Submission Problem Set 4

Nicholas R. Gonzalez

Worked on with Tanner Bentley.

Problem 1

(a)

```
pset4_function_noseed <- function(n, b1, b2, b3, usigma) {
  z <- rnorm(n, 5, 2)
  u <- rnorm(n, 0, usigma)
  x <- rbinom(n, 1, .5)
  y <- 50 + b1*x + b2*z + b3*x*z + u

  model <- lm(y ~ x * z)

  return(model)
}
```

(b)

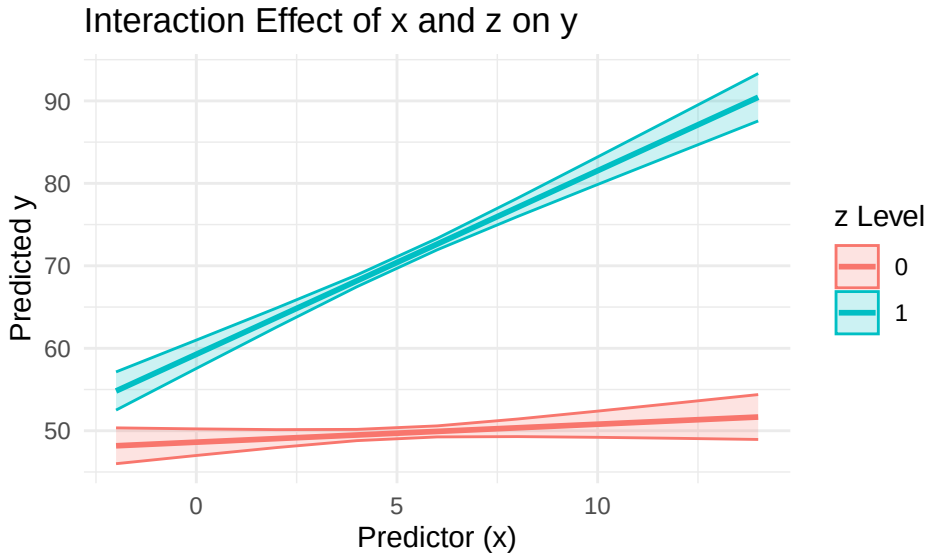
The results of our simulation suggest that there is a significant interaction between x and z, on y. The slope coefficient for x * z, 2.01, tells us that the effect of x on y relates to the value of z. In short, as z increases, the impact of x on y becomes stronger. We also see this within the plot created using *ggeffects*.

```
set.seed(4052025)

pset4_function <- function(n, b1, b2, b3, usigma) {
  z <- rnorm(n, 5, 2)
  u <- rnorm(n, 0, usigma)
  x <- rbinom(n, 1, .5)
  y <- 50 + b1*x + b2*z + b3*x*z + u

  model <- lm(y ~ x * z)
```

```
  return(model)
}

model <- pset4_function(n=500, b1= 10, b2= 0, b3= 2, usigma= 5)

tidy(model)
```

```
# A tibble: 4 x 5
  term        estimate std.error statistic   p.value
  <chr>          <dbl>     <dbl>     <dbl>     <dbl>
1 (Intercept)   48.6       0.821     59.2  5.48e-227
2 x             10.7       1.20       8.88 1.20e- 17
3 z              0.218     0.151      1.45 1.49e-  1
4 x:z            2.01      0.220      9.13 1.74e- 18
```

```
interaction_plot <- ggpredict(model, terms = c("z", "x"))


ggplot(interaction_plot, aes(x = x, y = predicted, color = factor(group))) +
  geom_line(size = 1) +
  geom_ribbon(aes(ymin = conf.low, ymax = conf.high, fill = factor(group)), alpha = 0.2) +
  labs(title = "Interaction Effect of x and z on y",
       x = "Predictor (x)",
       y = "Predicted y",
       color = "z Level",
       fill = "z Level") +
  theme_minimal()
```



(c)

```
set.seed(4052025)

beta_1_out <- c()
beta_2_out <- c()
beta_3_out <- c()
b1_lower <- c()
b1_upper <- c()
b2_lower <- c()
b2_upper <- c()
b3_lower <- c()
b3_upper <- c()


for(i in 1:5000){
  out <- pset4_function(n=500, b1= 10, b2= 0, b3= 2, usigma= 5)
  beta_1_out[i] <- out$coefficients["x"] #save the estimated coefficients
  beta_2_out[i] <- out$coefficients["z"]
  beta_3_out[i] <- out$coefficients["x:z"]

  tidy_out <- tidy(out, conf.int = TRUE)
  b1_lower[i] <- tidy_out$conf.low[2]
  b1_upper[i] <- tidy_out$conf.high[2]
  b2_lower[i] <- tidy_out$conf.low[3]
  b2_upper[i] <- tidy_out$conf.high[3]
  b3_lower[i] <- tidy_out$conf.low[4]
  b3_upper[i] <- tidy_out$conf.high[4]

}

pset4_loop <- as.data.frame(cbind(beta_1_out, beta_2_out, beta_3_out, b1_lower, b1_upper, b2_lo
```

(d)

(i)

The standard errors are almost exactly the same. Since they are the same, this suggests that the estimate is pretty accurate, and is centered around the true standard error.

```
sd_beta_1 <- sd(pset4_loop$beta_1_out)
sd_beta_2 <- sd(pset4_loop$beta_2_out)
sd_beta_3 <- sd(pset4_loop$beta_3_out)

sd_df <- data.frame(
```

```
  Coefficient = c("Beta_1", "Beta_2", "Beta_3"),
  Standard_Deviation = c(sd_beta_1, sd_beta_2, sd_beta_3)
)

print(sd_df)
```

```
  Coefficient Standard_Deviation
1      Beta_1          1.2158039
2      Beta_2          0.1593848
3      Beta_3          0.2265468
```

(ii)

I am not fully sure what to expect, or what to have expected. A proportion of 95% is higher than what I assume it is suppose to be, which is 5%, since for a 95% CI, only 5% of the data should cross zero.

```
cross_zero_b2 <- mean(pset4_loop$b2_lower <= 0 & pset4_loop$b2_upper >= 0)

print(cross_zero_b2)
```

```
[1] 0.9516
```

(iii)

Our research design has a false positive, or a Type I error. This suggests that our simulation has insufficient explanatory power. In sum, this simulation is likely not *sensitive* enough to detect *real* effects generated by the simulated data, which is a byproduct of the DGS. We also know this because we set β_2 to 2.

```
interaction_ci <- mean(pset4_loop$b3_lower <= 0 & pset4_loop$b3_upper >= 0)

print(interaction_ci)
```

```
[1] 0
```

(iv)

For β_3 the proportion of p-values is less than 0.005 is .048. For β_2, it is 1. We can find this out because confidence intervals are closely related to p-values. This is what allows us to subtract 1 from to get the proportion data that is within the designated p-value.

```r
b2_pvalue <- 1 - mean(pset4_loop$b2_lower <= 0 & pset4_loop$b2_upper >= 0)
b3_pvalue <- 1 - mean(pset4_loop$b3_lower <= 0 & pset4_loop$b3_upper >= 0)

print(b3_pvalue)
```

```
[1] 1
```

```r
print(b2_pvalue)
```

```
[1] 0.0484
```

(e)

```r
set.seed(4052025)

beta_1_out_1200 <- c()
beta_2_out_1200 <- c()
beta_3_out_1200 <- c()
b1_lower_1200 <- c()
b1_upper_1200 <- c()
b2_lower_1200 <- c()
b2_upper_1200 <- c()
b3_lower_1200 <- c()
b3_upper_1200 <- c()


for(i in 1:5000){
  increase <- pset4_function(n=1200, b1= 10, b2= 0, b3= 2, usigma= 5)
  beta_1_out[i] <- increase$coefficients["x"]
  beta_2_out[i] <- increase$coefficients["z"]
  beta_3_out[i] <- increase$coefficients["x:z"]

  tidy_increase <- tidy(increase, conf.int = TRUE)
  b1_lower[i] <- tidy_increase$conf.low[2]
  b1_upper[i] <- tidy_increase$conf.high[2]
  b2_lower[i] <- tidy_increase$conf.low[3]
  b2_upper[i] <- tidy_increase$conf.high[3]
  b3_lower[i] <- tidy_increase$conf.low[4]
  b3_upper[i] <- tidy_increase$conf.high[4]

}

pset4_increase <- as.data.frame(cbind(beta_1_out, beta_2_out, beta_3_out, b1_lower, b1_upper, b
```

(f)

(i)

They're lower, which makes sense because as we collect, or generate more data, the data becomes more consistent and has more explaining power.

```
b1_twelve_sd <- sd(pset4_increase$beta_1_out)
b2_twelve_sd <- sd(pset4_increase$beta_2_out)
b3_twelve_sd <- sd(pset4_increase$beta_3_out)

sd_df_1200 <- data.frame(
  Coefficient = c("Beta_1", "Beta_2", "Beta_3"),
  Standard_Deviation = c(b1_twelve_sd, b2_twelve_sd, b3_twelve_sd)
)

print(sd_df_1200)
```

```
  Coefficient Standard_Deviation
1      Beta_1          0.7886520
2      Beta_2          0.1032036
3      Beta_3          0.1463645
```

(ii)

They're the same. This is likely because the variance is low, and the sample size was already large enough. It also because of underlying theories, or realities of the data generating process. Which again speaks to low variance.

```
b2_twelve_ci <- mean(pset4_increase$b2_lower <= 0 & pset4_increase$b2_upper >= 0)
b3_twelve_ci <- mean(pset4_increase$b3_lower <= 0 & pset4_increase$b3_upper >= 0)

ci_df_1200 <- data.frame(
  Coefficient = c("Beta_2", "Beta_3"),
  Standard_Deviation = c(b2_twelve_ci, b3_twelve_ci)
)

print(ci_df_1200)
```

```
  Coefficient Standard_Deviation
1      Beta_2             0.9506
2      Beta_3             0.0000
```

(g)

```
set.seed(4052025)

beta_1_out <- c()
beta_2_out <- c()
beta_3_out <- c()
b1_lower <- c()
b1_upper <- c()
b2_lower <- c()
b2_upper <- c()
b3_lower <- c()
b3_upper <- c()


for(i in 1:5000){
  error_increase <- pset4_function(n=1200, b1= 10, b2= 0, b3= 2, usigma= 20)
  beta_1_out[i] <- error_increase$coefficients["x"]
  beta_2_out[i] <- error_increase$coefficients["z"]
  beta_3_out[i] <- error_increase$coefficients["x:z"]

  tidy_error_increase <- tidy(error_increase, conf.int = TRUE)
  b1_lower[i] <- tidy_error_increase$conf.low[2]
  b1_upper[i] <- tidy_error_increase$conf.high[2]
  b2_lower[i] <- tidy_error_increase$conf.low[3]
  b2_upper[i] <- tidy_error_increase$conf.high[3]
  b3_lower[i] <- tidy_error_increase$conf.low[4]
  b3_upper[i] <- tidy_error_increase$conf.high[4]

}

pset4_error_increase <- as.data.frame(cbind(beta_1_out, beta_2_out, beta_3_out, b1_lower, b1_u
```

(i)

These numbers/results are larger than those from our previous simulation, which is not surprising. This is due to an increase in the numerator in the standard error formula. As a result, the coefficients should indeed be larger, and they are.

```
b1_higherror_sd <- sd(pset4_error_increase$beta_1_out)
b2_higherror_sd <- sd(pset4_error_increase$beta_2_out)
b3_higherror_sd <- sd(pset4_error_increase$beta_3_out)


print(b1_higherror_sd)
```

```
[1] 3.154608
```

```
print(b2_higherror_sd)
```

```
[1] 0.4128146
```

```
print(b3_higherror_sd)
```

```
[1] 0.585458
```

(ii)

No, there is little difference. This comes even despite changing usigma, or the error term. For β_2 it likely stays the same since there is already a lot of uncertainty in the result. β_3 represents a small increase however, this is likely because it is more significant. Widen out the smaller CI's to begin with, is likely to contain some change. As opposed to β_2 which already had wide confidence intervals.

```
b2_ci_error <- mean(pset4_error_increase$b2_lower <= 0 & pset4_loop$b2_upper >= 0)
b3_ci_error <- mean(pset4_error_increase$b3_lower <= 0 & pset4_loop$b3_upper >= 0)

print(b2_ci_error)
```

```
[1] 0.9524
```

```
print(b3_ci_error)
```

```
[1] 0.0732
```

(iii)

The variance of our residuals mirrors the variance of the population error. Therefore, as the error variance changes, so in our case we upped it to 20, the residual variance would also change proportionally.