Gradient of Image

import library

In [ ]:
```
import numpy as np
import matplotlib.image as img
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.colors as colors
```

load input image ('test.jpeg')

In [ ]:
```
I0 = img.imread('test.jpeg')
```

In [ ]:
```
I0
```

Out[ ]:
```
array([[[192,  97,  53],
        [183,  87,  45],
        [189,  94,  48],
        ...,
        [142, 194,  84],
        [111, 195,  84],
        [  1,   0,   8]],

       [[190,  94,  52],
        [190,  95,  51],
        [180,  84,  42],
        ...,
        [142, 195,  81],
        [116, 199,  85],
        [  0,   0,   4]],

       [[179,  86,  43],
        [191,  98,  55],
        [179,  87,  40],
        ...,
        [152, 202,  87],
        [115, 195,  80],
        [  3,   1,   4]],

       ...,

       [[134, 139,  55],
        [129, 141,  55],
        [136, 145,  66],
        ...,
        [168, 198, 164],
        [202, 212, 178],
        [  3,   5,  17]],

       [[133, 131,  46],
        [137, 140,  59],
        [123, 133,  47],
        ...,
        [183, 214, 172],
        [194, 187, 141],
        [  0,   1,   0]],

       [[135, 125,  38],
        [131, 124,  54],
        [122, 134,  32],
        ...,
```

```
        [173, 203, 165],
        [193, 172, 129],
        [  2,   1,   0]]], dtype=uint8)
```

check the size of the input image

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#
num_row     = I0.shape[0]
num_column  = I0.shape[1]
num_channel = I0.shape[2]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I0 = ', num_row)
print('number of columns of I0 = ', num_column)
print('number of channels of I0 = ', num_channel)
```

```
number of rows of I0 =  510
number of columns of I0 =  512
number of channels of I0 =  3
```

convert the color image into a grey image

- take the average of the input image with 3 channels with respect to the channels into an image with 1 channel

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#

I = np.mean(I0,axis=2)


num_row     = I.shape[0]
num_column  = I.shape[1]
#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('number of rows of I = ', num_row)
print('number of columns of I = ', num_column)
```

```
number of rows of I =  510
number of columns of I =  512
```

In [ ]:
```
I
```

Out[ ]:
```
array([[114.        , 105.        , 110.33333333, ..., 140.        ,
        130.        ,   3.        ],
       [112.        , 112.        , 102.        , ..., 139.33333333,
        133.33333333,   1.33333333],
       [102.66666667, 114.66666667, 102.        , ..., 147.        ,
        130.        ,   2.66666667],
       ...,
       [109.33333333, 108.33333333, 115.66666667, ..., 176.66666667,
        197.33333333,   8.33333333],
       [103.33333333, 112.        , 101.        , ..., 189.66666667,
        174.        ,   0.33333333],
       [ 99.33333333, 103.        ,  96.        , ..., 180.33333333,
        164.66666667,   1.        ]])
```

normalize the converted image

- normalize the converted grey scale image so that its maximum value is 1 and its minimum value is 0

In [ ]:
```
# ++++++++++++++++++++++++++++++++++++++++++++++++++
# complete the blanks
#

I = (I-np.min(I))/(np.max(I)-np.min(I))

#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

print('maximum value of I = ', np.max(I))
print('minimum value of I = ', np.min(I))
```

```
maximum value of I =  1.0
minimum value of I =  0.0
```

In [ ]:
```
I.shape
```

Out[ ]:
```
(510, 512)
```

In [ ]:
```
for i in range(I.shape[0]):
    print(i)
```

```
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
```

define a function to compute the derivative of input matrix in x(row)-direction

- forward difference : $I[x+1, y] - I[x, y]$

In [ ]:
```
def compute_derivative_x_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
```

```
    #

    for i in range(I.shape[0]-1):
        for j in range(I.shape[1]):
            D[i,j] = I[i+1,j] - I[i,j]


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

In [ ]:
```
compute_derivative_x_forward(I).shape
```

Out[ ]:
```
(510, 512)
```

- backward difference : $I[x,y] - I[x-1,y]$

In [ ]:
```
def compute_derivative_x_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    for i in range(1, I.shape[0]):
        for j in range(I.shape[1]):
            D[i,j] = I[i,j] - I[i-1,j]


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- central difference : $\frac{1}{2}(I[x+1,y] - I[x-1,y])$

In [ ]:
```
def compute_derivative_x_central(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    for i in range(1, I.shape[0]-1):
        for j in range(I.shape[1]):
            D[i,j] = 0.5*(I[i+1,j] - I[i-1,j])


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

In [ ]:
```
compute_derivative_x_central(I)
```

Out[ ]:
```
array([[ 0.        ,  0.        ,  0.        , ...,  0.        ,
         0.        ,  0.        ],
       [-0.02225131,  0.01897906, -0.01636126, ...,  0.01374346,
         0.        , -0.00065445],
```

```
       [-0.0065445 ,  0.01832461,  0.01243455, ...,  0.0039267 ,
         0.00719895, -0.00065445],
       ...,
       [-0.00327225,  0.0091623 , -0.02028796, ...,  0.00850785,
        -0.01767016, -0.0039267 ],
       [-0.01963351, -0.0104712 , -0.03861257, ...,  0.00719895,
        -0.06413613, -0.01439791],
       [ 0.        ,  0.        ,  0.        , ...,  0.        ,
         0.        ,  0.        ]])
```

define a function to compute the derivative of input matrix in y(column)-direction

- forward difference : $I[x, y+1] - I[x, y]$

In [ ]:
```
def compute_derivative_y_forward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    for i in range(I.shape[0]):
        for j in range(I.shape[1]-1):
            D[i,j] = 0.5*(I[i,j+1] - I[i,j])

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

In [ ]:
```
compute_derivative_y_forward(I)
```

Out[ ]:
```
array([[-0.01767016,  0.0104712 ,  0.00327225, ..., -0.01963351,
        -0.24934555,  0.        ],
       [ 0.        , -0.01963351,  0.0104712 , ..., -0.0117801 ,
        -0.2591623 ,  0.        ],
       [ 0.02356021, -0.02486911,  0.01505236, ..., -0.03337696,
        -0.25      ,  0.        ],
       ...,
       [-0.00196335,  0.01439791, -0.05039267, ...,  0.04057592,
        -0.3710733 ,  0.        ],
       [ 0.01701571, -0.02159686, -0.00327225, ..., -0.03075916,
        -0.34096859,  0.        ],
       [ 0.00719895, -0.01374346,  0.00196335, ..., -0.03075916,
        -0.32133508,  0.        ]])
```

- backward difference : $I[x, y] - I[x, y-1]$

In [ ]:
```
def compute_derivative_y_backward(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    for i in range(I.shape[0]):
        for j in range(1, I.shape[1]):
            D[i,j] = 0.5*(I[i,j] - I[i,j-1])
```

```
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

- central difference : $\frac{1}{2}(I[x, y+1] - I[x, y-1])$

In [ ]:
```
def compute_derivative_y_central(I):

    D = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    for i in range(I.shape[0]):
        for j in range(1, I.shape[1]-1):
            D[i,j] = 0.5*(I[i,j+1] - I[i,j-1])


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return D
```

In [ ]:
```
compute_derivative_y_central(I).shape
```

Out[ ]:
```
(510, 512)
```

compute the norm of the gradient of the input image

- L_2^2-norm of the gradient $\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)$ is defined by $\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2$

In [ ]:
```
def compute_norm_gradient_central(I):

    norm_gradient = np.zeros(I.shape)

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    norm_gradient = compute_derivative_x_central(I)**2 + compute_derivative_y_central

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return norm_gradient
```

In [ ]:
```
compute_norm_gradient_central(I).shape
```

Out[ ]:
```
(510, 512)
```

---

functions for presenting the results

In [ ]:
```
def function_result_01():

    plt.figure(figsize=(8,6))
    plt.imshow(I0)
    plt.show()
```

In [ ]:
```
def function_result_02():

    plt.figure(figsize=(8,6))
    plt.imshow(I, cmap='gray', vmin=0, vmax=1, interpolation='none')
    plt.show()
```

In [ ]:
```
def function_result_03():

    D = compute_derivative_x_forward(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_04():

    D = compute_derivative_x_backward(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_05():

    D = compute_derivative_x_central(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_06():

    D = compute_derivative_y_forward(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_07():

    D = compute_derivative_y_backward(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_08():

    D = compute_derivative_y_central(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_09():

    D = compute_norm_gradient_central(I)

    plt.figure(figsize=(8,6))
    plt.imshow(D, cmap='gray')
    plt.show()
```

In [ ]:
```
def function_result_10():

    D = compute_norm_gradient_central(I)

    plt.figure(figsize=(8,6))
    im = plt.imshow(D, cmap=cm.jet, norm=colors.LogNorm())
    plt.colorbar(im)
    plt.show()
```

In [ ]:
```
def function_result_11():

    D = compute_derivative_x_forward(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_12():

    D = compute_derivative_x_backward(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_13():

    D = compute_derivative_x_central(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_14():

    D = compute_derivative_y_forward(I)
```

```
    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_15():

    D = compute_derivative_y_backward(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

In [ ]:
```
def function_result_16():

    D = compute_derivative_y_central(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

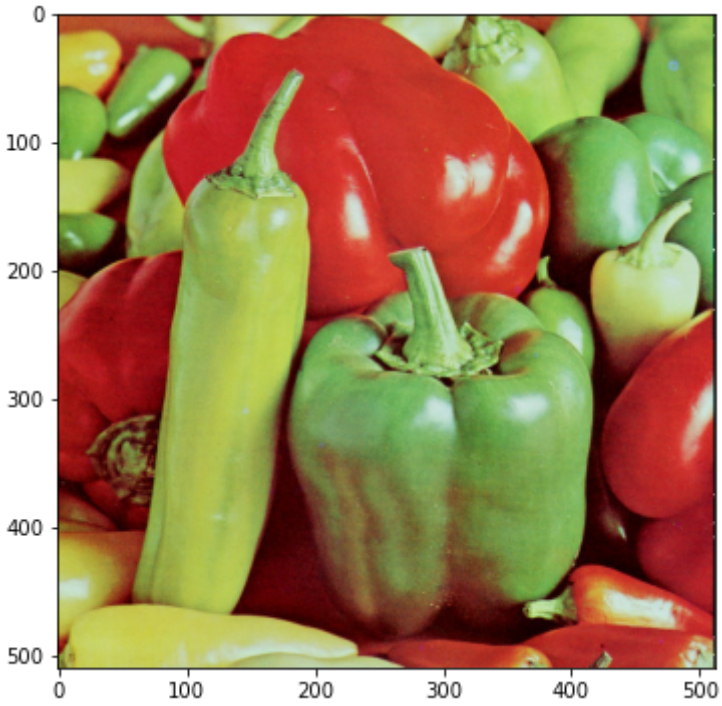
In [ ]:
```
def function_result_17():

    D = compute_norm_gradient_central(I)

    value1 = D[0, 0]
    value2 = D[-1, -1]
    value3 = D[100, 100]
    value4 = D[200, 200]

    print('value1 = ', value1)
    print('value2 = ', value2)
    print('value3 = ', value3)
    print('value4 = ', value4)
```

---

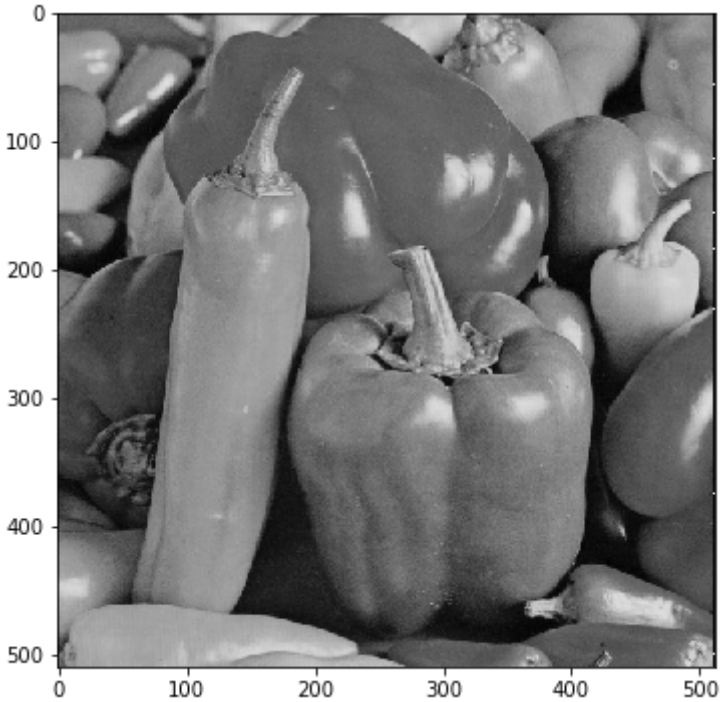
results

In [ ]:
```
number_result = 17

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('************************************************')
    print(title)
    print('************************************************')
    eval(name_function)
```

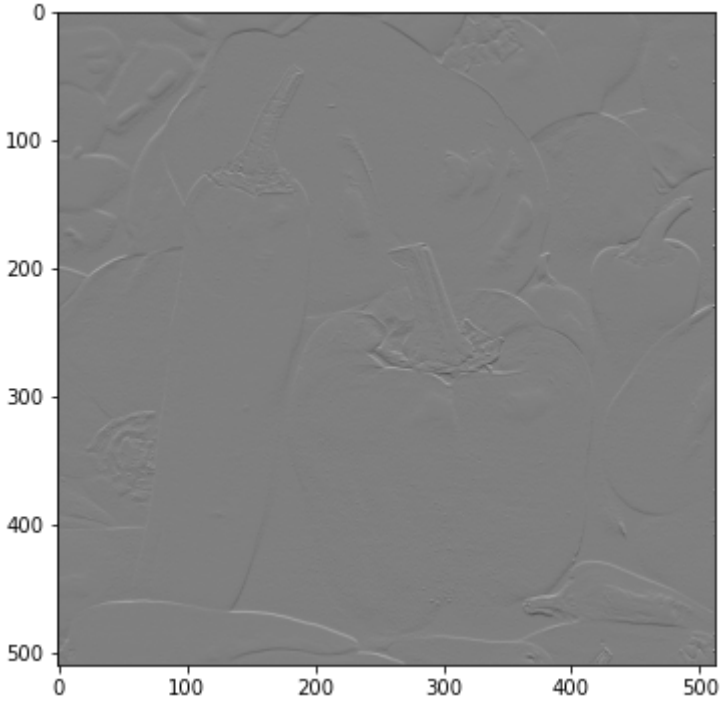
```
************************************************
## [RESULT 01]
************************************************
```



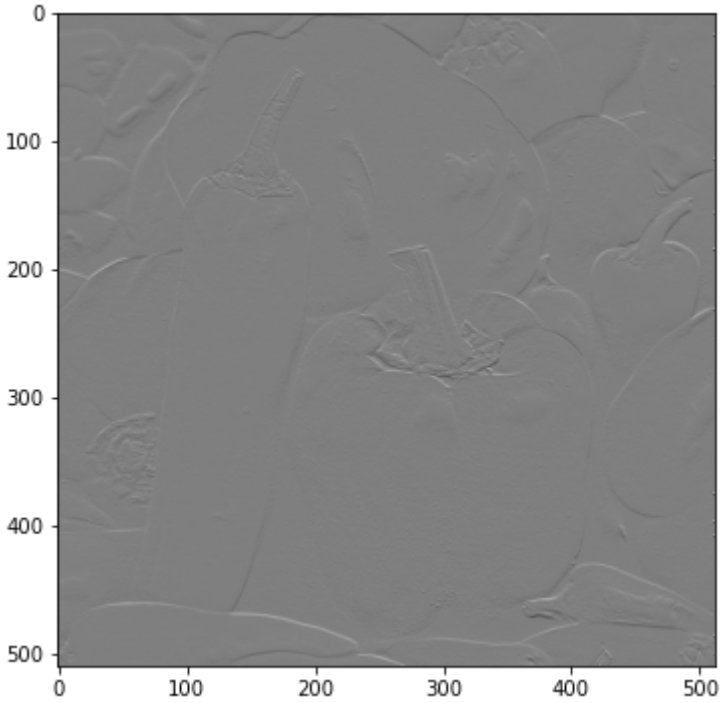

```
************************************************
## [RESULT 02]
************************************************
```



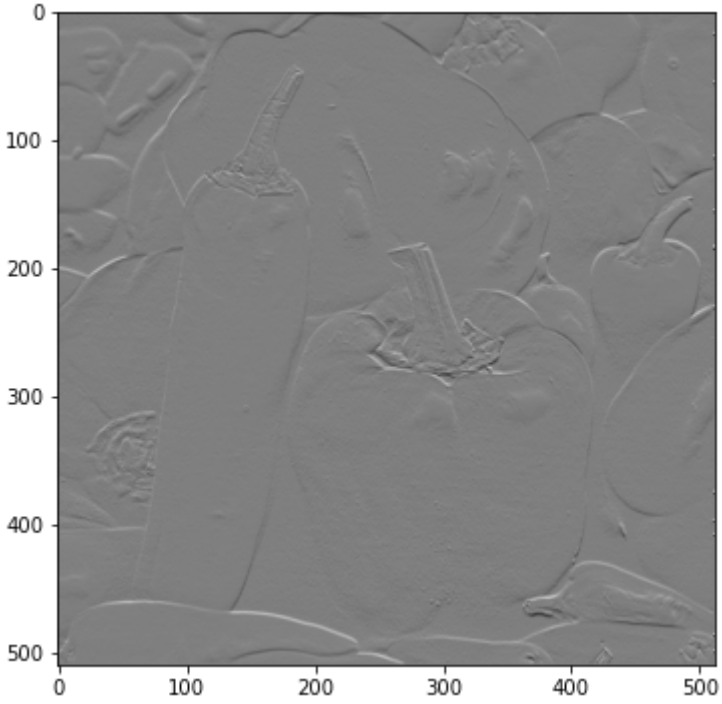

```
************************************************
## [RESULT 03]
************************************************
```



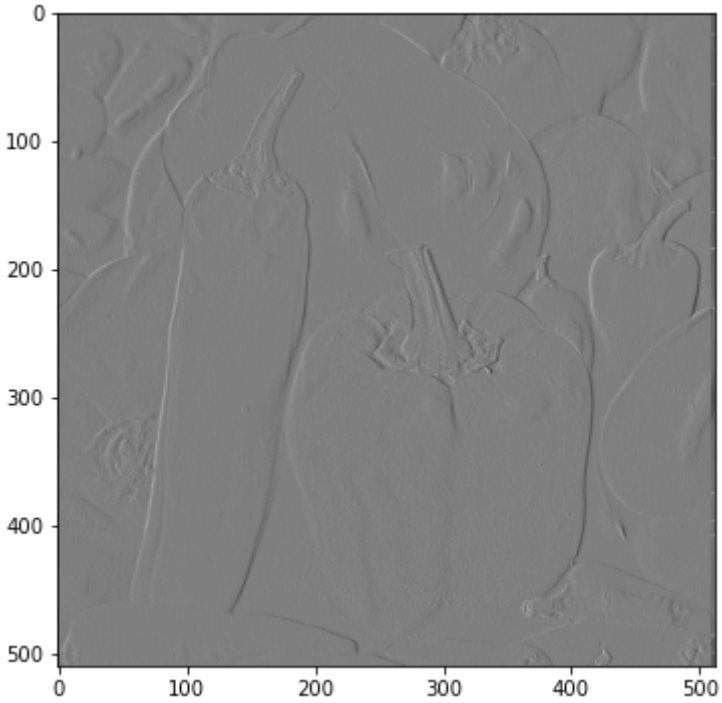
****
[RESULT 04]
****



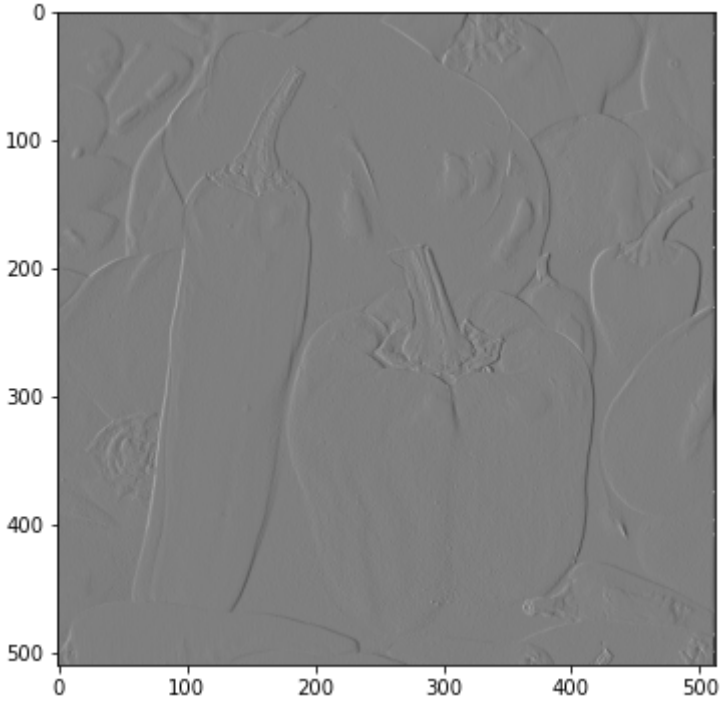
****
[RESULT 05]
****



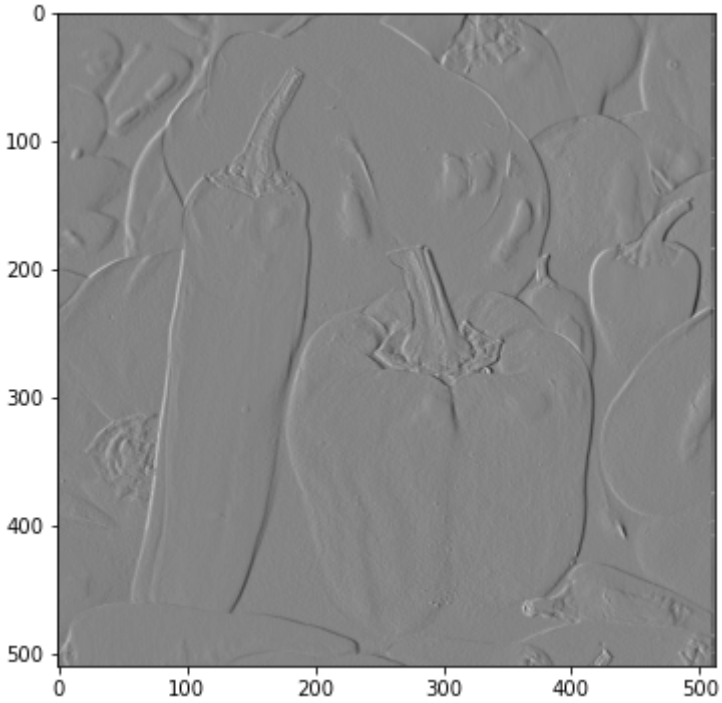
****
[RESULT 06]
****



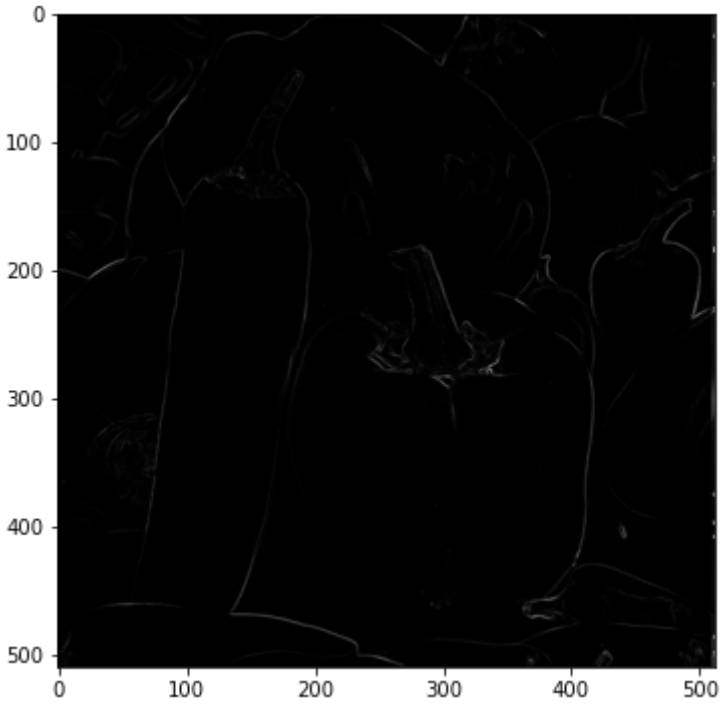
****
[RESULT 07]
****



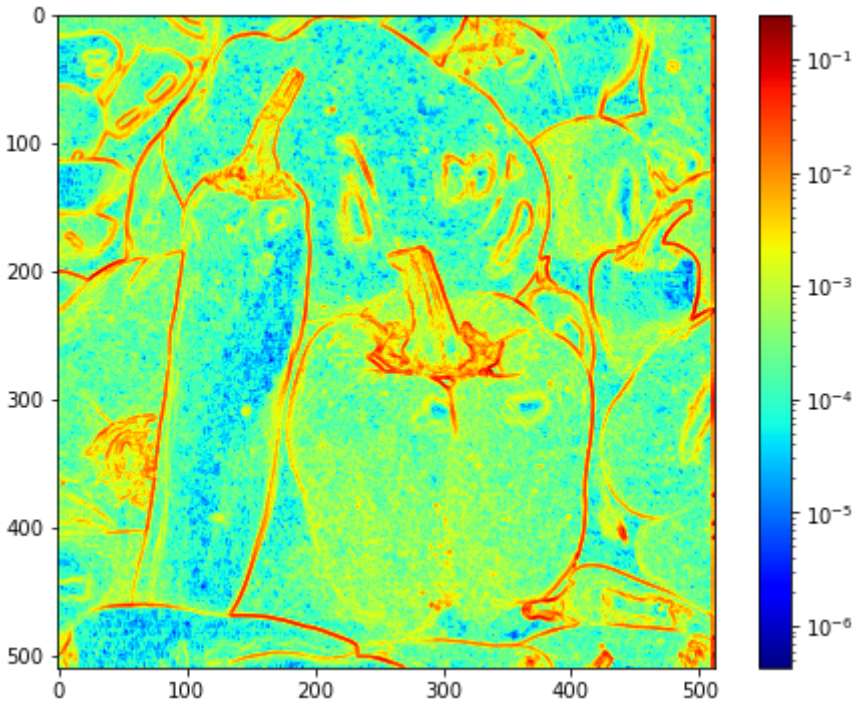
****
[RESULT 08]
****



****
[RESULT 09]
****



****
[RESULT 10]
****



****
[RESULT 11]
****
value1 = -0.007853403141361237
value2 = 0.0
value3 = -0.005235602094240843
value4 = 0.011780104712041883
****
[RESULT 12]
****
value1 = 0.0
value2 = 0.0026178010471204186
value3 = 0.01570680628272253
value4 = -0.013089005235602025
****
[RESULT 13]
****
value1 = 0.0

```
value2 =  0.0
value3 =  0.005235602094240843
value4 =  -0.0006544502617800707
**************************************************
## [RESULT 14]
**************************************************
value1 =  -0.017670157068062825
value2 =  0.0
value3 =  -0.008507853403141363
value4 =  0.0
**************************************************
## [RESULT 15]
**************************************************
value1 =  0.0
value2 =  -0.32133507853403137
value3 =  0.004581151832460745
value4 =  0.003926701570680646
**************************************************
## [RESULT 16]
**************************************************
value1 =  0.0
value2 =  0.0
value3 =  -0.0039267015706806185
value4 =  0.003926701570680646
**************************************************
## [RESULT 17]
**************************************************
value1 =  0.0
value2 =  0.0
value3 =  4.2830514514404736e-05
value4 =  1.5847290370329858e-05
```

In [ ]: