Principal Component Analysis

import library

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from matplotlib import cm
```

load data

In [ ]:

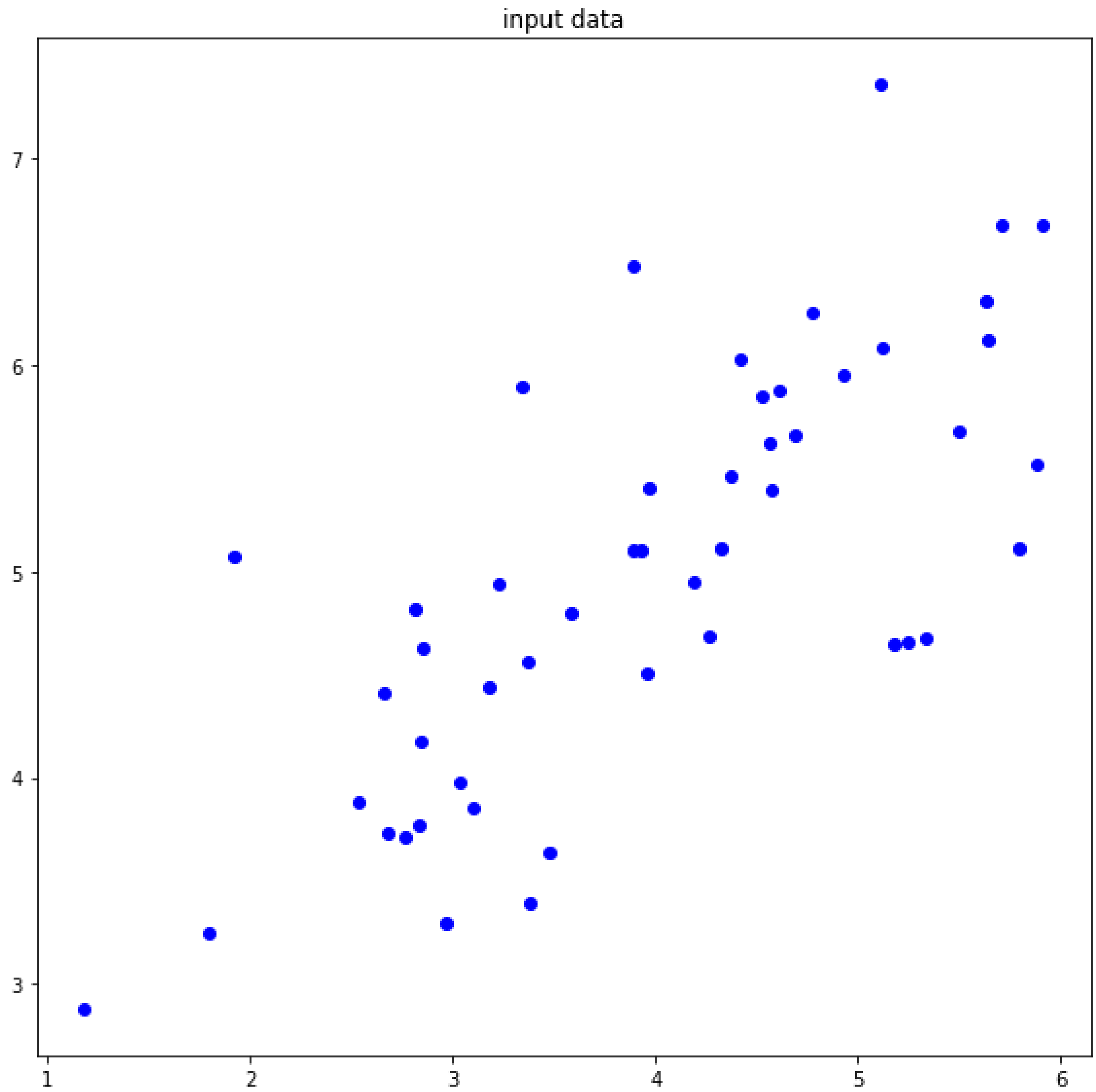
```
fname_data  = 'assignment_12_data.txt'
feature0    = np.genfromtxt(fname_data, delimiter=',')

number_data     = np.size(feature0, 0)
number_feature  = np.size(feature0, 1)

print('number of data : {}'.format(number_data))
print('number of feature : {}'.format(number_feature))
```

```
number of data : 50
number of feature : 2
```

plot the input data

In [ ]:

```
plt.figure(figsize=(8,8))
plt.title('input data')

x0 = feature0[:,0]
y0 = feature0[:,1]

plt.scatter(x0, y0, color='blue')

plt.tight_layout()
plt.show()
```



Normalization (Z-scoring)

- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features

In [ ]:
```
def normalize(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    mean = np.mean(feature,axis=0)
    std = np.std(feature0,axis=0)

    feature_normalize = (feature - mean)/std

    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return feature_normalize
```

In [ ]:
```
feature = normalize(feature0)

x    = feature[:, 0]
y    = feature[:, 1]
```

```python
min_x = np.min(x)
min_y = np.min(y)

max_x = np.max(x)
max_y = np.max(y)
```

compute covariance matrix

- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features

In [ ]:
```python
def compute_covariance(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    n = feature.shape[0]
    Sigma = np.divide(np.matmul(feature.T,feature),n)


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return Sigma
```

compute principal components

- `np.linalg.eig`
- `argsort()`
- return the eigenvalues and the eigenvectors in a decreasing order according to the eigenvalues

In [ ]:
```python
def compute_principal_component(feature):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    Sigma = compute_covariance(feature)

    w, v = np.linalg.eig(Sigma)

    w = w[w.argsort()[::-1]]
    v = v[:,w.argsort()[::-1]]

    principal_component_1 = w[0]*v[:,0]
    principal_component_2 = w[1]*v[:,1]



    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    return (principal_component_1, principal_component_2)
```

compute the projection of point onto the axis

- `np.matmul`

- `np.dot`
- shape of feature = $n \times m$ where n is the number of data and m is the dimension of features
- shape of vector = $m \times 1$ where m is the dimension of features

In [ ]:
```
def compute_projection_onto_line(feature, vector):

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    n = feature.shape[0]
    projection=np.zeros((n,2))

    for i in range(n):
        point = feature[i,:]
        proj_point = np.dot(vector,point)/np.dot(vector,vector)*vector

        projection[i]=proj_point





    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++


    return projection
```

compute the principal components and the projection of feature

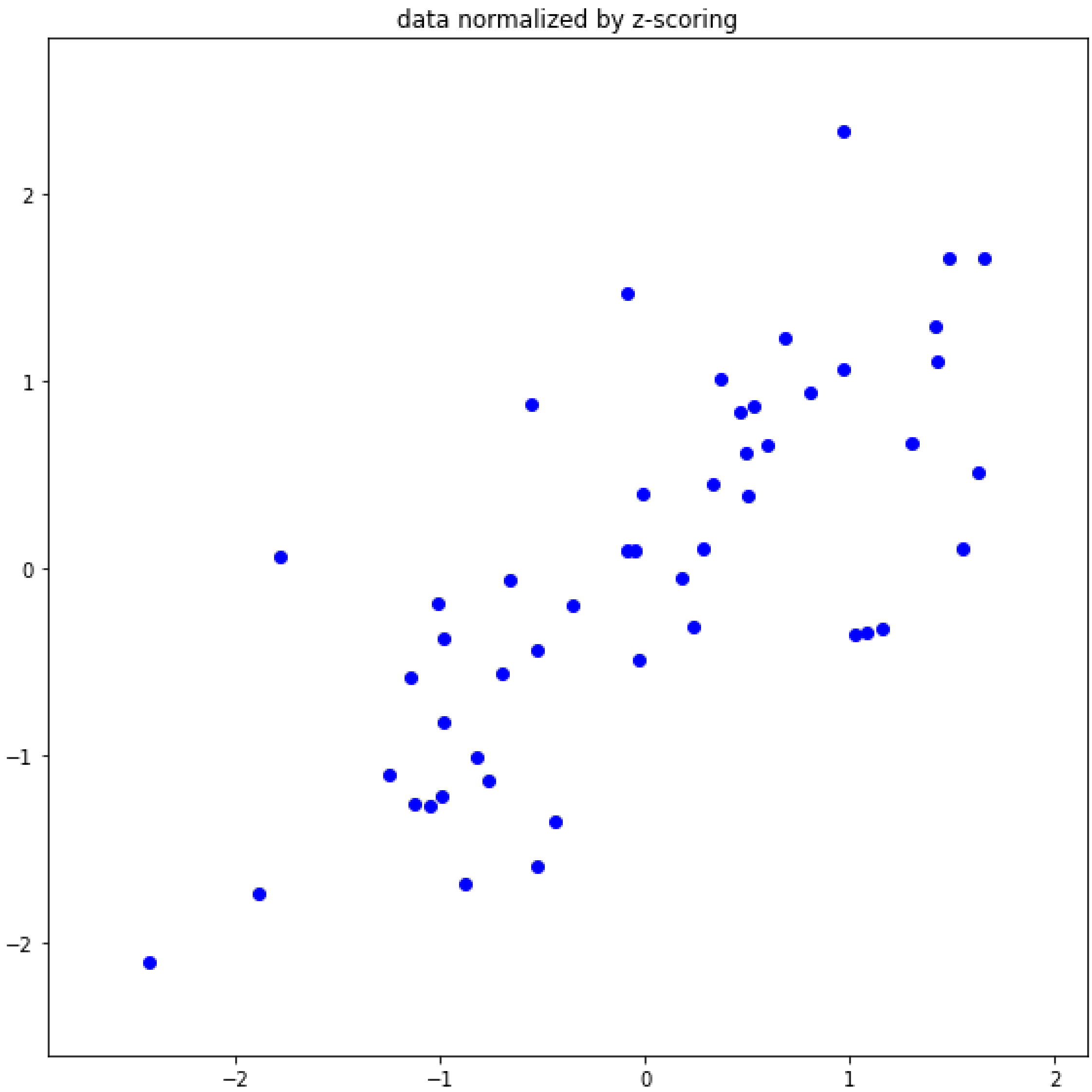
In [ ]:
```
(principal_component_1, principal_component_2) = compute_principal_component(feature)

projection1 = compute_projection_onto_line(feature, principal_component_1)
projection2 = compute_projection_onto_line(feature, principal_component_2)
```

---

functions for presenting the results

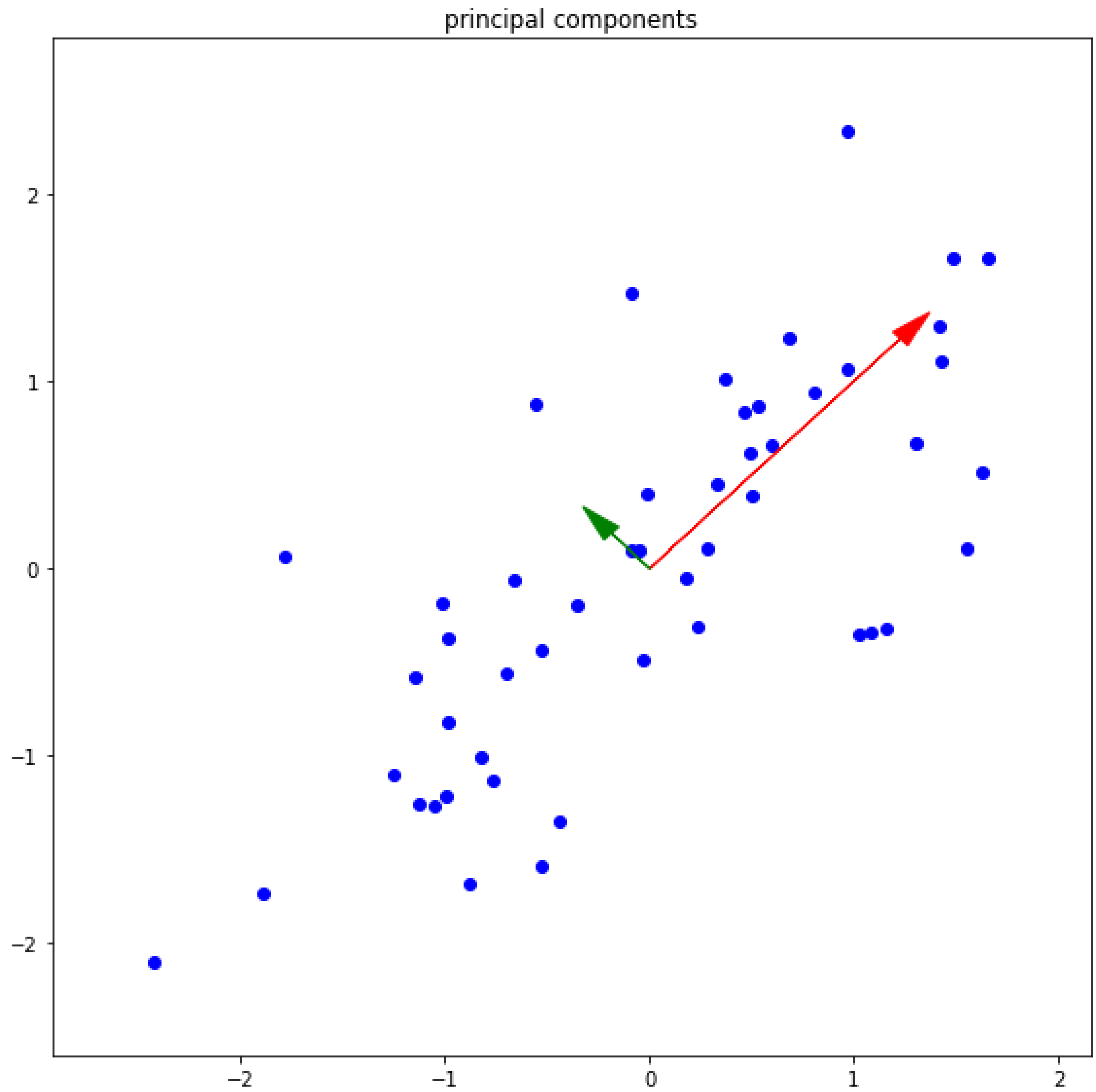
In [ ]:
```
def function_result_01():

    plt.figure(figsize=(8,8))
    plt.title('data normalized by z-scoring')
    plt.scatter(x, y, color='blue')

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:
```
function_result_01()
```



In [ ]:
```
def function_result_02():

    plt.figure(figsize=(8,8))
    plt.title('principal components')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    (principal_component_1, principal_component_2) = compute_principal_component(featu

    plt.arrow(0, 0, principal_component_1[0], principal_component_1[1], head_width =
    plt.arrow(0, 0, principal_component_2[0], principal_component_2[1], head_width =
    plt.scatter(x, y, color='blue')


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

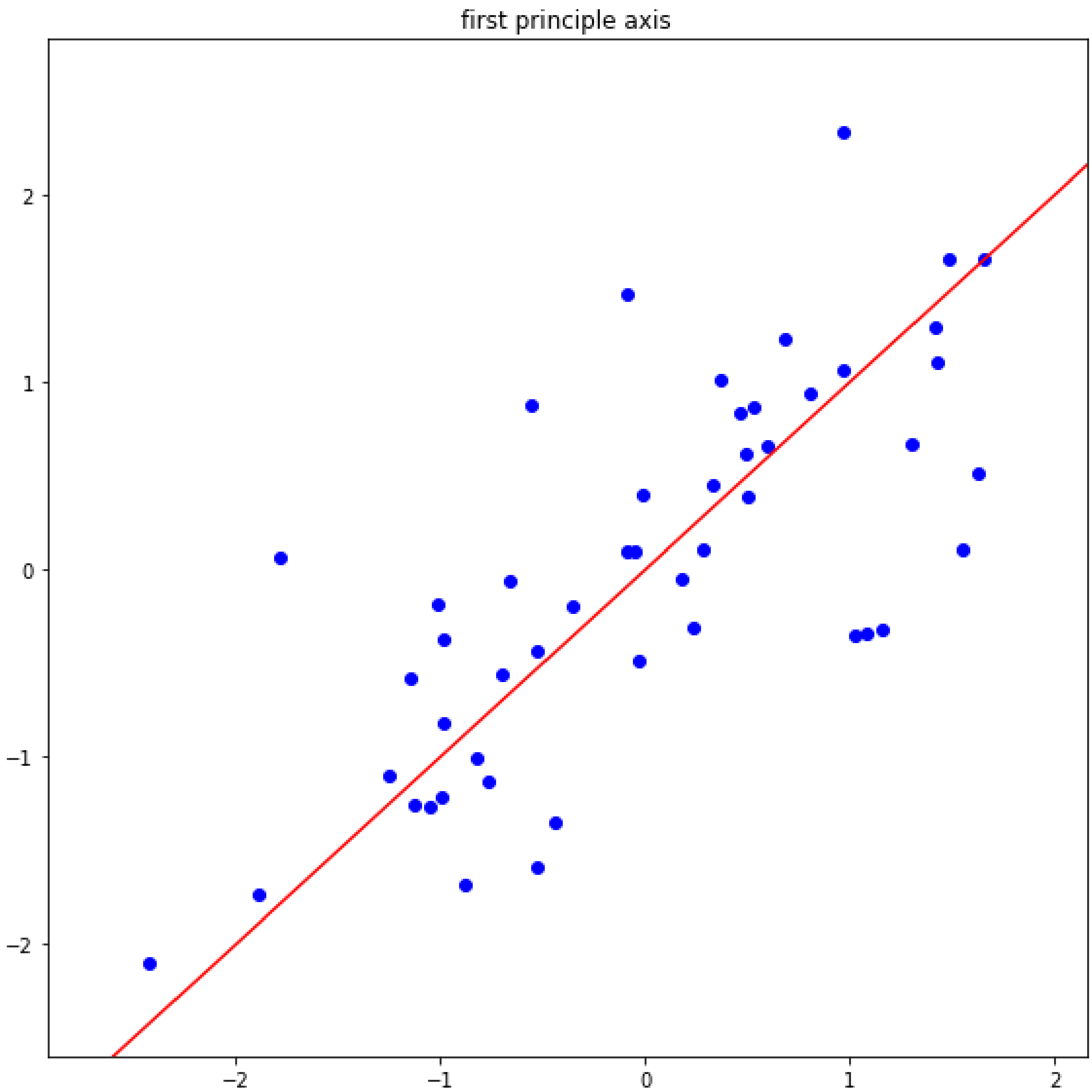
In [ ]:
```
function_result_02()
```



In [ ]:

```
def function_result_03():

    plt.figure(figsize=(8,8))
    plt.title('first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    (principal_component_1, principal_component_2) = compute_principal_component(featu


    X = np.arange(-3,3,0.1)

    Y = (principal_component_1[1]/principal_component_1[0])*X

    plt.plot(X,Y,'-',color='red')
    plt.scatter(x, y, color='blue')


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

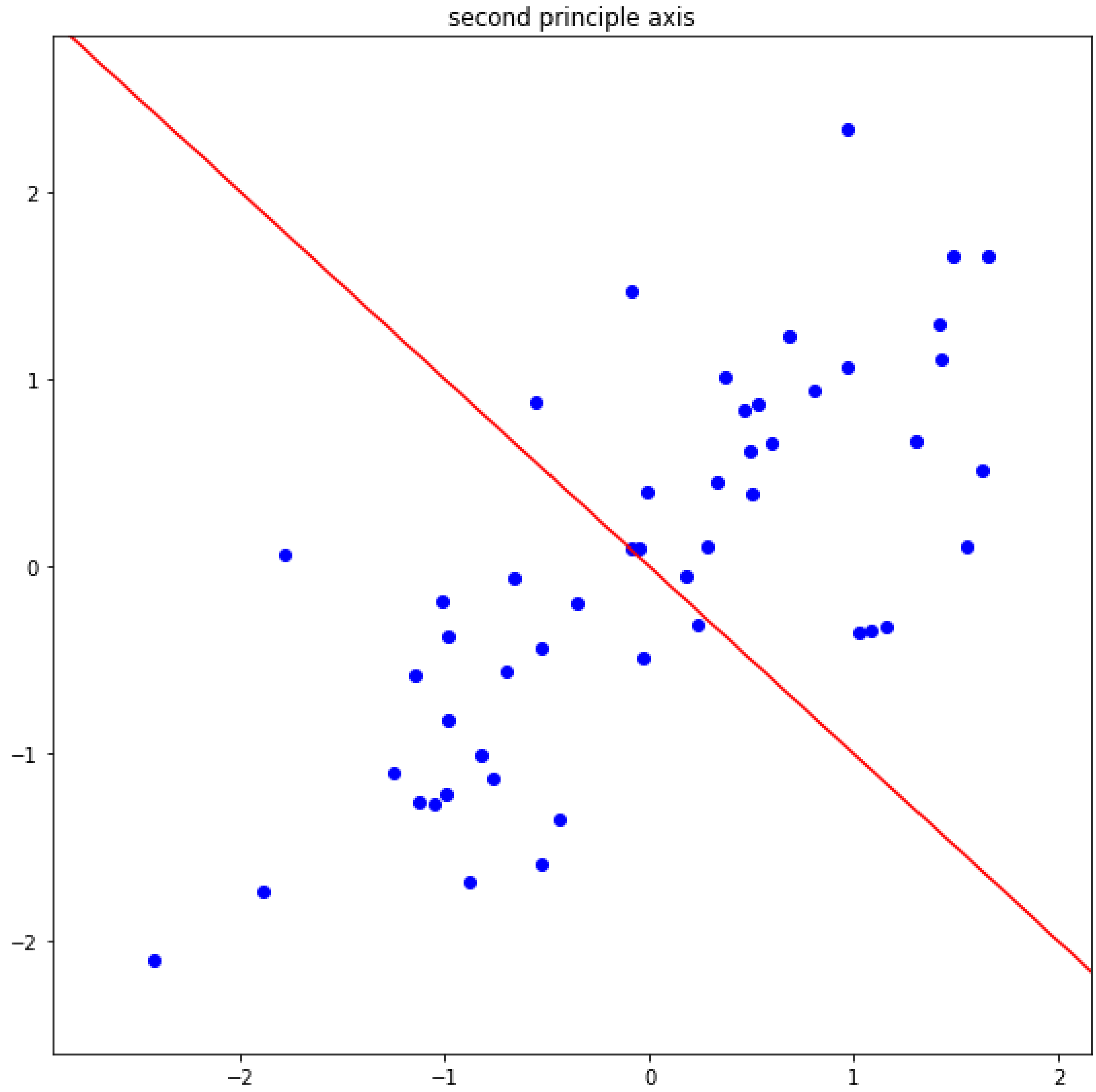
In [ ]:
```
function_result_03()
```



In [ ]:
```
def function_result_04():

    plt.figure(figsize=(8,8))
    plt.title('second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    (principal_component_1, principal_component_2) = compute_principal_component(featu


    X = np.arange(-3,3,0.1)

    Y = (principal_component_2[1]/principal_component_2[0])*X

    plt.plot(X,Y,'-',color='red')
    plt.scatter(x, y, color='blue')


    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)
```

```
    plt.tight_layout()
    plt.show()
```

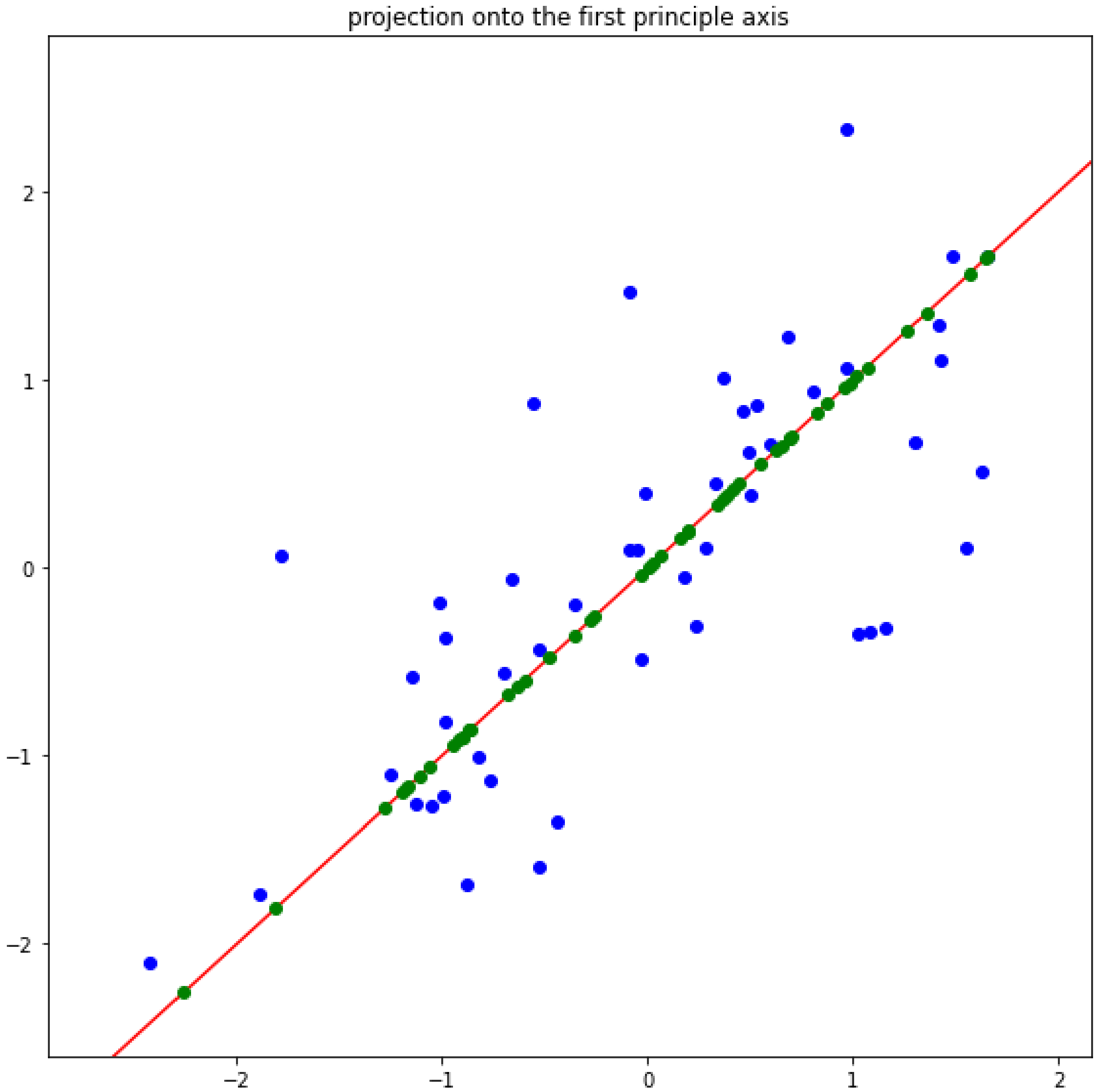
In [ ]:
```
function_result_04()
```



In [ ]:
```
def function_result_05():

    plt.figure(figsize=(8,8))
    plt.title('projection onto the first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #

    (principal_component_1, principal_component_2) = compute_principal_component(featu


    X = np.arange(-3,3,0.1)

    Y = (principal_component_1[1]/principal_component_1[0])*X


    plt.plot(X,Y,'-',color='red')
    plt.plot(projection1[:,0],projection1[:,1],'o',color='green')
    plt.scatter(x, y, color='blue')
```

```
    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

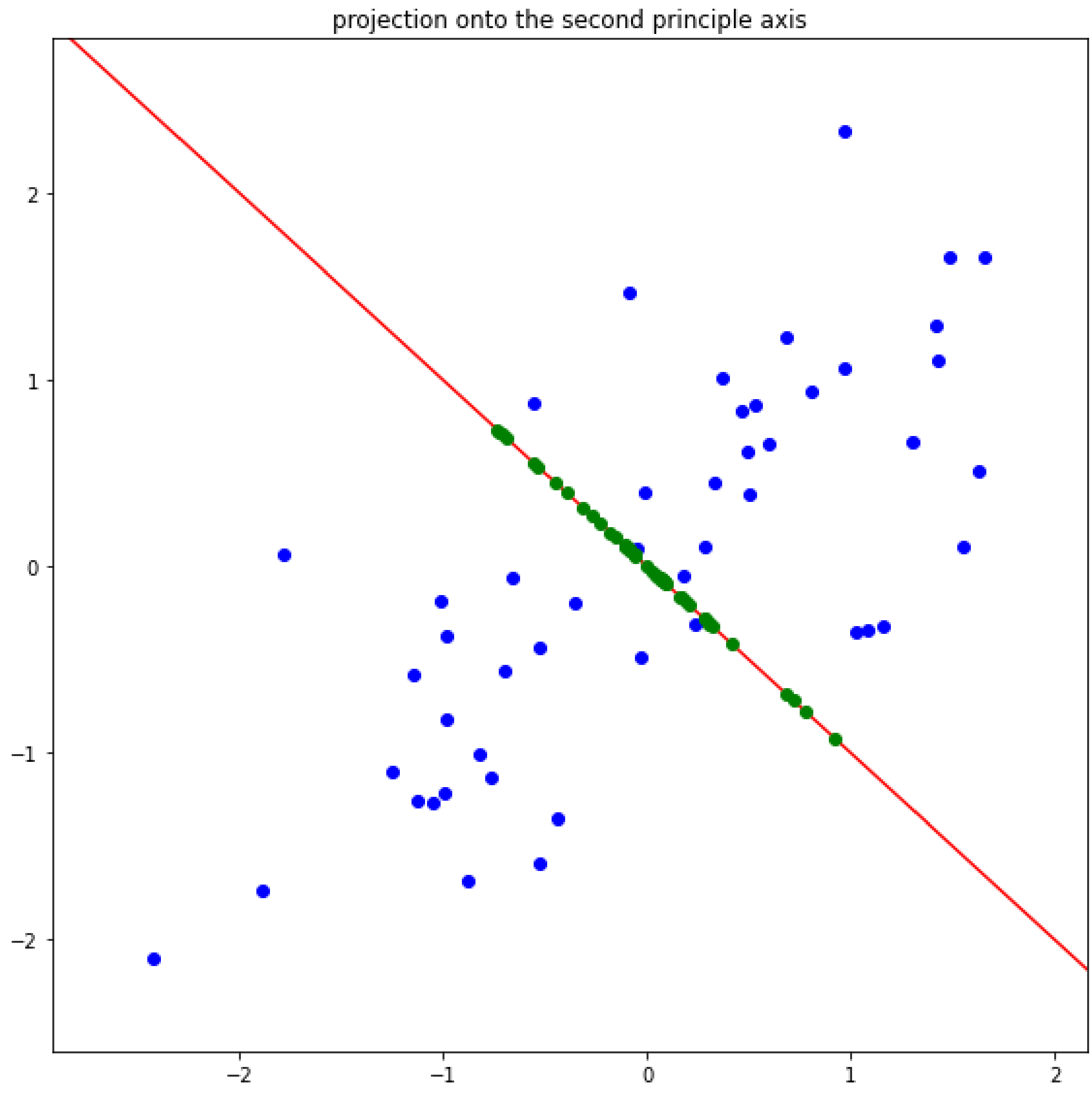
In [ ]:
```
function_result_05()
```



In [ ]:
```
def function_result_06():

    plt.figure(figsize=(8,8))
    plt.title('projection onto the second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    (principal_component_1, principal_component_2) = compute_principal_component(featu


    X = np.arange(-3,3,0.1)

    Y = (principal_component_2[1]/principal_component_2[0])*X
    plt.plot(X,Y,'-',color='red')
```

```python
    plt.plot(projection2[:,1],projection2[:,0],'o',color='green')
    plt.scatter(x, y, color='blue')



    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

In [ ]:
```python
function_result_06()
```

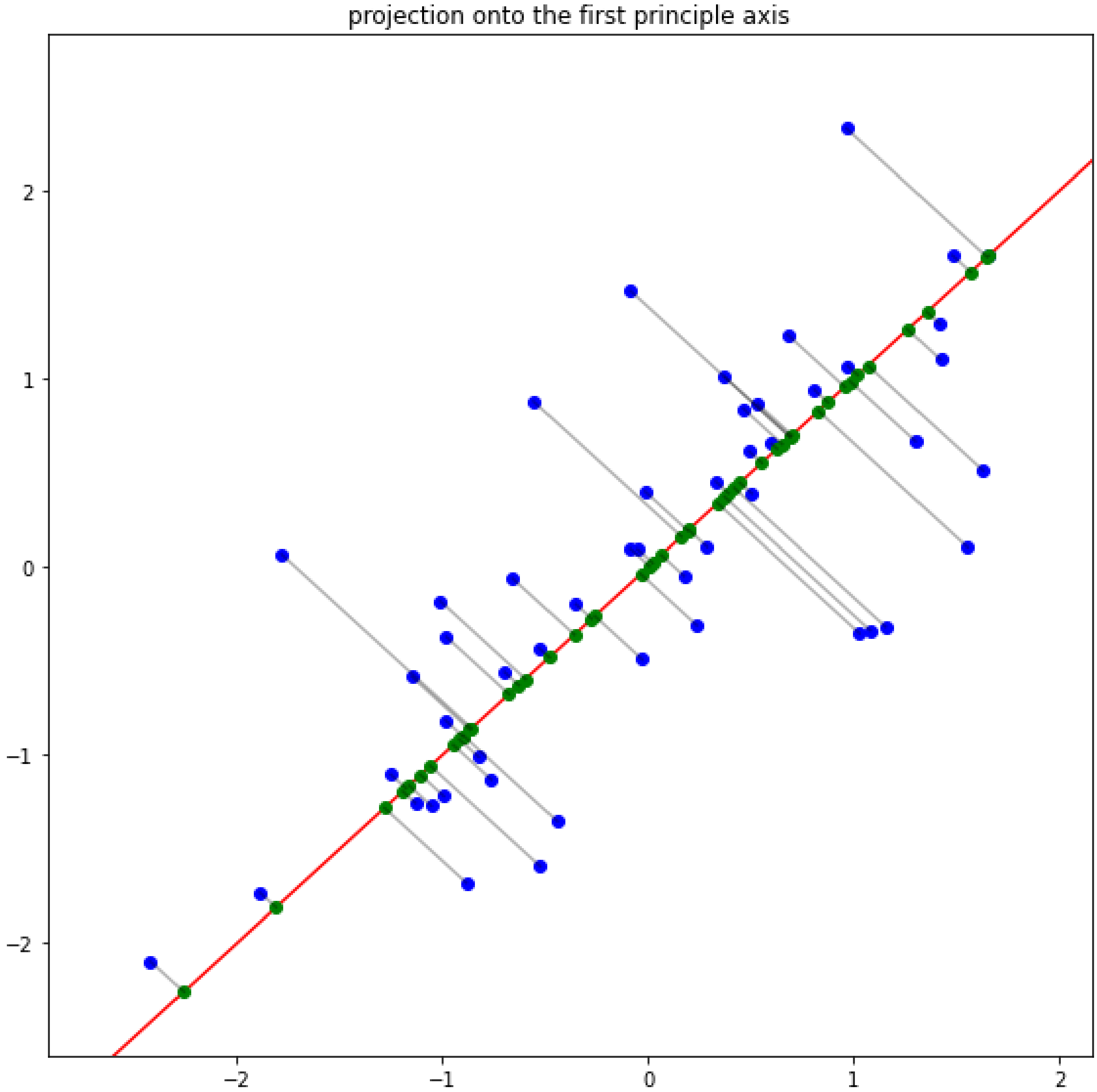


In [ ]:
```python
def function_result_07():

    plt.figure(figsize=(8,8))
    plt.title('projection onto the first principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    (principal_component_1, principal_component_2) = compute_principal_component(featu

    n = feature.shape[0]
```

```
X = np.arange(-3,3,0.1)

Y = (principal_component_1[1]/principal_component_1[0])*X


plt.plot(X,Y,'-',color='red')
plt.plot(projection1[:,0],projection1[:,1],'o',color='green')
plt.scatter(x, y, color='blue')

for i in range(n):
    x_line=[x[i],projection1[i,0]]
    y_line=[y[i],projection1[i,1]]
    plt.plot(x_line,y_line,'-',color='black',alpha=0.3)



#
# ++++++++++++++++++++++++++++++++++++++++++++++++++

plt.xlim(min_x - 0.5, max_x + 0.5)
plt.ylim(min_y - 0.5, max_y + 0.5)

plt.tight_layout()
plt.show()
```

In [ ]:

```
function_result_07()
```



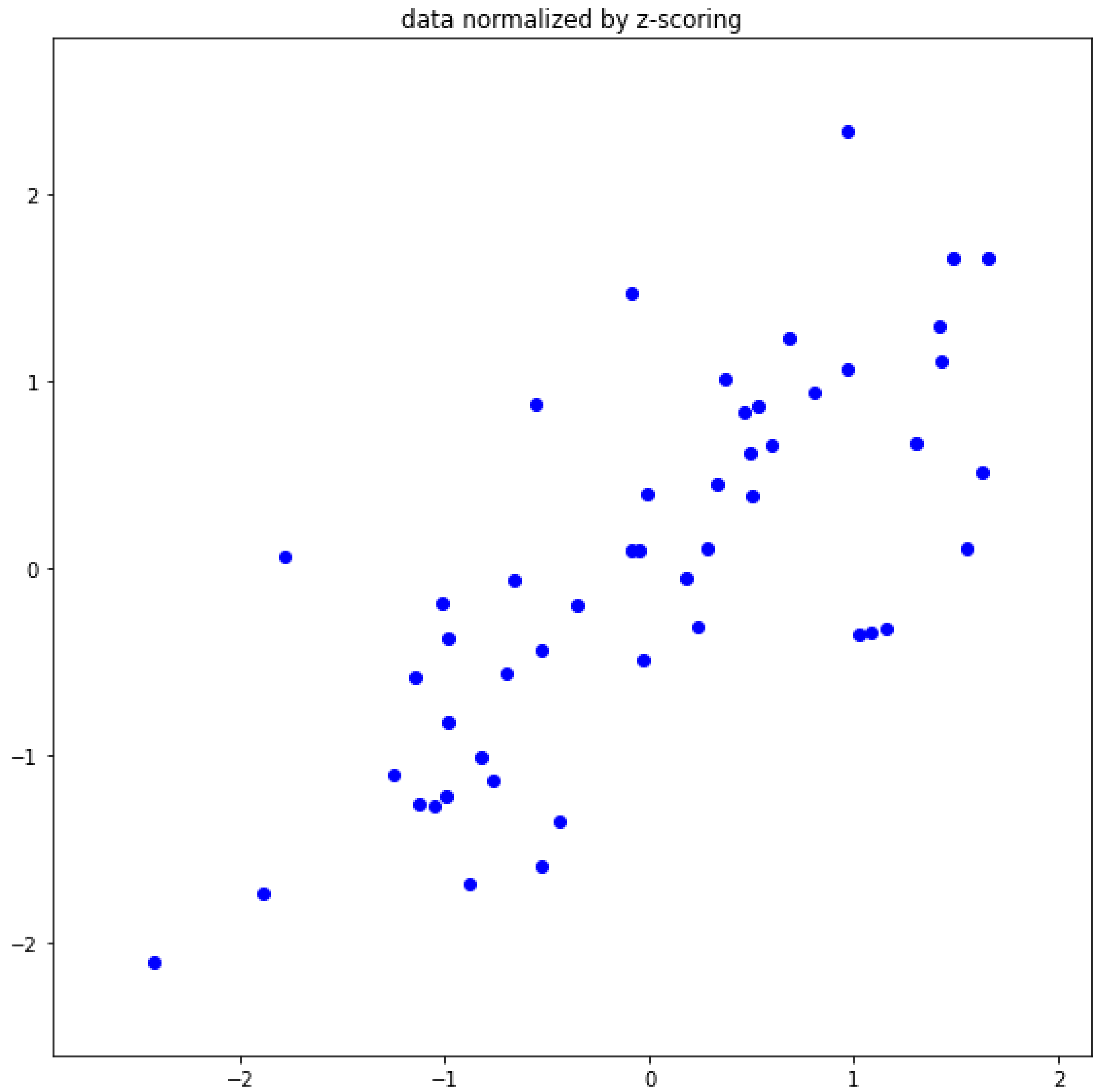
In [ ]:
```
def function_result_08():

    plt.figure(figsize=(8,8))
    plt.title('projection to the second principle axis')

    # ++++++++++++++++++++++++++++++++++++++++++++++++++
    # complete the blanks
    #
    (principal_component_1, principal_component_2) = compute_principal_component(featu

    n = feature.shape[0]
    X = np.arange(-3,3,0.1)

    Y = (principal_component_2[1]/principal_component_2[0])*X
    plt.plot(X,Y,'-',color='red')
    plt.plot(projection2[:,1],projection2[:,0],'o',color='green')
    plt.scatter(x, y, color='blue')


    for i in range(n):
        x_line2=[x[i],projection2[i,0]]
        y_line2=[y[i],projection2[i,1]]
        plt.plot(x_line2,y_line2,'-',color='black',alpha=0.3)



    #
    # ++++++++++++++++++++++++++++++++++++++++++++++++++

    plt.xlim(min_x - 0.5, max_x + 0.5)
    plt.ylim(min_y - 0.5, max_y + 0.5)

    plt.tight_layout()
    plt.show()
```

---

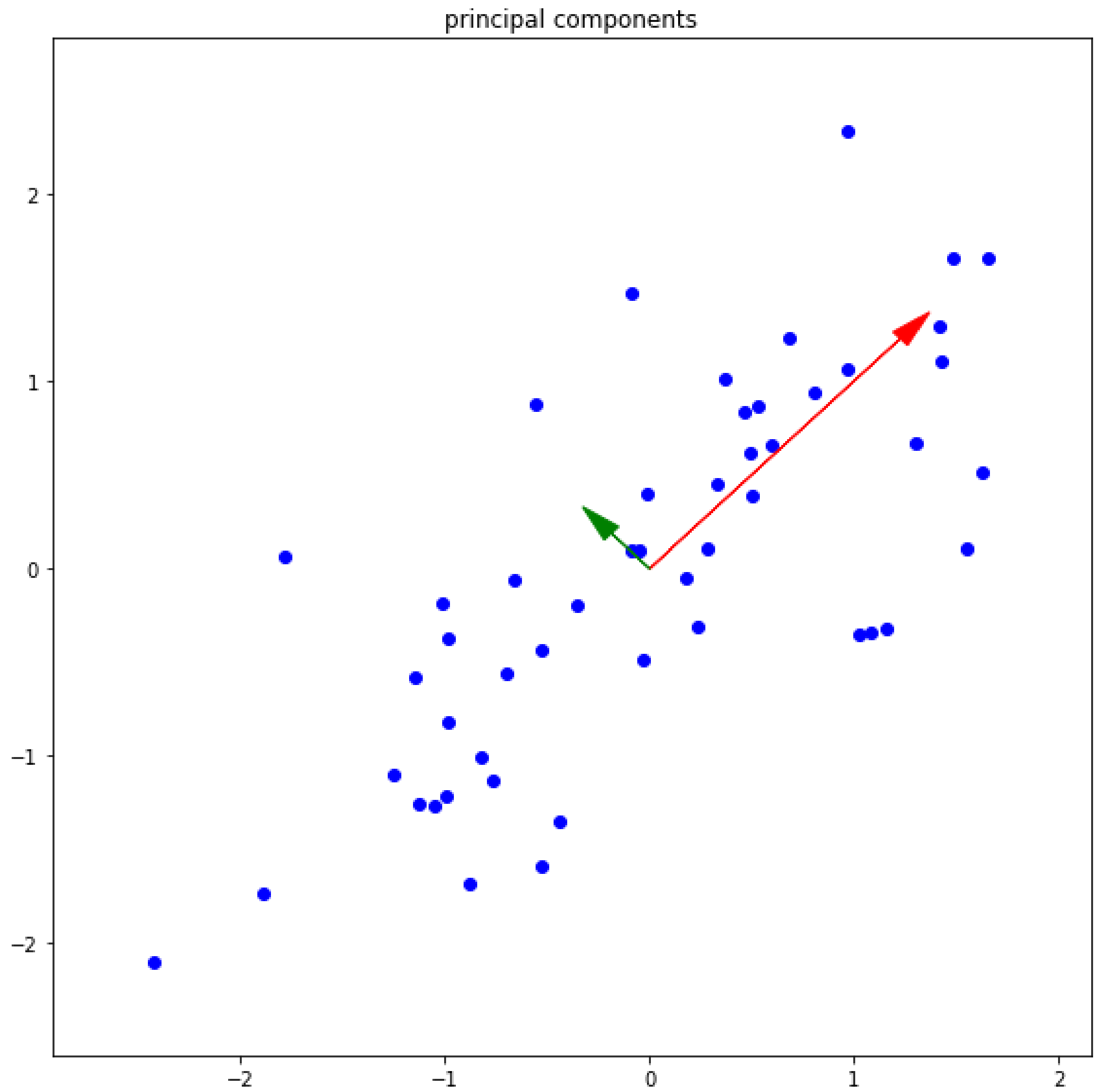
results

In [ ]:
```
number_result = 8

for i in range(number_result):
    title = '## [RESULT {:02d}]'.format(i+1)
    name_function = 'function_result_{:02d}()'.format(i+1)

    print('**************************************************')
    print(title)
    print('**************************************************')
    eval(name_function)
```

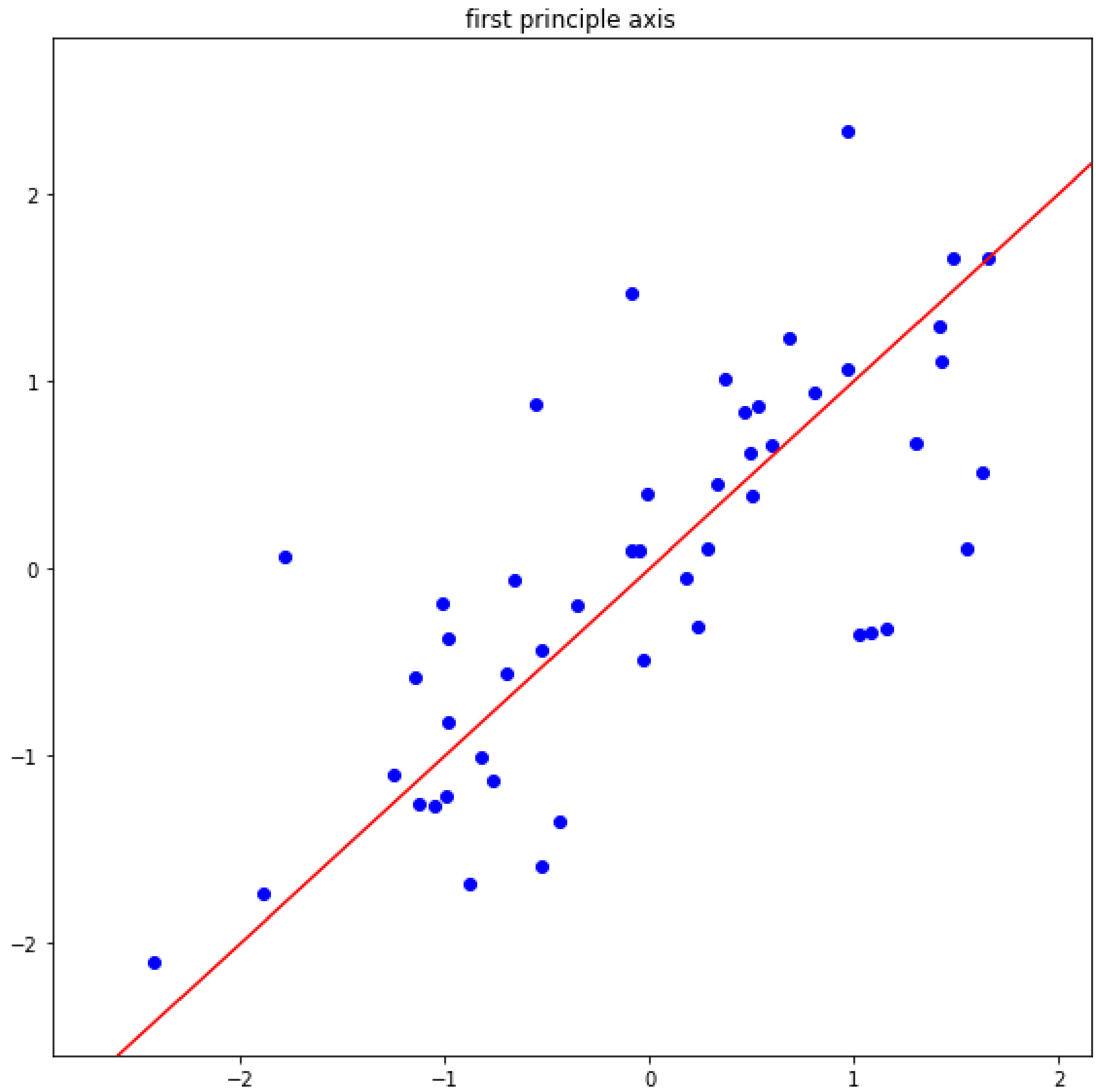
```
**************************************************
## [RESULT 01]
**************************************************
```



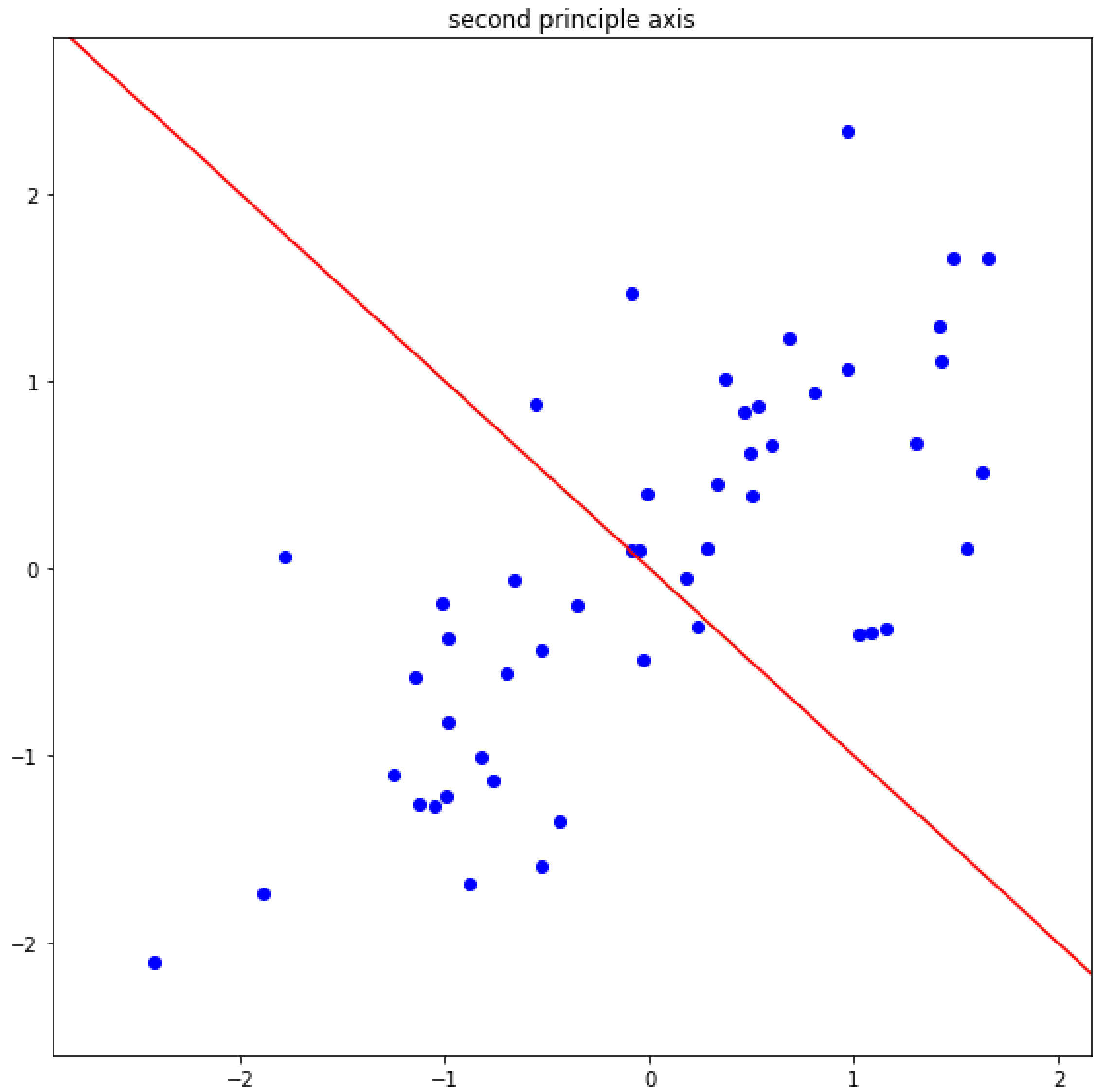

```
**************************************************
## [RESULT 02]
**************************************************
```



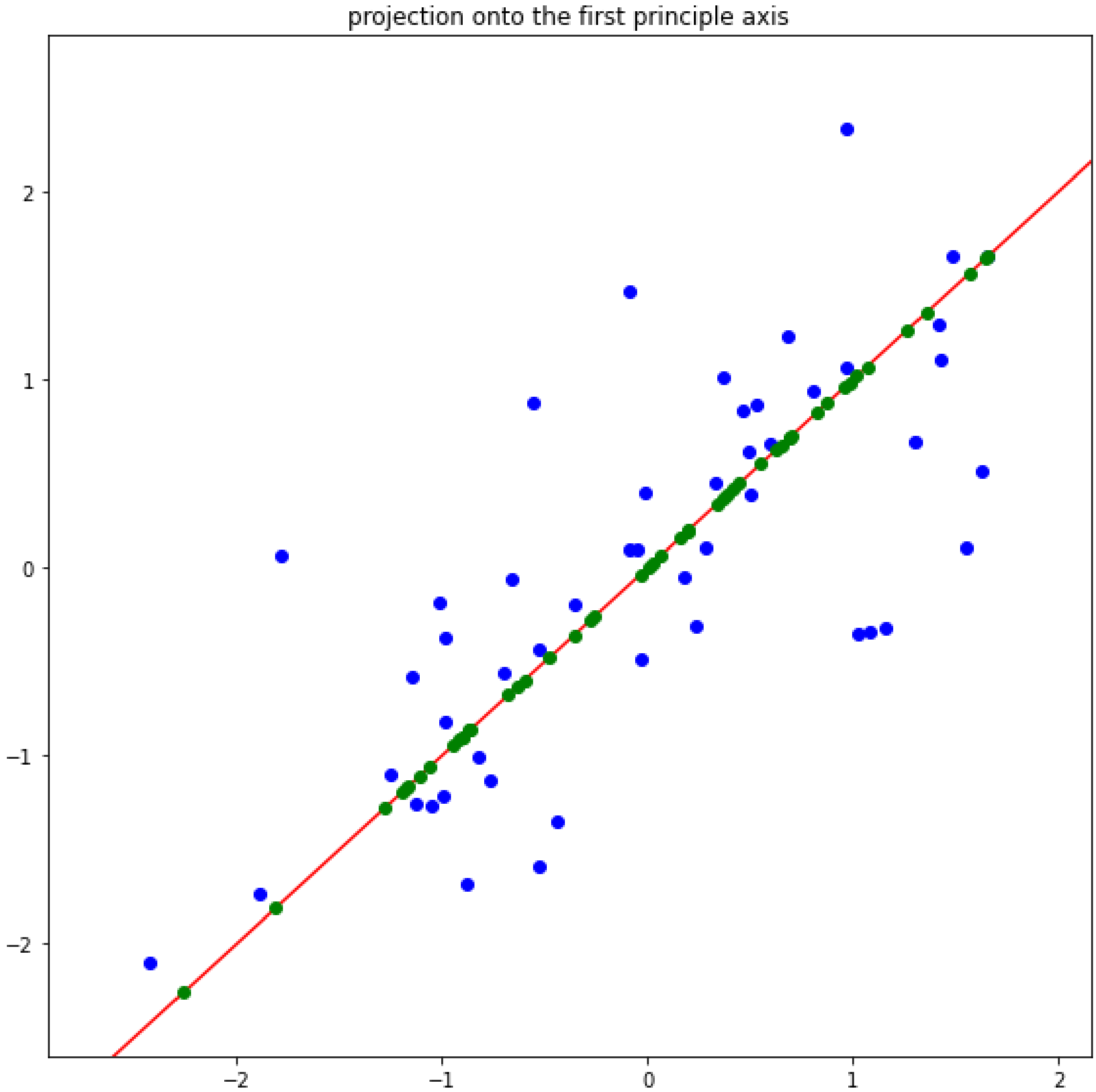

```
**************************************************
## [RESULT 03]
**************************************************
```



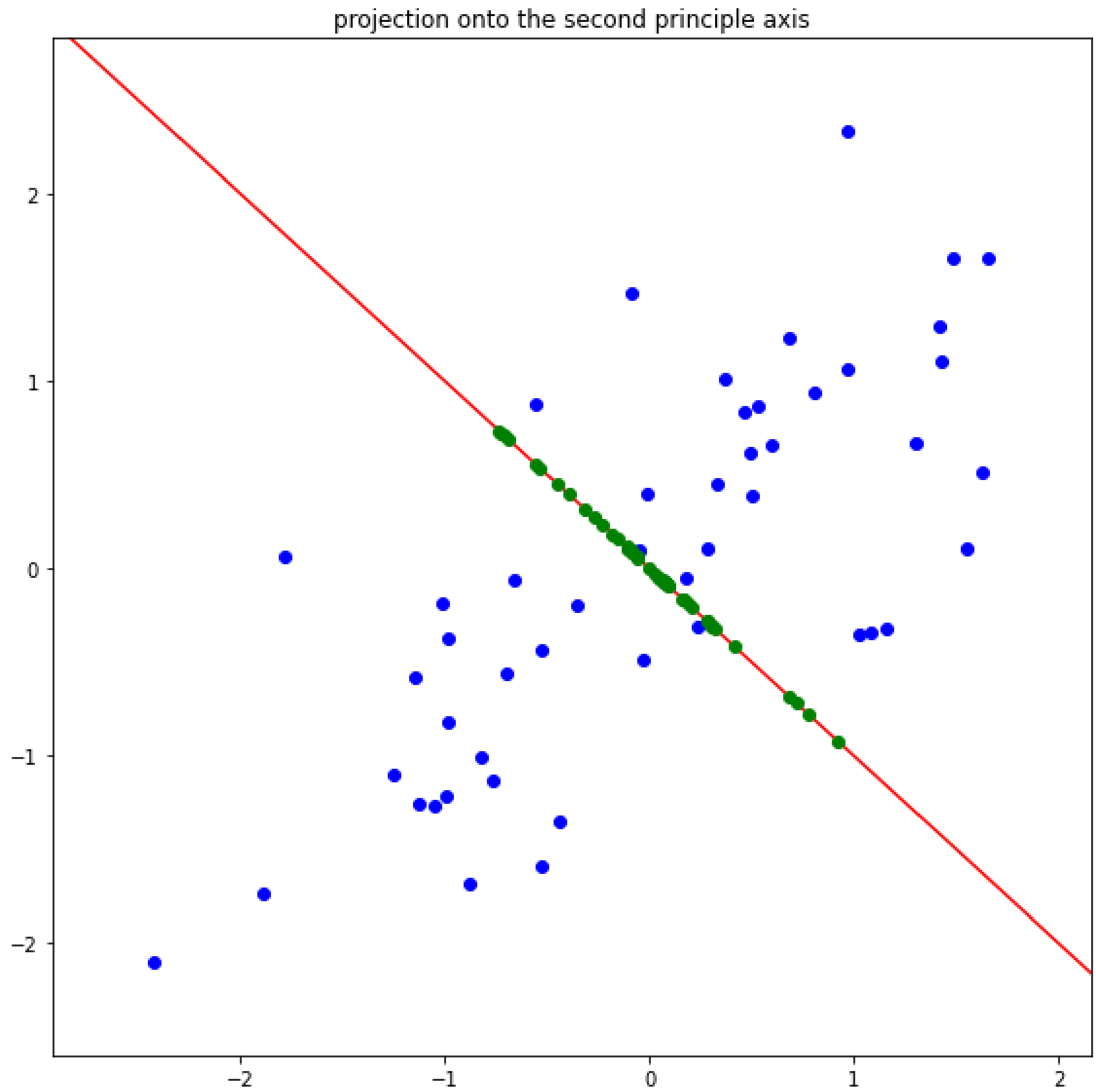

```
**************************************************
## [RESULT 04]
**************************************************
```



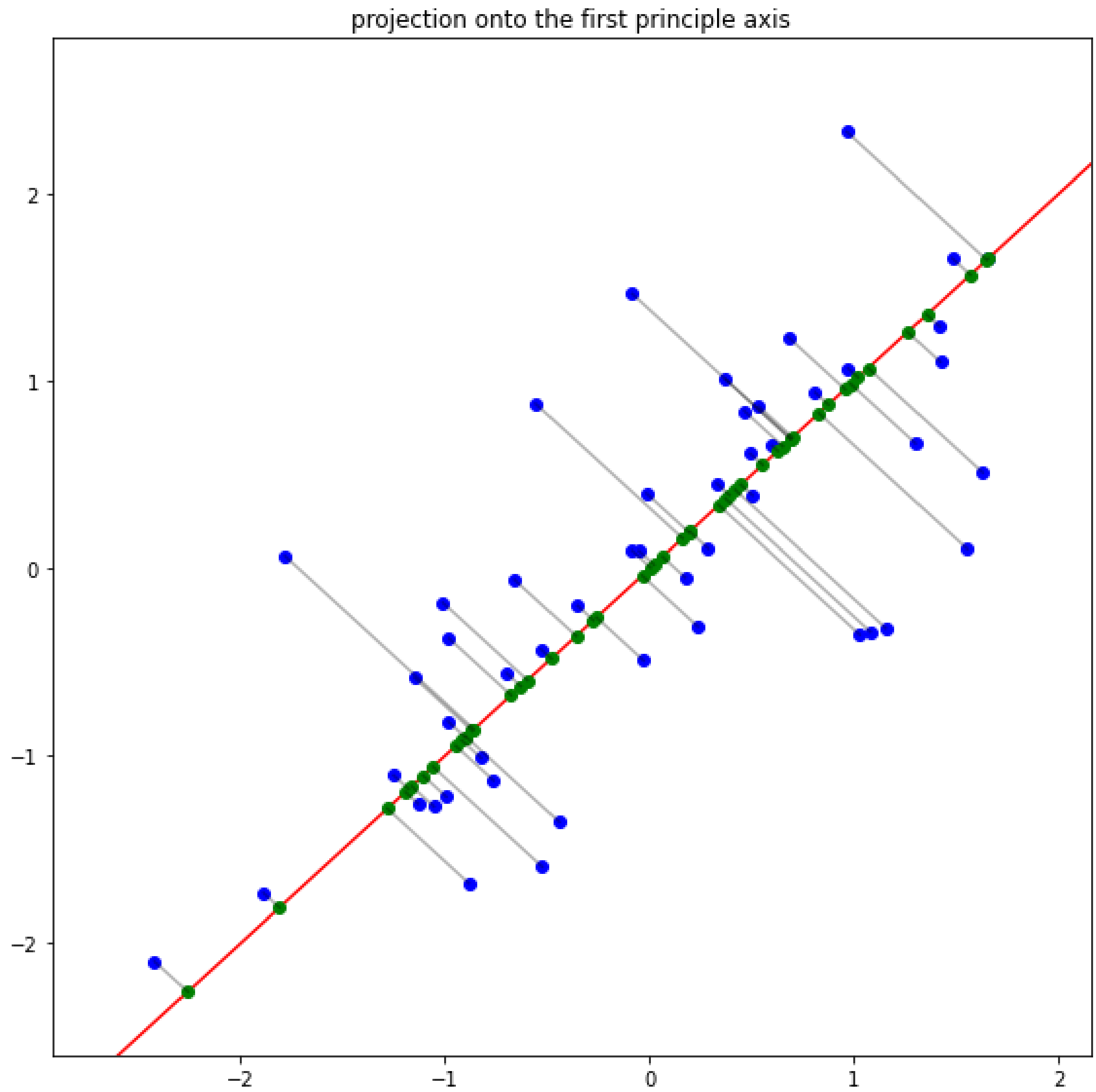

```
**************************************************
## [RESULT 05]
**************************************************
```



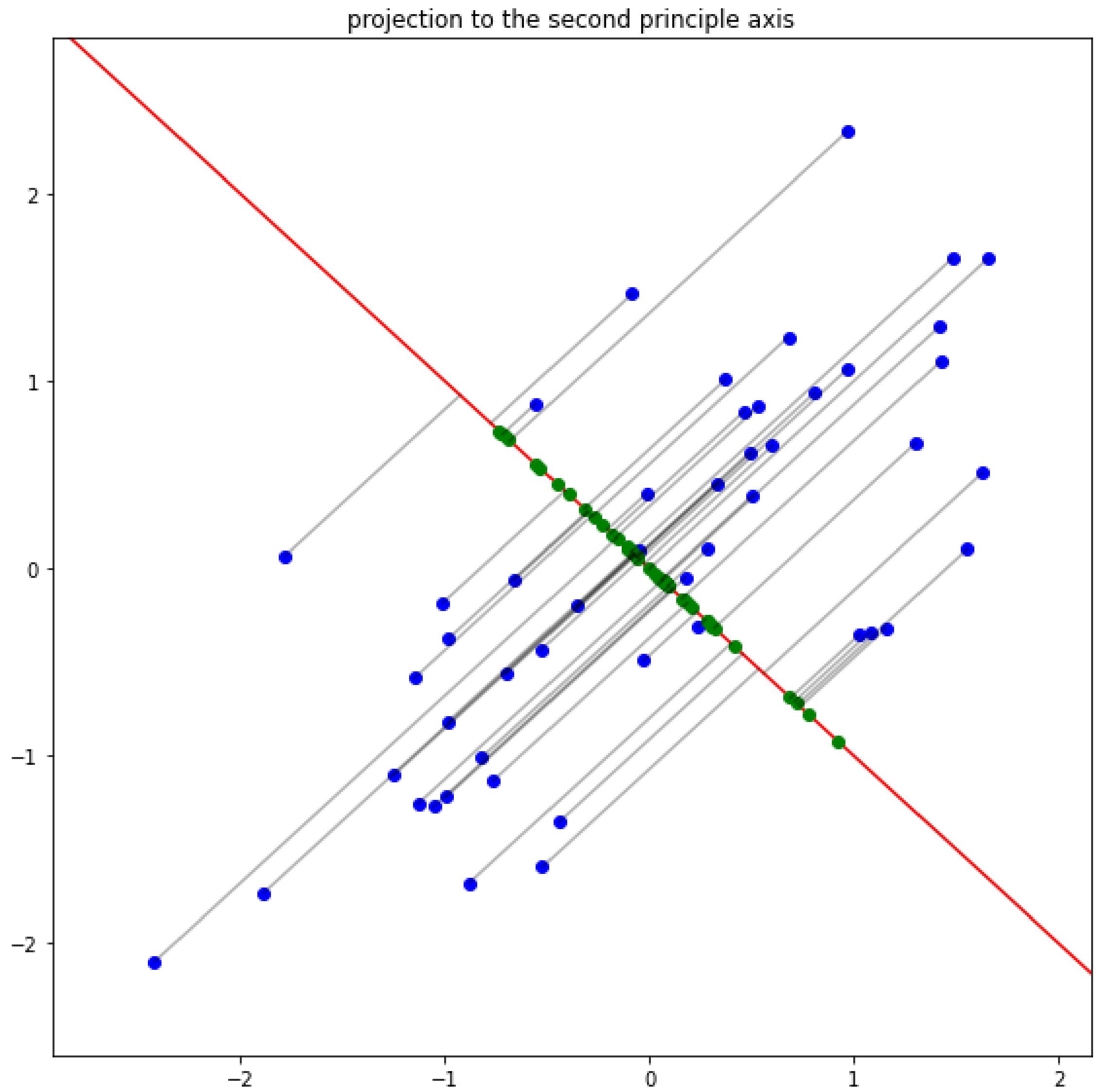

```
**************************************************
## [RESULT 06]
**************************************************
```




```
**************************************************
## [RESULT 07]
**************************************************
```




```
**************************************************
## [RESULT 08]
**************************************************
```



In [ ]: