) Obtain the reduction formula for $\int \cos^n x\,dx$ . Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^n x\,dx$ . 7 L2 5 1

) i) Evaluate $\int_0^{\infty} x^6 e^{-3x}\,dx$ using Gamma function

ii) Evaluate $\int_0^1 x^3 (1-x)^2\,dx$ using Beta and Gamma functions. 6 L1 5 1

) Find the volume of the solid generated by revolving the cardioid $r = a\,(1+\cos\theta)$ about the initial line 7 L2 5 2

) Find the surface area of the solid generated by revolving the astroid $x = a\sin^3 t$ , $y = a\cos^3 t$ about the x-axis 7 L2 5 1

oom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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c) Obtain the Maclaurin's expansion of $\log(1+x)$ upto three non-vanishing terms. 7 L1 2

## Unit – III

5\. a) Find the extreme values of $u = x^2+y^2+12x-6$ 6 L1 3

b) i) If $u$ is a homogeneous function of degree $n$ in $x$ and $y$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$

ii) If $U=\log\left(\frac{x^4+y^4}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$ 7 L2 3

c) If $U=e^x\sin(yz)$, where $x=t^2$, $y=t-1$, $z=1/t$, find $\frac{du}{dt}$ at $t=2$ 7 L2 3

6\. a) Find the directional derivative of $\phi = x^2yz+4xz^2$ at the point (1,-2,-1) in the direction of vector $2i-j-2k$ 6 L1 3

b) Find the angle between the surfaces $x^2+y^2+z^2=5$ and $z=x^2+y^2+1$ at (0,1,2) 7 L2 3

c) Establish the following identities:

(i) $\nabla\bullet\nabla\phi=\nabla^2\phi$ ii) $\nabla\bullet\left(\nabla\times\vec{F}\right)=0$ 7 L2 3

## Unit – IV

7\.

a) Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dx\,dy\,dz$ 6 L1 4

b) Change the order of integration and hence evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ 7 L2 4

c) Find the area of the Cardioid $r=a(1-\cos\theta)$. 7 L2 4

8\.

a) Evaluate $\int_0^1\int_0^{x^2} e^{\frac{y}{x}}\,dy\,dx$ 6 L1 4

b) Show that the area between the parabolas $y^2=4ax$ and $x^2=4ay$ is $\frac{16}{3}a^2$ 7 L2 4

c) By changing to polar co-ordinates evaluate $\int_0^a\int_0^{\sqrt{a^2-y^2}}(x^2+y^2)\,dx\,dy$ 7 L2 4

## Unit – V

9\.

a) Evaluate $\int_0^{\infty}\frac{1}{(1+x^2)^4}\,dx$ 6 L1 5

b) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ with usual notations 7 L2 5

USN

# NMAM INSTITUTE OF TECHNOLOGY, NITTE
(An Autonomous Institution affiliated to VTU, Belagavi)

## First Semester B.E. (Credit System) Degree Examinations
Supplementary Examinations – July 2019

### 18MA101 – ENGINEERING MATHEMATICS - I

ation: 3 Hours | Max. Marks: 100

*Note: Answer **Five full** questions choosing **One full** question from **each Unit***

| | | Marks | BT* | CO* | PO* |
|---|---|---|---|---|---|
| | **Unit – I** | | | | |
| a) | Find the rank of the following matrix using elementary row transformation $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & 2 & 3 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ | 6 | L*2 | 1 | 1 |
| b) | Test for consistency and solve the following system of equations by Gauss elimination method $x+y+z=3$; $x+2y+3z=4$; $x+4y+9z=6$ | 7 | L2 | 1 | 2 |
| c) | Find the eigen values and eigen vectors of the following matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ | 7 | L1 | 1 | 1 |
| a) | Reduce the quadratic form $8x^2+7y^2+3z^2$ into canonical form. | 6 | L2 | 1 | 1 |
| b) | Show that the transformation $y_1 = x_1 + x_2 + 3x_3$; $y_2 = x_1 + 3x_2 - 3x_3$; $y_3 = -2x_1 - 4x_2 - 4x_3$ represent a regular linear transformation. Find the inverse of this transformation. | 7 | L1 | 1 | 2 |
| c) | Diagonalize the matrix $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ | 7 | L2 | 1 | 1 |
| | **Unit – II** | | | | |
| a) | i) State Comparison test. ii) Test the convergence of the following series $\frac{1}{1+\sqrt{2}} + \frac{2}{1+2\sqrt{3}} + \frac{3}{1+3\sqrt{4}} + \dots\dots\dots \infty$ | 6 | L1 | 2 | 2 |
| b) | State Cauchy's root test and test the convergence of the series $\sum\left(\frac{nx}{n+2}\right)^n$ $x>0$ | 7 | L2 | 2 | 1 |
| c) | Obtain Taylor's series expansion of $\log x$ about $x=1$ upto fourth degree terms | 7 | L2 | 2 | 1 |
| a) | State Rolle's theorem, & verify the theorem for $f(x)=\frac{\sin x}{e^x}$ in $[0,\pi]$. | 6 | L2 | 2 | 2 |
| b) | State and prove Cauchy's mean value theorem. | 7 | L2 | 2 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 6. | a) | If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$ | 6 | L1 | 3 |
| | b) | Expand $x^2y + 3y - 2$ at the point $(1, -2)$ using Taylor's theorem upto terms of 2nd degree. | 7 | L2 | 3 |
| | c) | The period of a simple pendulum is $T = 2\pi\sqrt{l/g}$. Find the maximum error in T due to the possible error upto 1% in l and 2.5% in g. | 7 | L1 | 3 |

## Unit – IV

| | | | | | |
|---|---|---|---|---|---|
| 7. | a) | Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^3 + y^2)dxdy$ | 6 | L1 | 4 |
| | b) | Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integral. | 7 | L2 | 4 |
| | c) | Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing the polar coordinates. | 7 | L1 | 4 |
| 8. | a) | Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)dxdydz$ | 6 | L1 | |
| | b) | Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the 1st quadrant. | 7 | L2 | |
| | c) | Evaluate $\int_0^1 \int_x^{\sqrt{x}} xydydx$ by changing the order of integration. | 7 | L1 | |

## Unit – V

| | | | | | |
|---|---|---|---|---|---|
| 9. | a) | Find the volume of the solid generated by revolving one arch of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about the X -axis. | 6 | L1 | |
| | b) | Prove that $\beta(m, n) = \frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}$ with usual notations. | 7 | L1 | |
| | c) | Obtain the reduction formula for $\int \sin^n x dx$ and hence evaluate $\int_0^{\pi/2} \sin^n x dx$. | 7 | L1 | |
| 10. | a) | Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$ | 6 | L1 | |
| | b) | Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}dx$ by using Beta & Gamma functions. | 7 | L1 | |
| | c) | Find the surface area of the solid generated by the revolution of the cardioid $r = a(1 - \cos\theta)$ about the initial line. | 7 | L1 | |

BT* Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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# NMAM INSTITUTE OF TECHNOLOGY, NITTE

*(An Autonomous Institution affiliated to VTU, Belagavi)*

**First Semester B.E. (Credit System) Degree Examinations**

November - December 2019

## 19MA101 – ENGINEERING MATHEMATICS - I

Duration: 3 Hours Max. Marks: 100

*Note:* ***Answer Five full questions choosing One full question from each Unit.***

### Unit – I

| Q. | | Question | Marks | BT* | CO* | PO* |
|---|---|---|---|---|---|---|
| 1. | a) | Find the rank of the matrix using elementary row transformation. $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ | 6 | L*1 | 1 | 1 |
| | b) | Test for consistency and solve the system of equations by Gauss elimination method. $2x + y + 4z = 12,\ 4x + 11y - z = 33,\ 8x - 3y + 2z = 20.$ | 7 | L3 | 1 | 2 |
| | c) | Diagonalize the matrix $\begin{matrix} -19 & 7 \\ -42 & 16 \end{matrix}$ | 7 | L2 | 1 | 2 |
| 2. | a) | Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ | 7 | L3 | 1 | 2 |
| | b) | Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Find the inverse transformation. | 7 | L1 | 1 | 1 |
| | c) | Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. | 6 | L1 | 1 | 1 |

### Unit – II

| Q. | | Question | Marks | BT* | CO* | PO* |
|---|---|---|---|---|---|---|
| 3. | a) | i) State D'Alembert's ratio test. ii) Test the convergence of the series $1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \cdots \infty$ | 6 | L3 | 2 | 1 |
| | b) | State and prove Lagrange's mean value theorem. | 7 | L1 | 2 | 1 |
| | c) | Expand $\log_e(1 + x)$ by using Maclaurin's series upto 4th degree terms. | 7 | L1 | 2 | 1 |
| 4. | a) | Verify Cauchy's mean value theorem for the functions $f(x) = e^x, g(x) = e^{-x}$ in $[a \quad b]$ | 6 | L1 | 2 | 2 |
| | b) | Expand $tan^{-1}x$ at $x = 1$ upto 3rd degree term. | 7 | L1 | 2 | 1 |
| | c) | Discuss the convergence of the series $\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \cdots$ | 7 | L3 | 2 | |

### Unit – III

| Q. | | Question | Marks | BT* | CO* | PO* |
|---|---|---|---|---|---|---|
| 5. | a) | If $\tan u = \frac{x^3+y^3}{x-y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ | 6 | L1 | 3 | 1 |
| | b) | If $u = x + 3y^2,\ v = 4x^2yz,\ w = 2z^2 - xy$. Then find $J = \partial(u, v, w)/\partial(x, y, z)$ at $(1, -1, 0)$ | 7 | L1 | 3 | 1 |
| | c) | A rectangular box open at the top is to have a volume of 32 cubic units. What must be the dimensions of the box so that the total surface area of the box is a minimum? | 7 | L3 | 3 | 2 |

P.T.O.