Unit – V

9.	a)	Obtain a reduction formula for $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$ $(n > 0)$.	06	L1	5	:
	b)	Trace the curve $y^2(2a-x) = x^3$ (Cissoid).	07	L3	5	2
	c)	Find the area of the cardioid $r = a(1+\cos\theta)$	07	L2	5	2
10.	a)	Evaluate (i) $\int_0^{\frac{\pi}{6}} \cos^4 3x \sin^3 6x \, dx$ (ii) $\int_0^1 x^2 (1-x^2)^{\frac{3}{2}} \, dx$.	06	L1	5	2
	b)	Trace the curve $r^2 = a^2 \cos 2\theta$ (Lemniscate of Bernoulli).	07	L3	5	2
	c)	Find the volume of the spindle shaped solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.	07	L2	5	2

BT* Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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4. a) Examine the convergence of the series $\frac{\sqrt{2}-1}{3^3-1}+\frac{\sqrt{3}-1}{4^3-1}+\frac{\sqrt{4}-1}{5^3-1}+\cdots$. 07 L2 2 2

b) Find the nature of the series $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\cdots$. 06 L1 2 2

c) Expand $e^{\sin x}$ using Maclaurin's series expansion upto the term containing x^4. 07 L2 2 1

Unit – III

5. a) Prove that the pair of curves $r=a(1+\cos\theta)$, $r=b(1-\cos\theta)$ intersect each other orthogonally. 06 L2 3 1

b) Find the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ of the folium of De-Cartes $x^3+y^3=3axy$. 07 L1 3 1

c) State and prove Lagrange's mean value theorem. 07 L2 3 1

6. a) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$ 06 L2 3 1

b) If ρ be tha radius of curvature at any point ρ on the parabola $y^2=4ax$ and S be its focus, then show ρ^2 varies as $(sp)^3$ 07 L1 3 2

c) Show that the constant c of Cauchy's mean value theorem for the functions $\frac{1}{x^2}$ and $\frac{1}{x}$ in the interval (a, b) is the harmonic mean between a and $b\,(0<a<b)$. 07 L2 3 1

Unit – IV

7. a) If $u=\sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3\tan u$. 06 L1 4 1

b) In estimating the cost of a pile of bricks measured as $2m\times 15m\times 1.2m$, the tape is stretched 1% beyond the standard length. If the count is 450 bricks to 1 cu.m and bricks cost Rs. 530 per 1000, find the approximate error in the cost. 07 L1 4 2

c) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree. 07 L2 4 1

8. a) If $u=x^2+y^2+z^2$, $v=xy+yz+zx$ and $w=x+y+z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. 06 L1 4 1

b) If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. 07 L1 4 1

c) If $x+y+z=a$, show that the maximum value of $x^m y^n z^p$ is $m^m n^n p^p\left(\frac{a}{m+n+p}\right)^{m+n+p}$. 07 L2 4 2

NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester B.E. (Credit System) Degree Examinations

Make up Examinations - July - August 2021

20MA101 – ENGINEERING MATHEMATICS – I

Duration: 3 Hours | Max. Marks: 100

*Note: Answer **Five full** questions choosing **One full** question from each Unit.*

Unit – I

		Marks	BT*	CO*	PO*
1. a)	Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ using elementary row transformation.	06	L*1	1	1
b)	Apply the Gauss – Seidel iterative method to solve the system of equations $2x+10y+z=13$, $2x+2y+10z=14$, $10x+y+z=12$. Start with $x^{(0)}=y^{(0)}=z^{(0)}=0$ and carry out three iterations.	07	L2	1	
c)	Reduce the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form.	07	L1	1	
2. a)	Show that the linear transformation $y_1 = x_1+2x_2+5x_3$, $y_2 = 2x_1+4x_2+11x_3$, $y_3 = x_1 - x_2+2x_3$ is a regular linear transformation. Also, find the inverse of this transformation.	06	L2	1	
b)	Find the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by Rayleigh's power method.	07	L1	1	
c)	Reduce the quadratic form $6x^2+3y^2+3z^2-4xy+4zx-2yz$ into canonical form.	07	L1	1	

Unit – II

		Marks	BT*	CO*	PO*
3. a)	Test for convergence the series $\frac{4}{3}+\frac{4\cdot7}{3\cdot5}+\frac{4\cdot7\cdot10}{3\cdot5\cdot7}+\cdots$.	06	L2	2	
b)	Obtain Taylor's series expansion of $\log(\cos x)$ about the point $x=\frac{\pi}{3}$ up to the fourth degree term.	07	L1	2	
c)	Using Maclaurin's series, prove that $\sqrt{1+\sin 2x}=1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}+\cdots$.	07	L2	2	

10.	a)	Evaluate the following integrals: i) $\int_0^{\pi} \frac{\sin^4\theta}{(1+\cos\theta)^2}\, d\theta$ ii) $\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\, dx$	6	L1	5	1
	b)	Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1+\cos\theta)$.	7	L3	5	2
	c)	Find the volume of the solid generated by revolving the cardioid $r = a(1+\cos\theta)$ about the initial line.	7	L2	5	2

BT* Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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4. a) Using comparison test for convergence, test the convergence of the series: i) $\frac{1}{1.2}+\frac{2}{3.4}+\frac{3}{5.6}+\frac{4}{7.8}+\dots\dots\infty$. ii) $\sum\frac{n^2+5}{4n^5+7}$	6	L2	2	1
b) State the Cauchy's root test for convergence of an infinite series. Test the convergence of the series: $\sum\left(\frac{n+2}{n+3}\right)^n x^n;\quad x>0.$	7	L2	2	1
c) State the Taylor's theorem for a function of a single variable. Obtain the Taylor's expansion of $\log x$ about $x = 1$ up to the fourth-degree terms.	7	L1	2	1
5. a) State Cauchy's Mean value theorem. Verify Cauchy's mean value theorem for the pairs of functions: $f(x)=\sin x\,, g(x)=\cos x$ in $\left[0,\frac{\pi}{2}\right]$.	6	L1	3	1
b) Prove that the curves $r=\frac{a}{1+\cos\theta}$ and $r=\frac{b}{1-\cos\theta}$ intersect each other orthogonally.	7	L3	3	2
c) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$	7	L1	3	1
6. a) Find the angle between radius vector and the tangent for the curve: i) $r^2\cos2\theta = a^2$ ii) $r = a\,e^{\theta\cot\alpha}$	6	L2	3	2
b) Show that the radius of curvature at any point of the cardioid $r = a(1-\cos\theta)$ varies as $\sqrt{r}$.	7	L2	3	2
c) State and prove the Lagrange's Mean value theorem.	7	L1	3	1
7. a) If u is a homogeneous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = nu.$ Using this result show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = 2u\log u$ where $\log u = \frac{x^3+y^3}{3x+4y}$.	6	L1	4	1
b) If $x = r\cos\theta$ and $y = r\sin\theta$, find $J=\frac{\partial(x,\ y)}{\partial(r,\ \theta)}$ and $J'=\frac{\partial(r,\ \theta)}{\partial(x,\ y)}$. Hence prove that $JJ' = 1$.	7	L2	4	2
c) Find the extreme value of the function $2xy - 5x^2 - 2y^2 + 4x + 4y - 6$.	7	L3	4	2
8. a) Expand the function $f(x,y) = e^{2x}\cos 3y$ as a Maclaurin's series up to second degree terms.	6	L2	4	1
b) If $u = f(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0.$	7	L2	4	2
c) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1.$	7	L3	4	2
9. a) Find the surface area of the solid generated by revolving the asteroid $x = a\sin^3 t\ ,\ y = a\cos^3 t$ about the x- axis.	7	L2	5	2
b) Obtain the reduction formula for $\int\sin^n x\,dx$ where n is a positive integer. Hence evaluate $\int_0^{\frac{\pi}{2}}\sin^n x\ dx$.	6	L1	5	1
c) Trace the polar curve $r^2 = a^2\cos2\theta$.	7	L3	5	2

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First Semester B.E. (Credit System) Degree Examinations

Supplementary Examinations – September 2021

20MA101 – ENGINEERING MATHEMATICS - I

ation: 3 Hours | Max. Marks: 100

Note: Answer any Five full questions.

	Question	Marks	BT*	CO*	PO*
a)	Find the matrix of linear transformation that transforms (x_1, x_2, x_3) to $(x_1+2x_2+2x_3, 2x_1+x_2+2x_3, 2x_1+2x_2+x_3)$ i) Check whether this linear transformation is regular. ii) Is this transformation orthogonal?	6	L*2	1	2
b)	i) What is the necessary condition for a system of linear equation given by $AX = B$ to be consistent? ii) Check for consistency and hence solve the system of equations by the Gauss elimination method: $x+2y+2z=3$ $2x-y+z=5$ $3x-2y-2z=1$	7	L2	1	1
c)	Using Gauss - Seidel method solve the given system of linear equations: $\begin{matrix} 12x+3y-5z=1 \\ x+5y+3z=28 \\ 3x+7y+13z=76 \end{matrix}$, Take $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ as an initial approximation and carry out three iterations.	7	L3	1	2
a)	Using the power method, find the dominant Eigen value and corresponding Eigen vector of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ starting with an initial approximation to the Eigen value as $[1 \ \ 0 \ \ 0]^T$. Perform 5 iterations.	6	L1	1	1
b)	Find the Eigen values and its corresponding Eigen vectors of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	7	L2	1	1
c)	Reduce the quadratic form $2x^2+2y^2+z^2-8xy$ into canonical form.	7	L3	1	2
a)	i) Prove that if $\sum_{n=1}^{\infty} u_n$ is convergent then $\lim_{n\to\infty} u_n = 0$. ii) Is the converse true? Justify your answer with an example.	6	L1	2	1
b)	State the D 'Alembert's ratio test for convergence of an infinite series. Test the convergence of the series: $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \cdots\cdots\cdots \infty$.	7	L2	2	2
c)	Obtain the Maclaurin's Series expansion of the function $e^x \cos x$. Expand up to four non vanishing terms.	7	L1	2	1