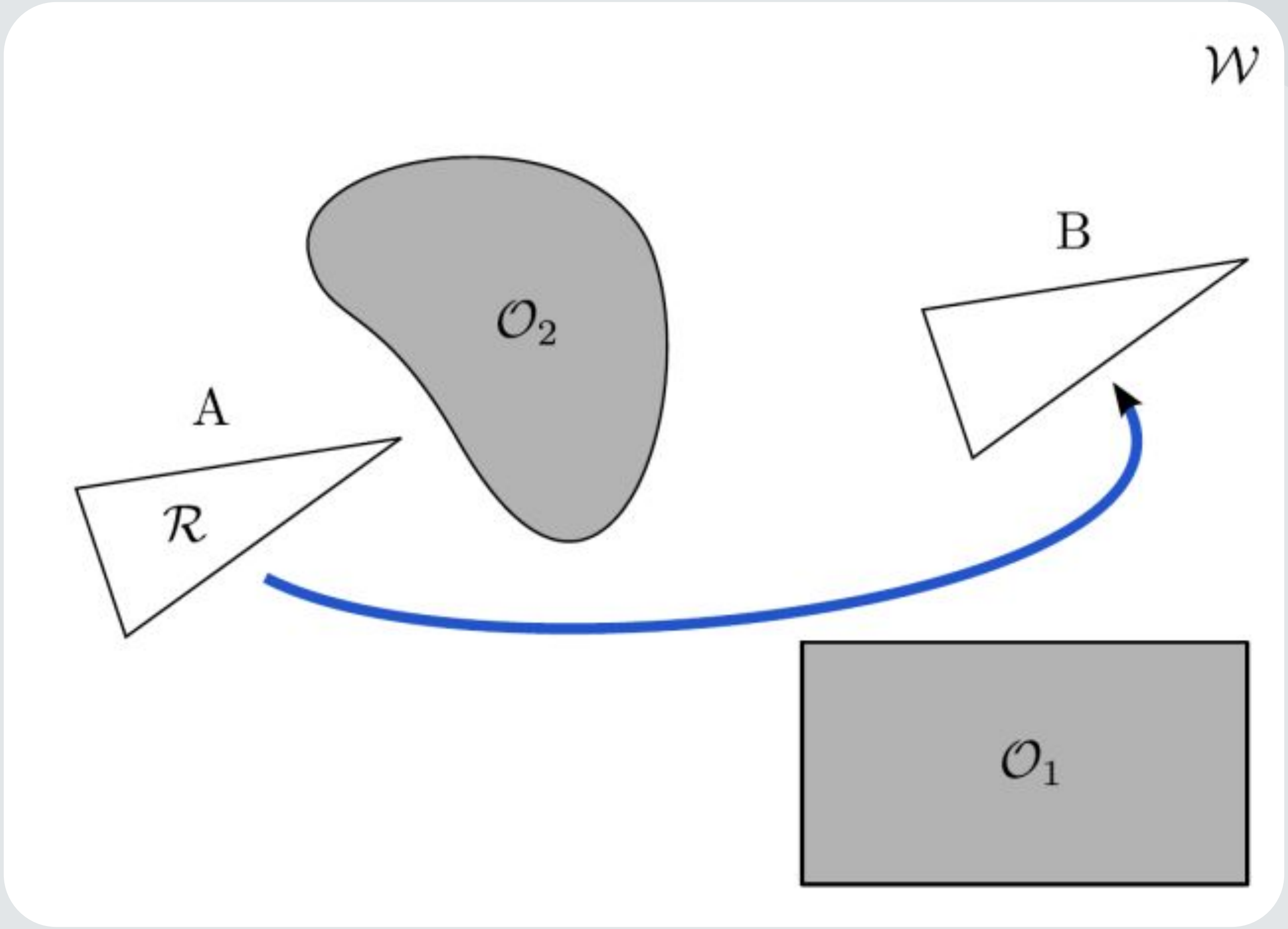
Configuration Free Space

- The subset $\mathcal{F} \subseteq \mathcal{C}$ of all collision free configurations is the **free space**

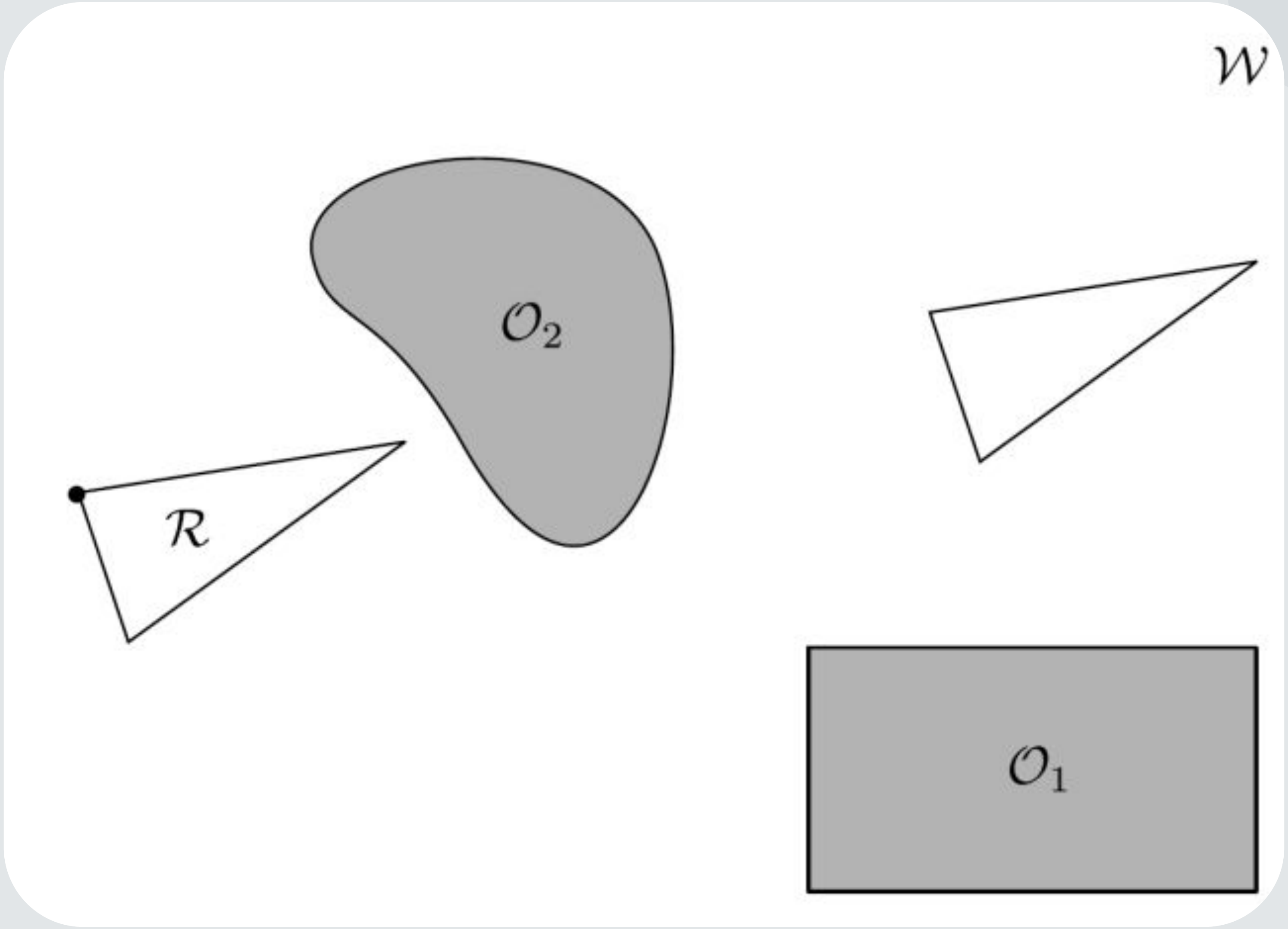




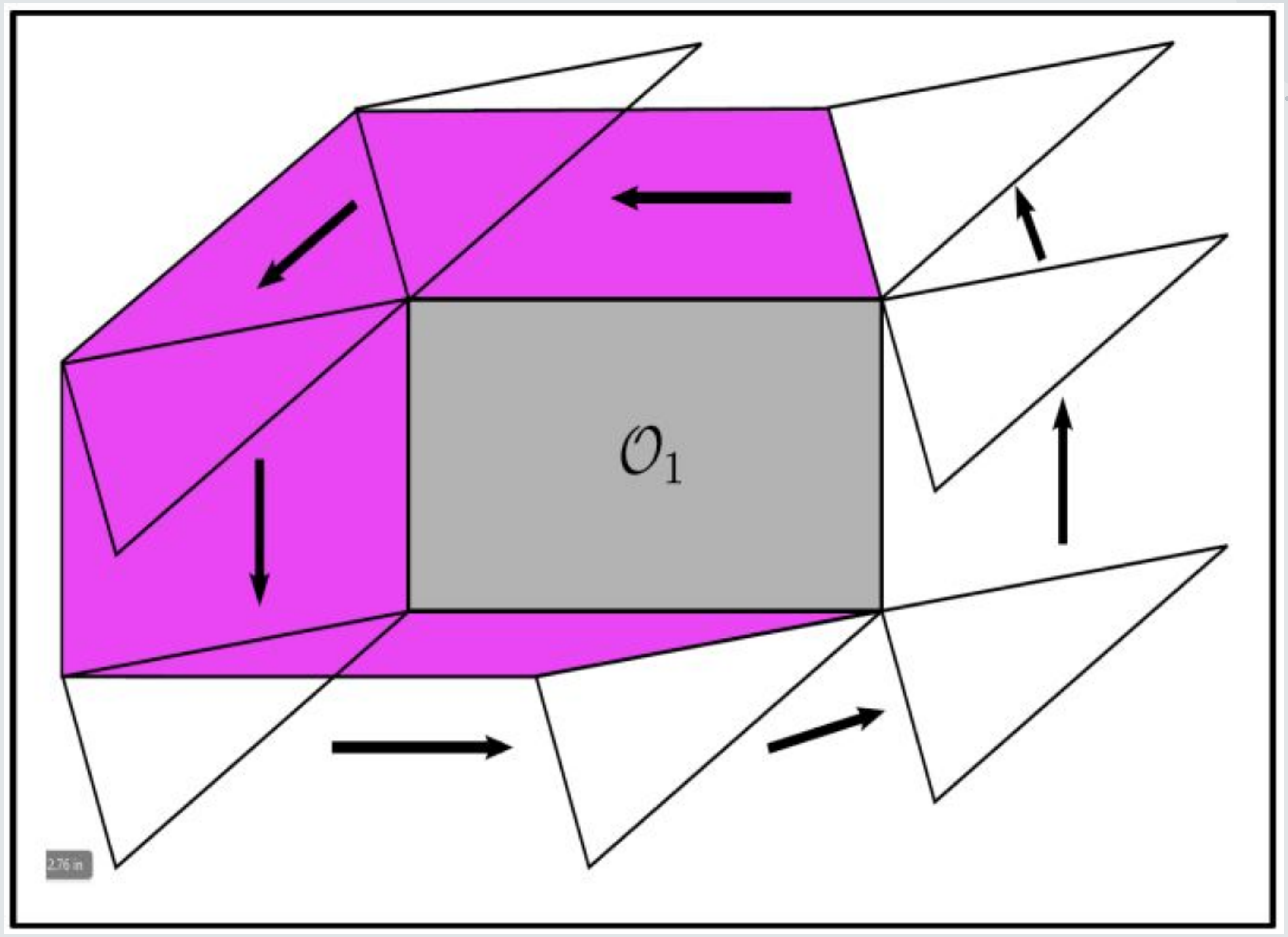
Configuration Free Space



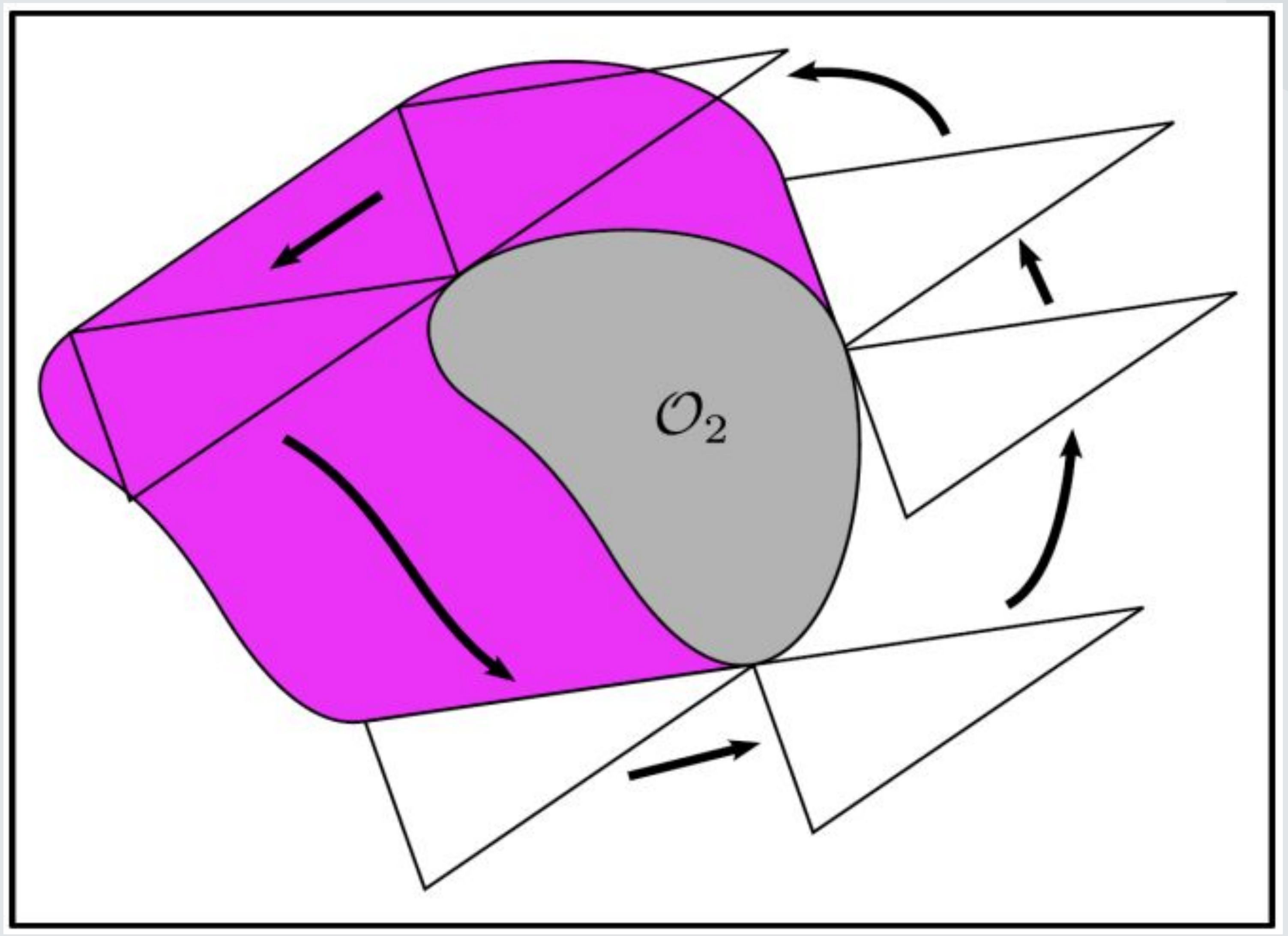
Configuration Free Space



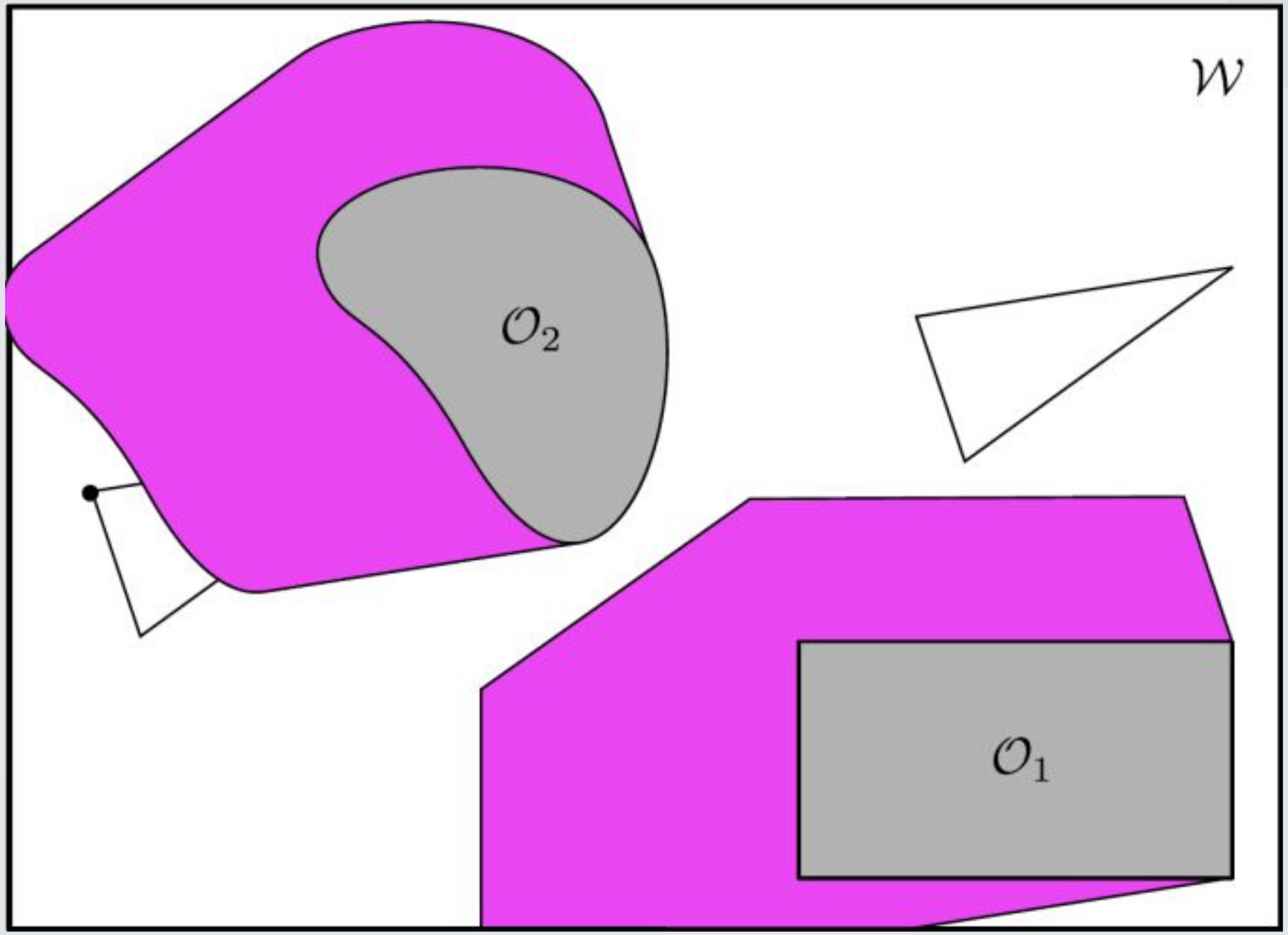
Configuration Free Space



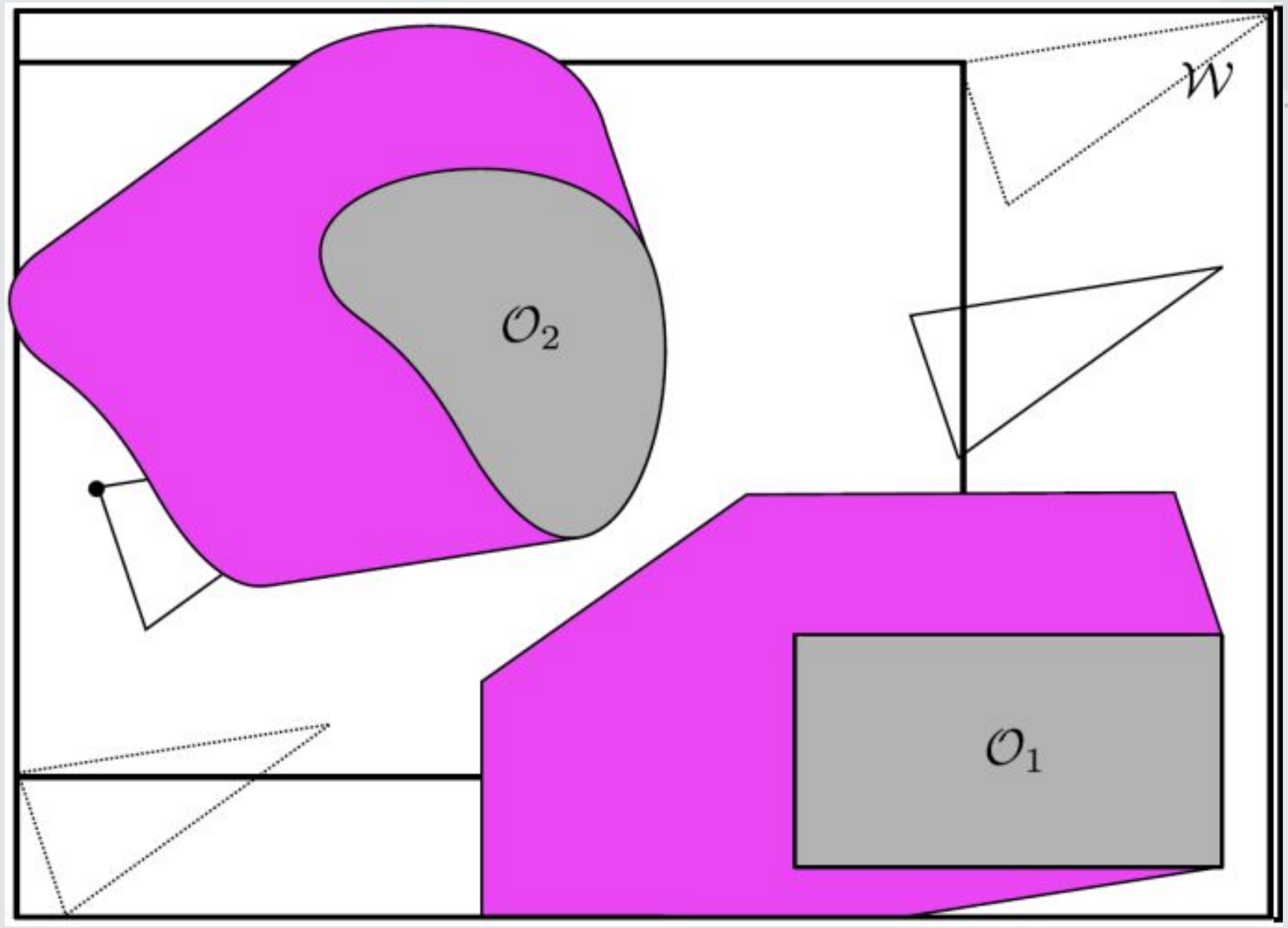
Configuration Free Space



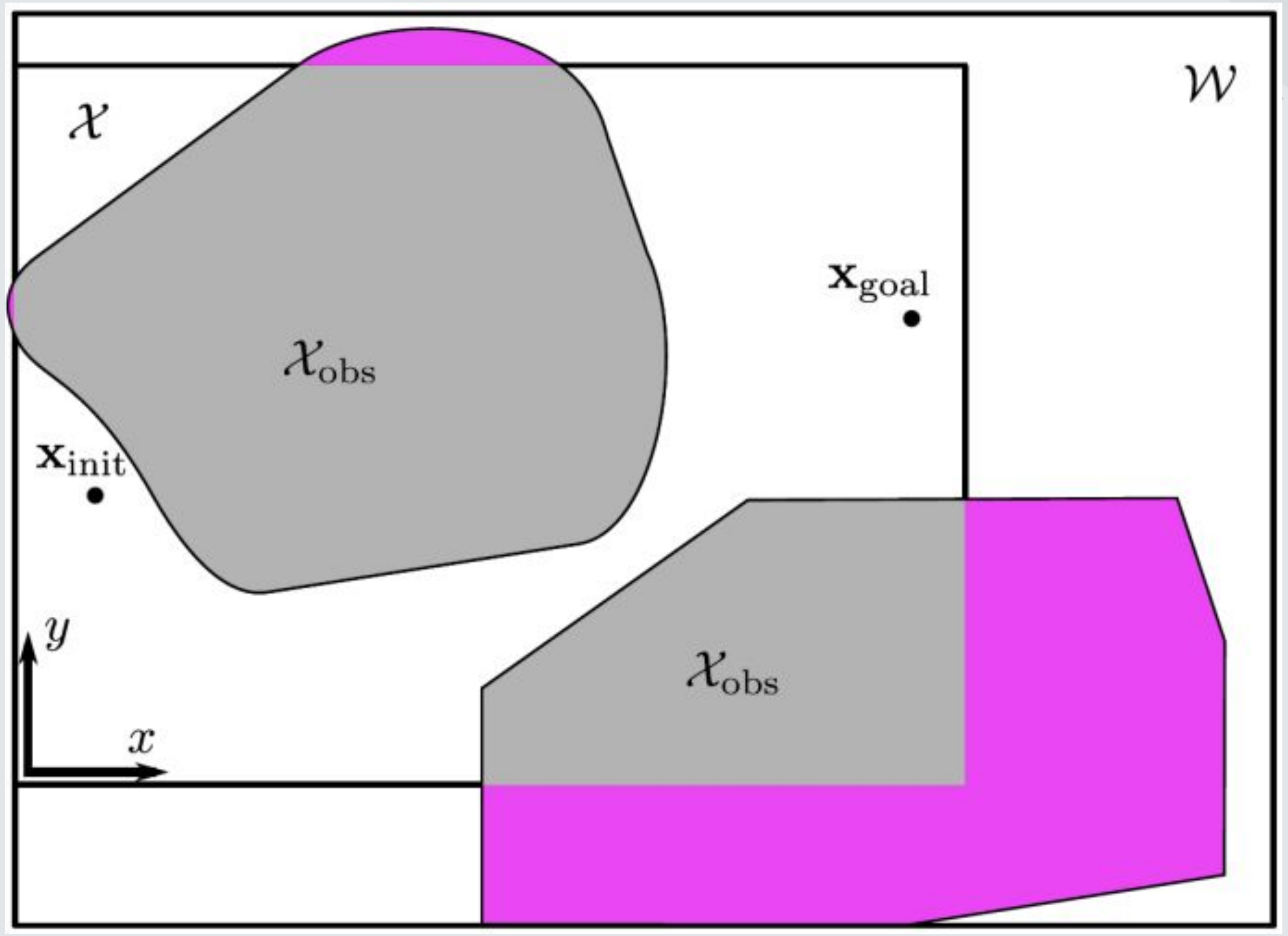
Configuration Free Space



Configuration Free Space

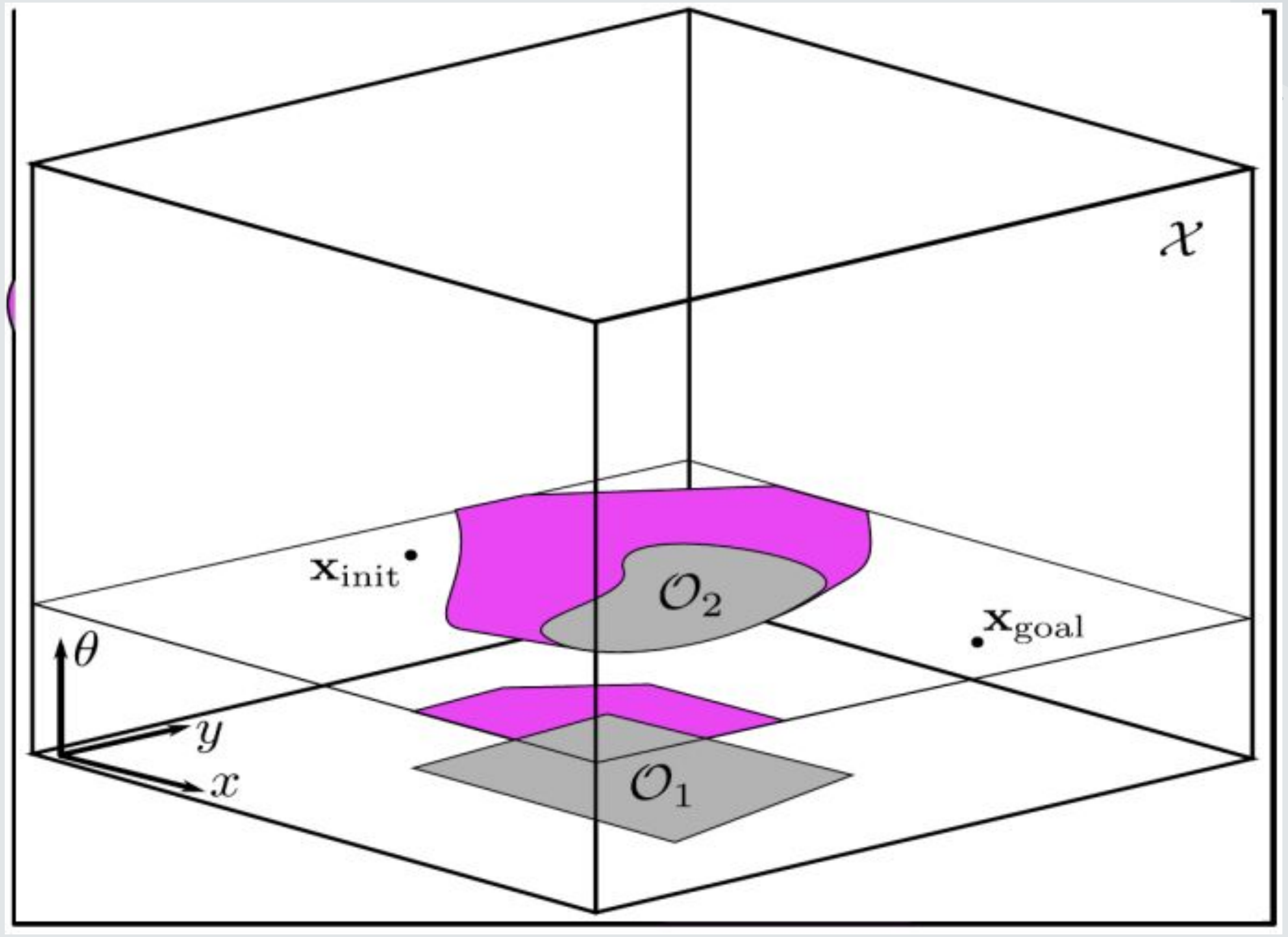


Configuration Free Space

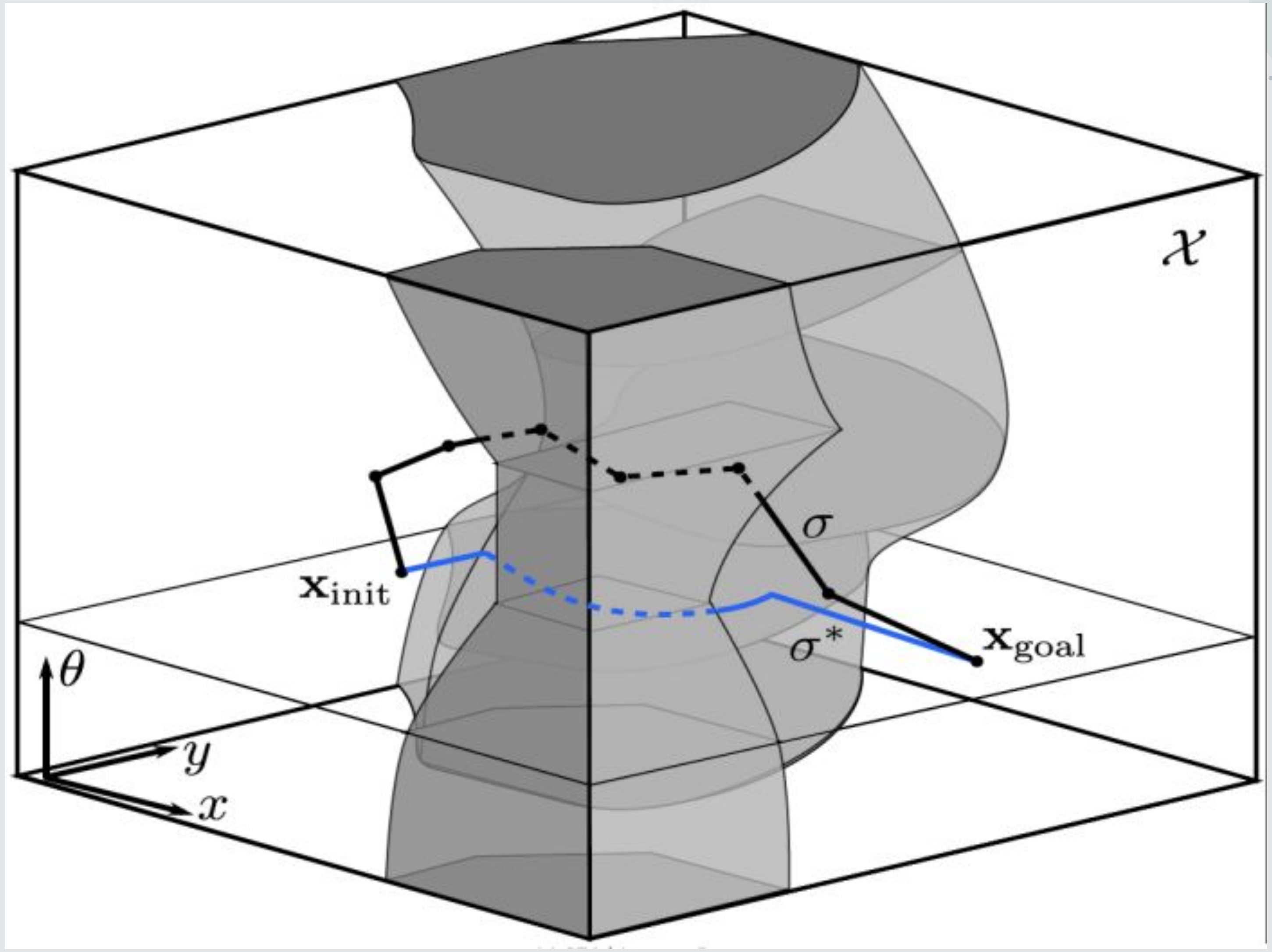


$\mathcal{W}$
$\mathcal{X}$
$\mathcal{X}_{\text{obs}}$
$\mathbf{x}_{\text{goal}}$
$\mathbf{x}_{\text{init}}$
y
x
$\mathcal{X}_{\text{obs}}$

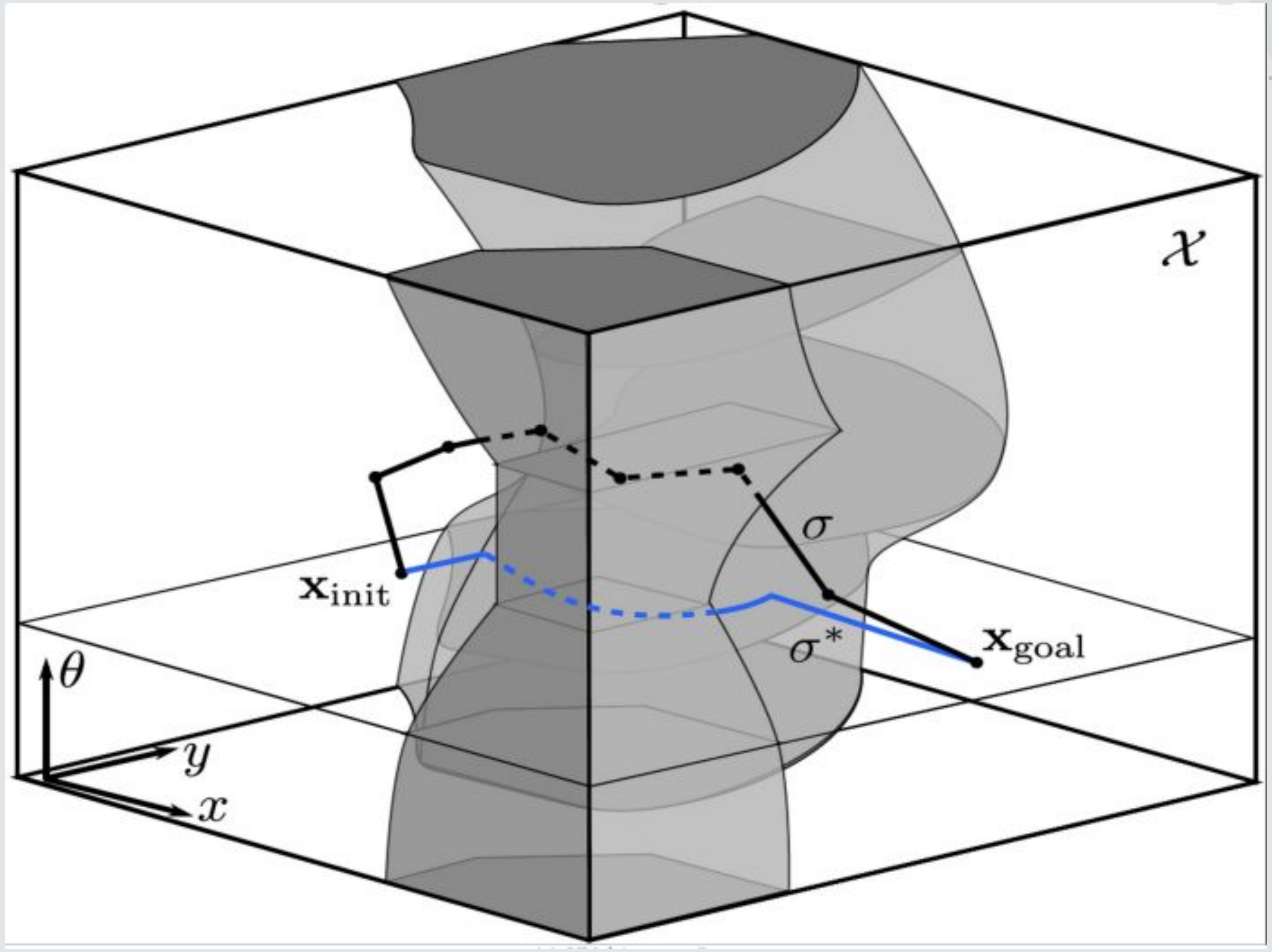
Configuration Free Space



Configuration Free Space

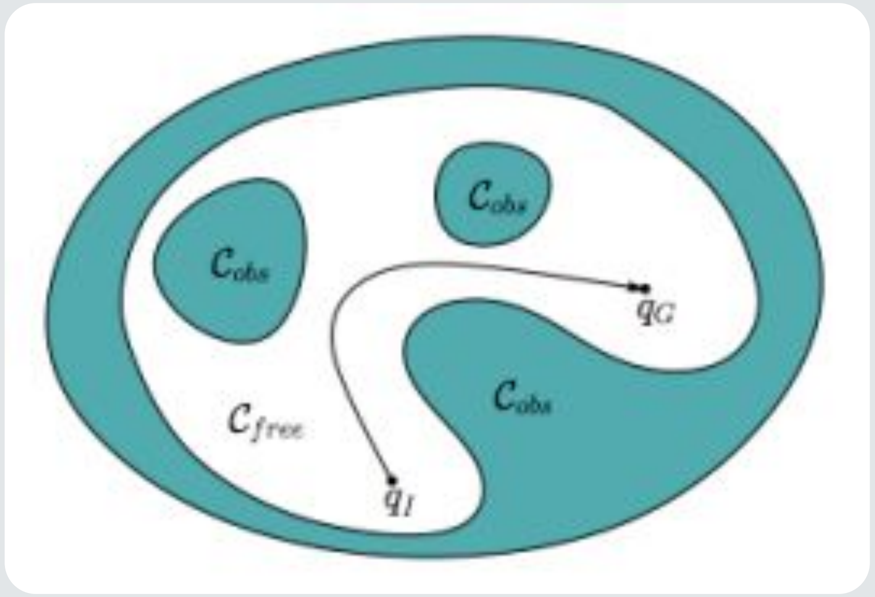


Configuration Free Space



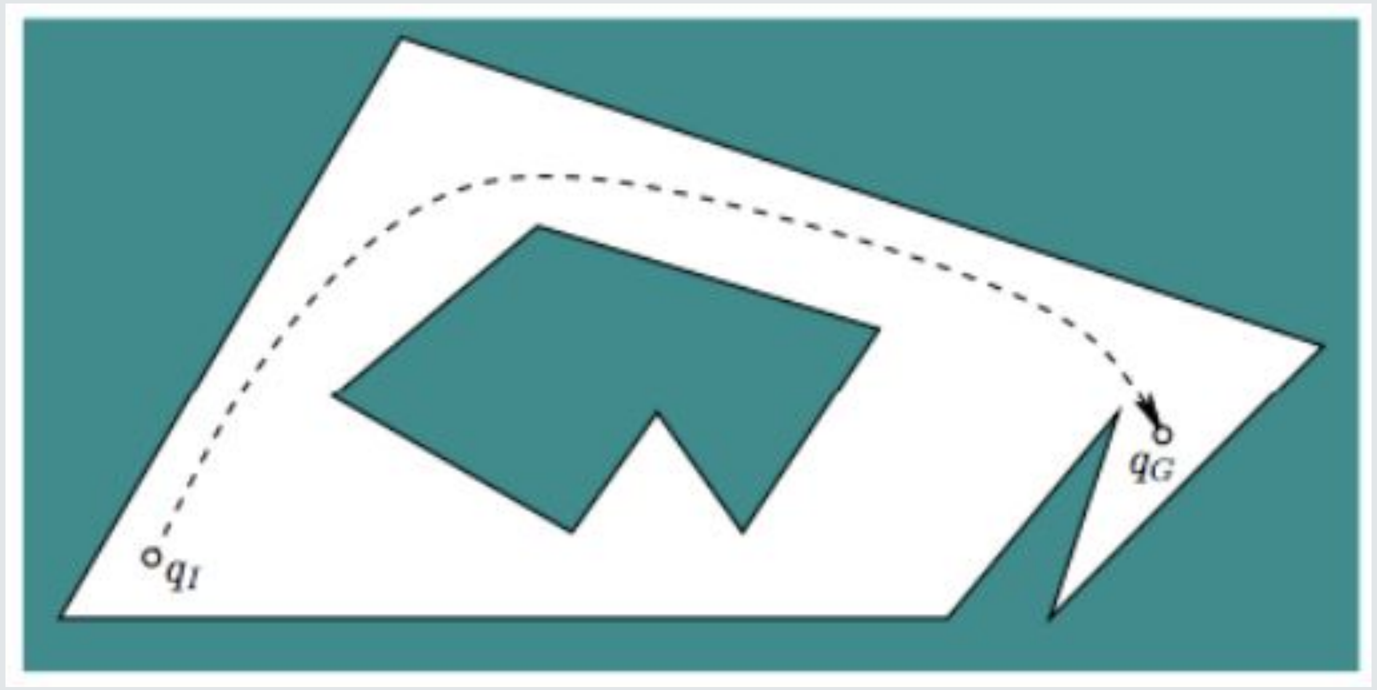
Planning in C-space

- Let $R(q) \subset \mathcal{W}$ denote the set of points in the world occupied by the robot when in configuration q
- Robot in collision $\Leftrightarrow R(q) \cap O \neq \phi$
- Accordingly, *free* space is defined as: $C_{free} = \{q \in C | R(q) \cap O = \phi \}$
- Path planning problem in C-space: compute a **continuous** path: $\tau: [0,1] \rightarrow C_{free}$, with $\tau(0) = q_I$ and $\tau(1) = q_G$



Combinatorial planning

- Key idea: compute a roadmap, which is a graph in which each vertex is a configuration in C_{free}, and each edge is a path through C_{free}, that connects a pair of vertices



Free-space Roadmaps

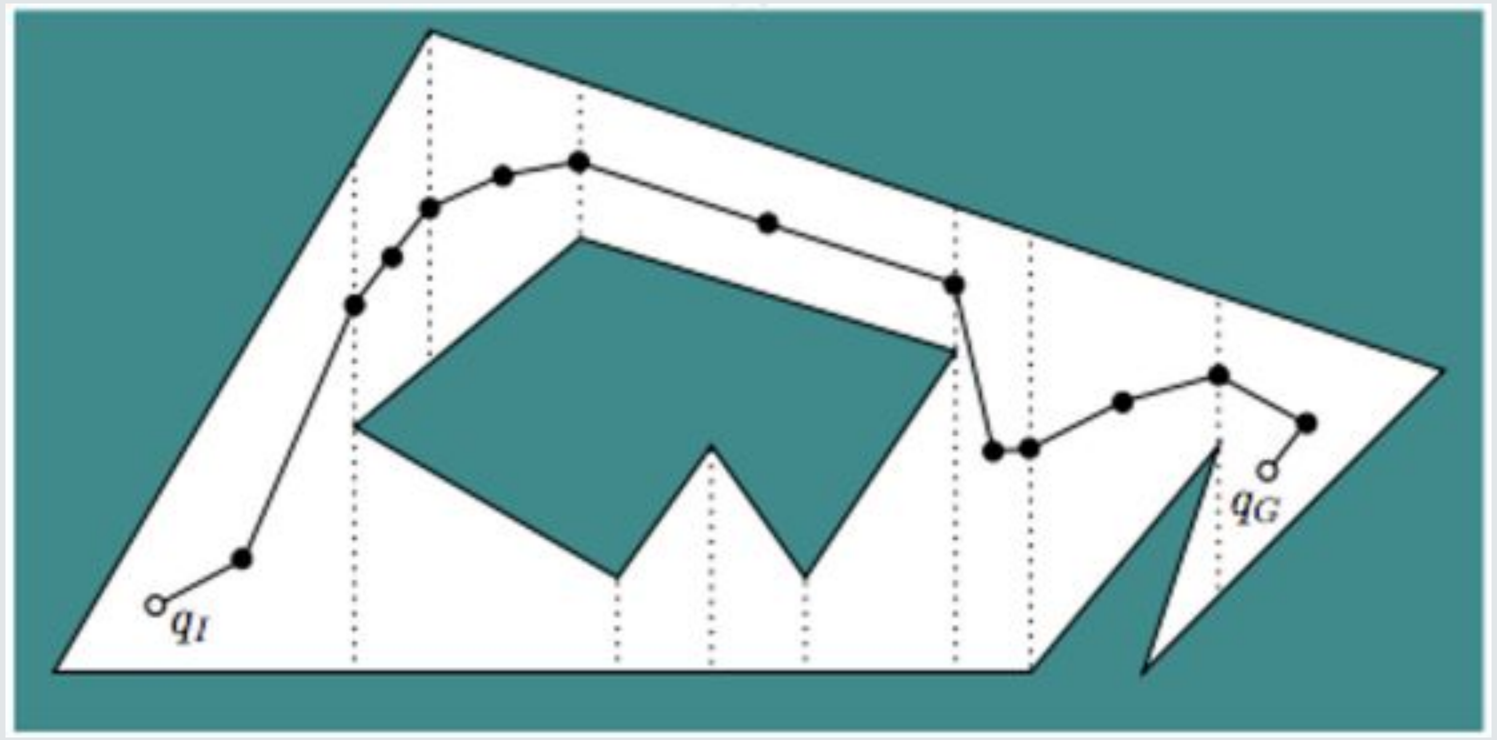
- Given a complete representation of the free space, we compute a roadmap that captures its connectivity

- **A roadmap should preserve:**
  - **Accessibility:** it is always possible to connect some $\boldsymbol{q}$ to the roadmap (e.g., $q_1 \rightarrow s_1$, $q_G \rightarrow s_2$)
  - **Connectivity:** if there exists a path from q_1 to q_G, there exists a path on the roadmap from s_1 to s_2

- **Main point:** a roadmap provides a discrete representation of the continuous motion planning problem *without losing* any of the original connectivity information needed to solve it

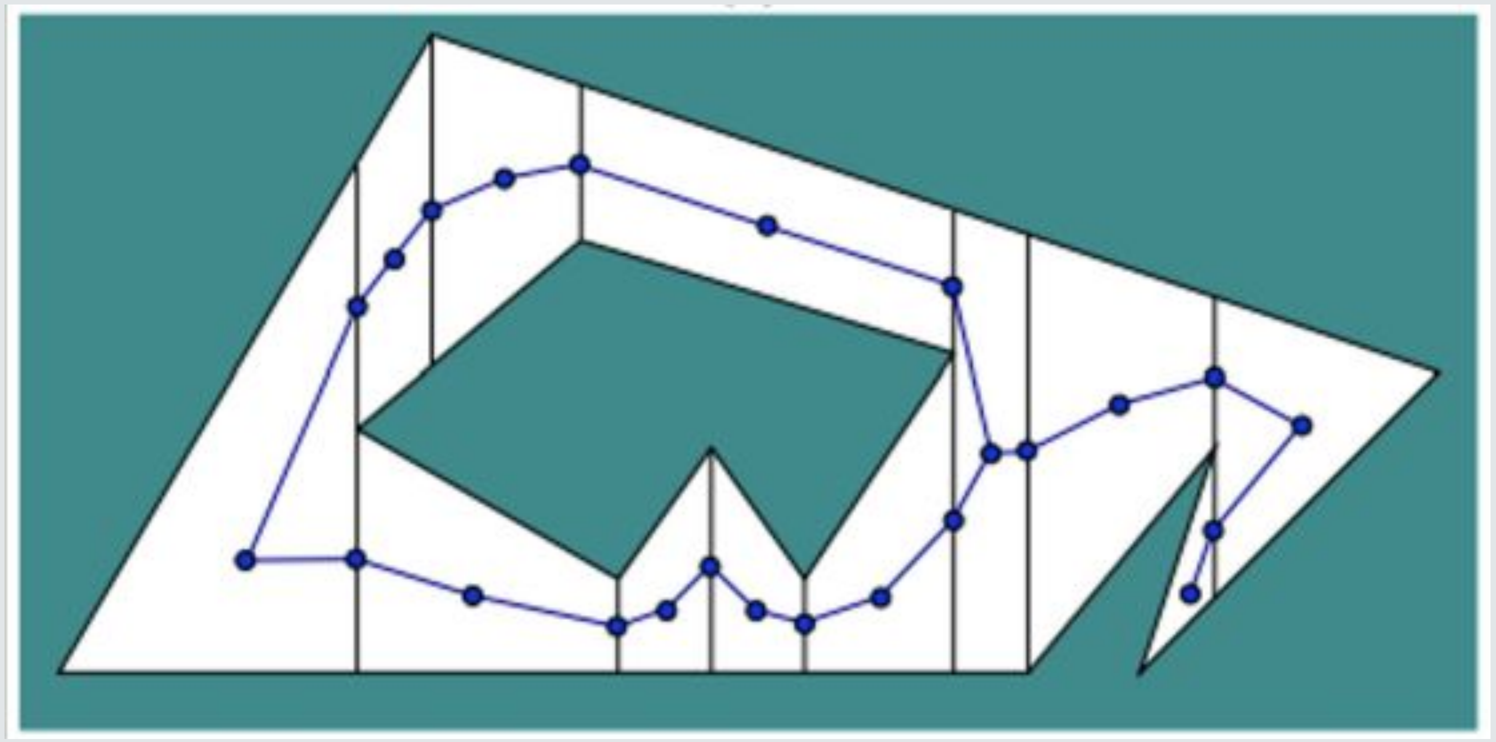
Cell decomposition

- Typical approach: cell decomposition. General requirements:
  - Decomposition should be easy to compute
  - Each cell should be easy to traverse (ideally convex)
  - Adjacencies between cells should be straightforward to determine

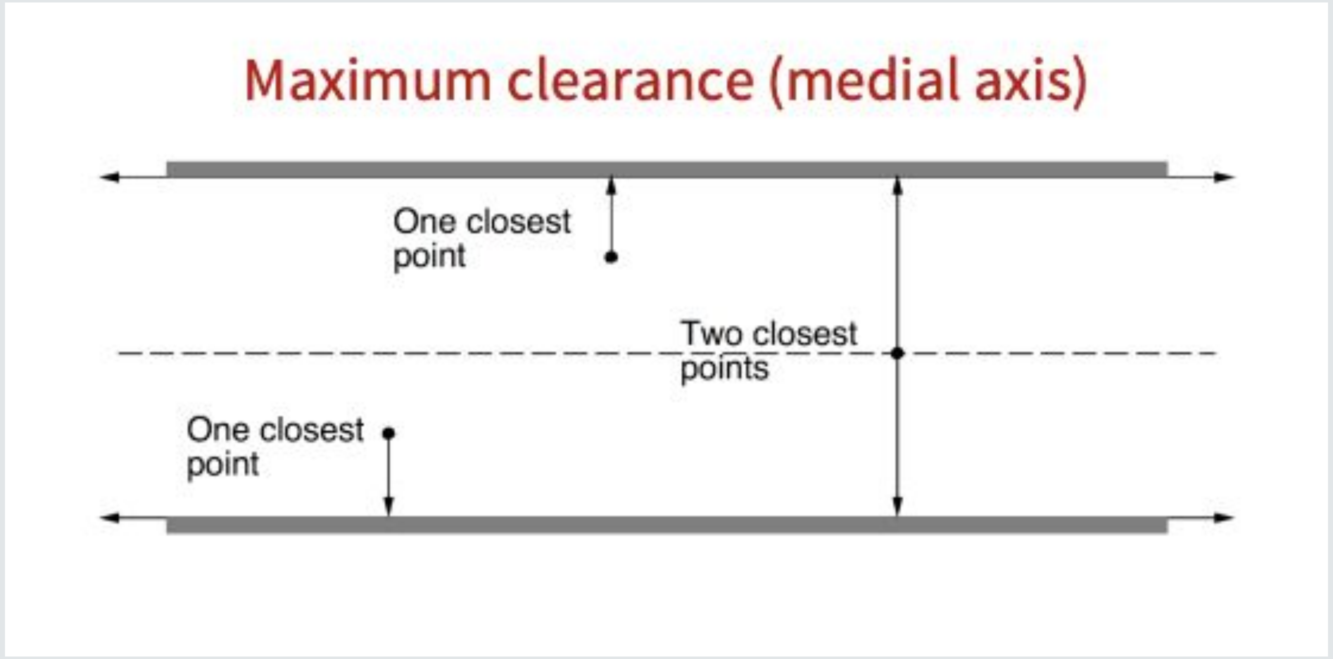


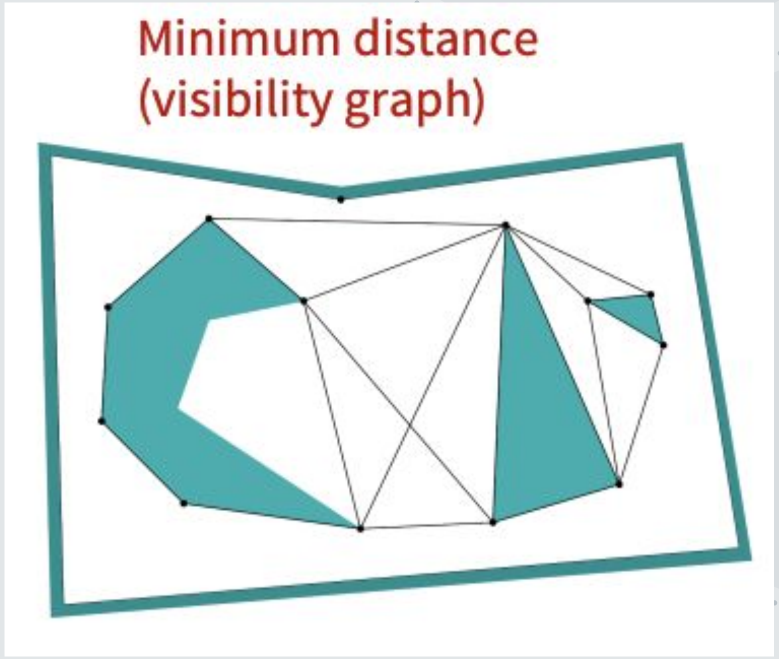
Computing a trapezoidal cell decomposition

- For every vertex (corner) of the forbidden space:
- Extend a vertical ray until it hits the first edge from top and bottom
- Compute intersection points with all edges, and take the closest ones
- More efficient approaches exists



Other Roadmaps

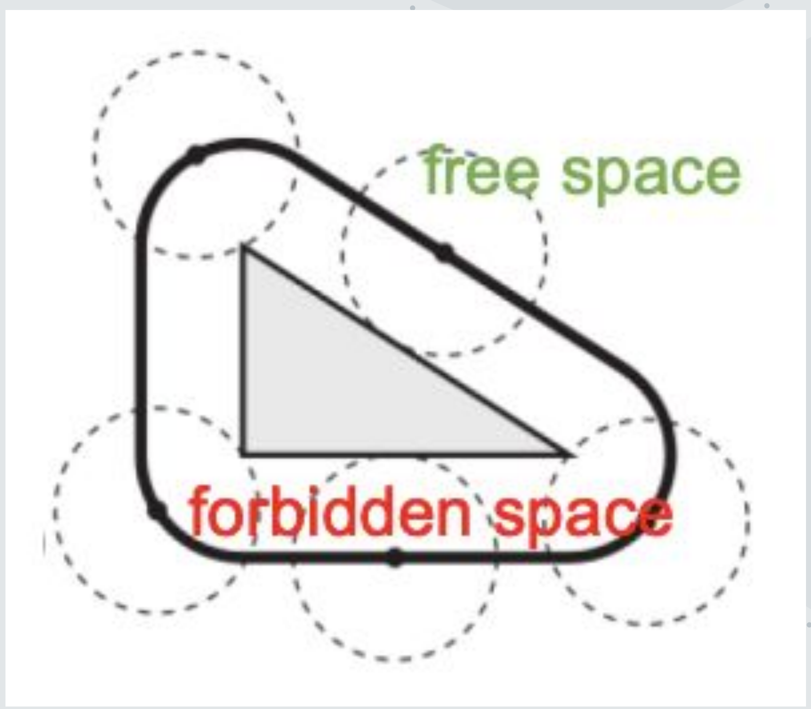




Note: No loss in optimality for a proper choice of discretization

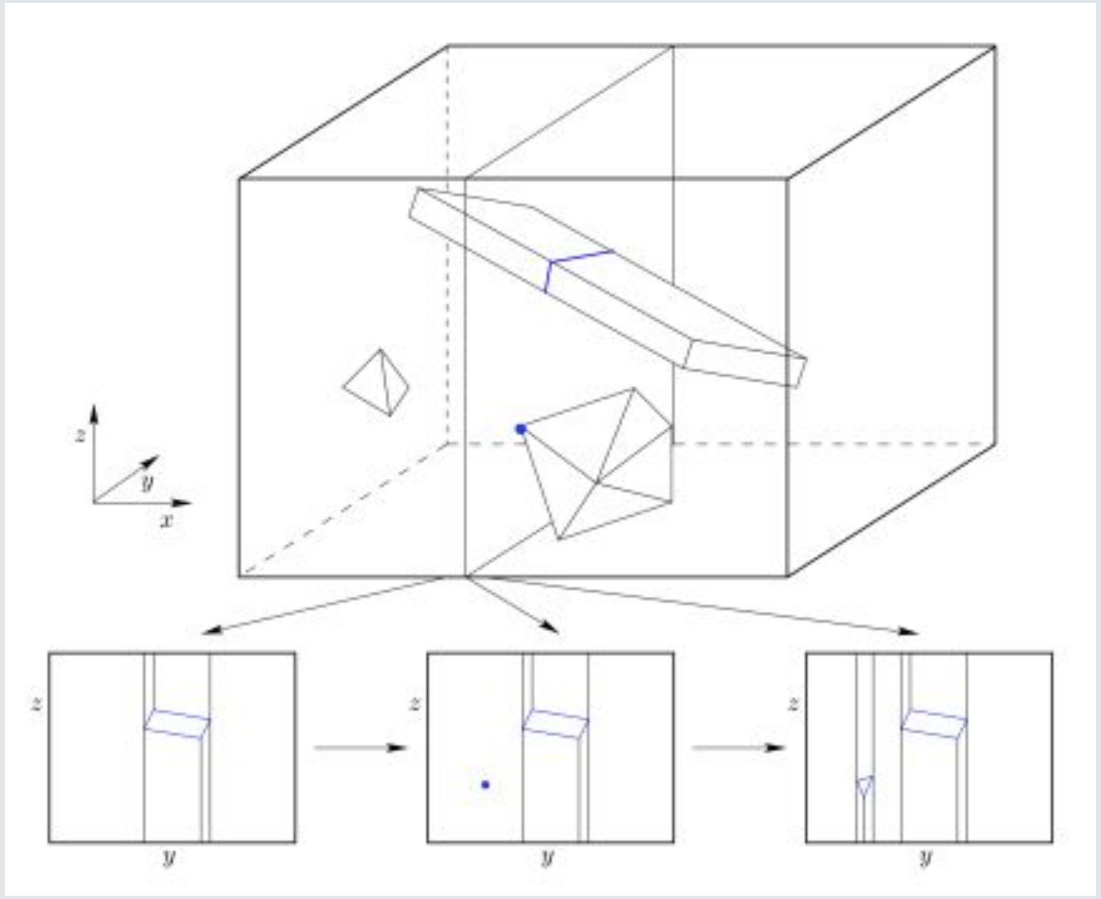
Caveat: Free-space Computation

- The free space is **not known** in advance
- We need to compute this space given the ingredients
  - Robot representation, i.e., its shape (polygon, polyhedron, …)
  - Representation of obstacles
- To achieve this, we do the following:
  - Contract the robot into a point
  - In return, inflate (or stretch) obstacles by the shape of the robots



Higher Dimensions

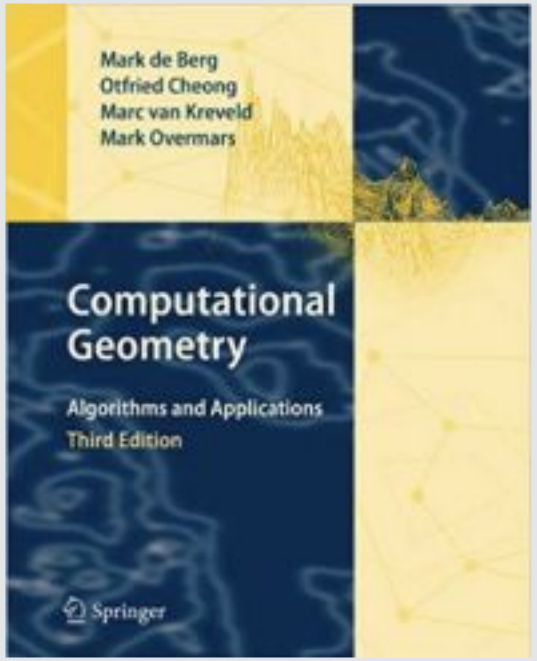
- Extensions to higher dimensions is challenging ⇒ algebraic decomposition methods



Additional resources on combinatorial planning

- Visualization of C-space for polygonal robot: https://www.youtube.com/watch?v=SBFwgR4K1Gk
- Algorithmic details for Minkowski sums and trapezoidal decomposition: de Berg et al., “Computational geometry: algorithms and applications”, 2008
- Implementation in C++: Computational Geometry Algorithms Library

CGAL


Mark de Berg
Otfried Cheong
Marc van Kreveld
Mark Overmars

Computational Geometry

Algorithms and Applications
Third Edition

Springer

Combinatorial Planning: Summary

- **These approaches are complete and even optimal in some cases**
    - Do not discretize or approximate the problem
- **Have theoretical guarantees on the running time**
    - I.e., computational complexity is known

- **Usually limited to small number of DOFs**
    - Computationally intractable for many problems
- **Problem specific: each algorithm applies to a specific type of robot/problem**
- **Difficult to implement; requires special software to reason about geometric data structures (CGAL)**

Next Lecture : Sampling-based Planning