Predicting COVID-19 deaths and recoveries, a q-exponential model approach.

Freddy H. Marín-Sánchez
fmarinsa@eafit.edu.co
Department of Mathematical Science
Universidad EAFIT

Jhon Alexanders Miranda Echeverri
jmirand1@eafit.edu.co
Department of Mathematical Science
Universidad EAFIT

Nicolas Rengifo Campo
nrengifoc@eafit.edu.co
Department of Mathematical Science
Universidad EAFIT

Juan David Rengifo Castro
jdrengifoc@eafit.edu.co
Department of Mathematical Science
Universidad EAFIT

January 30, 2021

Abstract

In this research we propose a mathematical model based on q-exponential family of functions. As will be presented in the results section, we obtained a great performance when predicting variables with exponential or logistic behaviour, such as confirmed deaths by COVID-19 in different countries. Depending of the behaviour, rolling windows methodology is employed. In a similar way, q-exponential parameters are estimated minimizing the prediction error.

Keywords: COVID-19 modelling, COVID-19 prediction, q-exponential function, moving windows.

1 Introduction

Since the COVID-19 was declared a pandemic in March by [Cucinotta & Vanelli, 2020] the world that we used to know had changed drastically. It is not only due to the uncertainty of when will it end, but by all the side effects of the lockouts around the world.

Despite the COVID-19 has a clear impact in multiple areas, one of the most affected areas is the economic. As result of its importance, [Ozili & Arun, 2020] explained how is that a health crisis is translated to an economic one. Otherwise, the health systems around the world are being the top priority emergency to treat. It is shown by [Velavan & Meyer, 2020] how this coronavirus influences the health of an average person. Apart from COVID-19 infections, [Pfefferbaum & North, 2020] states that the health crisis its magnified by the development of mental health problems. In consequence the world is facing a global problematic like none other in the recent story.

The sooner hope is the development of the vaccine against COVID-19. It is shown by [Lurie *et al.*, 2020] and [Le *et al.*, 2020], how difficult can be this process but also how useful. A certain fact is that we will have to get used to a different world, with new regulations.

This work will be based on mathematical models, which are used to describe some interest variables in COVID-19 context, such as reported deaths and recoveries. Forecasting these variables can be useful in order to take adequate decisions and planning prevention strategies by governments around the world. Since we are considering variables as mathematical models itself, many approaches can be used. A simple but powerful model is worked on [Petropoulos & Makridakis, 2020], where predictions on the reported cases are done based on exponential smoothing and logistic behaviours.

On the other hand, [Shoukat *et al.*, 2020] analyzed a situation in intense care units using agent based modelling. Other perspective is the time series approach. For example, [Maleki *et al.*, 2020] use auto regressive models to model reported cases and recoveries. Furthermore, [Ma *et al.*, 2020] use generalized

additive models in order to find how exogenous variables influence the deaths, particularly the weather. The machine learning area had also contributed, [Chimmula & Zhang, 2020] proposes a LTSM network model to represent the dynamics of death and recoveries rate in Canada.

One of the most used approaches is the dynamical systems one, they are mainly based in differential equations systems which allow a great versatility of the models. Traditionally, the most recognized epidemic model is based in dynamical systems, they had been studied deeply through the years and each time they are more complex and able to represent more realistic situations. This can be illustrated by [Anastassopoulou *et al.*, 2020], where epidemic models are used to represent deaths, cases and confirmed recoveries. Besides, [Sarkar *et al.*, 2020] support this kind of approach specifically in India.

Traditionally, the epidemic models (dynamical systems) considers level variables such as susceptible, infected, recoveries and deaths; we will focus our research in recoveries and deaths. [Sarkar *et al.*, 2020] illustrates us how the traditional models can be modified to include different variables such as the quarantine state. Considering these new variables makes easier the scenarios analysis through simulation and offers a more trustworthy representation of the reality.

In this research we are going to study the COVID-19 deceases in different countries and recoveries in the Colombian case. Accumulated levels of the variable are considered in order to obtain exponential type variables. An exponential response is desired since our methodology is strongly bounded to this particular behaviour. We select these variables due to its importance and in the case of deaths, its robust behaviour in contrast to COVID-19 detected cases.

The variables that we are searching to model, deaths and recoveries, had been focused from different approaches. An example is the research done by [Anastassopoulou *et al.*, 2020], where thanks to a SIRD model the pandemic is modelled, great conclusions in terms of prevention strategies can be found. Other approach is developed by [Chimmula & Zhang, 2020], where the death and recoveries rate are modelled, it is interesting that is not an obligation to work with the level variable.

It is now well established from the work developed by [Vasconcelos *et al.*, 2020], how useful the researches are in the prevention strategies. It is an interest article for our proposal since it is based in exponential models, specifically Richards growth model (RGM) [Pienaar & Turnbull, 1973]. Particularly, one of our interest variables is modelled, the confirmed deaths. In the literature these approaches are often seen as prevention strategies.

In another major investigation, [Tsallis & Tirnakli, 2020] applied the q-exponential approach to the active cases and deaths in different countries. We pretend to improve this approach by considering the rolling windows methodology, which offers a higher variability that helps in modelling a more realistic evolving variable than a single q-exponential trajectory. This is clearly observed when the curve flattens itself, a q-exponential curve is no longer a good fit, but when considering a rolling window this behaviour can yet be adjusted.

We propose to use a simple methodology based on the q-exponential family of curves and rolling windows. Even if it is simple, we would see that it has a great performance in the practice, which is valuable considering that it is a simple and parsimonious model. We define the mathematical model in the following section.

2 q-exponential model and parameters estimation

An exponential function with respect to the parameter q or q-exponential function, for a variable x is defined as:

$$exp_q(x) = e_q^x :\equiv \begin{cases} [1+(1-q)x]^{\frac{1}{1-q}} & \text{if } 1+(1-q)x > 0 \\ 0, & \text{if } 1+(1-q)x \leq 0. \end{cases}$$

Where $x, q \in \mathbb{R}$. Note that $e_1^x :\equiv \lim_{q\to 1^+} e_q^x = \lim_{q\to 1^-} e_q^x = e^x \ \ \forall x$.
Furthermore, if $q < 1$ the q-exponential function fades for $x \leq \frac{1}{q-1}$ and when x increases from $\frac{-1}{1-q}$ to ∞, it continuously and monotonously increases from 0 to ∞.
On the other hand, if $q > 1$ the q-exponential function turns out to be divergent for $x > \frac{1}{q-1}$ and when it

increases from $-\infty$ to $\frac{1}{q-1}$, it increases continuously and monotonously from 0 to ∞.
The properties of the q-exponential function and other details can be consulted in [McAnally, 1995, Tsallis, 1994, Tsallis, 2001].

2.1 q-exponential model

2.1.1 Basic Assumptions

Let the dynamic behavior of the number of daily reported deaths and recoveries of COVID-19. For an infected geographic, information about daily reports, deaths and recoveries, is used.

The proposed model is an uni-variate one, which means that only the daily historical data of reported deaths and recoveries are considered and excludes other dynamics of the classic epidemiological models associated with number of infections, exposed, isolated, such as the SEIR model with stochastics parameters, SEIR without quarantine and SIRU [Ashutosh Simha & Narayana, 2020], [Dandekar & Barbastathis, 2020],[Z. Liu & Webb, 2020]. So the statistical inference and the estimation of the parameters are extracted exclusively from the data of the available sample.

Since the q-exponential function is continuous and the observed data for deaths and recoveries are in discrete time, the approximation of the reported deaths and recoveries curves are performed at discrete points of the q-exponential function.

2.1.2 Model building

Suppose there are constants $f(\tau_n)$, $q(\tau_n)$ y $m(\tau_n)$ that explain "better" the behavior of a curve $C(\tau_t)$ up to time τ_n using the expression

$$C(\tau_t) \approx f(\tau_n) exp_{q(\tau_n)}(m(\tau_n)\tau_t)$$

For observation $C(\tau_{n+1})$ we would have new parameters $f(\tau_{n+1})$, $q(\tau_{n+1})$ and $m(\tau_{n+1})$ that would explain the curve updated up to time τ_{n+1} and so on, so that for each new observation the curve $C(\tau_t)$ is updated.

2.2 Parameters estimation and Iterative procedure

Consider the q-exponential model 2.1.2 with the initial condition $X(\tau_0 = 1)$ and an unknown parameters vector $\Theta = [f(\tau_t), m(\tau_t), q(\tau_t)]$; $t = 1, 2, \cdots, n$. Consider the finite partition of the time interval $[0, T]$, $0 = \tau_0 \leq \tau_1 \leq \ldots \leq \tau_{n-1} \leq \tau_n = T$ with $\delta t = \tau_t - \tau_{t-1} \geq 0$; where $t = 0, 1, 2, \ldots, n$ and a daily report curve (deaths or recoveries) $C(\tau_t) \in [0, T]$ observed at discrete points for the times $\{\tau_0, \tau_1, \ldots, \tau_{n-1}, \tau_n\}$. The objective is to estimate the unknown parameters vector Θ given the historical data of the sample $C = [C\tau_0, C\tau_1, \cdots C\tau_n]$ with equally spaced data.

The estimation of the number of total cases $C(\tau_n)$ for a day τ_n is given by its estimation $\hat{C}(\tau_n)$ in the q-exponential model

$$\hat{C}(\tau_n) = \hat{f}(\tau_n)(1 + (1 - \hat{q}(\tau_n))\hat{m}(\tau_n)\tau_n)^{\frac{1}{1-\hat{q}(\tau_n)}}$$

If we extend the last equation to the interval $[\tau_0, \tau_n]$, the difference between real reported data and estimated data is given by the Sum of Squared Errors $(SSE) = g(\epsilon_t)$; where $\epsilon_t = C(\tau_t) - \hat{C}\tau_t$.

$$\begin{aligned} g(\epsilon_t) &= \sum_{i=0}^{n} \Big(C(\tau_t) - \hat{C}(\tau_t)\Big)^2 \\ &= \sum_{i=0}^{n} \Big(C(\tau_t) - \hat{f}(\tau_n)(1 + (1 - \hat{q}(\tau_n)\hat{m}(\tau_n)\tau_t)^{\frac{1}{1-\hat{q}(\tau_n)}}\Big)^2 \end{aligned}$$

If $g(\epsilon_t)$ has a minimum it happens for values of $\hat{f}$, $\hat{m}$ and $\hat{q}$ that satisfy the equation

$$\frac{\partial g(\epsilon_t)}{\partial \Theta} = \vec{0} \quad \text{i.e., } \hat{\Theta} = \underset{\Theta}{Arg\ min}(g(\epsilon_t)) \tag{1}$$

The parameter estimation $\hat{f}$ $\hat{m}$ and $\hat{q}$ are given by the solution of the equation 1, considering the next partial derivatives:

$$\frac{\partial g(\epsilon_t)}{\partial \hat{f}} = -\sum_{t=0}^{n} 2(C(\tau_t) - \hat{f}(\tau_n)(v_t)^{\frac{1}{1-\hat{q}(\tau_n)}})(v_t)^{\frac{1}{1-\hat{q}(\tau_n)}}$$

$$\frac{\partial g(\epsilon_t)}{\partial \hat{m}} = \sum_{t=0}^{n}(-2\hat{f}(\tau_n)(v_t)^{\frac{\hat{q}(\tau_n)}{1-\hat{q}(\tau_n)}})(C(\tau_t) - \hat{f}(\tau_n)(v_t)^{\frac{1}{1-\hat{q}(\tau_n)}})\tau_t$$

$$\frac{\partial g\left(\epsilon_t\right)}{\partial \hat{q}} = \sum_{t=0}^{n} -\frac{2}{\left(1-\hat{q}\left(\tau_n\right)\right)^2} u_i \hat{f}\left(\tau_n\right) v_t^{\frac{1}{1-\hat{q}(\tau_n)}} \left(\frac{(1-\hat{q})m\left(\tau_n\right)}{v_t} + \ln\left(v_t\right)\right)$$

Where:

$$v_t = 1 + (1 - \hat{q}(\tau_n))\hat{m}(\tau_n)\tau_t$$

$$w_t = \frac{\ln(v_t)}{1 - \hat{q}(\tau_n)}$$

Now, lets define an iterative procedure for the parameters vector $\hat{\Theta}(\tau_n) = (\hat{f}, \hat{m}, \hat{q})$, with the initial condition $\hat{\theta_0} = (\hat{f}_0, \hat{m}_0, \hat{q}_0)$ is realized. Define $\mathbf{J}_t = \frac{\partial \hat{C}_{\tau_t}}{\partial \Theta}$ and the first order estimate given by a new estimate $\theta + \delta$

$$SSE(\theta + \delta) \approx \sum_{t=0}^{n}(C_{\tau_t} - \hat{C}_{\tau_t} - \mathbf{J}_t\delta)^2$$

Taking the derivative respect to δ and setting to zero, in vectorial form

$$\mathbf{J}^T\mathbf{J}\delta = J^T[C_{\tau_t} - \theta)]$$

Iterative step given a damping factor λ

$$(\mathbf{J}^T J + \lambda I)\delta = J^T[C_{\tau_t} - \hat{C}_{\tau_t}(\theta)]$$

Daily forecasts are made by estimating the parameters $f(\tau_n)$, $q(\tau_n)$ and $m(\tau_n)$ of the daily daily report curve (deaths or recoveries) with the data reported up to time τ_n. To obtain greater precision in the forecast, it is very useful to calibrate the parameter $\hat{f}(\tau_n)$ by adjusting it to the last data in the sample using the expression:

$$\hat{\hat{f}}(\tau_n) = \frac{C(\tau_n)}{exp_{\hat{q}(\tau_n)}(\hat{m}(\tau_n)\tau_n)}$$

So that the forecast for the next day

$$\hat{C}(\tau_{n+1}) = \hat{\hat{f}}(\tau_0)exp_{\hat{q}(\tau_n)}(\hat{m}(\tau_n)\tau_{n+1})$$

has a lower relative error.

When the curve of reported infections is in its growth phase (before some flattening), forecasts can be made for several days ahead using the expression $\hat{C}(\tau_{n+k}) = \hat{\hat{f}}(\tau_n)exp_{\hat{q}(\tau_n)}(\hat{m}(\tau_n)\tau_{n+k})$ for $k = 1, 2, \cdots, p$.

Estimations and predictions are subject to errors in the fitted model, parameter deviations and external uncertainties. That is why an interval of values is required instead of a punctual one so that given a confidence level between 0 and 1, the possible values that might take a curve of cases at a specific time are contained with that probability. The procedure to generate confidence bands for predictions is described below.

1. For a time τ_{n+k}, find the prediction $\hat{C}_{n+k} = \hat{C}(\tau_{n+k})$.
2. Specify a confidence level.
3. Compute the SSE using equation (3), the degrees of freedom as DOF= $\tau_{n+k} - p$ where p is the number of estimated parameters (3 for this case) and the inverse of the covariance matrix of the parameters R.
4. Set RMSE= $\sqrt{\frac{\text{SSE}}{\text{DOF}}}$
5. Get the derivative with respect to parameters at the value τ_{n+k} and find the numerical gradient $\mathbf{d}$ and set E = $\mathbf{d}R$.
6. Compute the critical value using a T distribution as crit= $-t_{(1-\text{level})/2,\text{DOF}}$
7. Compute the amplitude of the band $b = (\sqrt{\sum E \odot E})(\text{RMSE}(\text{crit}))$
8. For the estimation $\hat{c}_i$ the confidence band is $[\hat{c}_{\tau_{n+k}} - b, \hat{c}_{\tau_{n+k}} + b]$.

2.3 Description of moving windows

It is possible to estimate advanced stages of a curve using a sample that gives priority to new available data. In other words, for a day τ_n with T observations, a time window w is specified to estimate with data of the last $T - w$ days. The value of w is between 4 (to ensure valid degrees of freedom) and T. The number of predictions can be extended up to 5 days.

A good choice of the moving window should take into consideration the phases of the growth curve. In the first phase, the value of w is near to T because the curve has an accelerated behavior. During the second phase, the size might be around 7 and 15 days given a reduced growth rate. While for the third phase of curve flattening the size might vary between 4 and 21 days.

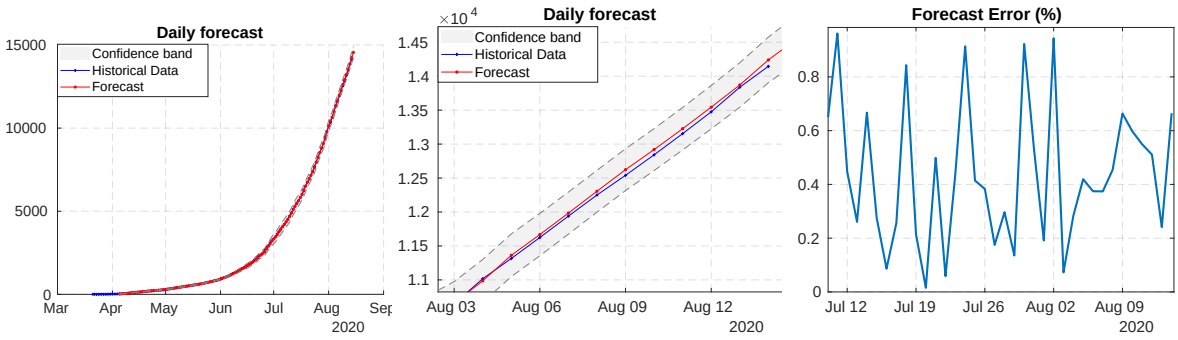
Most of the data are updated daily and hence it can not be explicitly established at which stage is a curve of cumulative deaths or recoveries. To deal with this issue, exploration algorithms as Grid Search can be used to find the appropriate size of the time window. This requires an objective function based on estimations or predictions: Mean Squared Error (MSE), the amplitude of confidence bands or proximity to a statistic (mean, median, standard deviation).

3 Model analysis and prediction results

3.1 Data description

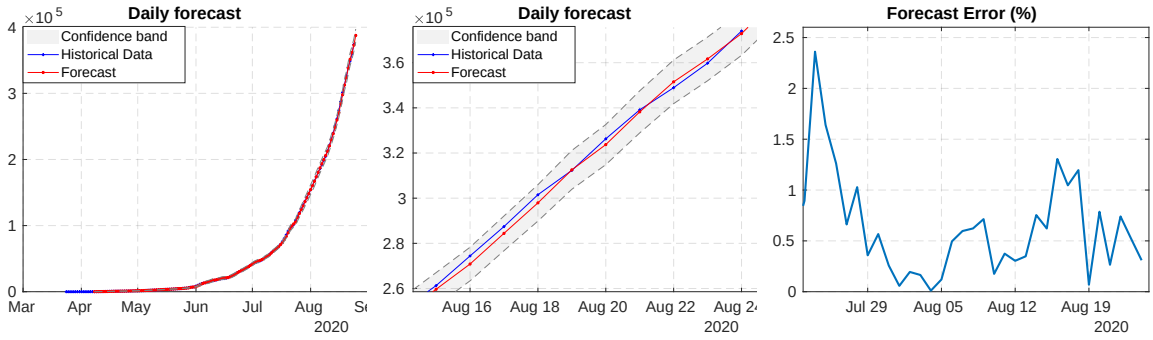
The data sets that we consider, for the Colombian variables, are the official reports of the Colombian Ministry of Health through [Institute of National Health, 2020], we are considering the accumulated of daily reported deaths and recoveries. For non Colombian deaths reports, the data is taken from [Johns Hopkins, 2020].

3.2 Pure q-exponential Model



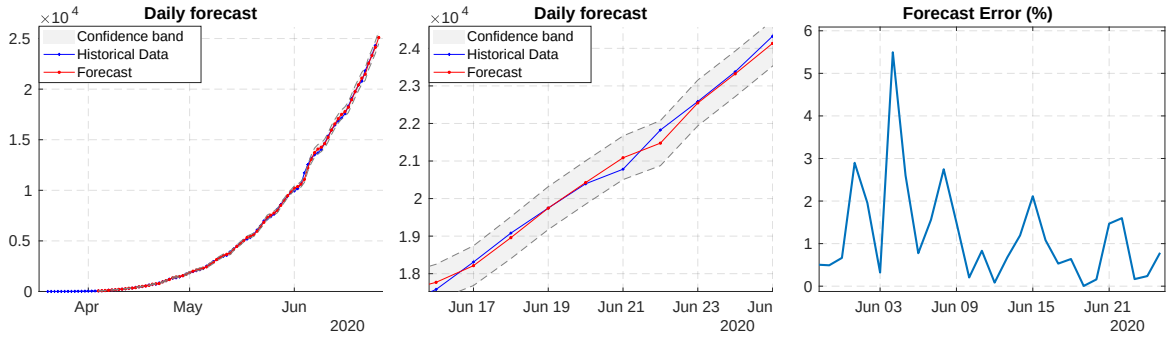
(a) (b) (c)

Figure 1: (a) Comparison between historical data of deaths for Colombia and their forecasts. (b) Detail of the forecast of 10 days and confidence band. (c) Absolute relative error of the last 30 forecasts.



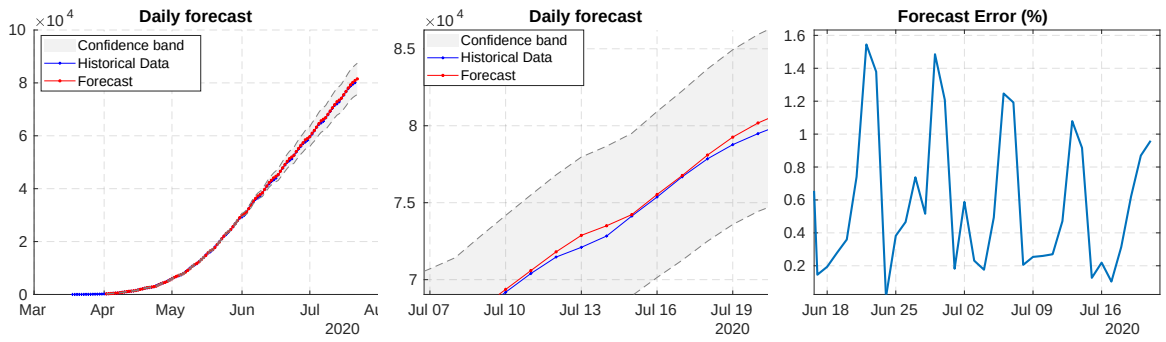
(a) (b) (c)

Figure 2: (a) Comparison between historical data of recovered for Colombia and their forecasts. (b) Detail of the forecast of 10 days and confidence band. (c) Absolute relative error of the last 30 forecasts.



(a) (b) (c)

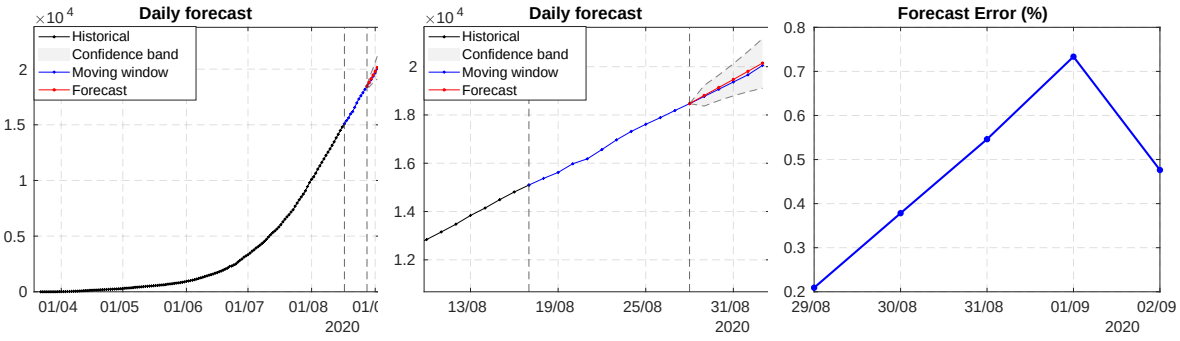
Figure 3: (a) Comparison between historical data of deaths for Mexico and their forecasts. (b) Detail of the forecast of 10 days and confidence band. (c) Absolute relative error of the last 30 forecasts.



(a) (b) (c)

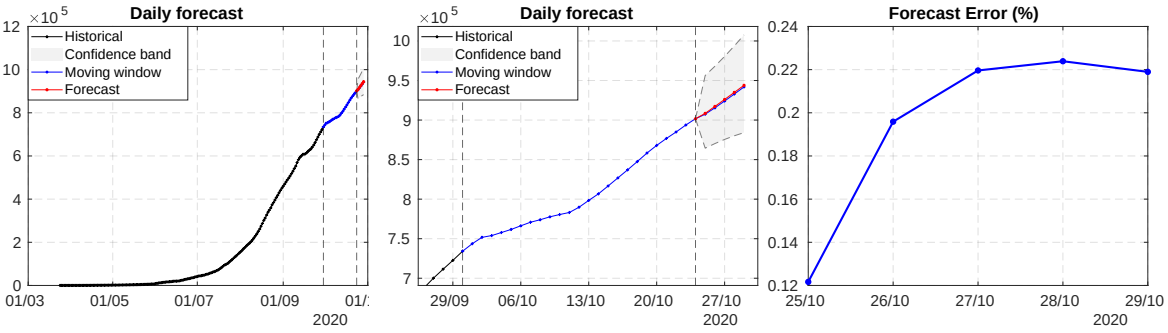
Figure 4: (a) Comparison between historical data of deaths for Brazil and their forecasts. (b) Detail of the forecast of 10 days and confidence band. (c) Absolute relative error of the last 30 forecasts.

3.3 Rolling windows q-exponential Model



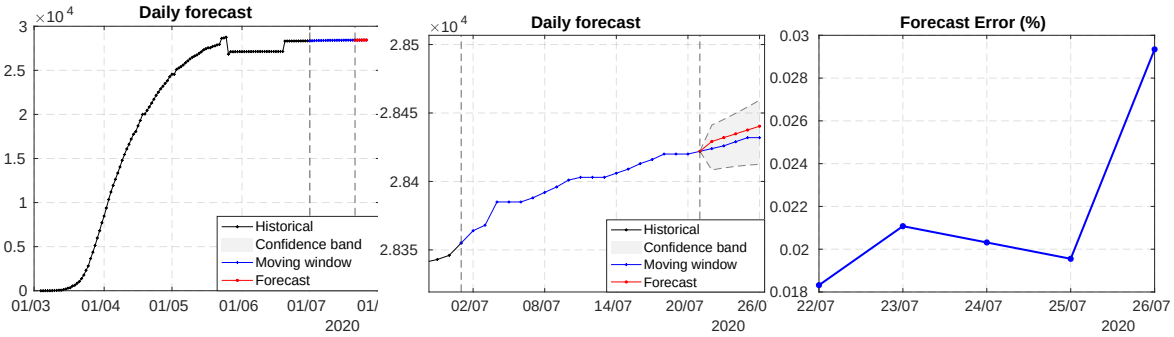
(a) (b) (c)

Figure 5: (a) Comparison between historical data of deaths for Colombia and their forecasts using a moving window of 11 days. (b) Detail of the forecast of 5 days and confidence band. (c) Absolute relative error of the last 5 forecasts.



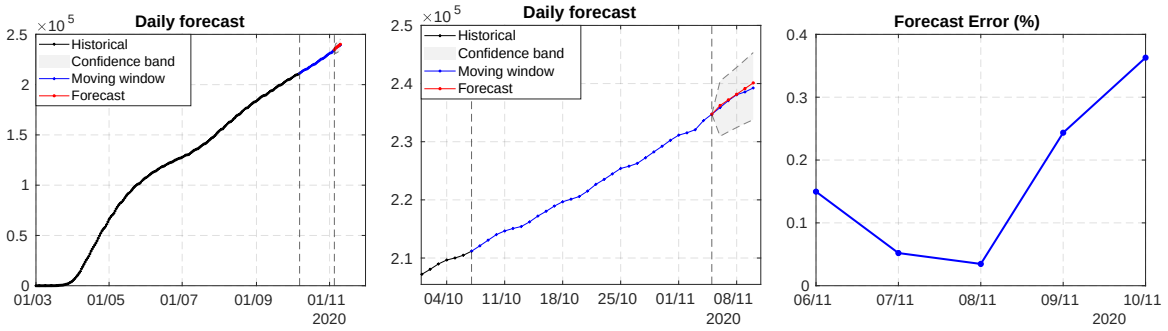
(a) (b) (c)

Figure 6: (a) Comparison between historical data of recovered for Colombia and their forecasts using a moving window of 24 days. (b) Detail of the forecast of 5 days and confidence band. (c) Absolute relative error of the last 5 forecasts.



(a) (b) (c)

Figure 7: (a) Comparison between historical data of deaths for Spain and their forecasts using a moving window of 11 days. (b) Detail of the forecast of 5 days and confidence band. (c) Absolute relative error of the last 5 forecasts.



(a) (b) (c)

Figure 8: (a) Comparison between historical data of deaths for United States and their forecasts using a moving window of 29 days. (b) Detail of the forecast of 5 days and confidence band. (c) Absolute relative error of the last 5 forecasts.

4 Conclusions

Thanks to the pure q-exponential model we were able to predict 10 days ahead of the last report. Nonetheless, this model only presents good results when the time-series are in exponential growing. This made us to consider a rolling windows q-exponential model, that is able to represent other kind of behaviours, for instance, the logistic one. When using this model, better results were obtained and we could include in our analysis other countries where the reported deaths were not an exponential trajectory. The forecast done in this research can be used for prevention strategies politics. For future work we recommend to extrapolate this model to other research areas such as economic one.

5 Acknowledgements

This research has been carried out in the framework of the project "Plataforma web para la recolección de datos, visualización, análisis, predicción y evaluación de estrategias de control de la enfermedad producida por SARS-CoV-2 mediante herramientas de modelación matemática, simulación e inteligencia artificial" which has been funded by the program MinCienciaTón (Covid-19 2020) of MinCinencias Colombia and EAFIT University through the agreement n°1010-000003.

References

[Anastassopoulou *et al.*, 2020] Anastassopoulou, Cleo, Russo, Lucia, Tsakris, Athanasios, & Siettos, Constantinos. 2020. Data-based analysis, modelling and forecasting of the COVID-19 outbreak. *PloS one*, **15**(3), e0230405.

[Ashutosh Simha & Narayana, 2020] Ashutosh Simha, R. Venkatesha Prasad, & Narayana, Sujay. 2020. A simple Stochastic SIR model for COVID-19 Infection Dynamics for Karnataka after interventions – Learning from European Trends. *arXiv preprint: 2003.11920*, **3**.

[Chimmula & Zhang, 2020] Chimmula, Vinay Kumar Reddy, & Zhang, Lei. 2020. Time series forecasting of COVID-19 transmission in Canada using LSTM networks. *Chaos, Solitons & Fractals*, 109864.

[Cucinotta & Vanelli, 2020] Cucinotta, Domenico, & Vanelli, Maurizio. 2020. WHO declares COVID-19 a pandemic. *Acta Bio Medica: Atenei Parmensis*, **91**(1), 157.

[Dandekar & Barbastathis, 2020] Dandekar, Raj, & Barbastathis, George. 2020. Neural Network aided quarantine control model estimation of global Covid-19 spread. *arXiv preprint: 2004.02752*, **1**.

[Institute of National Health, 2020] Institute of National Health, (INS). 2020. *COVID-19 en Colombia*. `https://www.ins.gov.co/Noticias/paginas/coronavirus.aspx`.

[Johns Hopkins, 2020] Johns Hopkins, University of Medicine. 2020. *Coronavirus resource center*. `https://coronavirus.jhu.edu/data`.

[Le *et al.*, 2020] Le, T Thanh, Andreadakis, Zacharias, Kumar, Arun, Roman, R Gomez, Tollefsen, Stig, Saville, Melanie, & Mayhew, Stephen. 2020. The COVID-19 vaccine development landscape. *Nat Rev Drug Discov*, **19**(5), 305–306.

[Lurie *et al.*, 2020] Lurie, Nicole, Saville, Melanie, Hatchett, Richard, & Halton, Jane. 2020. Developing Covid-19 vaccines at pandemic speed. *New England Journal of Medicine*, **382**(21), 1969–1973.

[Ma *et al.*, 2020] Ma, Yueling, Zhao, Yadong, Liu, Jiangtao, He, Xiaotao, Wang, Bo, Fu, Shihua, Yan, Jun, Niu, Jingping, Zhou, Ji, & Luo, Bin. 2020. Effects of temperature variation and humidity on the death of COVID-19 in Wuhan, China. *Science of The Total Environment*, 138226.

[Maleki *et al.*, 2020] Maleki, Mohsen, Mahmoudi, Mohammad Reza, Wraith, Darren, & Pho, Kim-Hung. 2020. Time series modelling to forecast the confirmed and recovered cases of COVID-19. *Travel Medicine and Infectious Disease*, 101742.

[McAnally, 1995] McAnally, DS. 1995. q-exponential and q-gamma functions. I. q-exponential functionsa. *Journal of Mathematical Physics*, **36**(1), 546–573.

[Ozili & Arun, 2020] Ozili, Peterson K, & Arun, Thankom. 2020. Spillover of COVID-19: impact on the Global Economy. *Available at SSRN 3562570*.

[Petropoulos & Makridakis, 2020] Petropoulos, Fotios, & Makridakis, Spyros. 2020. Forecasting the novel coronavirus COVID-19. *PloS one*, **15**(3), e0231236.

[Pfefferbaum & North, 2020] Pfefferbaum, Betty, & North, Carol S. 2020. Mental health and the Covid-19 pandemic. *New England Journal of Medicine*.

[Pienaar & Turnbull, 1973] Pienaar, Leon V, & Turnbull, Kenneth J. 1973. The Chapman-Richards generalization of Von Bertalanffy's growth model for basal area growth and yield in even-aged stands. *Forest Science*, **19**(1), 2–22.

[Sarkar *et al.*, 2020] Sarkar, Kankan, Khajanchi, Subhas, & Nieto, Juan J. 2020. Modeling and forecasting the COVID-19 pandemic in India. *Chaos, Solitons & Fractals*, **139**, 110049.

[Shoukat *et al.*, 2020] Shoukat, Affan, Wells, Chad R, Langley, Joanne M, Singer, Burton H, Galvani, Alison P, & Moghadas, Seyed M. 2020. Projecting demand for critical care beds during COVID-19 outbreaks in Canada. *CMAJ*, **192**(19), E489–E496.

[Tsallis, 1994] Tsallis, Constantino. 1994. What are the numbers that experiments provide. *Quimica Nova*, **17**(6), 468–471.

[Tsallis, 2001] Tsallis, Constantino. 2001. I. nonextensive statistical mechanics and thermodynamics: Historical background and present status. *Pages 3–98 of: Nonextensive statistical mechanics and its applications*. Springer.

[Tsallis & Tirnakli, 2020] Tsallis, Constantino, & Tirnakli, Ugur. 2020. Predicting COVID-19 peaks around the world. *Frontiers in Physics*, **8**, 217.

[Vasconcelos *et al.*, 2020] Vasconcelos, Giovani L, Macêdo, Antônio MS, Ospina, Raydonal, Almeida, Francisco AG, Duarte-Filho, Gerson C, Brum, Arthur A, & Souza, Inês CL. 2020. Modelling fatality curves of COVID-19 and the effectiveness of intervention strategies. *PeerJ*, **8**, e9421.

[Velavan & Meyer, 2020] Velavan, Thirumalaisamy P, & Meyer, Christian G. 2020. The COVID-19 epidemic. *Tropical medicine & international health*, **25**(3), 278.

[Z. Liu & Webb, 2020] Z. Liu, P. Magal, O. Seydi, & Webb, G. 2020. Predicting the cumulative number of cases for the COVID-19 epidemic in China from early data. *arXiv preprint: 2002.12298*, **1**.