Name____________________________________

Math 2150

Homework 7 Due Aug 14

1. (2 points) Convert 33471 to base 2. Then use the base 2 representation to give base 4, base 8 and base 16 representations. You don't need to show your work.

   Base 2: 1000001010111111

   Base 4: 20022333

   Base 8: 101277

   Base 16: 82BF

2. Greatest Common Divisor

   a. (1 point) Give the prime factorizations of 158 and 58. Then use those to find gcd(158,58).
   158 = 2*79
   58 = 2 * 29
   gcd(158,58) = 2

   b. (1 point) Compute gcd(158,58) using the recursive algorithm gcd(a,b) = gcd(b, a mod b), gcd(a,0) = a.

   gcd(158, 58) = gcd(58, (158 mod 58 = 42))
   = gcd(42, (58 mod 42 = 16))
   = gcd(16, (42 mod 16 = 10))
   = gcd(10, (16 mod 10 = 6))
   = gcd(6, (10 mod 6 = 4))
   = gcd(4, (6 mod 4 = 2))
   = gcd(2, (4 mod 2 = 2))
   = gcd(2, (2 mod 2 = 0)) = 2

   c. (2 points) Show the steps in the Euclidean Algorithm to find gcd(158, 58)

   gcd(58, 158) :
   158 = (2 * 58) + 42
   58 = (1 * 42) + 16
   42 = (2 * 16) + 10
   16 = (1 * 10) + 6
   10 = (1 * 6) + 4
   6 = (1 * 4) + 2
   4 = (1 * 2) + 2
   2 = (1 * 2) + 0
   gcd is 2

   d. (2 points) Using your work in Part c. and a series of backward substitutions find Bézout coefficients of 158 and 58. (see page 270 and lecture notes for examples). Show your work.
   2 = 4 - (1 * 2)
   2 = 4 - (1 * (6 - (1 * 4))) = (2 * 4) - (1 * 6)
   2 = (2 * ((1 * 10) - 6)) - (1 * 6) = (2 * 10) - (3 * 6)
   2 = (2 * 10) - (3 * (1 * 16) - 10)) = (5 * 10) - (3 * 16)
   2 = (-3 * 16) + (5 * 10) = (-3 * 16) + (5 * (42 - (2 * 16))) = (5 * 42) - (13 * 16)
   2 = (5 * 42) - (13 * (58 - 42)) = (-13 * 58) + (18 * 42)
   2 = (-13 * 58) + (18 * (158 - (2 * 58))
   2 = (18 * 158) - (49 * 58) // Bézout Coefficients are 18 and -49

   e. (2 points) Solve the Diophantine equation 158s + 58t = 2. That means Find all integer pairs (s,t) such that 158s + 58t = 2. Justify your answer.

f. (2 points) Solve the Diophantine equation $158s + 58t = 43200$. Hint: find one solution, then proceed as in part e.

g. (2 points) A contractor will purchase two types of board for a project. The high-grade boards cost \$15.80 each and the low-grade boards cost \$5.80. If he must spend exactly \$4320, how many low-grade and how many high-grade boards can he buy? Give all solutions.

3. Modular Inverses

a. (2 points) Find the smallest positive inverse of 29 modulo 50. You must use the approach shown on page 276 and show your work.

29^(-1)

b. (2 points) Solve the linear congruence $29x \equiv 7 \pmod{50}$ (See page 277 for an example.) Show your work.

c. (1 point) Does the linear congruence $4x \equiv 12 \pmod{64}$ have a solution? If it does find all solutions. If it doesn't, explain why there can be no solution.

d. (1 point) Does the linear congruence $15x \equiv 7 \pmod{9}$ have a solution? If it does find all solutions. If it doesn't, explain why there can be no solution.

4. (3 points) Use the Chinese Remainder Theorem to solve the system $x \equiv 3 \pmod{5}$, $x \equiv 2 \pmod{3}$, $x \equiv 1 \pmod{4}$. Show your work. (See Example 5 on page 278).

5. (2 points) Use Fermat's little theorem to find the remainder when $6^{1975875393}$ is divided by 11. Show your work.