

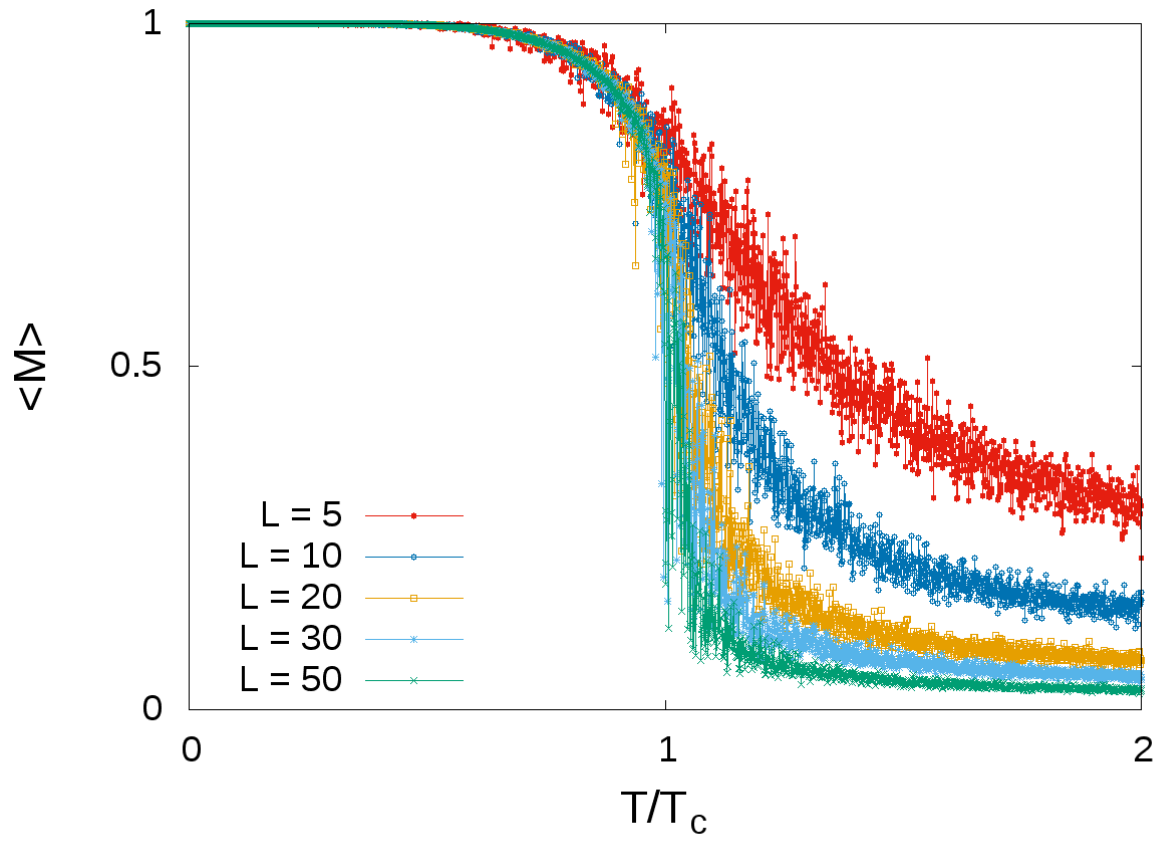
Answers to Homework 6

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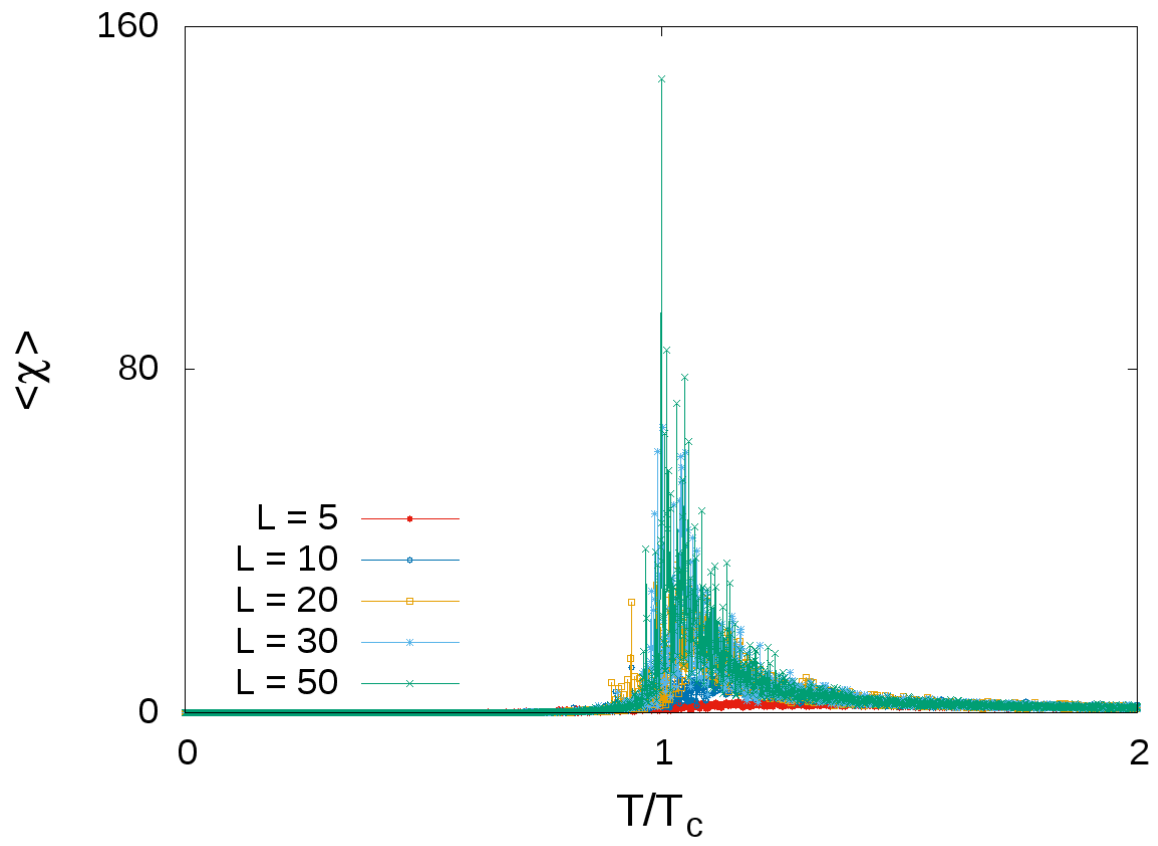
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1 Question 1

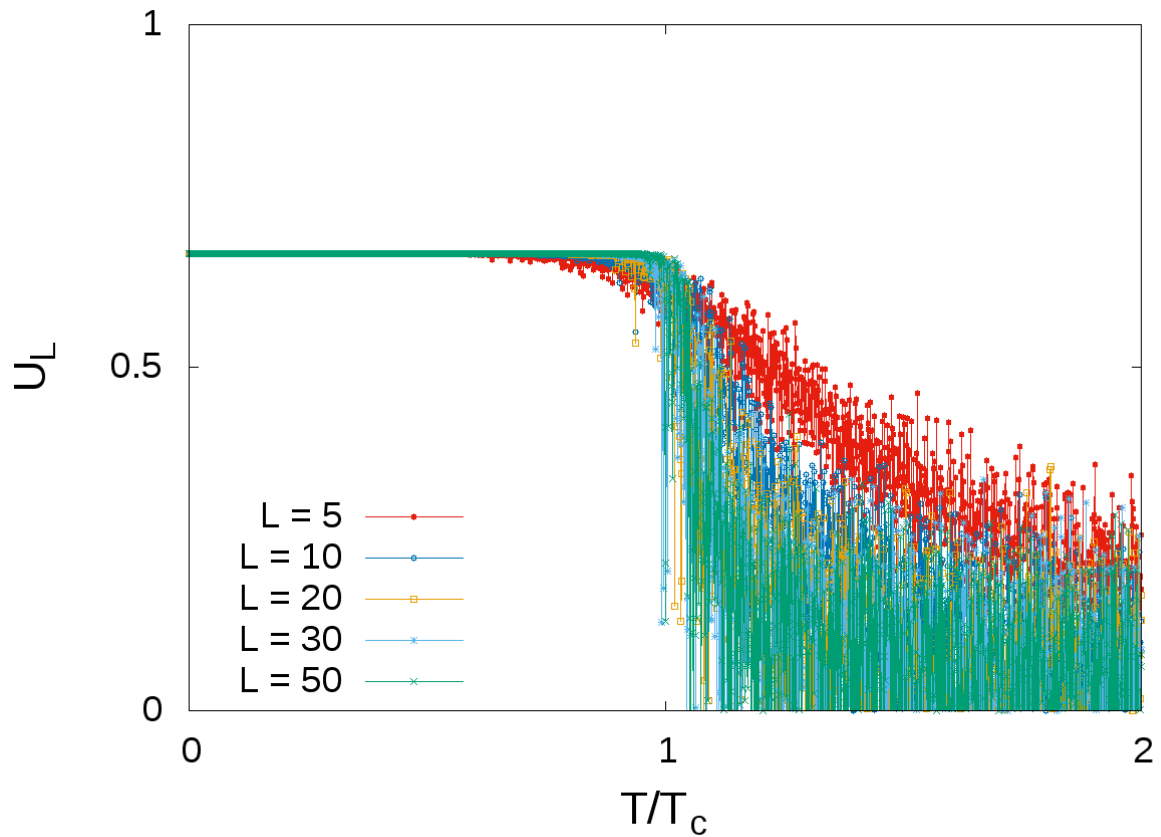
1.1 Ensemble average of the magnetization per lattice site



1.2 Ensemble average of the susceptibility per lattice site



2 Question 2



Remarks: The fourth order cumulant is given by

$$U_L = 1 - \frac{\langle M_L^4 \rangle}{3\langle M_L^2 \rangle^2} \quad (1)$$

As expected, U_L 's for various lattice sizes intersect at the point $T/T_c = 1$, i.e., at the critical temperature. Using the exact solution $J/kT_c = (1/2)\ln(1 + \sqrt{2})$ and setting $T/T_c = x$, we can write

$$\begin{aligned} \frac{J}{kT} &= \frac{J}{kT_c \cdot T/T_c} \\ &= \frac{J}{kT_c x} \\ &= \frac{1}{2x} \ln(1 + \sqrt{2}) \end{aligned} \quad (2)$$

Note that J/kT has been replaced with $(1/2x)\ln(1 + \sqrt{2})$ in the code. If we set $J = K_B = 1$, then the exact solution tells us that the critical temperature is $T = 2.27K$. If we didn't know the exact solution, we could plot U_L 's for various L against T and identify the value of the critical temperature as the point of intersection of various U_L 's.

Note: I discussed the solution with Rasika, Kemal, Lukasz and Phillip. The greatest help I received was from Rasika - I was able to identify the mistake in my code only after I got a chance to look at his code.