

e by the method of variation of parameters: $(D^2 + 1)y = \sec x \cdot \tan x$

- b) A spring is such that it would be stretched by 19.6 cm by a weight of 4.9 kg. Let the weight be attached to the spring and pulled down 15 cm below the equilibrium position. If the weight is started with an upward velocity of 9.8 cm per second describe the motion. No damping or impressed force is present.
- c) Solve: $(D^2 - 2D + 5)y = e^{2x} \sin x$

Unit - IV

7. a) Find i) $L\left\{\int_0^t \frac{e^t \sin t}{t} dt\right\}$ ii) $L\{e^{-3t}(2 \cos 5t - 3 \sin 5t)\}$

- b) If $f(t)$ is a periodic function with period T , then prove that

$$L\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

- c) Using unit step function find the Laplace transform of $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4, & 2 < t < 4 \\ 0, & t > 4 \end{cases}$

8. a) Find i) $L^{-1}\left\{\frac{s^2}{(s+1)^3}\right\}$ ii) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$

- b) Using Laplace transform technique solve the initial value problem $x''(t) + x(t) = 6 \cos 2t$; $x(0) = 3$, $x'(0) = 1$.

- c) State convolution theorem for Laplace Transform and use it to obtain

$$L^{-1}\left\{\frac{1}{(s^2 + 1)(s+1)}\right\}$$

Unit - V

9. a) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dz dy dx$

- b) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing to polar co-ordinates.

- c) Evaluate $\iint_R y dx dy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

10. a) Using differentiation under integral sign evaluate $\int_0^\infty \frac{e^{-x} \sin \alpha x}{x} dx$.

- b) i) Show that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$. ii) Using Gamma function evaluate $\int_0^1 (\log x)^4 dx$

- c) Evaluate $\int_0^1 \int_0^1 x e^{-x^2-y} dy dx$ by changing the order of integration.

3 Hours

Note: Answer **Five full** questions choosing **One full** question from **each Unit**.

Unit - I

Use Gauss - Seidel iteration method to solve

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72 \quad \text{start with } x^{(0)} = y^{(0)} = z^{(0)} = 0 \quad \text{Carry out three iterations.}$$

$$x + y + 54z = 110$$

- i) Define rank of a matrix. Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 1 & -2 & 3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$ by

- reducing it to row echelon form.
c) Find the numerically largest Eigen value and corresponding Eigen vector of the matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

by taking the initial approximation to the Eigen vector as $[1, 0, 0]^T$.
Carry out five iterations.

- i) Define a basis of a vector space V.
ii) If $\{u_1, u_2, \dots, u_n\}$ is a basis for a vector space V then prove that any vector in V can be expressed as a unique linear combination of vectors in the basis.
b) i) Define linear dependence and linear independence of a set of vectors.
ii) Check whether the set of vectors $\{(1, 0, -1, 2); (4, 2, 0, -1); (6, 4, -2, 3)\}$ are linearly dependent.
c) Check whether $V = \{(x, y) / x, y \in \mathbb{R}\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication defined by $k(x, y) = (kx, ky)$ is a vector space.

Unit - II

- a) Solve the differential equation $y(x + y + 1) dx + x(x + 3y + 2) dy = 0$
b) Solve the differential equation $xdy - ydx = \sqrt{x^2 + y^2} dx$
c) Determine the orthogonal trajectories for the family of curves $r = \frac{2a}{1 + \cos \theta}$

- i. a) Solve the differential equation $x^3 \frac{dy}{dx} - x^2 y = -y^4 \cos x$.
b) A bacterial population is known to grow at a rate proportional to the amount present. After one hour the population of bacteria is 1000 and after 4 hours the population is 3000. Find the number of bacteria present at any time t and initially.
c) Solve the differential equation $(ye^{xy})dx + (xe^{xy} + 2y)dy = 0$

Unit - III

5. a) Solve by the method of undetermined coefficients $(D^2 + 2D + 4)y = 2x^2 + 3x + 1$
b) Solve: $(D^2 + 2D + 2)y = 1 + 3x + x^2$
c) Solve: $x^2 y'' - 2xy' - 4y = x^4$

P.T.O.

13MA201

- c) Solve by the method of variation of parameters $y'' + a^2y = \sec ax$
6. a) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1+x)^2$
- b) Solve $(D^2 - 2D + 3)y = x^2 + \cos x$ by the method of undetermined coefficients.
- c) The differential equation of a simple pendulum is $\frac{d^2x}{dt^2} + \omega^2x = F \sin nt$ where ω and F are constants. If at $t=0$, $x=0$ and $\frac{dx}{dt} = 0$. Determine the motion when $n=\omega$

Unit - IV

7. a) Find the Laplace transform of i) $(t+2)^2 e^t$ ii) $t^5 e^{4t} \cosh 3t$

b) Find i) $L \left\{ \frac{e^{-at} - e^{-bt}}{t} \right\}$

ii) If $L\{f(t)\} = F(s)$ then prove that $L \left\{ \int_0^t f(u) du \right\} = \frac{1}{s} F(s)$.

- c) Express $f(t) = \begin{cases} t-1, & 0 \leq t < 2 \\ 3-t, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$ in terms of unit step function and hence find its Laplace Transform.

a) Find $L^{-1} \left\{ \frac{4s+5}{(s-1)^2(s+2)} \right\}$

b) Using convolution theorem evaluate $L^{-1} \left\{ \frac{1}{(s^2 + a^2)^2} \right\}$

- c) Solve $x''(t) + 4x(t) = 2t - 8$; $x(0) = 1$, $x'(0) = 0$ by Laplace Transform method.

Unit - V

9. a) Change the order of integration in $\int_1^2 \int_1^{x^2} (x^2 + y^2) dy dx$ and evaluate it.

b) Using differentiation under integral sign, evaluate $\int_0^1 \frac{x^{\alpha-1}}{\log x} dx$, $\alpha \geq 0$

c) Show that $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta = \pi$

10. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$

b) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

c) Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$

NMAM/INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belgaum)
Second Semester B.E. (Credit System) Degree Examinations
Make up / Supplementary Examinations - July 2014

13MA201 – ENGINEERING MATHEMATICS - II

Max. Marks: 100

Note: Answer Five full questions choosing One full question from each Unit.

Unit – I

- Unit - I
- a) Check whether the following of vectors is linearly dependent.
 $\{(1,0,-1,2); (4,2,0,-1); (6,4,-2,3)\}$
- b) i) Define basis and dimension of a vector space V .
ii) Find the dimension of the subspace of \mathbb{R}^3 spanned by the vectors $\{(3,1,0); (2,1,3); (1,1,-2)\}$
- c) Check whether $V = \{(x,y) | x,y \in \mathbb{R}\}$ with vector addition defined by $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and scalar multiplication defined by $c(x,y) = (cx,y)$ is a vector space or not.

- a) Determine the value of 'a' so that the system of equations

$$x+y-z=1$$

$$2x+3y+az=3$$

$$x + ay + 3z = 2$$

has

- i) no solution ii) more than one solution iii) a unique solution.

Find the Rank of matrix A by elementary row transformation given that

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

- c) Use Rayleigh's Power method to find numerically the largest Eigen value and the corresponding Eigen vector of the matrix.

Q.10. Find Eigen value and Eigen vector of $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking initial Eigen vector as $[1, 0.8, 0.8]^T$. Carry out 4 iterations.

Unit – II

- a) Solve: $xy \frac{dy}{dx} = 1 + x + y + xy$
b) Solve $y(x + y + 1)dx + x(x + 3y + 2)dy = 0$
c) Solve: $\frac{dy}{dx} = \frac{x+y-1}{x-y+1}$

- a) Solve $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$
b) Solve $(1+y^2)dx + (x - \tan^{-1}y)dy = 0$

- c) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ where λ is a parameter

Unit – III

- a) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x + 4$
b) Solve $y'' - 2y' + y = x \cos x$

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