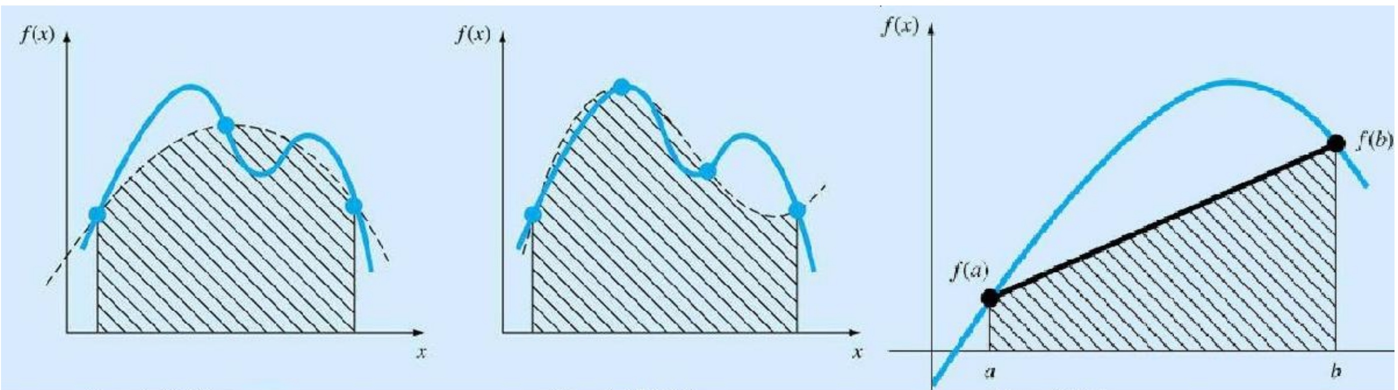
Numerical Integration:

Most common numerical integration rules are:

1.Trapezoidal Rule

2.Simpson's 1/3rd Rule

3.Simpson's 3/8th Rule



Simpson's 1/3 Rule   Simpson's 3/8th Rule   Trapezoidal Rule

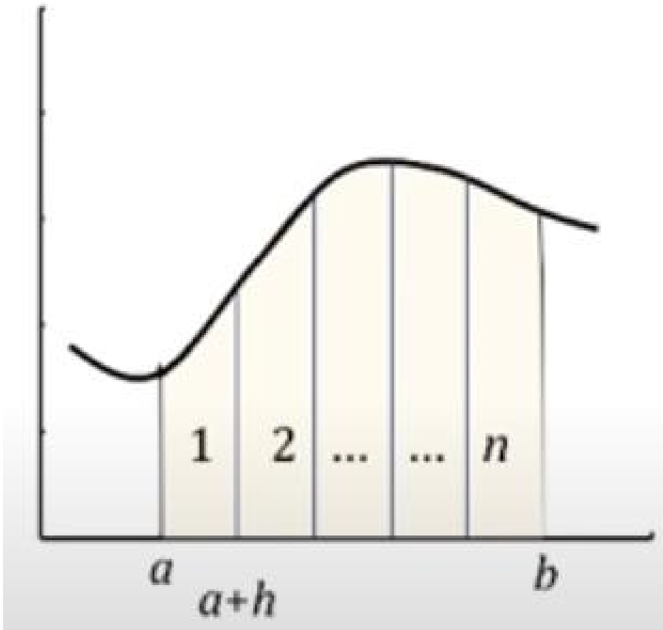
For single application

Trapezoidal Rule: $\int_a^b f(x)\,\mathrm{dx} = \frac{h}{2}\left[f(a) + f(a+h)\right]$, where $h = b - a$

Simpson's 1/3rd Rule: $\int_a^b f(x)\,\mathrm{dx} = \frac{h}{3}\left[f(a) + 4f(a+h) + f(a+2h)\right]$, where $h = \frac{b-a}{2}$

Simpson's 3/8th Rule: $\int_a^b f(x)\,\mathrm{dx} = \frac{3h}{8}\left[f(a) + 3f(a+h) + 3f(a+2h) + f(a+3h)\right]$, where $h = \frac{b-a}{3}$

For multiple segments direct formula:



Trapezoidal Rule: If we write, $f(a) = f_0; f(a+h) = f_1; \ldots.. f(a+\mathrm{nh}) = f_n$; where a+nh=b

Then, $\int_a^b f(x)\mathrm{dx} = \frac{h}{2}\left[f_0 + 2(f_1 + f_2 + \ldots + f_{n-1}) + f_n\right]$

Simpson's 1/3 Rule: If we write, $f(a) = f_0; f(a+h) = f_1; \ldots.. f(a+\mathrm{nh}) = f_n$; where a+nh=b & n=even number

Then, $\int_a^b f(x)\mathrm{dx} = \frac{h}{3}\left[f_0 + 4(f_1 + f_3 + \ldots + f_{n-1}) + 2(f_2 + f_4 + \ldots + f_{n-2}) + f_n\right]$

Simpson's 3/8 Rule: If we write, $f(a) = f_0; f(a+h) = f_1; \ldots.. f(a+\mathrm{nh}) = f_n$; where a+nh=b & n is multiple of 3

Then, $\int_a^b f(x)\mathrm{dx} = \frac{3h}{8}\left[f_0 + 3(f_1 + f_2 + f_4 + f_5 + \ldots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \ldots + f_n)\right]$

Example:

Find the Numerical integration of $f(x) = 2 - x + \ln x$;In the limit a=1 & b=2. consider intervals n={5,10,15,20} & make a log log plot of error

Using: Trapezoidal Rule & Simpson's 1/3 Rule

Solution:

First create a function file to evalute the given function

Give name of the function file as $myFunc$.m

edit myFunc.m

```
function f=myFunc(x)

f=2-x+log(x);
```

end

Now calculate the true value of the integration

$$\int_a^b [2 - x + \log(x)]\,\mathrm{dx} = \left[x - \frac{x^2}{2} + \mathrm{xlnx}\right]_a^b$$

```
a=1;
b=2;
trueVal=(b-b^2/2+b*log(b))-(a-a^2/2+a*log(a))
```

```
trueVal = 0.8863
```

Trapezoidal rule:

```
n=20;% You can change the values of n and check the error or you can assign n as an array and
h=(b-a)/n
```

```
h = 0.0500
```

```
x=a:h:b
```

```
x =

    1.0000    1.0500    1.1000    1.1500    1.2000    1.2500    1.3000    1.3500
```

```
f=myFunc(x)
```

```
f =

    1.0000    0.9988    0.9953    0.9898    0.9823    0.9731    0.9624    0.9501
```

Method 1-Trapezoidal: Using single application formula at each segment and then sum all the integrals

```
numValue_trap_intrvl=zeros(1,n);
for i=1:n
    numValue_trap_intrvl(i)=(h/2)*(f(i)+f(i+1));
end
numValue_trap=sum(numValue_trap_intrvl)
```

```
numValue_trap = 0.8862
```

Method 2-Trapezoidal: Using direct formula $\int_a^b f(x)\mathrm{dx} = \frac{h}{2}[f_0 + 2(f_1 + f_2 + ... + f_{n-1}) + f_n]$

```
numValue_trap_direct=(h/2)*(f(1)+2*sum(f(2:n))+f(n+1))
```

```
numValue_trap_direct = 0.8862
```

Error in Trapezoidal Rule

```
err_trap=abs(trueVal-numValue_trap)
```

```
err_trap = 1.0415e-04
```

Simpson's 1/3 Rule:

```
n=20;% Make sure n is even
h=(b-a)/n
```

```
h = 0.0500
```

```
x=a:h:b
```

```
x =

    1.0000    1.0500    1.1000    1.1500    1.2000    1.2500    1.3000    1.3500
```

```
f=myFunc(x)
```

```
f =

    1.0000    0.9988    0.9953    0.9898    0.9823    0.9731    0.9624    0.9501
```

Method 1-Simpson's 1/3 Rule: Using single application formula at each segment and then sum all the integrals

```
numValue_simp1_3_intrvl=zeros(1,n);
for i=1:2:n-1
    numValue_simp1_3_intrvl(i)=(h/3)*(f(i)+4*f(i+1)+f(i+2));
end
numValue_simp1_3=sum(numValue_simp1_3_intrvl)
```

```
numValue_simp1_3 = 0.8863
```

Method 2-Simpson's 1/3 Rule: Using direct formula $\int_a^b f(x)\mathrm{dx} = \frac{h}{3}\left[f_0 + 4(f_1 + f_3 + ... + f_{n-1}) + 2(f_2 + f_4 + ... + f_{n-2}) + f_n\right.$

```
numValue_simp1_3_direct=(h/3)*(f(1)+4*sum(f(2:2:n))+2*sum(f(3:2:n-1))+f(n+1))
```

```
numValue_simp1_3_direct = 0.8863
```

Error in Simpson's 1/3 Rule

```
err_trap=abs(trueVal-numValue_simp1_3)
```

```
err_trap = 6.0526e-08
```

Inbuilt MATLAB function integral

Steps:

Create a function file

then use $integral(@(x)functionFileName(x), a, b)$ to calculate integral in the range a to b

Example:

It is known that when hot oil is flowing over a flat plate, wall shear stress is varying along the length of the plate $\tau_w(x) = 0.332u_\infty\sqrt{\rho\mu u_\infty}\,x^{-1/2}$, where $\rho = 900\frac{\text{kg}}{m^3}$, $u_\infty = 1\frac{m}{s}$, $\mu = 0.06\text{Pa}.s$, Length of the plate is l=2m. Find the total drag force acting on the plate.

Solution:

Create the function file with name $wallShear.m$

Use integral function to calculate Drag= $\int_0^l \tau_w(x)\,\text{dx}$

```
l=2; %length of the plate in m
D=integral(@(y) wallShear(y),0,l)
```

```
D = 6.9005
```

Assignment:

1. $\int_a^b [2 - x + \log(x)]\,\text{dx} = \left[x - \frac{x^2}{2} + \text{xlnx}\right]_a^b$ Use Simpson's 3/8 rule to solve the the problem & find the error take n=20, a=1,b=2
2. It is known that when hot oil is flowing over a flat plate, wall shear stress is varying along the length of the plate $\tau_w(x) = 0.332u_\infty\sqrt{\rho\mu u_\infty}\,x^{-1/2}$, where $\rho = 900\frac{\text{kg}}{m^3}$, $u_\infty = 1\frac{m}{s}$, $\mu = 0.06\text{Pa}.s$, Length of the plate is l=2m. Find the total drag force acting on the plate. Use numerical integration method to solve this problem.

Solution 2:

```
l=2;
n=100;% Make sure n is even for Simpson's 1/3 rule
a=.001;
b=l;
h=(b-a)/n
```

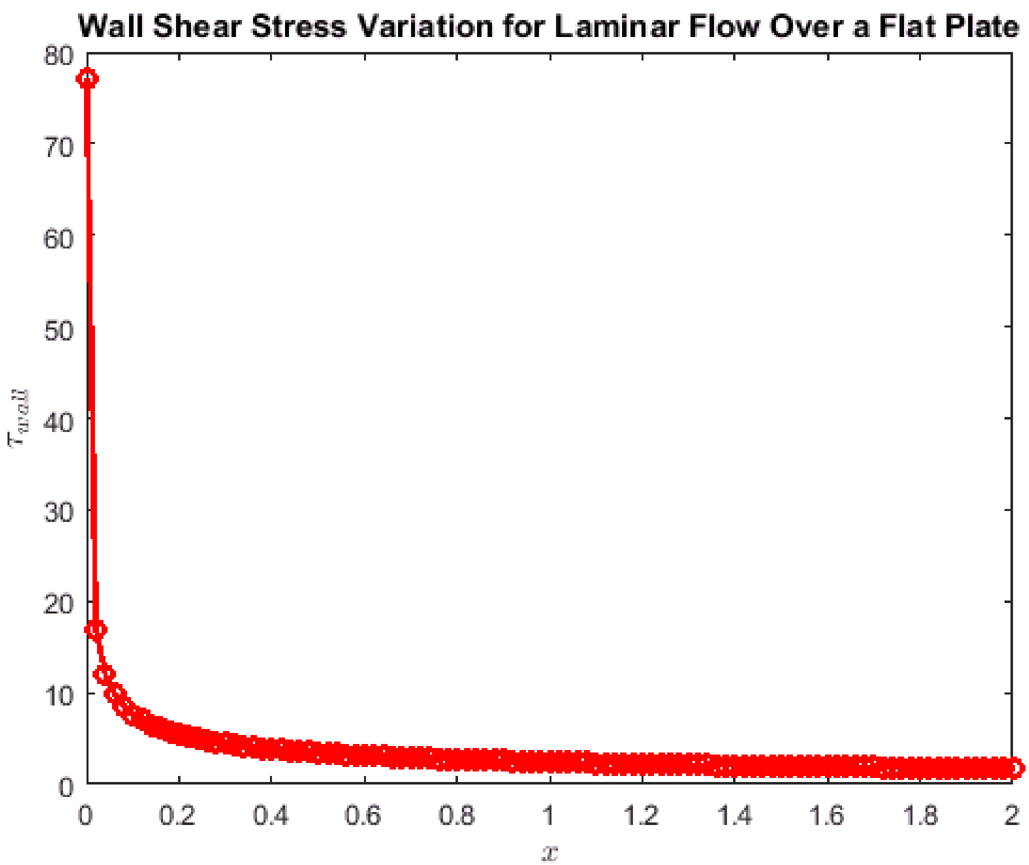
```
h = 0.0200
```

```
x=a:h:b
```

```
x =

    0.0010    0.0210    0.0410    0.0610    0.0810    0.1010    0.1209    0.1409
```

```
% Wall Shear Stress
tau=wallShear(x);
plot(x,tau,'r-o','LineWidth',2)
xlabel('$x$','Interpreter','latex');
ylabel('$\tau_{wall}$','FontSize',12,'Interpreter','latex');
title('Wall Shear Stress Variation for Laminar Flow Over a Flat Plate');
```




```
% To find drag per unit width on the plate we have integrate tau along x

% Using Simpson's 1/3 rule
numValue_simp1_3_intrvl=zeros(1,n);
for i=1:2:n-1
    numValue_simp1_3_intrvl(i)=(h/3)*(tau(i)+4*tau(i+1)+tau(i+2));
end
numValue_simp1_3=sum(numValue_simp1_3_intrvl)
```

```
numValue_simp1_3 = 6.9562
```

```
%Using Trapizoidal Rule
numValue_trap_intrvl=zeros(1,n);
for i=1:n
    numValue_trap_intrvl(i)=(h/2)*(tau(i)+tau(i+1));
```

```
end
numValue_trap=sum(numValue_trap_intrvl)
```

```
numValue_trap = 7.1457
```