

VECTOR CALCULUS AND VECTOR INTEGRATION:

1. If $\phi = x^2 + y - z - 1$, then $|\nabla \phi|$ at $(1, 0, 0)$ is
(a) $\sqrt{2}$ (b) $\sqrt{6}$ (c) $\sqrt{5}$ (d) 1
2. The directional derivative of $\phi = 3x^2 + 2y - 3z$ at $(1, 1, 1)$ in the direction of $2\hat{i} + 2\hat{j} - \hat{k}$ is
(a) $\frac{13}{3}$ (b) 4 (c) $\frac{19}{3}$ (d) none of these
3. The unit normal vector to the surface $x^2 + y^2 - 2z + 3 = 0$ at $(1, 2, -1)$
(a) $\frac{2\hat{i}+4\hat{j}+2\hat{k}}{\sqrt{24}}$ (b) $\frac{2\hat{i}+4\hat{j}}{\sqrt{20}}$ (c) $\frac{2\hat{i}+2\hat{j}}{\sqrt{20}}$ (d) $\frac{2\hat{i}+4\hat{j}-2\hat{k}}{\sqrt{24}}$
4. If $f = \tan^{-1}\left(\frac{y}{x}\right)$ then $\text{div}(\text{grad } f)$ is equal to
(a) 1 (b) -1 (c) 0 (d) 2
5. The value of $\text{curl}(\text{grad } f)$, where $f = 2x^2 - 3y^2 + 4z^2$ is
(a) $4x - 6y + 8z$ (b) $4x\hat{i} - 6y\hat{j} + 8z\hat{k}$ (c) 0 (d) 3
6. What is the value of $\nabla \times (xy\hat{i} + yz\hat{j} + zx\hat{k})$ is
(a) $-y\hat{i} + z\hat{j} - x\hat{k}$ (b) $-y\hat{i} - z\hat{j} - x\hat{k}$
(c) $-y - z - x$ (d) $-y + z - x$
7. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $\text{div}\vec{r} = \underline{\hspace{2cm}}$ and $\text{curl}\vec{r} = \underline{\hspace{2cm}}$
(a) $\hat{i} + \hat{j} + \hat{k}$ and 0 (b) 3 and $\vec{0}$ (c) 3 and 0 (d) none of these
8. The angle between the vectors $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - 9\hat{j} + 6\hat{k}$ is
(a) $\theta = 1.4143$ (b) $\theta = 0.897$ (c) $\theta = 0$ (d) $\theta = 0.1558$
9. If $\vec{F} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$, then at $(2, -1, 1)$ $\nabla \cdot \vec{F} = \underline{\hspace{2cm}}$
(a) $-\hat{i} + 12\hat{j} + 3\hat{k}$ (b) $-\hat{i} + 12\hat{j} + 5\hat{k}$ (c) 16 (d) 14
10. Find 'a' such that $(-x^2 + yz)\hat{i} + (4ay - z^2x)\hat{j} + (2xz - 4z)\hat{k}$ is solenoidal
(a) -1 (b) 1 (c) 0 (d) none of these
11. Find 'a' such that the vector $\vec{F} = (x + y + az)\hat{i} + (x + 2y - z)\hat{j} + (-x - y + 2z)\hat{k}$ is irrotational
(a) -1 (b) 1 (c) 0 (d) none of these

12. If $\varphi = xy^3z^2 = 4$, then $\nabla \varphi$ at the point $(1, 1, -1)$ is

- (a) $\hat{i} + 3\hat{j} + 2\hat{k}$ (b) $\hat{i} + 3\hat{j} - 2\hat{k}$
(c) $\hat{i} - 3\hat{j} + 2\hat{k}$ (d) $\hat{i} - 3\hat{j} - 2\hat{k}$

13. The unit directional derivative to the curve $x = t, y = t^2, z = t^3$ at the point $(-1, 1, -1)$ is

- (a) $\frac{1}{\sqrt{14}}(\hat{i} - 2\hat{j} + 3\hat{k})$ (b) $\frac{1}{\sqrt{14}}(\hat{i} + 2\hat{j} + 3\hat{k})$
(c) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

14. A vector field which has a vanishing divergence is called as

- (a) Solenoidal field (b) irrotational field (c) rotational field (d) scalar field

15. $\vec{F} = (x + 2y + 4z)\hat{i} + (2ax - 3y - z)\hat{j} + (4x - y + 2z)\hat{k}$ is

- (a) Solenoidal (b) irrotational
(c) rotational (d) both solenoidal and irrotational

16. A unit tangent vector to the surface $x = t, y = e^t, z = -3t^2$ at $t = 0$ is

- (a) $\hat{i} + \hat{j}$ (b) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ (c) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j}$ (d) none of these

17. Unit normal vector to the surface $z = 2xy$ at the point $(2, 1, 4)$ is

- (a) $\frac{1}{\sqrt{20}}(2\hat{i} + 4\hat{j})$ (b) $\frac{1}{\sqrt{20}}(2\hat{i} - 4\hat{j})$
(c) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} - \hat{k})$ (d) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} + \hat{k})$

18. Maximum value of the directional derivative of $\varphi = xyz^2$ at the point $(1, 0, 3)$ is

- (a) 9 (b) 10 (c) 0 (d) none of these

ANSWERS:

1. b
2. c
3. d
4. c
5. c

6. b
7. b
8. a
9. d
10. b
11. a
12. b
13. a
14. b
15. a
16. b
17. c
18. a