

1) Construct an algorithm that finds both the largest and smallest entries among $a[0] \dots a[n-1]$. A straightforward method might require roughly $2n$ element comparisons. Instead, try to find a method that does roughly $1.5n$ element comparisons.

Hint: Two standard approaches involve

i) working with 2 elements at a time ii) breaking the list in half and working recursively.

2) Consider the following algorithm to evaluate $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where the values are all real numbers (floats).

```
p = a[0];
xpower = 1;
for (int i = 1; i <= n; i++) {
    xpower = x * xpower;
    p = p + a[i] * xpower;
} //endfor
```

How many float multiplications are performed? How many float additions are performed? Can you improve on this algorithm?

1) Suppose $a[0]..a[n-1]$ is an array of n elements with the first m elements ($a[0]..a[m-1]$) in ascending order and the final $n-m$ elements ($a[m]..a[n-1]$) in no particular order.

Searching for an entry will be done as follows:

- i) Perform a binary search on $a[0]..a[m-1]$.
- ii) If not found, perform a sequential search on $a[m]..a[n-1]$.

Assume the following: We have $m = 2^k$ for some nonnegative integer k and the binary search algorithm always performs $(\lg m) + 1$ comparisons for both successful and unsuccessful searches. (Note that $\lg x$ is standard notation for $\log_2 x$.)

- a) Determine the number of array entry comparisons for an unsuccessful search.
- b) Determine the average number of array entry comparisons for a successful search.

Assume that all n outcomes are equally likely.

Your answers should be precise functions of n and m in closed form.

Note: Always show the work you did to obtain your solutions.

Some rules for all graded problems:

You must submit work that is totally your own. Do not get help from anyone else and do not share your work with other students. Academic dishonesty will result in course failure and the filing of a report with the Student Affairs Office.

These problems are due at the beginning of class on the date listed above. Late problems will not be accepted. If you cannot attend class that day, be sure to email your solutions by the beginning of class.

The problems will be graded not only for correctness, but also for clarity and conciseness.

a) $(\log_2(m) + 1) + (m-n)$

b) $(\log_2(m) - 1) + (1/n)$