Linear regression

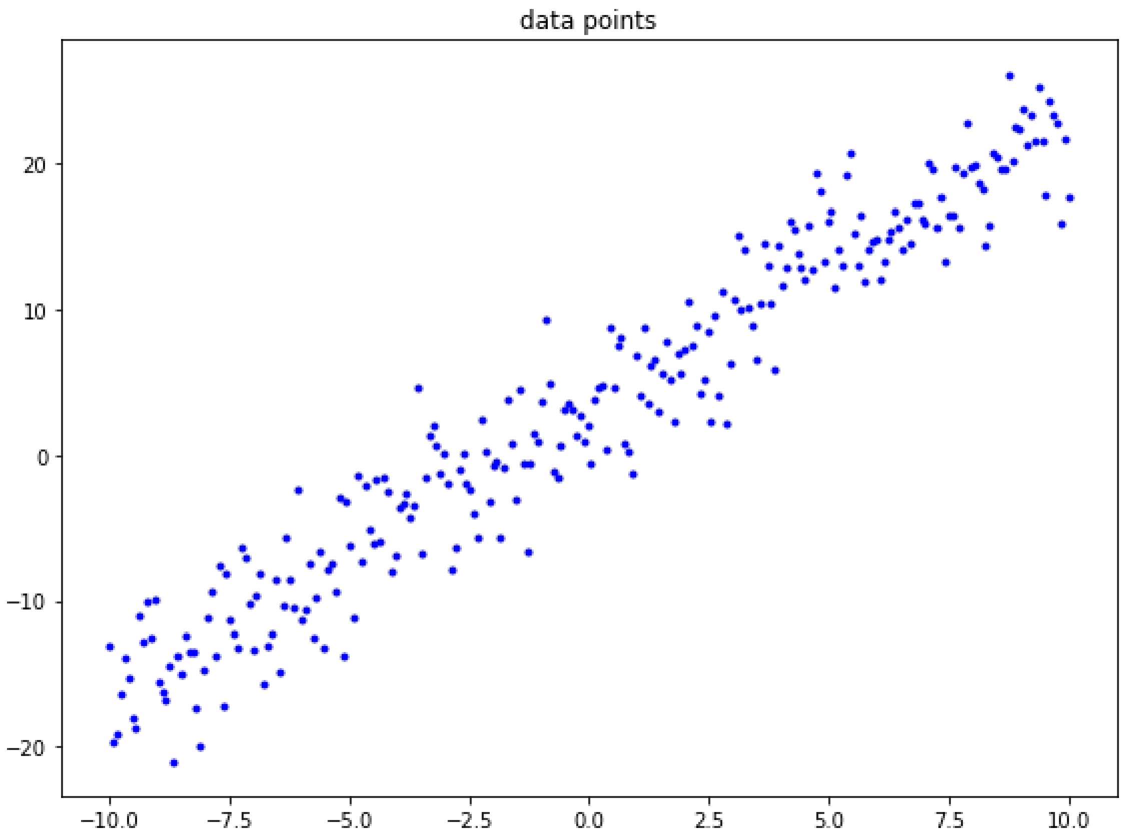
import library

In [ ]:
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from mpl_toolkits.mplot3d import Axes3D
```

load data points

- $\{(x_i, y_i)\}_{i=1}^n$

In [ ]:
```
filename      = 'assignment_06_data.csv'
data          = np.loadtxt(filename, delimiter = ',')

x             = data[0, :]     # independent variable
y             = data[1, :]     # dependent variable

plt.figure(figsize=(8,6))

plt.plot(x, y, '.', color = 'blue')
plt.title('data points')

plt.tight_layout()
plt.show()
```



In [ ]:
```
len(x)
```

Out[ ]: 256

compute the residual

In [ ]:
```
def compute_residual(x, y, theta0, theta1):

    n           = len(x)
    residual    = np.zeros(n)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    y_hat = theta0 + theta1*x

    residual = y_hat - y

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return residual
```

compute the loss

- useful functions: `np.inner`

In [ ]:
```
def compute_loss(x, y, theta0, theta1):

    n           = len(x)
    loss        = 0

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    residual = compute_residual(x, y, theta0, theta1)

    loss = np.divide(np.inner(residual,residual),2*n)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return loss
```

In [ ]:
```
np.inner([1,2,3],[1,2,3])
```

Out[ ]: 14

compute the gradient with respect to θ_0

- useful functions: `np.inner`

In [ ]:
```
def compute_gradient_theta0(x, y, theta0, theta1):

    n           = len(x)
    derivative  = 0

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    residual = compute_residual(x, y, theta0, theta1)
```

```python
    derivative = np.divide(np.inner(residual,np.ones(len(residual))),n)

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return derivative
```

In [ ]:
```python
np.inner(np.array([1,2,3]),np.ones(3))
```

Out[ ]: 6.0

In [ ]:
```python
np.ones(3)
```

Out[ ]: array([1., 1., 1.])

compute the gradient with respect to θ_1

- useful functions: `np.inner`

In [ ]:
```python
def compute_gradient_theta1(x, y, theta0, theta1):

    n           = len(x)
    derivative  = 0

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    residual = compute_residual(x, y, theta0, theta1)
    derivative = np.divide(np.inner(residual,x),n)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return derivative
```

gradient descent for the optimization

In [ ]:
```python
number_iteration    = 500
learning_rate       = 0.01

theta0              = 0
theta1              = 0

list_theta0         = np.zeros(number_iteration)
list_theta1         = np.zeros(number_iteration)
list_loss           = np.zeros(number_iteration)

for i in range(number_iteration):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    theta0  = theta0 - learning_rate*compute_gradient_theta0(x, y, theta0, theta1)
    theta1  = theta1 - learning_rate*compute_gradient_theta1(x, y, theta0, theta1)
    loss    = compute_loss(x, y, theta0, theta1)
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    list_theta0[i]  = theta0
```

```
    list_theta1[i]  = theta1
    list_loss[i]    = loss
```

---

functions for presenting the results

In [ ]:
```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.plot(x, y, '.', color='blue')
    plt.title('data points')
    plt.show()
```

In [ ]:
```
def function_result_02():

    plt.figure(figsize=(8,6))
    ax = plt.gca()
    plt.plot(list_theta0, '-', color='blue', label=r'$\theta_0$')
    plt.plot(list_theta1, '-', color='red', label=r'$\theta_1$')
    plt.title('model parameters')
    ax.legend()
    plt.show()
```

In [ ]:
```
def function_result_03():

    plt.figure(figsize=(8,6))
    plt.plot(list_loss, '-', color='blue')
    plt.title('loss curve')
    plt.show()
```

In [ ]:
```
def function_result_04():

    f = theta0 + theta1 * x

    plt.figure(figsize=(8,6))
    ax = plt.gca()
    plt.plot(x, y, '.', color='blue', label='data point')
    plt.plot(x, f, '-', color='red', label='regression')
    plt.title('regression')
    ax.legend()
    plt.show()
```

In [ ]:
```
def function_result_05():

    X0  = np.arange(-10, 10, 0.1)
    X1  = np.arange(-10, 10, 0.1)

    grid_theta0, grid_theta1 = np.meshgrid(X0, X1)

    grid_loss   = np.zeros(grid_theta0.shape)

    for i, t0 in enumerate(X0):
        for j, t1 in enumerate(X1):

            grid_loss[j, i] = compute_loss(x, y, t0, t1)
```

```
    fig = plt.figure(figsize=(8,6))

    ax  = fig.add_subplot(111, projection='3d')
    plt.title('loss surface')

    ax  = plt.axes(projection='3d')

    ax.set_xlabel(r'$\theta_0$')
    ax.set_ylabel(r'$\theta_1$')
    ax.set_zlabel('loss')

    ax.plot_surface(grid_theta0, grid_theta1, grid_loss, rstride=1, cstride=1, cmap='

    plt.tight_layout()
    plt.show()
```

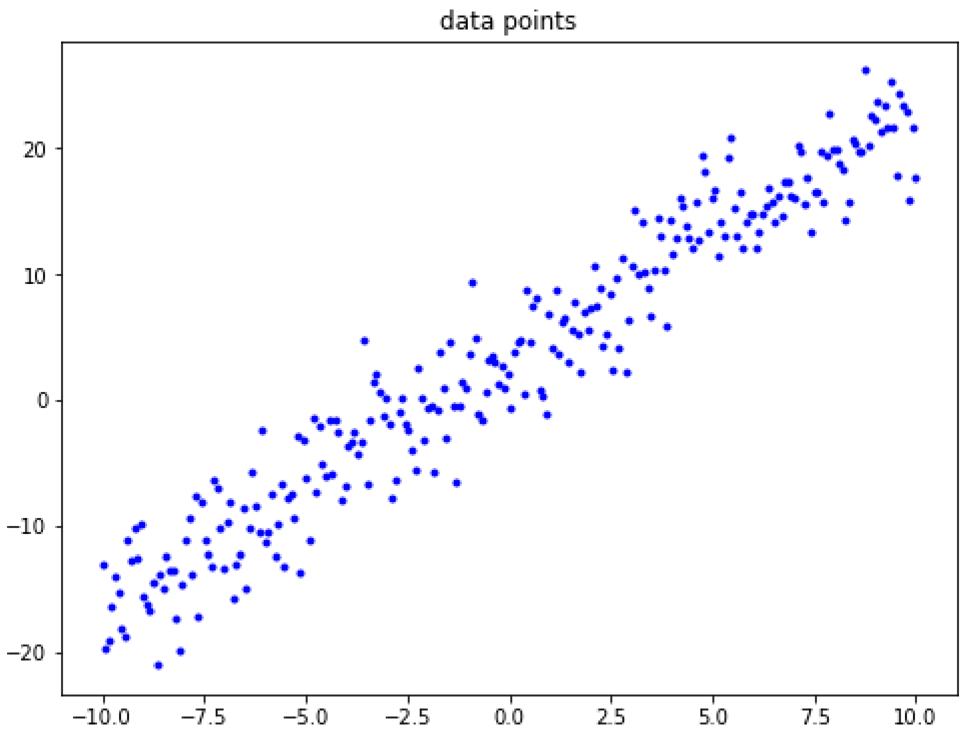
---

results

In [ ]:

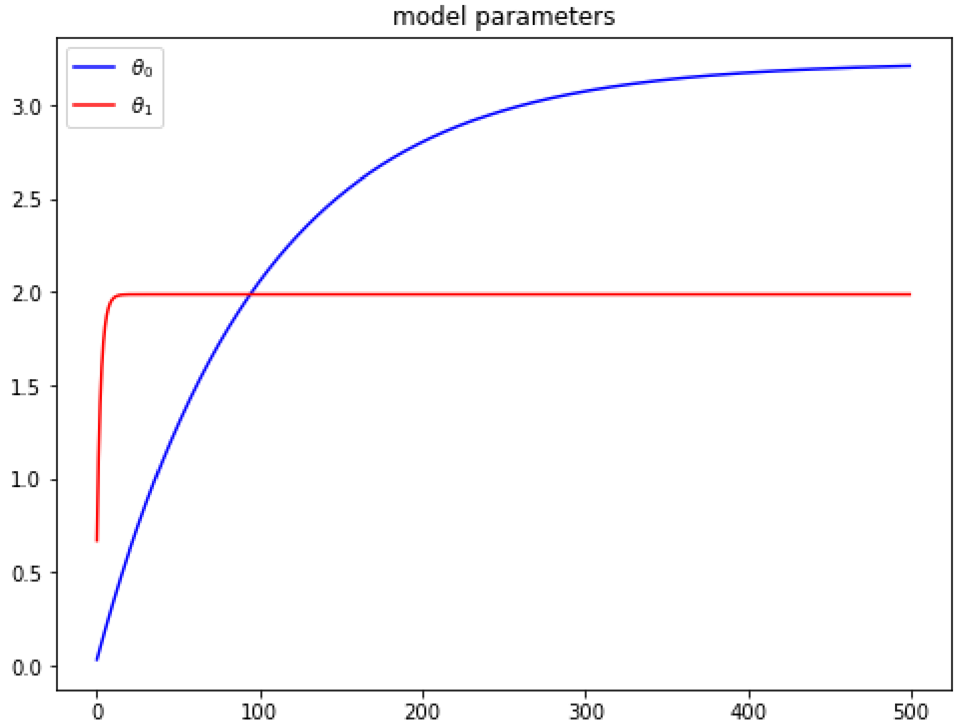
```
number_result = 5

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

```
**************************************************
## [RESULT 01]
**************************************************
```

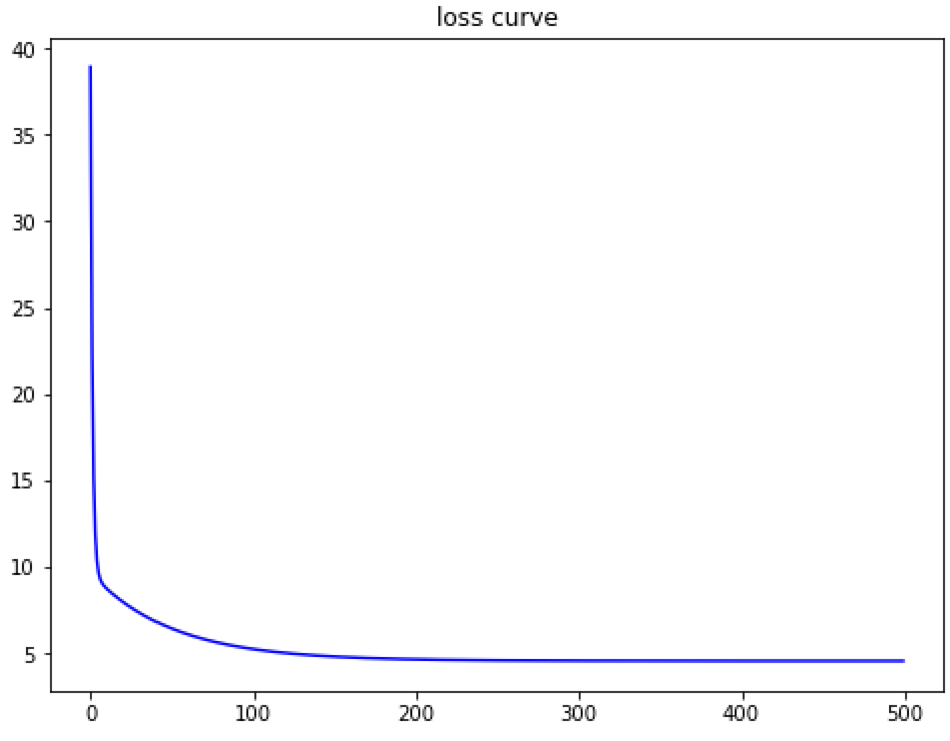



```
**************************************************
```

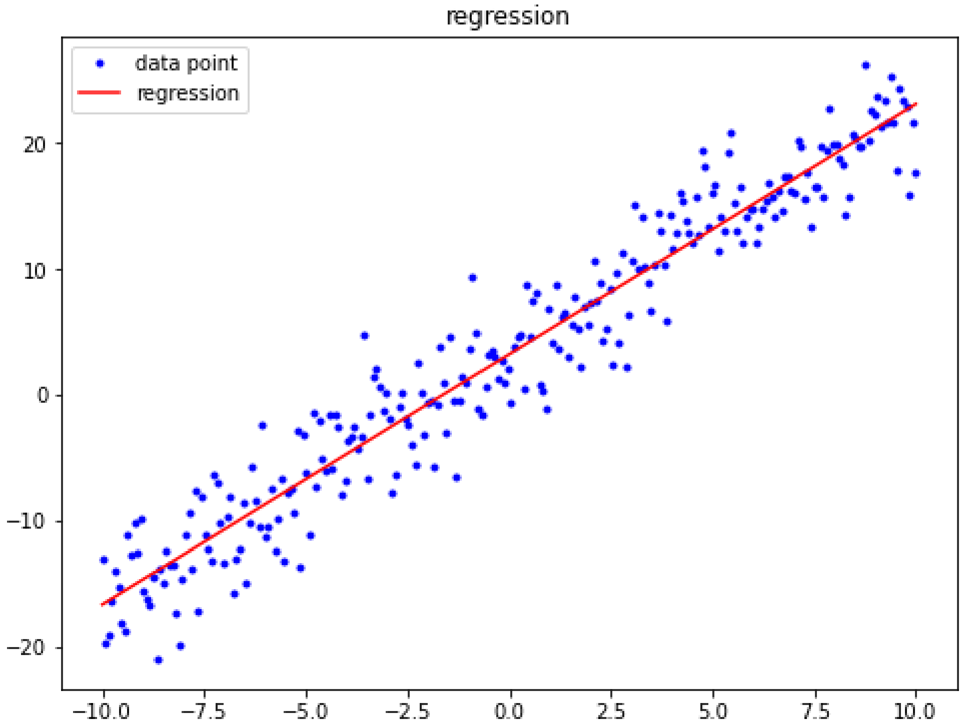
[RESULT 02]
****



****
[RESULT 03]
****

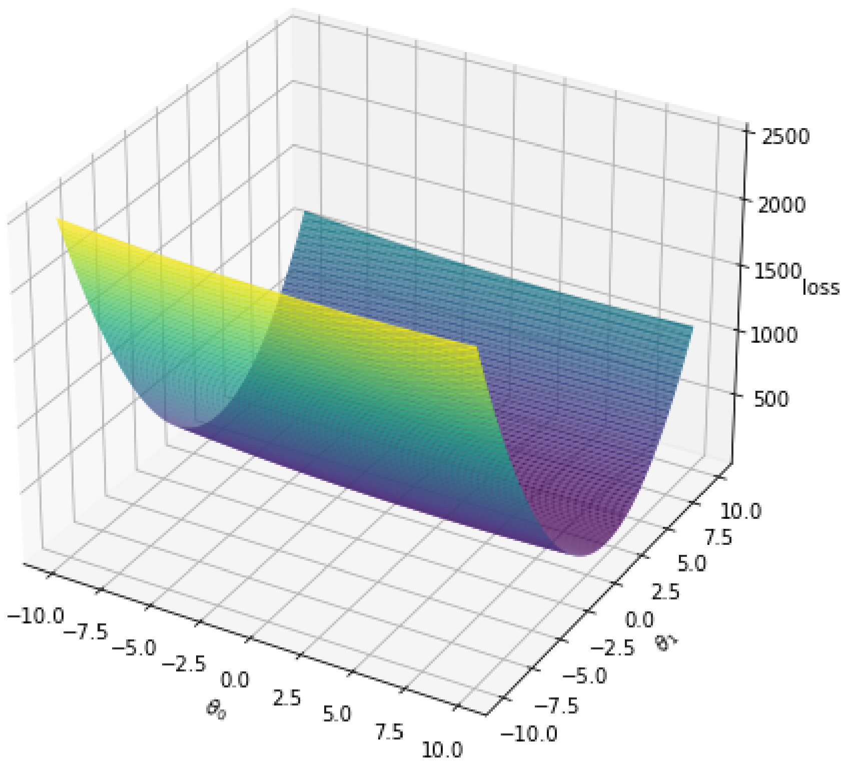


****
[RESULT 04]
****



regression
data point
regression
20
10
0
−10
−20
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


```
**************************************************
## [RESULT 05]
**************************************************
```



2500
2000
1500
loss
1000
500
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
θ_1
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
θ_0

In [ ]: