Exploration of Polynomials

Topic 9 Homework

JavaScript

```
  function isPositive(number) {
    return number > 0;
  };

  function sameParity(a,b) {
    return (a-b)%2 == 0;
  };

  caseInsensitive = function(a,b) {
    return a.toLowerCase() == b.toLowerCase();
  };

// sameDerivative checks to see if the derivative with respect to x and c are equal.
  sameDerivative = function(a,b) {
    return (a.derivative('x').equals( b.derivative('x') ) && a.derivative('C').equals( b.der
  };

  slowOdd = function(a) {
    return new Promise( function(resolve, reject) {
      if (a == 0)
        reject('I do not like zero.');
      else
        setTimeout(function(){
          resolve(a % 2 == 1);
        }, 1000);
    });
  };
```

Problem 1 *Consider the polynomial $p(x) = x^2 + 15x + 56$. How many real roots does this polynomial have counting multiplicity?* $\boxed{2}$

Problem 1.1 *What are the zeros of $p(x)$? Order from smallest to largest.*

$\boxed{-8}$, $\boxed{-7}$

Learning outcomes:
Author(s):

Problem 1.1.1 *What is the fully factored form of the polynomial? Enter the root corresponding to the smallest zero first.*

$(\boxed{x+8})(\boxed{x+7})$

Problem 1.1.2 *What are the zeros of $p(2x+1)$? Order from smallest to largest.*

$\boxed{\frac{-9}{2}}, \boxed{-4}$

Problem 2 *Consider the function $f(x) = \frac{3}{4}(-x-2)^3 + 1$.*

What is the parent function of f? $g(x) =$ $\boxed{x^3}$

Problem 2.1 *Which of the following graphical transformations occurs? (Select all that apply.)*

Select All Correct Answers:

(a) *The graph is horizontally stretched by a factor of $\frac{3}{4}$.*

(b) *The graph is horizontally stretched by a factor of $\frac{4}{3}$.*

(c) *The graph is vertically stretched by a factor of $\frac{3}{4}$.* ✓

(d) *The graph is vertically stretched by a factor of $\frac{4}{3}$.*

(e) *The graph is reflected over the y-axis.* ✓

(f) *The graph is reflected over the x-axis.*

Problem 2.1.1 *Which of the following translations occur? (Select all that apply.)*

Select All Correct Answers:

(a) *The graph is shifted 2 units left.*

(b) *The graph is shifted 2 units right.* ✓

(c) *The graph is shifted 2 units up.*

(d) *The graph is shifted 2 units down.*

(e) *The graph is shifted 1 unit left.*

(f) *The graph is shifted 1 unit right.*

(g) *The graph is shifted 1 unit up.* ✓

(h) *The graph is shifted 1 unit down.*

Problem 2.1.1.1 *Which of the transformations occurs first?*

Multiple Choice:

(a) *The graph is vertically stretched by a factor of* $\frac{3}{4}$.

(b) *The graph is shifted right by 2 units.*

(c) *The graph is shifted up by 1 unit.*

(d) *The graph is reflected over the y-axis.* ✓

Problem 2.1.1.2 *Which of the transformations occurs second?*

Multiple Choice:

(a) *The graph is vertically stretched by a factor of* $\frac{3}{4}$.

(b) *The graph is shifted right by 2 units.* ✓

(c) *The graph is shifted up by 1 unit.*

(d) *The graph is reflected over the y-axis.*

Problem 2.1.1.3 *Which of the transformations occurs third?*

Multiple Choice:

(a) *The graph is vertically stretched by a factor of* $\frac{3}{4}$. ✓

(b) *The graph is shifted right by 2 units.*

(c) *The graph is shifted up by 1 unit.*

(d) *The graph is reflected over the y-axis.*

Problem 2.1.1.4 *Which of the transformations occurs last?*

Multiple Choice:

(a) *The graph is vertically stretched by a factor of* $\frac{3}{4}$.

(b) *The graph is shifted right by 2 units.*

(c) *The graph is shifted up by 1 unit.* ✓

(d) *The graph is reflected over the y-axis.*

Problem 3 *Write an expression to describe the following transformations: The function* $f(x)$ *is horizontally stretched/compressed by a factor of* $\frac{1}{3}$*, reflected over the x-axis and then shifted up 2 units.*

$g(x) = \boxed{-f(3x) + 2}$

Problem 3.1 *Determine the new coordinates of the given points under this set of transformations. (That is, the given points are on the graph of* f*. Use them to find points on the graph of* g*.)*

A = (2,3) *B = (-1,0)*

C = (0, 5) *D = (-3, -3)*

$A' = (\boxed{\frac{2}{3}}, \boxed{-1})$ $B' = (\boxed{-\frac{1}{3}}, \boxed{2})$

$C' = (\boxed{0}, \boxed{-3})$ $D' = (\boxed{-1}, \boxed{5})$

Problem 3.1.1 *Determine four points on the graph of $g^{-1}(x)$. Order from smallest x-coordinate to largest x-coordinate.*

$(\boxed{-3}, \boxed{0})$, $(\boxed{-1}, \boxed{\frac{2}{3}})$, $(\boxed{2}, \boxed{-\frac{1}{3}})$, $(\boxed{5}, \boxed{-1})$

Problem 4 *Perform the given operations given $f(x) = \dfrac{1}{x+1}$, $g(x) = 3x^2 - 6$ and $h(x) = x - 3$.*

$(f \circ g)(x) = \boxed{\dfrac{1}{3x^2 - 5}}$

$(h - g)(x) = \boxed{-3x^2 + x + 3}$

$(f \circ f)(2) = \boxed{\dfrac{3}{4}}$

$(f \cdot h)(-3) = \boxed{3}$

Problem 5 *Fully factor $x^6 - 64$. (Hint: $x^6 = (x^3)^2$.)*

How many terms does the fully factored form have? $\boxed{4}$

Problem 5.1 *What are the (real-valued) zeros of this polynomial? Order from smallest to largest.*

$\boxed{-2}, \boxed{2}$

Problem 5.1.1 *Which of the following is the fully factored form of this polynomial?*

Multiple Choice:

(a) $(x^2 - 4)(x^4 + 4x^2 + 16)$

(b) $(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$ ✓

(c) $(x - 2)(x + 2)(x^2 + 2x + 4)^2$

(d) $(x - 2)(x + 2)(x^2 - 2x + 4)^2$

Problem 6 *Consider the polynomial* $p(x) = 6x^3 + 28x^2 + 26x - 16$. *How many roots (real or complex) does this polynomial have, counting multiplicity?* $\boxed{3}$

Problem 6.1 *Which of the following is not a possible zero of p according to the rational root theorem?*

Multiple Choice:

(a) *-8*

(b) $\frac{4}{3}$

(c) *4*

(d) $\frac{1}{16}$ ✓

(e) *-16*

Problem 6.2 *Factor p fully to determine the number of real zeros. How many real-valued zeros does* $p(x)$ *have?* $\boxed{1}$

Problem 6.2.1 *What is the real-valued zero of p?* $x = \boxed{-\frac{8}{3}}$

Problem 7 *Consider the polynomials* $p(x) = 2x^5 + x^4 - 6x + 9$ *and* $q(x) = x^2 - 3x + 1$.

What is the quotient when $p(x)$ *is divided by* $q(x)$*?* $\boxed{2x^3 + 7x^2 + 19x + 50}$

What is the remainder? $\boxed{125x - 41}$