14MA101

4. a) (i) State Cauchy's root test.

(ii) Test for the convergence of the series $\frac{2}{1}+\frac{2.5}{1.5}+\frac{2.5.8}{1.5.9}+........$

b) If $y=(x^2-1)^n$ then prove that $(1-x^2)y_{n+2}-2xy_{n+1}+n(n+1)y_n=0.$

c) i) Obtain the Maclaurin's series expansion of $f(x)=\tan^{-1}x$ up to the terms containing x^3.

ii) State Leibnitz theorem for the n^{th} derivative of y=uv where u and v are differentiable functions of x.

Unit – III

5. a) If $z=e^{ax+by}f(ax-by)$ then prove that $b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$

b) If $u=\frac{yz}{x}, v=\frac{zx}{y}, w=\frac{xy}{z}$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$.

c) Find the percentage error in the calculated value of the volume of a rectangular parallelopiped when errors of 2%,-1% and 1% are made in measuring the length, breadth and height respectively.

6. a) If u is a homogeneous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu.$

Hence deduce that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u.$

b) Using Maclaurin's series expand $f(x,y)=e^{ax+by}$ upto second degree terms in x and y.

c) Find the maximum value of $x^2+y^2+z^2$ when $x+y+z=3a$ by Lagrange's multipliers method.

Unit – IV

7. a) If ρ_1 and ρ_2 are the radii of curvature at the extremities of any chord of the cardioid $r=a(1+\cos\theta)$ which passes through the pole, then show that $\rho_1^2+\rho_2^2=\frac{16a^2}{9}$.

b) Prove that the curves $r=a(1+cos\theta)$ $r=b(1-cos\theta)$ intersect each other orthogonally.

c) With usual notations prove that $\rho=\frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$.

8. a) Verify Cauchy's mean value theorem for the functions $f(x)=\frac{1}{x^2}$, $g(x)=\frac{1}{x}$ in [a,b], b>a>0

b) State and prove Lagrange's mean value theorem.

c) Verify Rolle's theorem for the function $f(x)=\log\left(\frac{x^2+12}{7x}\right)$ in $[3,4]$

Unit – V

9. a) Obtain the reduction formula for $\int\cos^n x dx$. Hence evaluate $\int_0^{\frac{\pi}{2}}\cos^n x dx$.

b) Find the surface area of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the x-axis.

c) Trace the curve $r=a(1+\cos\theta)$

10. a) Find the volume of the solid generated by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line.

b) Evaluate (i) $\int_0^{2a}\frac{x^2}{\sqrt{2ax-x^2}}dx$ ii) $\int_0^{\pi}Sin^4 x\, dx$.

c) Find the area bounded by the parabola $y^2=4ax$ and its latus rectum.

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NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belgaum)

First Semester B.E. (Credit System) Degree Examinations

Make up Examinations - January 2015

14MA101 – ENGINEERING MATHEMATICS - I

Duration: 3 Hours Max. Marks: 100

Note: Answer Five full questions choosing One full question from each Unit.

Unit – I

1. a) Find the rank of the following matrix using elementary row transformations.

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ -1 & 7 & 4 & 9 \\ 7 & -7 & 6 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ 6

b) Using Rayleigh's power method, obtain the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$, select $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as the initial eigen vector and carry out five iterations. 7

c) Use Gauss-Seidel iteration method to solve the following system of linear equations:

$20x + y - 2z = 17$

$3x + 20y - z = -18$ start with $x^{(0)} = y^{(0)} = z^{(0)} = 0$ and carry out three iterations.

$2x - 3y + 20z = 25$ 7

2. a) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to canonical form. 6

b) (i) If A is an orthogonal matrix then prove that A^T is also orthogonal.

(ii) Prove that the matrix $\begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{bmatrix}$ is an orthogonal matrix. 7

c) Diagonalise the matrix $\begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. 7

Unit – II

3. a) Prove that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges for $p \le 1$ using Cauchy's integral test. 6

b) State comparison test. Test for convergence of the series $\frac{1}{1^2} + \frac{1+2}{1^2+2^2} + \frac{1+2+3}{1^2+2^2+3^2} + \ldots\ldots$ 7

c) If $y = \tan^{-1}x$, then prove that $(1 + x^2)y_{n+2} + [2(n+1)x]y_{n+1} + n(n+1)y_n = 0$. 7

P.T.O.

Supplementary – July 2015

b) (i)Test for convergence of the series $\sum_{n=1}^{\infty}(1+\frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$

(ii) Test for convergence of the series $\sum_{n=1}^{\infty}3^n(\frac{n}{n+1})^{n^2}$ 7

c) Prove that the harmonic series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges for $p>1$ and diverges for $p\le 1$ using Cauchy's integral test. 7

Unit – III

5. a) If u is a homogeneous function of degree n in x and y, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$.
Hence deduce that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$. 7

b) If $z=f(u,v)$ where $u=x^2-y^2, v=2xy$ then prove that $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=2(x^2+y^2)\frac{\partial z}{\partial u}$ 7

c) Expand $f(x,y)=e^{xy}$ about (1,1) in Taylor's series up to terms of second degree. 6

6. a) The temperature T at any point (x,y,z) in space is $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. 7

b) Find the possible error in computing the resistance r from the formula , $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}$ if r_1, r_2 are both in error by 2%. 7

c) If $x+y+z=u, y+z=uv$ and $z=uvw$, find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. 6

Unit – IV

7. a) With usual notation prove that $\tan\phi=r\frac{d\theta}{dr}$. 7

b) Find the angle of intersection between the curves $r=2\sin\theta$ and $r=\sin\theta+\cos\theta$. 7

c) Show that the radius of curvature at any point of the cardioid $r=a(1-\cos\theta)$ varies as $\sqrt{r}$. 6

a) State Rolle's theorem. Verify Rolle's theorem for the function $f(x)=2x^3+x^2-4x-2$ in $[-\sqrt{2},\sqrt{2}]$. 7

b) State and prove Lagrange's mean value theorem 7

c) Verify Cauchy's mean value theorem for the functions $f(x)=x^3, g(x)=x^2$ in [1,2]. 6

Unit – V

a) Obtain the reduction formula for $\int\sin^n x dx$. Hence evaluate $\int_0^{\frac{\pi}{2}}\sin^n x dx$. 7

b) Find the volume of the spindle shaped solid generated by the revolution of the astroid $x^{2/3}+y^{2/3}=a^{2/3}$ about the x-axis. 7

c) Evaluate $\int_0^{2a}\frac{x^{\frac{7}{2}}}{\sqrt{2a-x}}dx$. 6

a) Trace the curve $y^2=x^2(\frac{a-x}{a+x}), a>0$ 7

b) Find the area bounded by one arch of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ and its base 7

c) Find the surface area of the solid generated by revolving the astroid $x=a\sin^3 t \quad y=a\cos^3 t$ about the x-axis. 6

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NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester B.E. (Credit System) Degree Examinations

Supplementary Examinations – July 2015

14MA101 – ENGINEERING MATHEMATICS - I

Duration: 3 Hours Max. Marks: 100

Note: Answer ***Five full*** *questions choosing* ***One full*** *question from each Unit.*

Unit – I

1. a) Find the rank of the following matrix using elementary row transformations.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$ 6

b) Using Rayleigh's power method, obtain the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, select $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ as the initial eigen vector and carry out five iterations. 7

c) Use Gauss-Seidel iteration method to solve the following system of linear equations:

$20x + y - 2z = 17$

$3x + 20y - z = -18$ start with $x^{(0)} = y^{(0)} = z^{(0)} = 0$ and carry out three iterations.

$2x - 3y + 20z = 25$ 7

a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to canonical form. 6

b) (i) If A is an orthogonal matrix then prove that A^T is also orthogonal.

(ii) Prove that the matrix $\begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{bmatrix}$ is an orthogonal matrix. 7

c) Diagonalise the matrix $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. 7

Unit – II

a) (i) State Cauchy's root test.

(ii) Test for convergence of the series $\sum_{n=1}^{\infty} \sqrt{n^2+1} - n$ 6

b) (i) State Cauchy's integral test.

(ii) Using Cauchy's integral test, test the following series for convergence $\sum_{2}^{\infty} \frac{1}{n \log n}$. 7

c) Obtain the Taylor's series expression of $\sin x$ up to fourth degree terms. 7

a) If $y = \tan^{-1} x$ then prove that

$\left(1 + x^2\right) y_{n+2} + 2(n+1) x y_{n+1} + n(n+1) y_n = 0.$ 6

P.T.O.

State and prove Cauchys Mean value theorem. 7 L2

Find the angle of intersection between the curves $r=a(1-\cos\theta)$ and $r=2a\cos\theta$ 6 L1 L3

With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$ 7 L2

State Lagranges mean value theorem.Using Lagranges Mean value theorem determine c in the function $f(x)=(x-1)(x-2)(x-3)$ in [0,4] 7 L1 L2

Find the pedal equation for the curve $\frac{2a}{r}=1-\cos\theta$ 6 L1 L3

Unit – V

Obtain the reduction formula $\int \sin^n x\,dx$. Hence evaluate $\int_0^{\frac{\pi}{2}} \sin^n x\,dx$ where n is a positive integer. 7 L2 L3

Find the length of the cardioid r=a(1+cosθ) also show that the upper half is bisected by θ=π/3 7 L1 L3

Evaluate i) $\int_0^a \frac{x^7}{\sqrt{(a^2-x^2)}}dx$

ii) $\int_0^{\frac{\pi}{2}} \cos^6 x\,dx$ 6 L3 L3

7 L3

Trace the curve x=acos³t , y=asin³t. 7 L3

Find the area included between the curve $y^2(2a-x)=x^3$ and its asymptote.

Find the surface of the solid formed by revolving the cardioid r = a(1+cosθ) about the initial line. 6 L

...m's Taxonomy, L* Level *************

b) Test for the convergence of the series (i) $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\ldots\infty$

(ii) $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\ldots\infty$ 7

c) (i) State Leibnitz theorem.

(ii) If $y=(\sin^{-1}x)^2$,show that $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-n^2 y_n=0$. 7

4. a) (i) State Cauchy's root test.

(ii) Test for the convergence of the series $\left(\frac{1}{3}\right)^2+\left(\frac{1.2}{3.5}\right)^2+\left(\frac{1.2.3}{3.5.7}\right)^2+\ldots\infty$ 6

b) Obtain the Maclaurin's series of $f(x)=e^{(\sin 2x)}$ upto terms containing x^3 7

c) State Cauchy's integral test. Using Cauchy's integral test prove that $\sum_{1}^{\infty}\frac{1}{n^p}$ converges for p>1 and diverges for $p\le 1$. 7

Unit – III

5. a) State and prove the Euler's theorem. Verify Euler's theorem for the function $u=\sqrt{x^2+y^2}$. 7

b) If $x=e^u\cos v,\ y=e^u\sin v$ then find $J=\frac{\partial(x,y)}{\partial(u,v)}$ and $J'=\frac{\partial(u,v)}{\partial(x,y)}$. Hence show that $JJ'=1$. 7

c) Prove that, if the perimeter of a triangle is constant, the triangle has maximum area when it is equilateral. 7

6. a) If $u=\frac{e^{x+y}}{e^x+e^y}$ then show that $u_x+u_y=u$. 6

b) If $z=f(u,v)$ where $u=x^2-y^2,\ v=2xy$, prove that

i) $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=2(x^2+y^2)\frac{\partial z}{\partial u}$

ii) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=4(x^2+y^2)\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]$. 7

c) Expand the function $f(x,y)=x^y$ about the point (1,1) upto third degree terms. 7

Unit – IV

7. a) Show that radius of curvature at any point of the cardioid $r=a(1-\cos\theta)$ varies as $\sqrt{r}$ 7

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First Semester B.E. (Credit System) Degree Examinations

November - December 2015

15MA101 – ENGINEERING MATHEMATICS - I

on: 3 Hours

Max. Marks: 100

Note: Answer Five full questions choosing One full question from each Unit.

Unit – I | Marks | BT*

) Define the rank of a matrix. Find the rank of the matrix

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$ using elementary row transformations. 7 L*2

) Using Gauss Seidel iteration method solve the system of equations
$20x+y-2z=17$
$3x+20y-z=-18$
$2x-3y+20z=25$ start with $x^{(0)}=y^{(0)}=z^{(0)}=0$ and carry out three iterations. 7 L3

) Determine the values of 'a' and 'b' for which the system
$x+2y+3z=6$
$x+3y+5z=9$
$2x+5y+az=b$ has i) no solution ii) unique solution. 6 L3

) Using Rayleigh's power method, obtain the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$, select $\begin{bmatrix} 1 \\ 0.8 \\ 0.8 \end{bmatrix}$ as the initial eigen vector and carry out five iterations. 7 L3

) Diagonalize the matrix $A=\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ 7 L5

) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to canonical form 6 L2

Unit – II

) Test for the convergence of the series (i) $\sum_{1}^{\infty}\left(\frac{n+1}{3n}\right)^n$. L1

(ii) $\frac{4}{18}+\frac{4.12}{18.27}+\frac{4.12.20}{18.27.36}+\dots\infty$ 6 L3