

# 중력파 데이터 분석

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2022 수치상대론 및 중력파 겨울학교 / 계산천체물리 경진대회

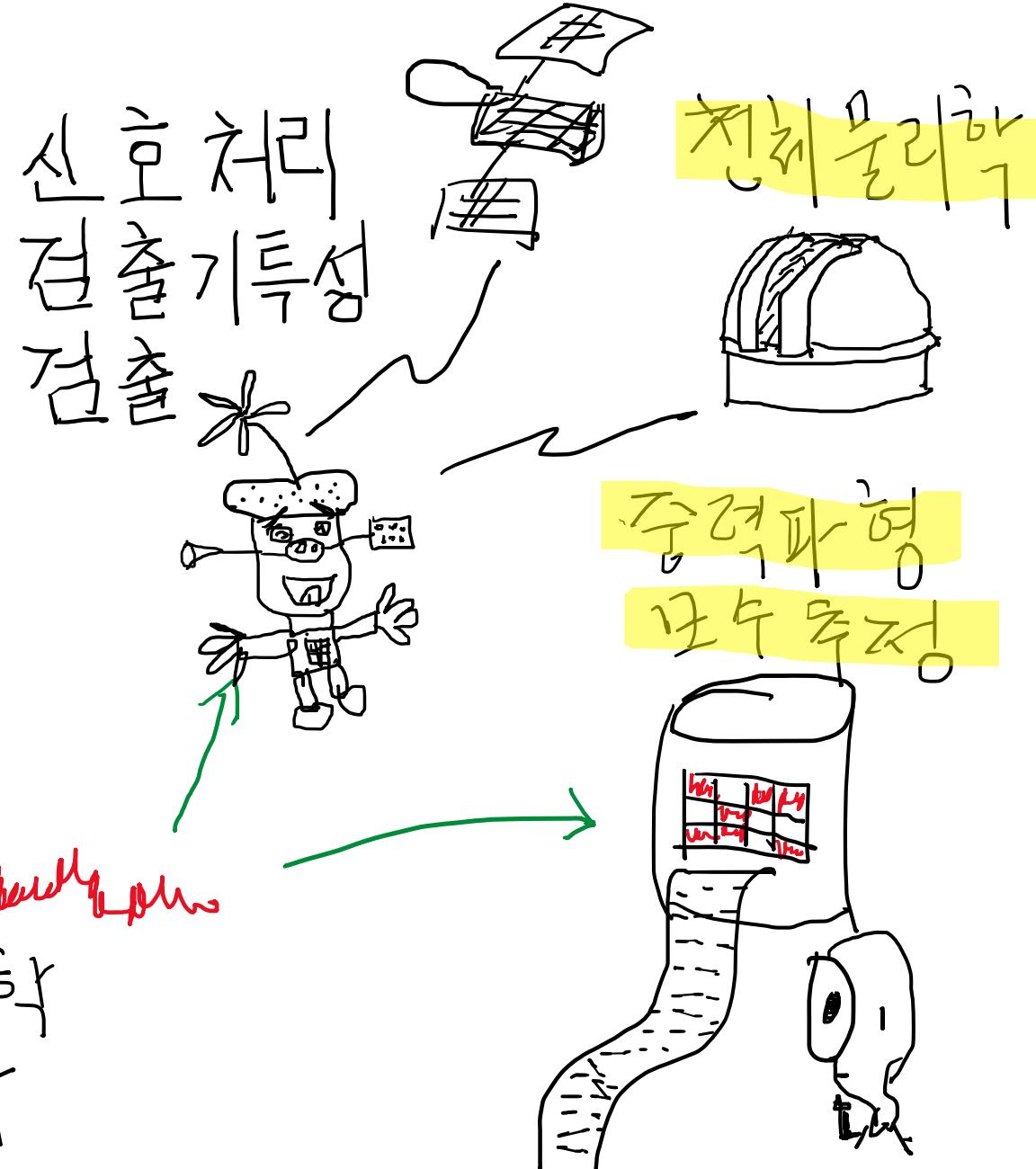
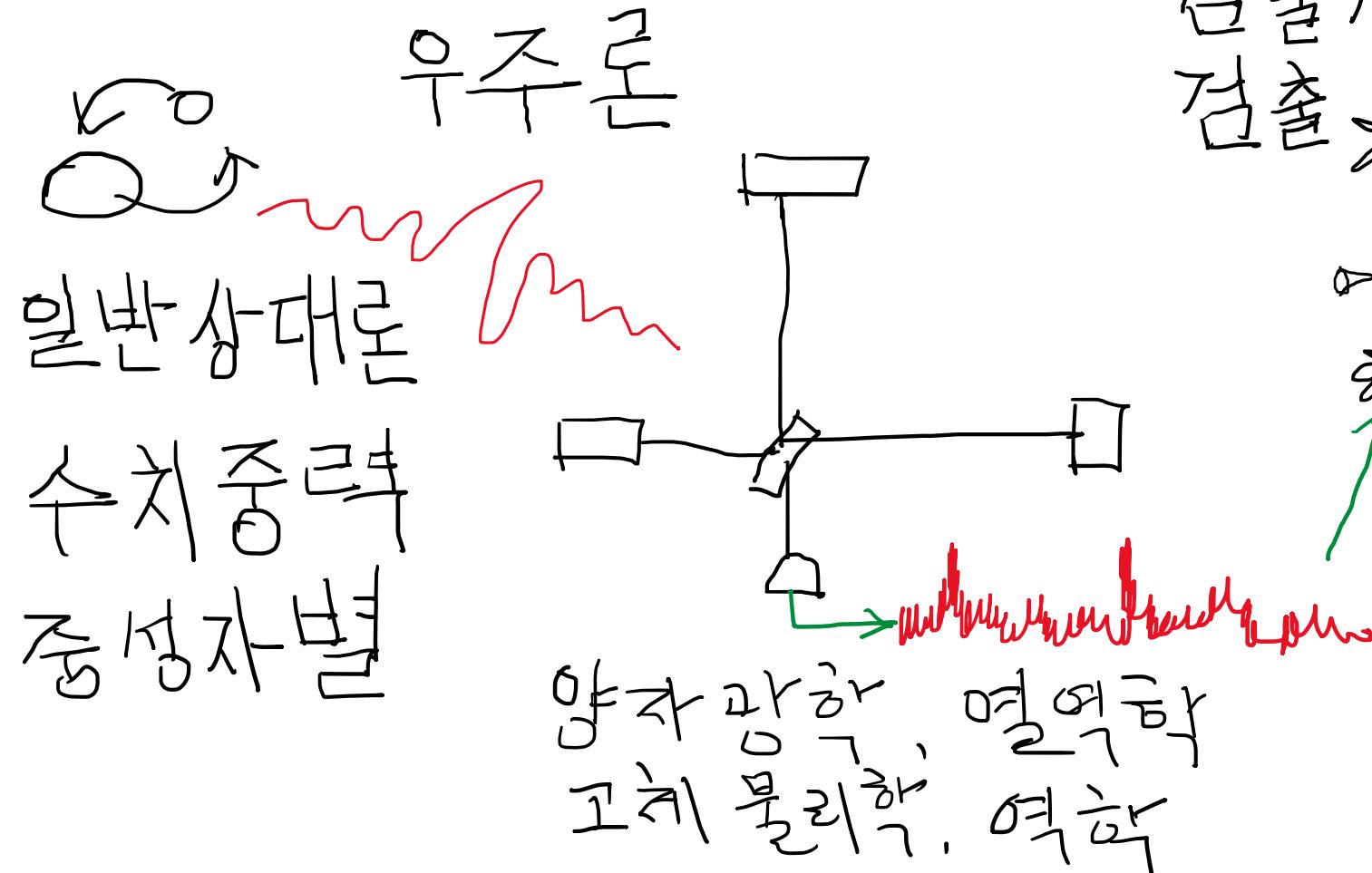
2022년 1월 18일

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- 후처리 도구
- 맷음 말

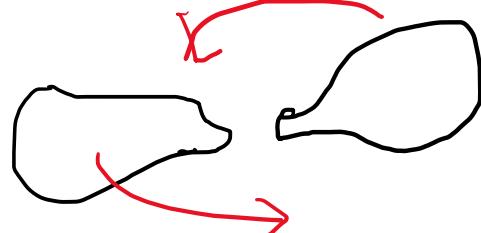
# 큰 그림

# 큰 그림(중력파 프로젝트)



# 중력파 발생

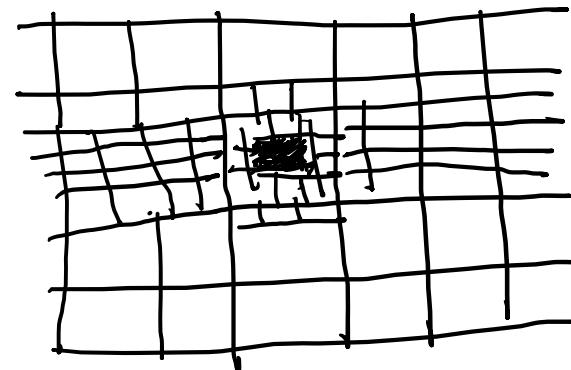
- 아인스타인 방정식
- 선형근사
- 포스트뉴턴리안 근사
- 수치중력
- 중성자별 상태 방정식



$$1916 \quad G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\frac{GM}{Rc^2} \sim \frac{1}{c^2} \ll 1$$



계산천체물리 강의 2  
(김진호)

# 우주론

우주론 수치모의 실험  
(김용휘)

- 공변거리(comoving distance)
- 고유거리(proper distance)
- 광도거리(luminosity distance)

FRW 경량

$$c^2 dt^2 = c^2 dt^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r_d^2 d\Omega^2 \right)$$

$$\chi = \int_{t_{src}}^{t_{obs}}$$

$$\frac{cdt'}{a(t')} = c \int_0^z \frac{dz'}{H(z')}$$

$$D = a(t_{obs}) \chi$$

$$d_L = (1+z) D$$

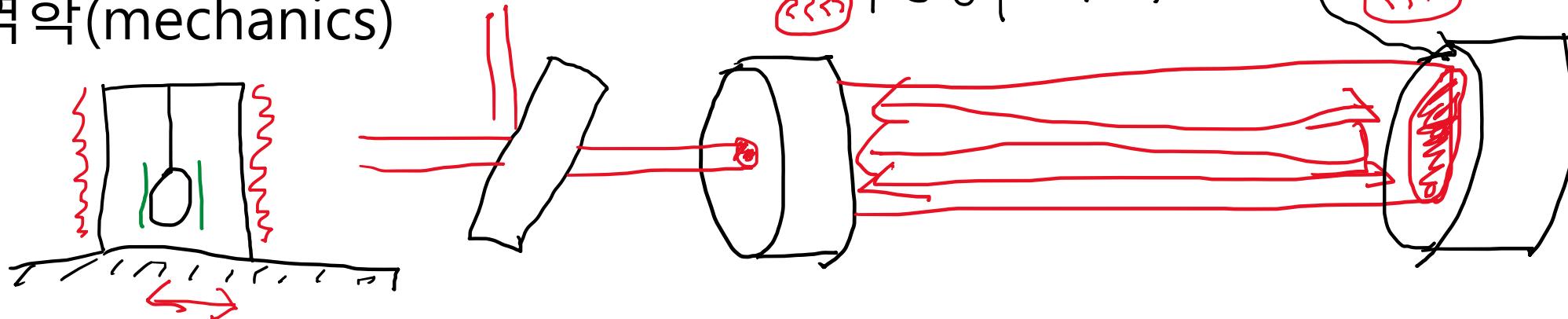
$$\tilde{h}(f_{obs}) = (1+z) \tilde{h}(f_{src})$$

$$\approx \frac{1}{d_L} \left( \frac{(1+z) M}{M_{det}} \right)^{5/6} f^{-\frac{7}{6}} e^{i4(f)} \rightarrow f_{obs}$$

$$H^2 = \frac{8\pi G}{3} \rho$$

# 검출기

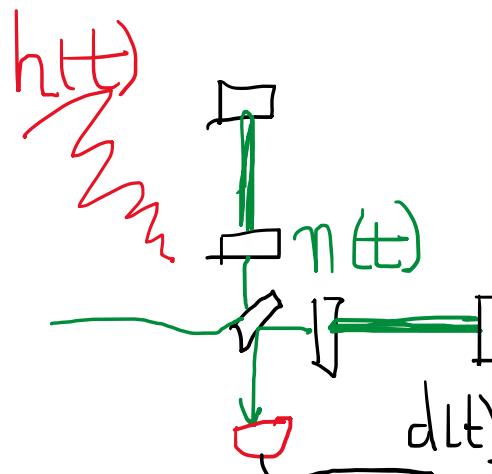
- 양자광학(quantum optics)
- 광공진기(optical cavity)
- 스퀴즈(squeezed light)
- 고체물리학(solid state physics)
- 열역학(thermal physics)
- 역학(mechanics)



# 신호처리

기계학습을 이용한 중력파 데이터 분석  
(김경민)

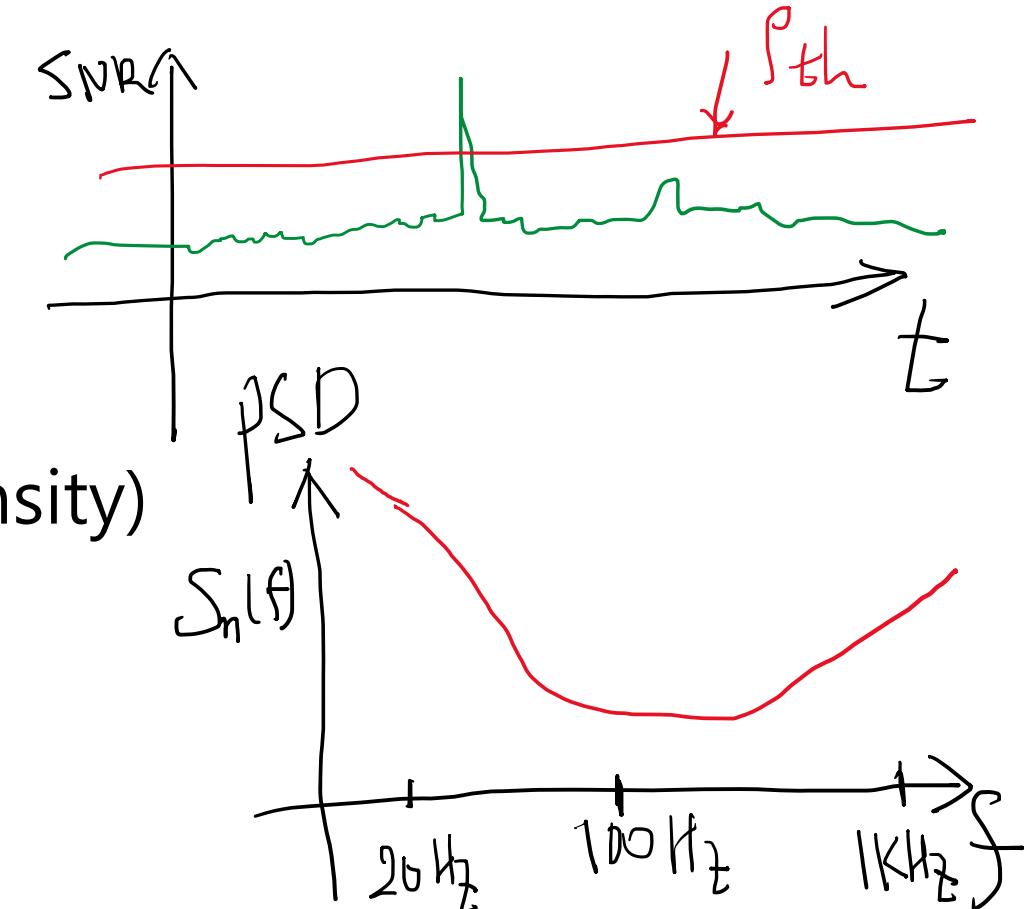
- 정합필터(Matched Filter)
- 전력스펙트럼밀도(Power Spectral Density)
- 검출(Search)
- 신호대잡음비(Signal to Noise Ratio)



$$\Lambda(h, s) = e^{(s, h) - \frac{1}{2} \|h\|^2}$$

$$(s, h) = 4 \operatorname{Re} \int_0^{\infty} \frac{\hat{s}(f) \hat{h}^*(f)}{S_n(f)} df$$

$$\sigma^2 = (g, g)$$



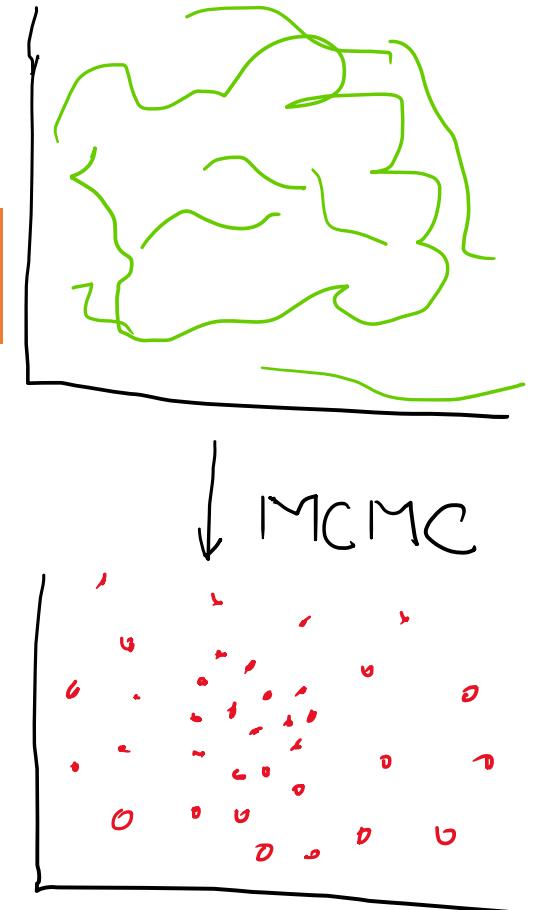
$$\rho^2 = (h, h), \quad \rho = \frac{\rho^2}{\sigma^2}, \quad \text{template}$$

$$h(t) = Ag(t), \quad x = (s, g)$$

# 모수추정(Parameter Estimation)

- 중력파형(Waveforms)
- 통계처리(Statistical Analysis)
- 베이지언 추정(Bayesian Inference)
- 몬테카를로(Markov Chain Monte Carlo)

중력파 데이터 분석 계산천체물리 강의 1  
(이형원) (김영민)



?

$L$

$h(t) \sim \int \frac{d\vec{r}}{r} d\vec{P} \sim \frac{\vec{I}}{r}$

$\downarrow \text{FFT}$

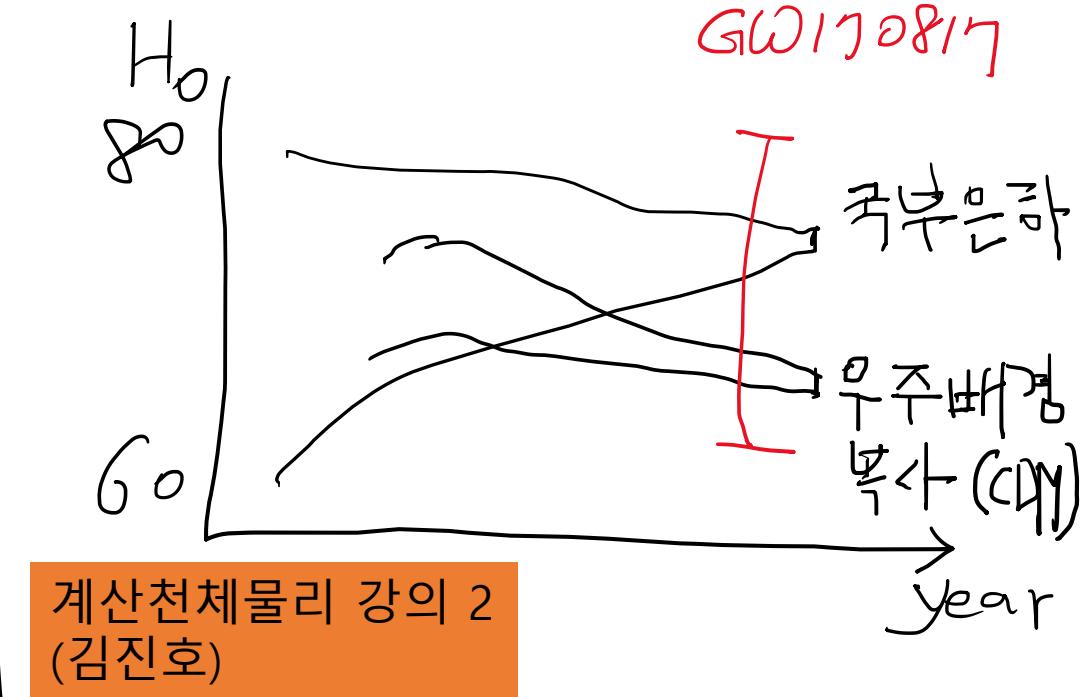
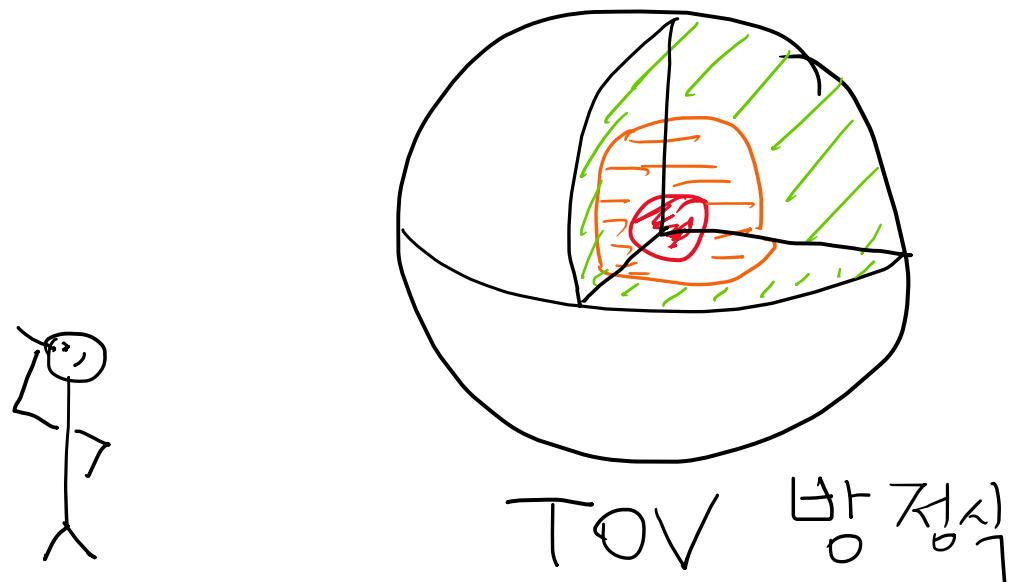
$\tilde{h}(f) \sim \frac{M_c^{5/6}}{r} f^{-\frac{7}{6}} e^{i\phi(f)}$

$$d(t) = n(t) + h(t)$$
$$\mathcal{L} \sim e^{-\frac{1}{2} (d - h, d - h)}$$
$$p(\vec{\theta} | \vec{d}) \sim \mathcal{L}(\vec{d} | \vec{\theta}) p(\vec{\theta})$$

# 천체물리학(Astrophysics)

중성자별의 내부 구조 및 상태 방정식  
(이창환)  
성단 역학 및 블랙홀 쌍성의 형성  
(배영복)

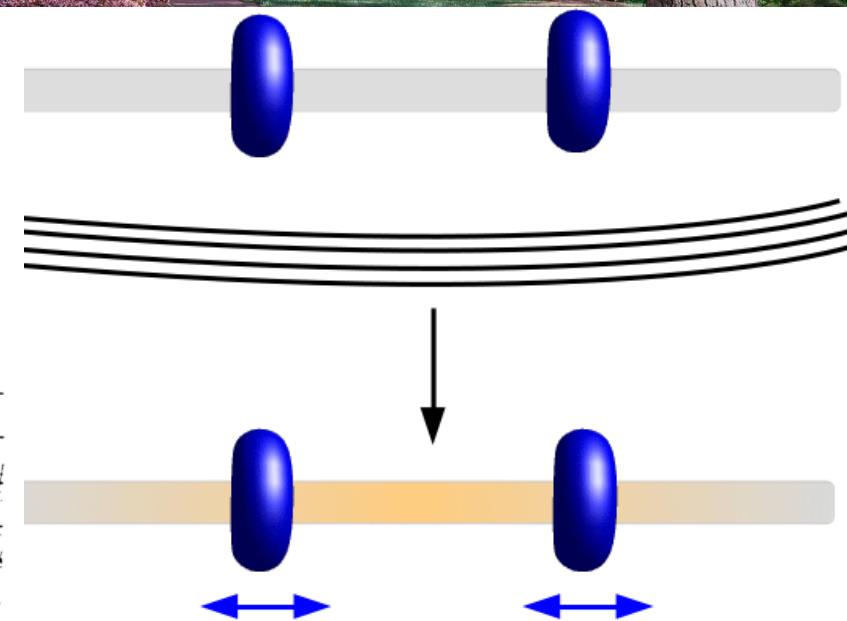
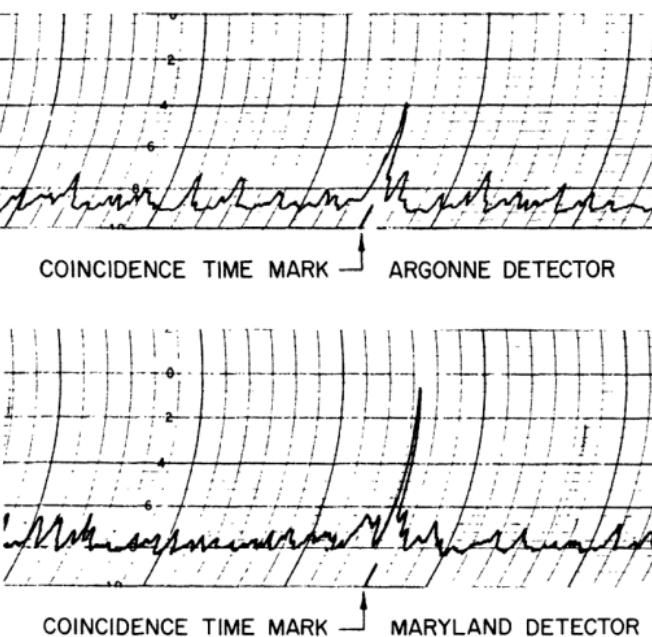
- 블랙홀 병합율(Blackhole Merger rate)
- 중성자별 상태 방정식(Equation of State of NS)
- 허블 상수(Hubble Constant)



# 지금까지의 성과

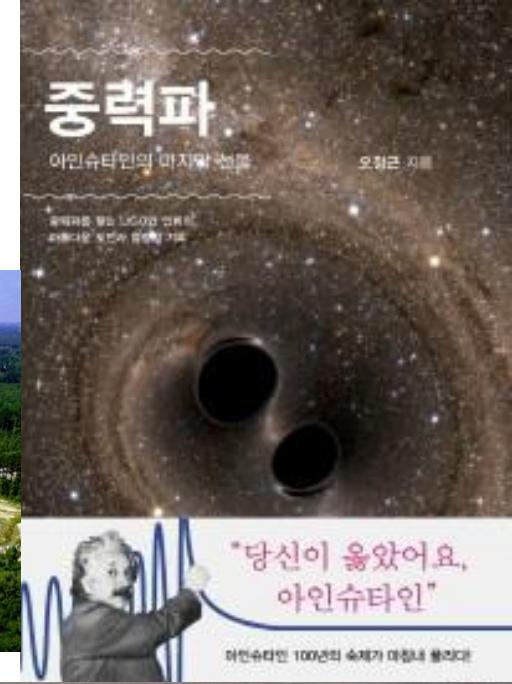
# 초기 검출기(Weber Bar)

- Chapel Hill Conference, 1957
- Feynmann's Sticky bids
- Weber bar(1966)
- Braginsky, Thorne(1972)



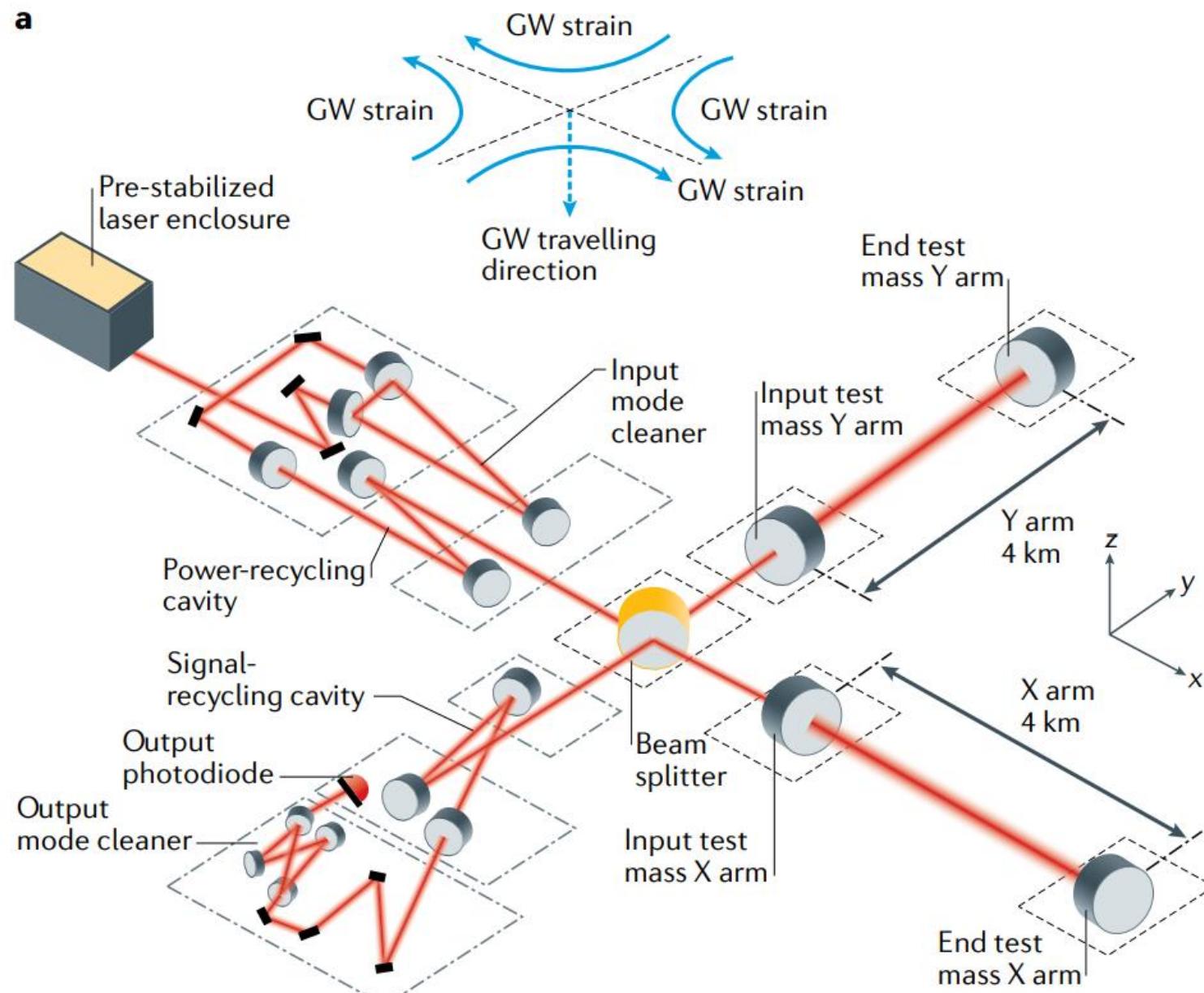
# 간섭계(Laser Interferometer)

- Gertsenshtein and Pustovoit, 1962
- Weber and Weiss, 1967
- Weiss, noise analysis, 1972
- Drever and Whitcomb, Caltech, 1980
- Drever, Weiss and Thorne, LIGO, 1984
- Barish, LIGO Director, 1997
- Virgo joined, 2007
- KAGRA joined, 2019



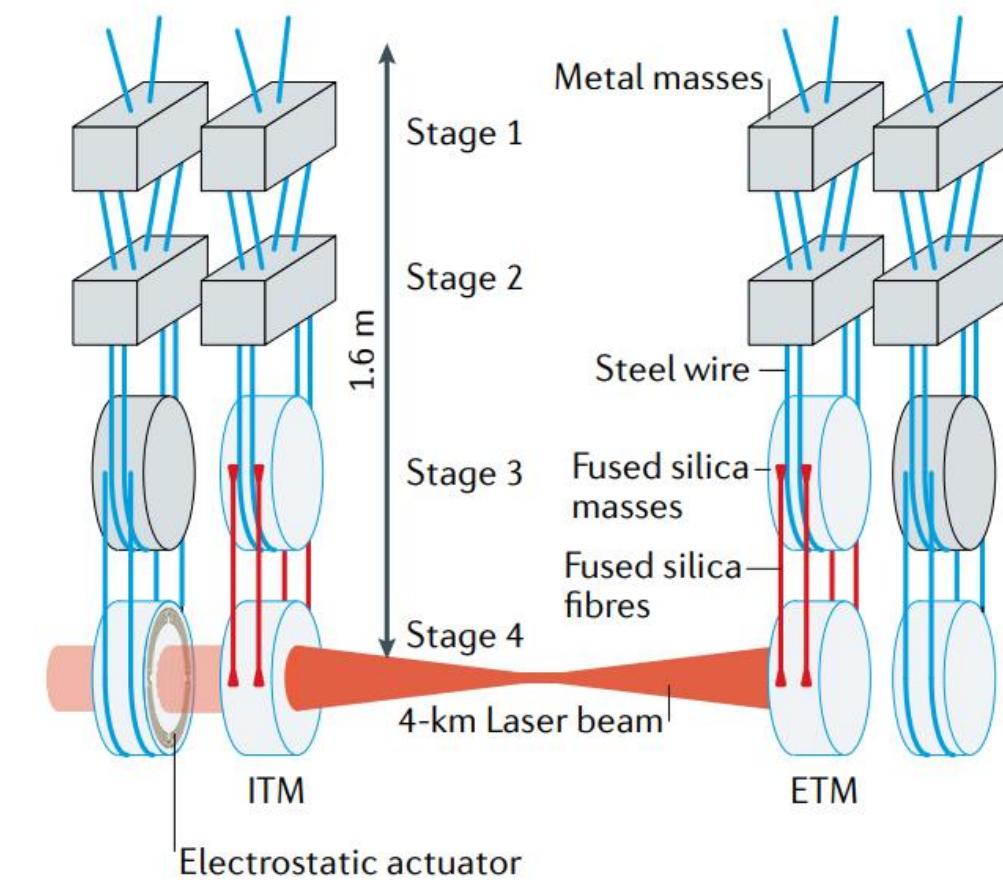
# LIGO 검출기

a



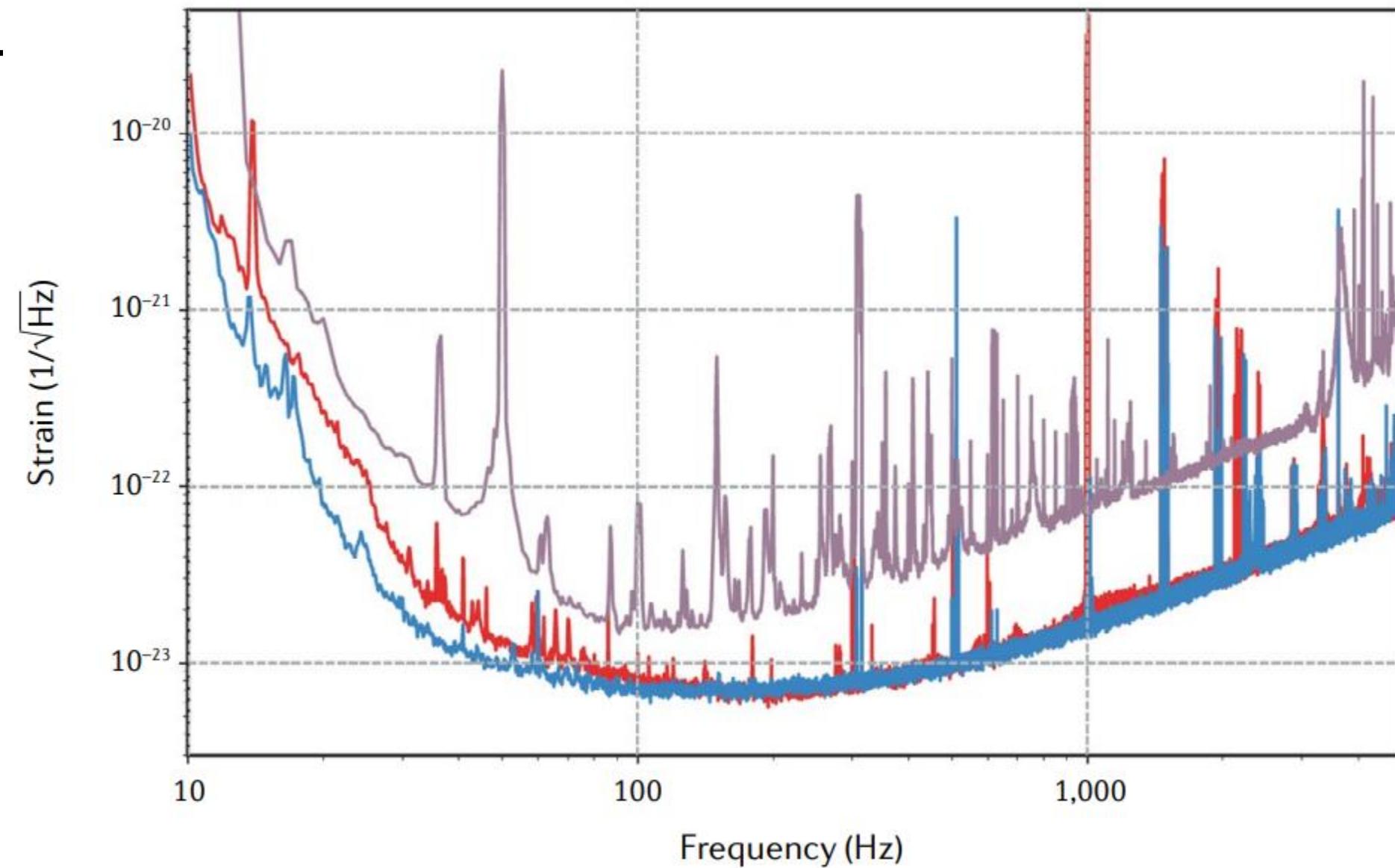
b

[Nat. Rev. Physics, May 2021](#)



# LIGO 감도

- 지진잡음
- 열 잡음
- 중력경사 잡음
- 양자 잡음
- 기술 잡음
  - 레이저 주파수
  - 레이저 세기
  - 레이저 산란
  - 감지기, 작동기
  - 전자기 잡음
  - 입자(우주선)



# 공공 알림(Public Alerts)

← → ⌂ https://gcn.gsfc.nasa.gov/gcn3\_archive.html

## GCN Circulars Archive (in serial number order)

This page changes after each Circular submission, so hit the <reload> button NOW.

The processing of Circulars to this page was stopped (from ~01:30 to ~18:00 UT 01 Dec 2020).

The Circulars page has been fixed as of ~18:00 UT 01 Dec 2020.

(The Circulars WERE ALWAYS BEING DISTRIBUTED -- it is only this archive web page (and sub-pages) that had problems.)

1. [The Latest Circulars](#)
2. [Older Circulars](#)
3. [Tarfile of all Circulars](#)
4. [Circulars grouped by each Event](#)
5. [All Circulars on the GRB source type](#)
6. [All Circulars on the GW source type](#)
7. [All Circulars on the SGR source type](#)
8. [All Circulars on the misc source type \(ie not in any of the above 3 pages\)](#).



# VIRGO Public Alerts User Guide

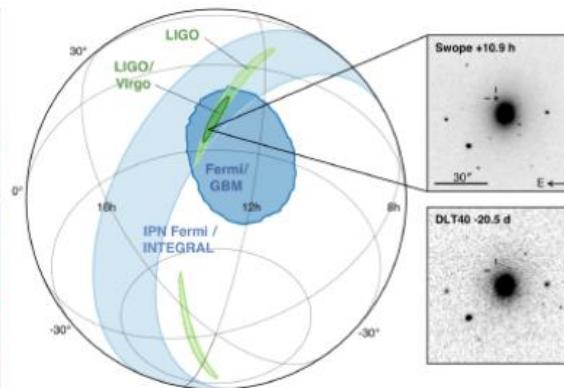
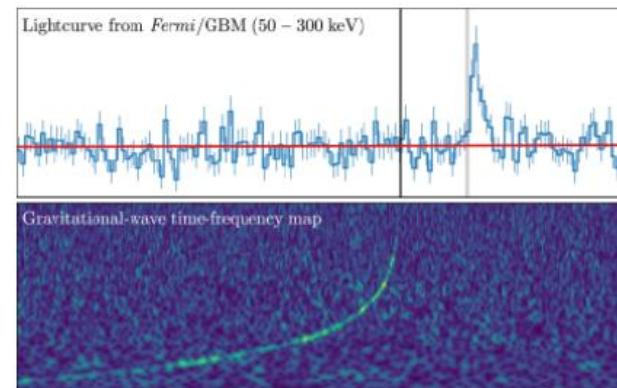
Primer on public alerts for astronomers from the LIGO and Virgo gravitational-wave observatories.

## Navigation

- [Getting Started Checklist](#)
- [Observing Capabilities](#)
- [Data Analysis](#)
- [Alert Contents](#)
- [Sample Code](#)
- [Additional Resources](#)
- [Early-Warning Alerts](#)

[Getting Started Checklist →](#)

# LIGO/Virgo Public Alerts User Guide



Welcome to the LIGO/Virgo Public Alerts User Guide! This document is intended for both professional astronomers and science enthusiasts who are interested in receiving alerts and real-time data products related to gravitational-wave (GW) events.

Three sites ([LHO](#), [LLO](#), [Virgo](#)) together form a global network of ground-based GW

# GraceDB

https://gracedb.ligo.org/latest/

GraceDB Public Alerts Latest Search Documentation Login

Please log in to view full database contents.

Latest as of 22 December 2021 19:21:47 UTC

Test and MDC events and superevents are not included in the search results by default; see the [query help](#) for information on how to search for events and superevents in those categories.

Query:

Search for:

Tap on entry for detailed information

UID	Labels	FAR (Hz)	Created ▾
S200316bj	EM_READY PE_READY ADVOK EM_Selected SKYMAP_READY EMBRIGHT_READY PASTRO_READY DQOK GCN_PRELIM_SENT	7.098e-11	2020-03-16 21:58:12 UTC
S200311bg	EM_READY PE_READY ADVOK EM_Selected SKYMAP_READY EMBRIGHT_READY PASTRO_READY DQOK GCN_PRELIM_SENT	8.939e-26	2020-03-11 11:59:09 UTC

# 검색엔진

- **파형의존**

- GstLAL: Gstreamer from LALSuite
- MBTA: Multi-Band Template Analysis
- PyCBC Broad
- PyCBC BBH

알려진 CBC중력파형 사용  
정합필터와 파형집합 사용  
검출기 사이의 동시성 확인  
GstLAL은 한 검출기 검색 가능

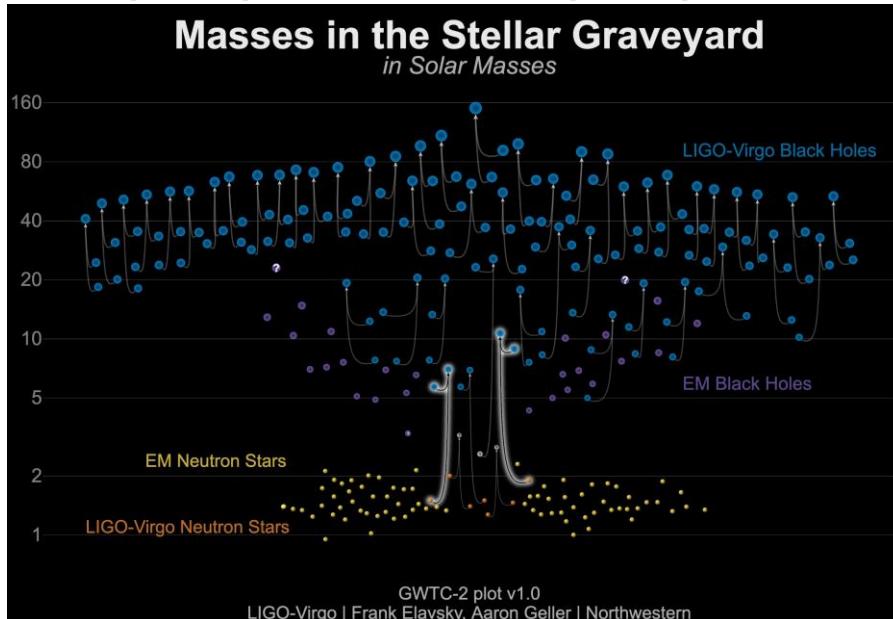
- **파형독립**

- cWB: Coherent Wave-Burst

CBC이외의 중력파 검출 가능  
정합필터 사용하지 않음  
검출기 사이의 동시성 확인  
초과 에너지 계산

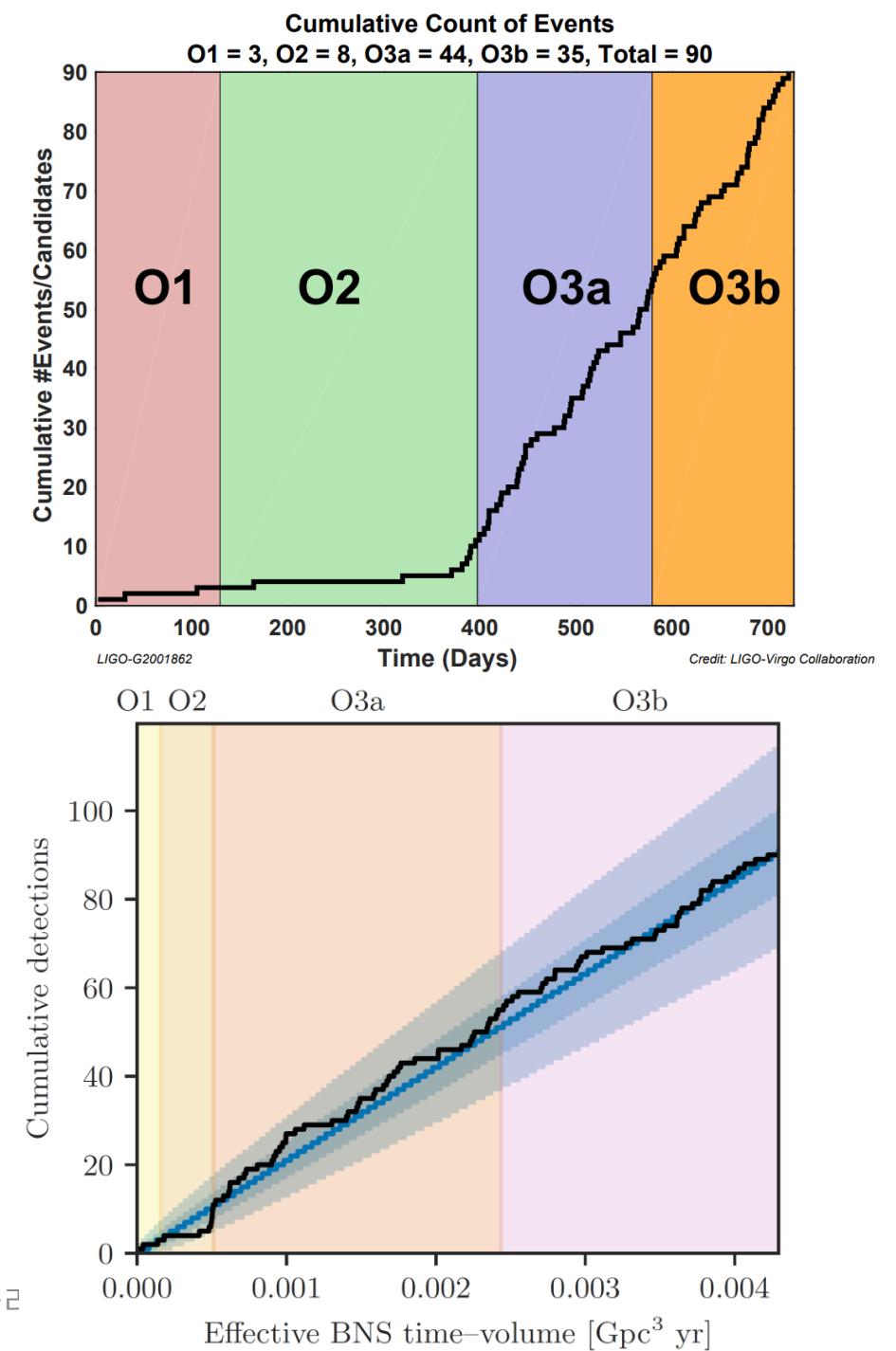
# 관측 결과(Observations)

- O1: 2015/09/12~2016/01/19
- O2: 2016/11/30~2017/08/25
- O3a : 2019/04/01 ~ 2019/09/30
- O3b : 2019/11/01 ~ 2020/03/27



2022-01-15

2022 수치상대론 및 중력파 겨울학교 / 계산천체물리



# 관측결과(90)

[GWTC-3](#)

1916 Einstein predicts gravitational waves in general relativity

1974 First indirect evidence of gravitational waves from binary pulsars

2015 First observation of gravitational waves at the start of O1

## Observing runs

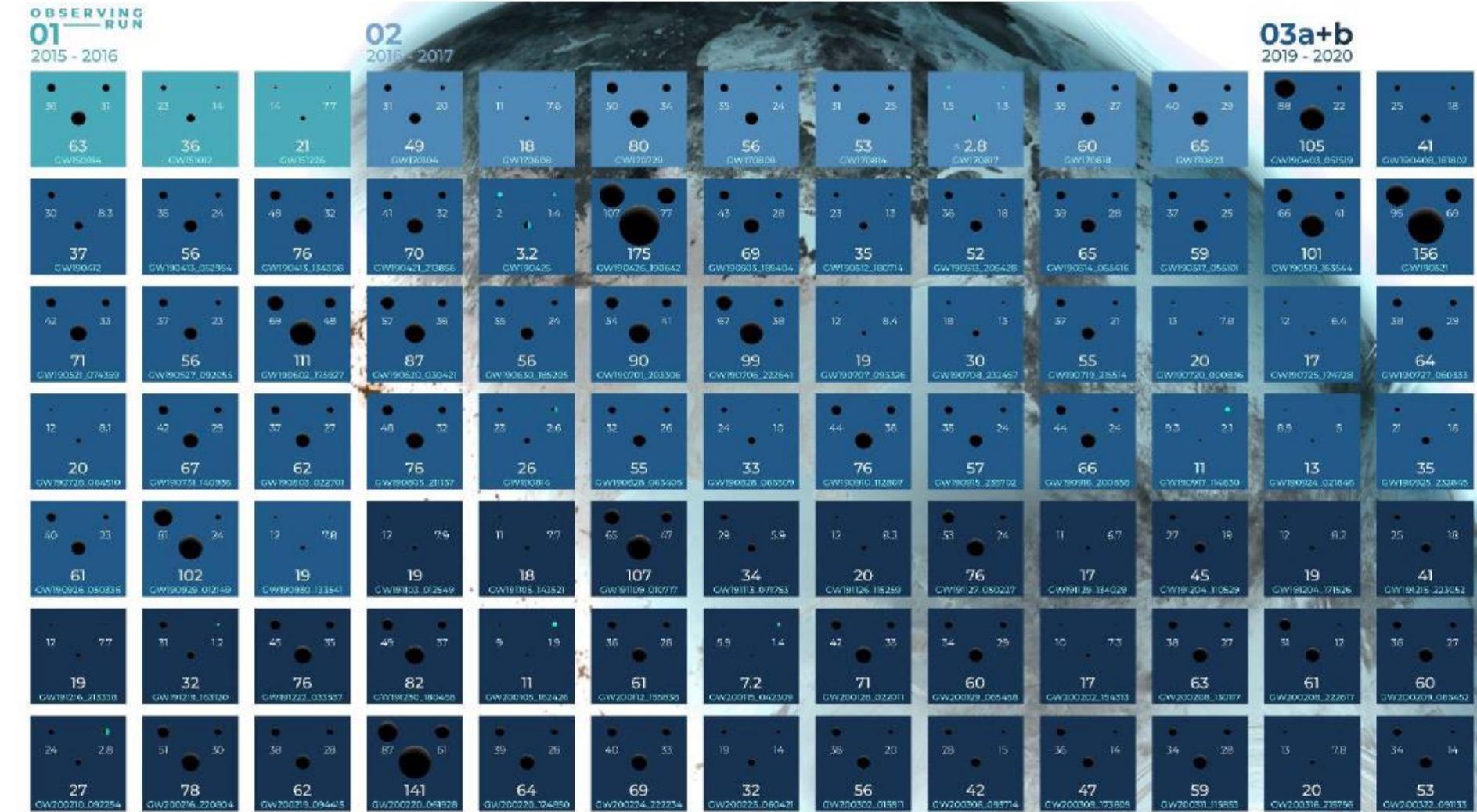
O1: 2015-2016

O2: 2016-2017

O3: 2019-2020

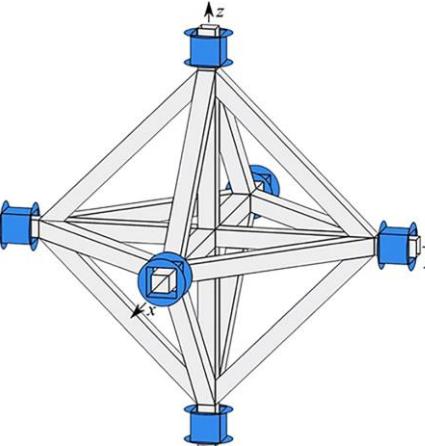
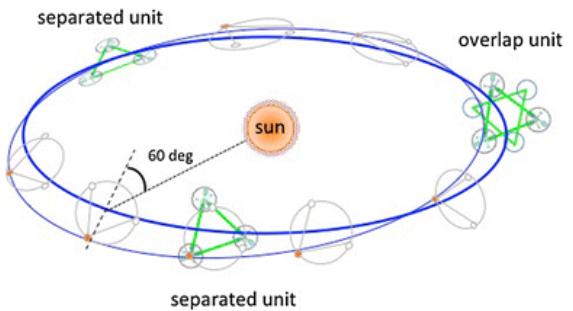
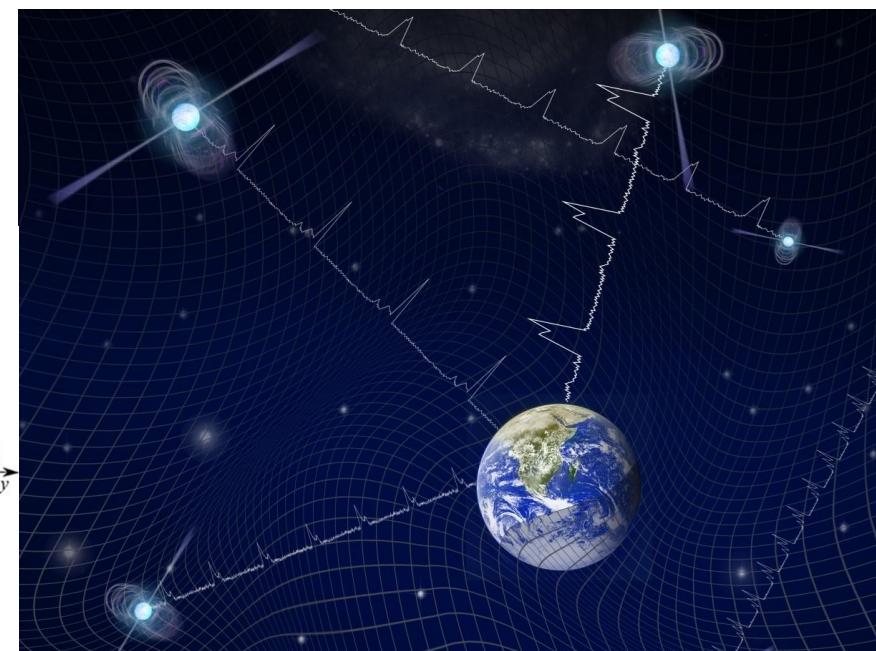
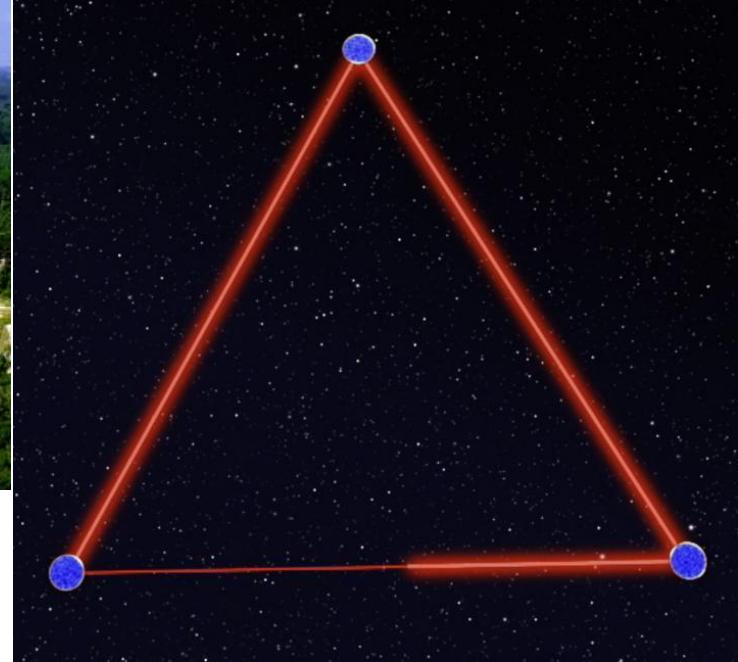
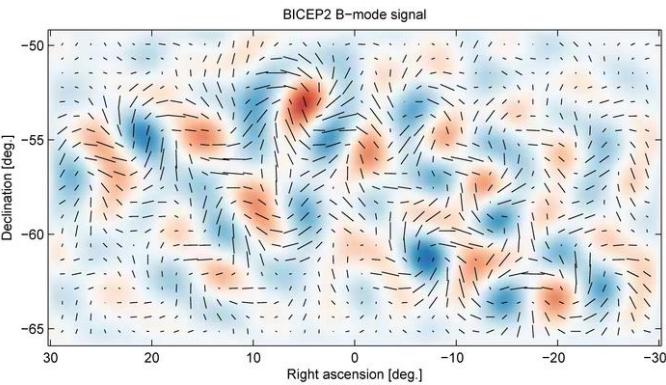
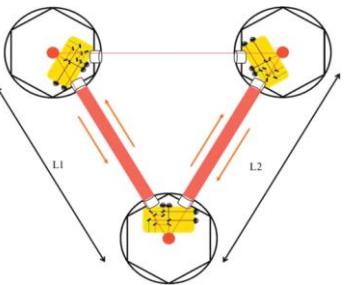
O4: ~2022-2023

# The gravitational-wave story



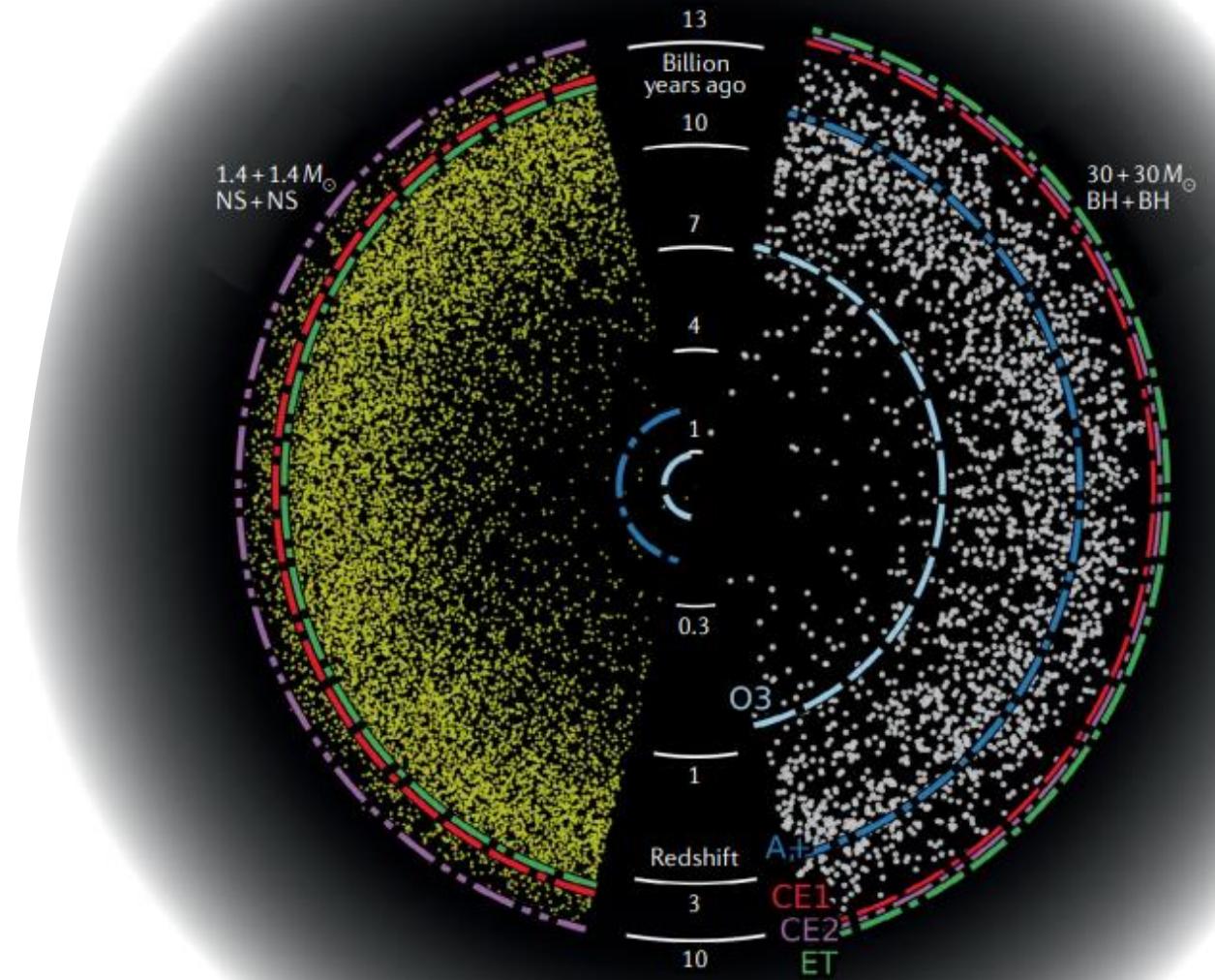
# 여러가지 검출기

- 공명기
- 지상간섭계
- 우주간섭계
- 펄사타이밍어레이
- 우주배경복사 편광



# 검출기 미래

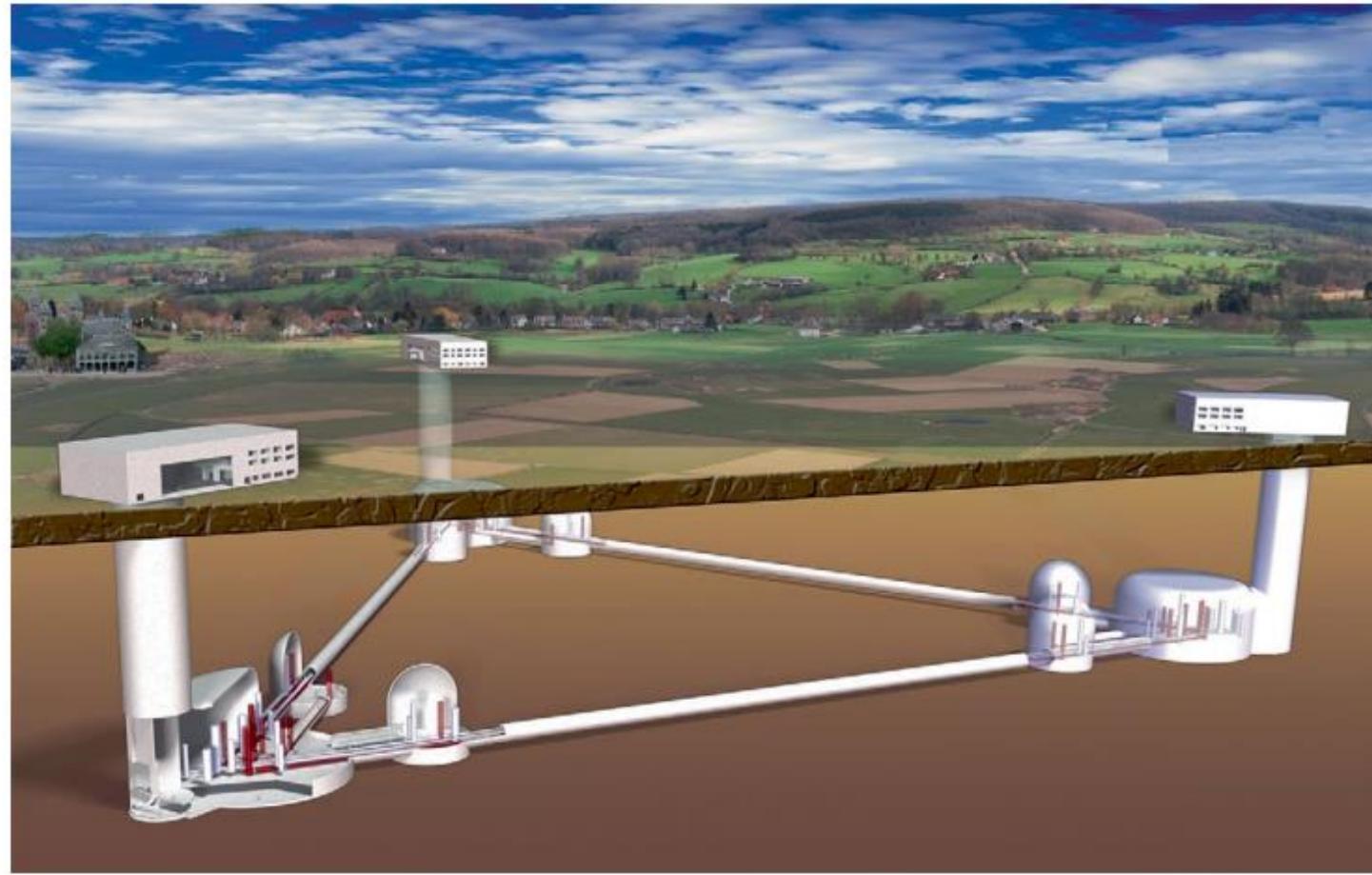
- O3/O4/O5
- A+(Advanced LIGO)
- CE1, CE2(Cosmic Explorer)
- ET(Einstein Telescope)



[Nat. Rev. Physics, May 2021](#)

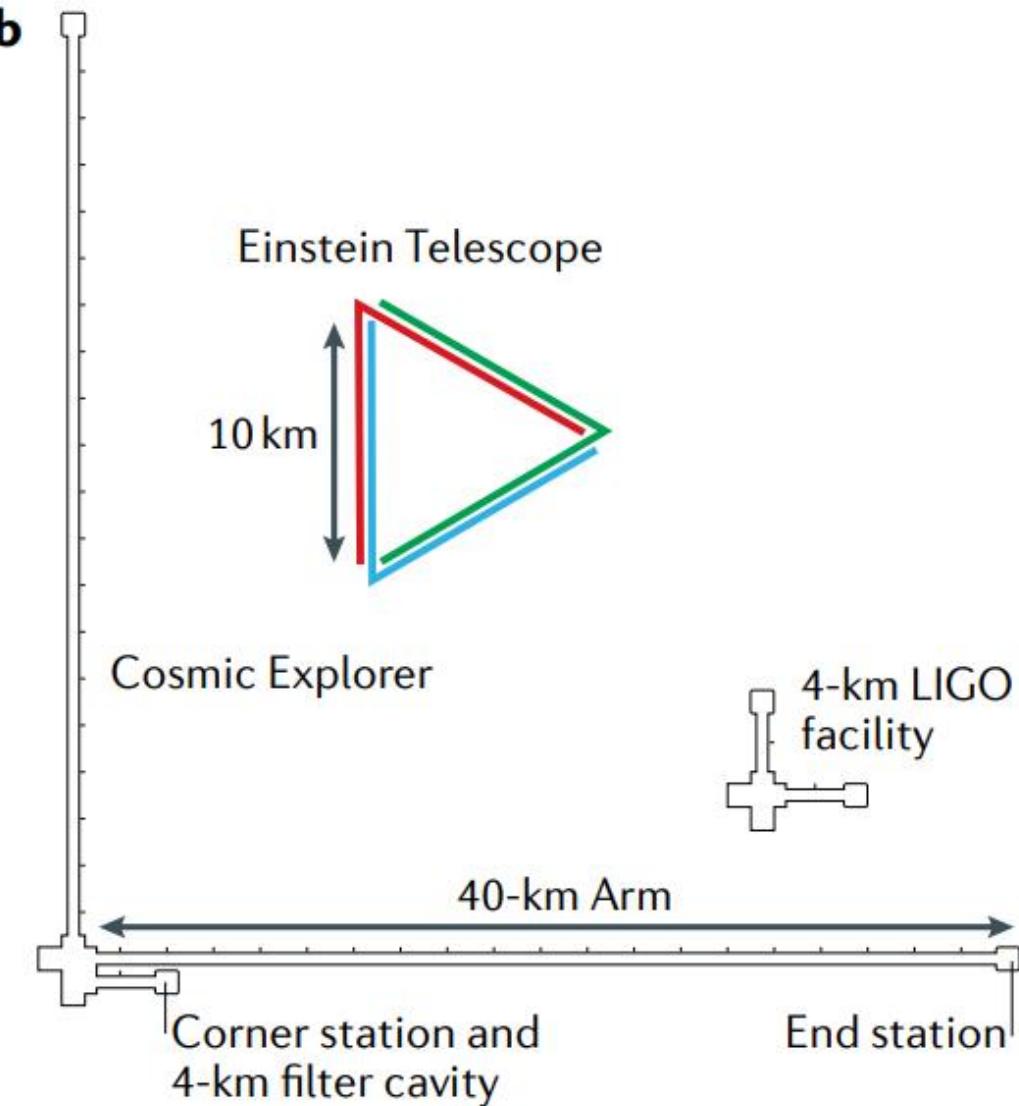
# 3세대 검출기

a



[Nat. Rev. Physics, May 2021](#)

b



# 중력파 천체물리학

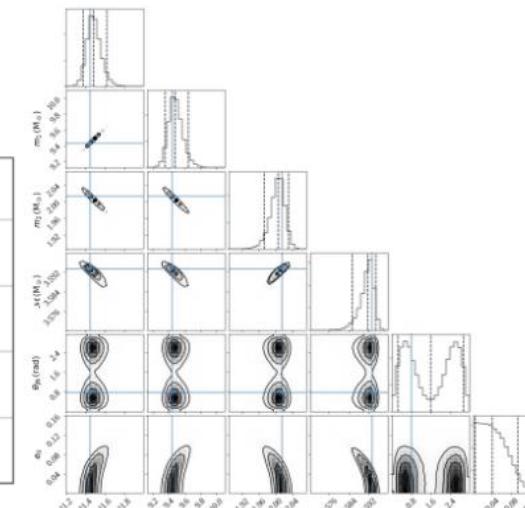
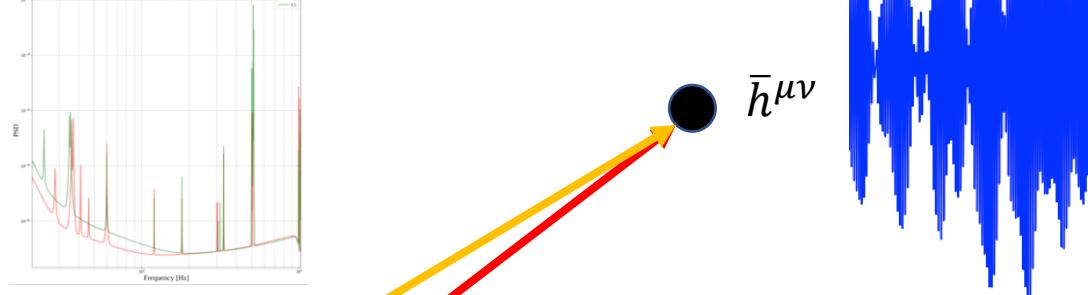
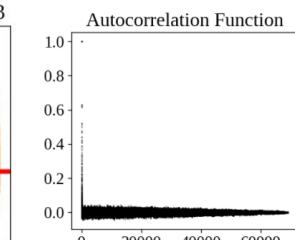
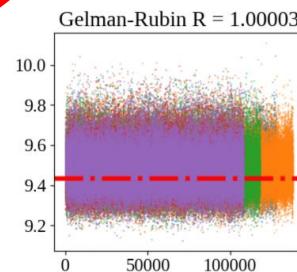
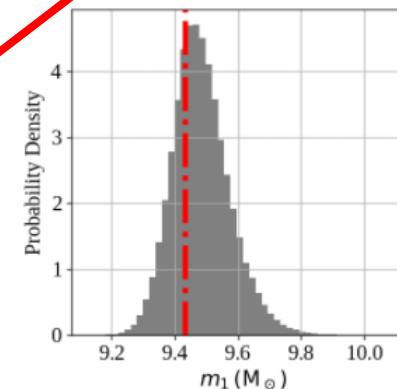
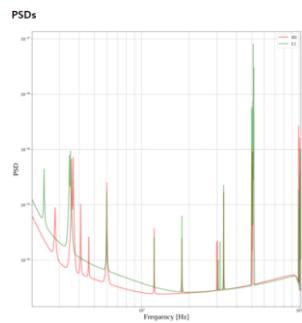
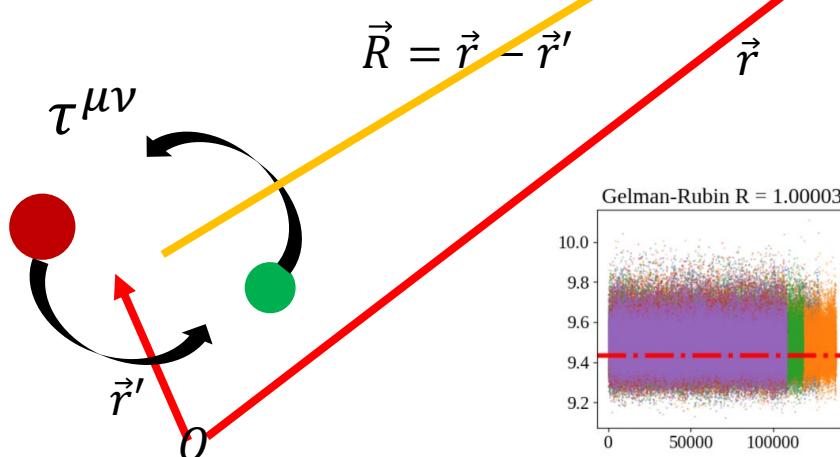
## Box 1 | Fundamental questions addressed through gravitational-wave observations

- What is the physics of stellar core collapse? How often do core-collapse supernovae occur<sup>257,258</sup>?
- What is the equation of state, and what are the radii, of neutron stars<sup>120,121</sup>?
- What are the multi-messenger emission mechanisms of high-energy transients (gamma-ray bursts and kilonovae)<sup>259</sup>?
- How do binary black holes of tens of solar masses form and evolve<sup>260,261</sup>?
- How did super-massive black holes at the cores of galactic nuclei form and evolve, and what were their seeds and demographics<sup>262</sup>?
- Are black hole spacetimes as predicted by general relativity<sup>136</sup>?
- Are there any signatures of horizon structure or other manifestations of quantum gravity accessible to gravitational-wave observations<sup>263</sup>?
- Is dark matter composed, in part, of primordial black holes, or must it be composed solely from exotic matter such as axions or dark fermions<sup>263</sup>?
- What is the expansion rate of the Universe<sup>264</sup>?
- What is the nature of dark energy<sup>264</sup>?
- Is there a measurable gravitational-wave stochastic background due to phase transitions in the early Universe? If so, what were its properties<sup>265–267</sup>?
- How does gravity behave in the strong/highly dynamical regime<sup>99–102</sup>?
- Do we live in a Universe with large extra dimensions<sup>116</sup>?
- Are black holes, neutron stars and white dwarfs the only compact objects in our Universe, or are there even more exotic objects<sup>133</sup>?

# 중력파 데이터 분석

# 데이터분석

- 고유특성
  - $m_1, m_2, \vec{S}_1, \vec{S}_2, \Lambda_1, \Lambda_2, e_0$
- 외부특성
  - $t_c, \varphi_c, \alpha, \delta, D, \theta_{JN}, \psi$



적분  
매개변수 분포

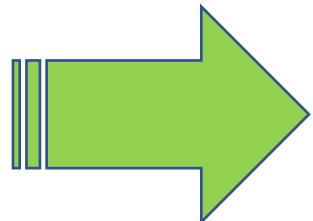
얼마나 비슷한가?  
**Likelihood**

베이지언 추론  
**Posterior**

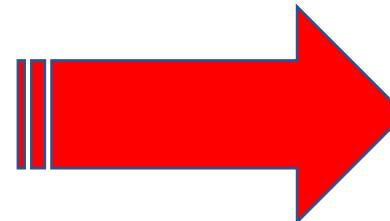
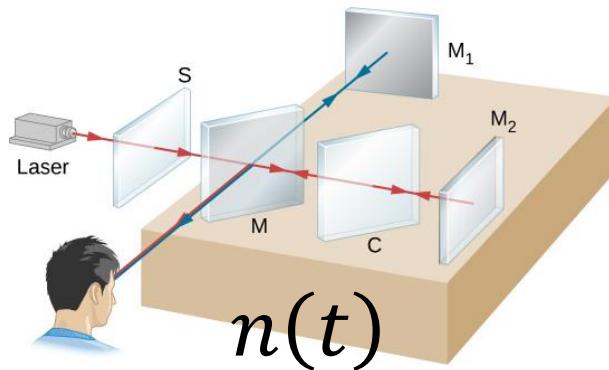
샘플생성  
**MCMC, Nested, RIFT**

# Linear Optimal Filter

$h(t)$



$$d(t) = h(t) + n(t)$$



$$\varepsilon(t) = \mathcal{H}(d(t)) - h(t)$$

minimize  $\overline{\varepsilon^2}$

$$K(f) \propto \frac{h(f)}{S_n(f)}$$

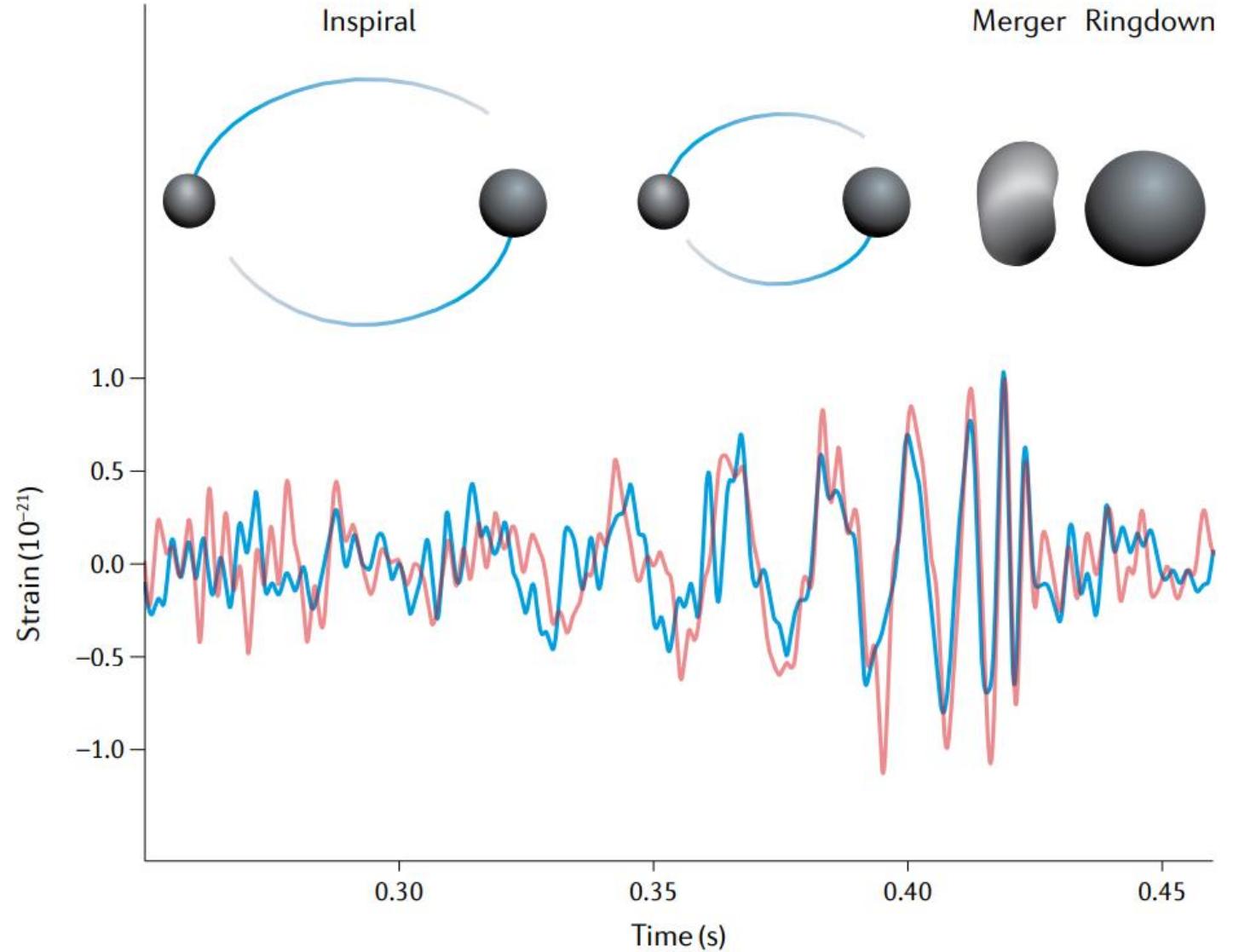
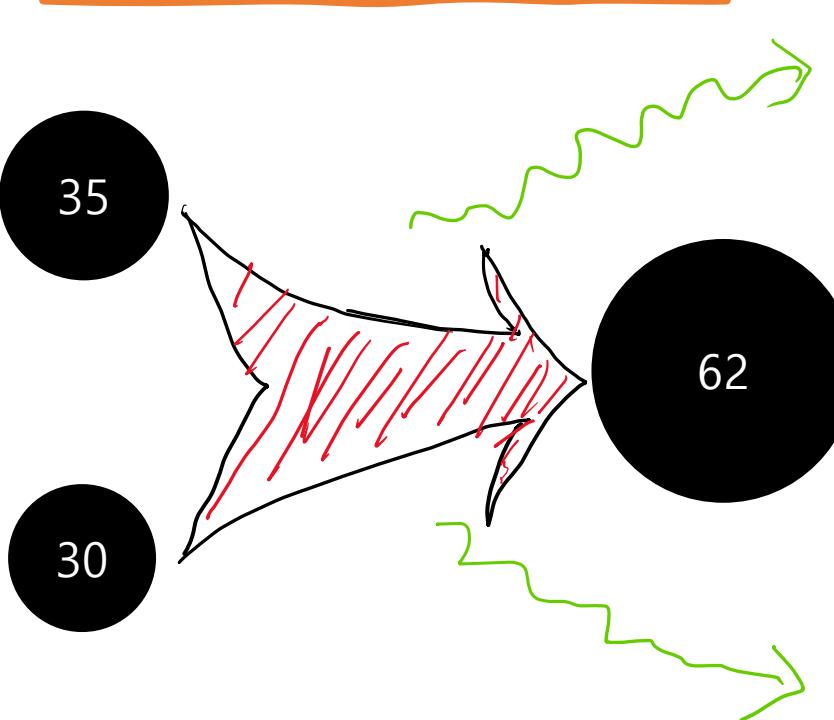
# 중력파형

- 107 Waveforms
- Post-Newtonian
- Time-domain
- Frequency-domain
- Tidal effects
- Spin effects
- NR approximation
- Effective One Body

```
enum Approximant {  
    TaylorT1, TaylorT2, TaylorT3, TaylorF1,  
    EccentricFD, TaylorF2, TaylorF2Ecc, TaylorF2NLTides,  
    TaylorR2F4, TaylorF2RedSpin, TaylorF2RedSpinTidal, PadeT1,  
    PadeF1, EOB, BCV, BCVSpin,  
    SpinTaylorT1, SpinTaylorT2, SpinTaylorT3, SpinTaylorT4,  
    SpinTaylorT5, SpinTaylorF2, SpinTaylorFrameless, SpinTaylor,  
    PhenSpinTaylor, PhenSpinTaylorRD, SpinQuadTaylor, FindChirpSP,  
    FindChirpPTF, GeneratePPN, BCVC, FrameFile,  
    AmpCorPPN, NumRel, NumRelNinja2, Eccentricity,  
    EOBNR, EOBNRv2, EOBNRv2HM, EOBNRv2_ROM,  
    EOBNRv2HM_ROM, TEOBResum_ROM, SEOBNRv1, SEOBNRv2,  
    SEOBNRv2_opt, SEOBNRv3, SEOBNRv3_pert, SEOBNRv3_opt,  
    SEOBNRv3_opt_rk4, SEOBNRv4, SEOBNRv4_opt, SEOBNRv4P,  
    SEOBNRv4PHM, SEOBNRv2T, SEOBNRv4T, SEOBNRv1_ROM_EffectiveSpin,  
    SEOBNRv1_ROM_DoubleSpin, SEOBNRv2_ROM_EffectiveSpin, SEOBNRv2_ROM_DoubleSpin, SEOBNRv2_ROM_DoubleSpin_HI,  
    Lackey_Tidal_2013_SEOBNRv2_ROM, SEOBNRv4_ROM, SEOBNRv4HM_ROM, SEOBNRv4_ROM_NRTidal,  
    SEOBNRv4_ROM_NRTidalv2, SEOBNRv4_ROM_NRTidalv2_NSbh, SEOBNRv4T_surrogate, HGimri,  
    IMRPhenomA, IMRPhenomB, IMRPhenomFA, IMRPhenomFB,  
    IMRPhenomC, IMRPhenomD, IMRPhenomD_NRTidal, IMRPhenomD_NRTidalv2,  
    IMRPhenomNSBH, IMRPhenomHM, IMRPhenomP, IMRPhenomPv2,  
    IMRPhenomPv2_NRTidal, IMRPhenomPv2_NRTidalv2, IMRPhenomFC, TaylorEt,  
    TaylorT4, EccentricTD, TaylorN, SpinTaylorT4Fourier,  
    SpinTaylorT5Fourier, SpinDominatedWf, NR_hdf5, NRSur4d2s,  
    NRSur7dq2, NRSur7dq4, SEOBNRv4HM, NRHybSur3dq8,  
    IMRPhenomXAS, IMRPhenomXHM, IMRPhenomPv3, IMRPhenomPv3HM,  
    IMRPhenomXP, IMRPhenomXPHM, TEOBResumS, IMRPhenomT,  
    IMRPhenomTHM, IMRPhenomTP, IMRPhenomTPHM, NumApproximants  
}
```

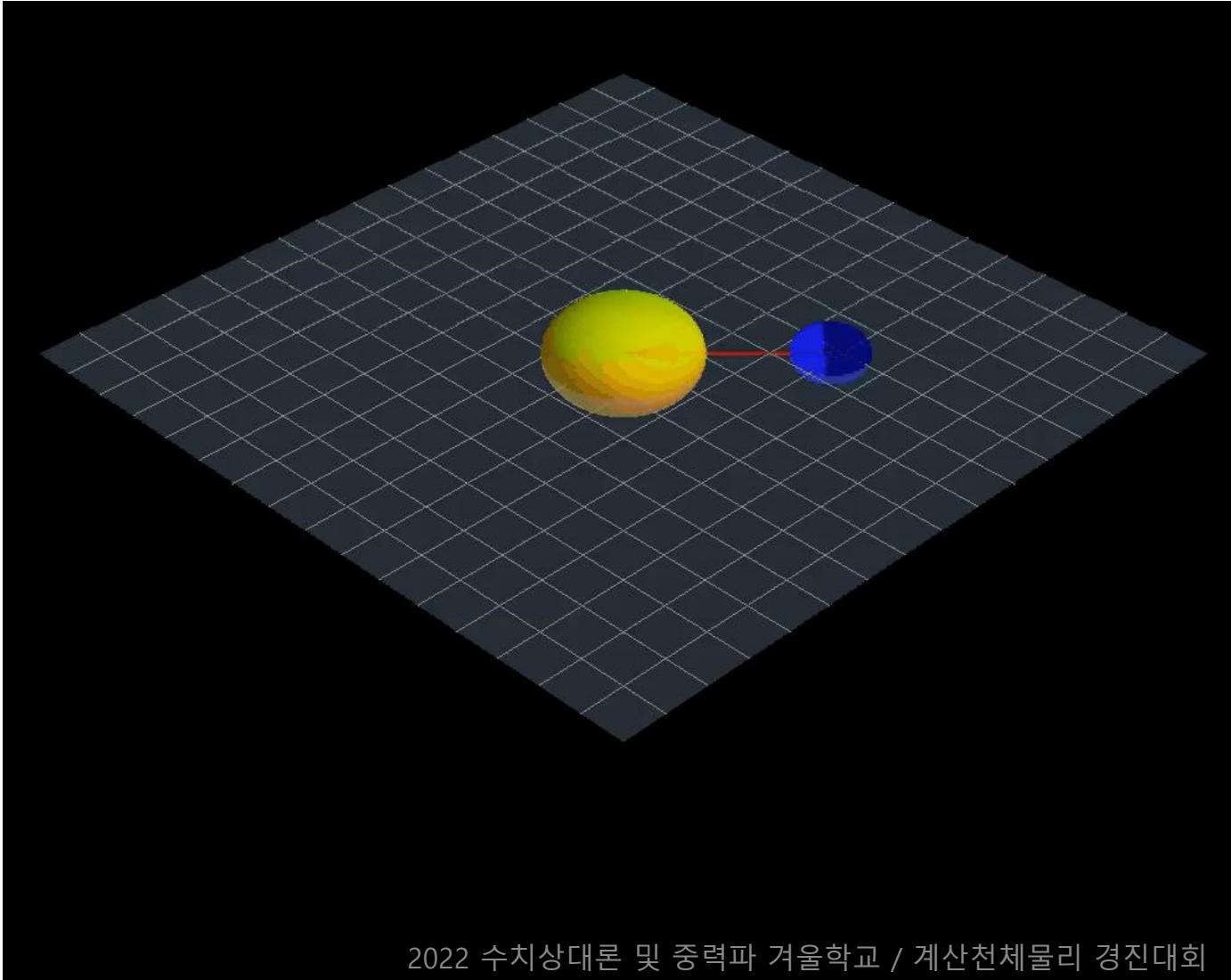
LALSimInspiral.h

# GW150914



# Binary orbit in Newtonian Theory

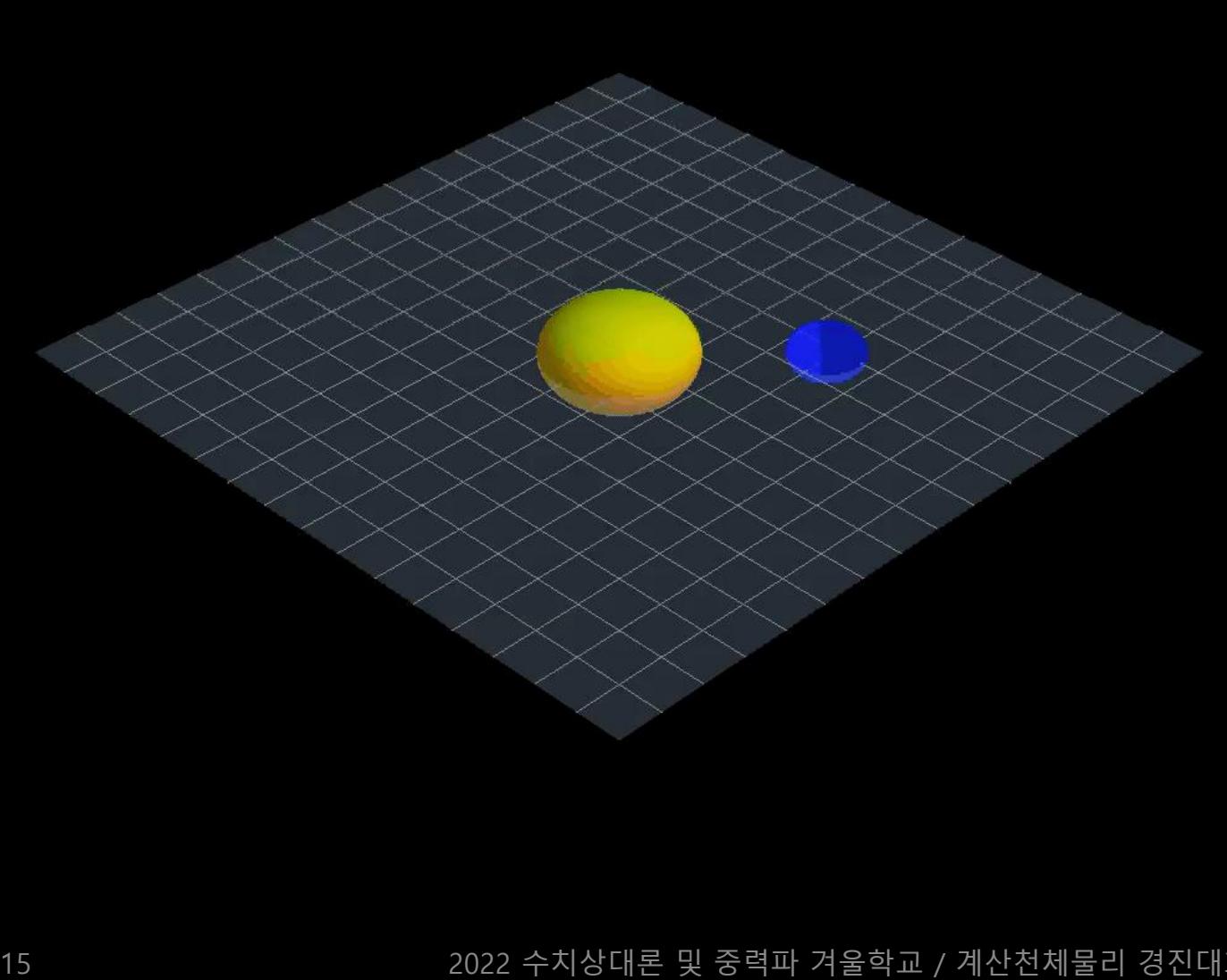
Orbital motion does not change space-time



Credit: Carl Rodriguez

# General Relativity

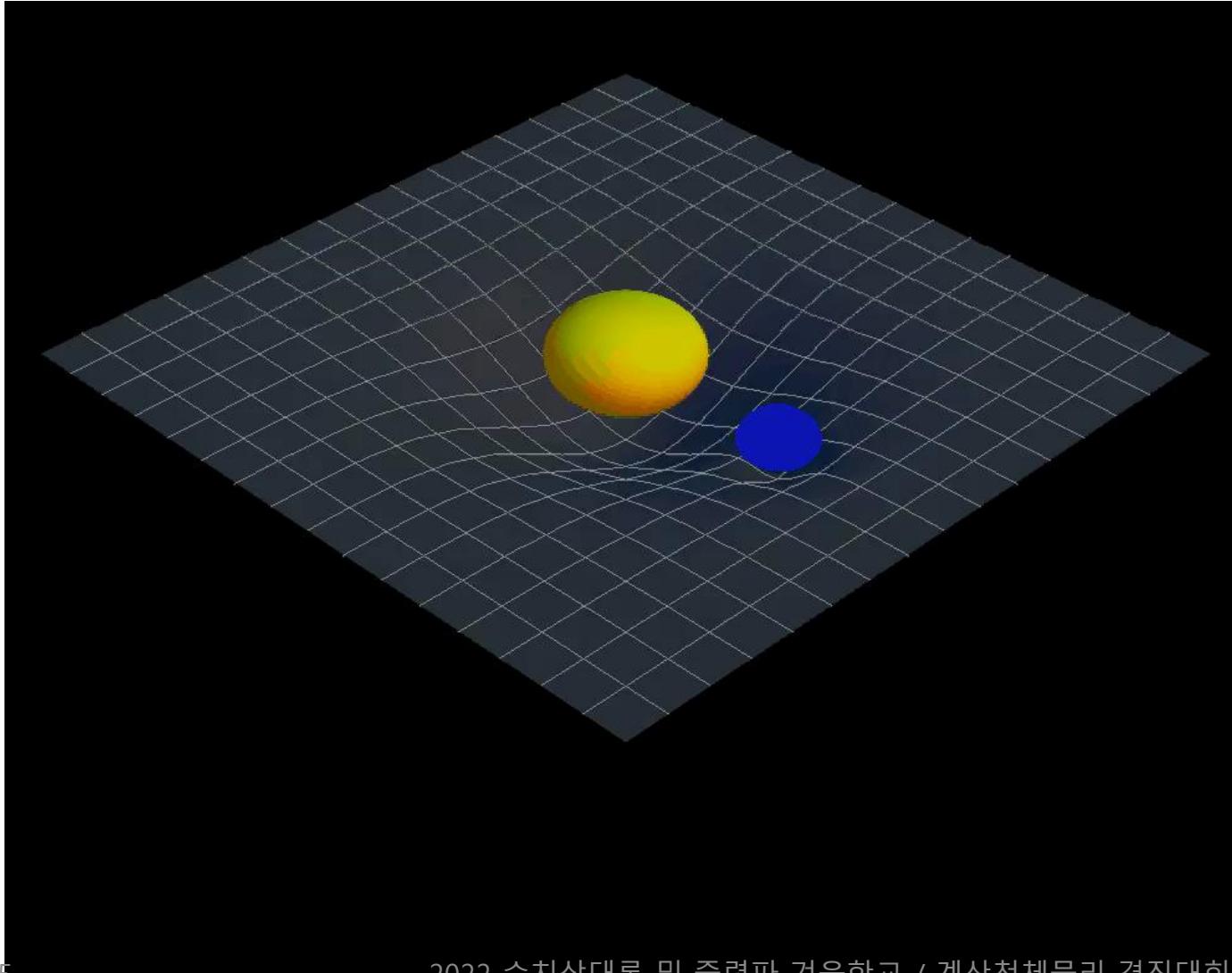
Mass, Energy ~ curved space-time



Credit: Carl Rodriguez

# 아인슈타인의 상대성 이론과 쌍성의 궤도 운동

Change in space-time caused by binary motion propagate in speed of light(GW)



Credit: Carl Rodriguez

# 중력파 자료 예

KAGALI h(t) data

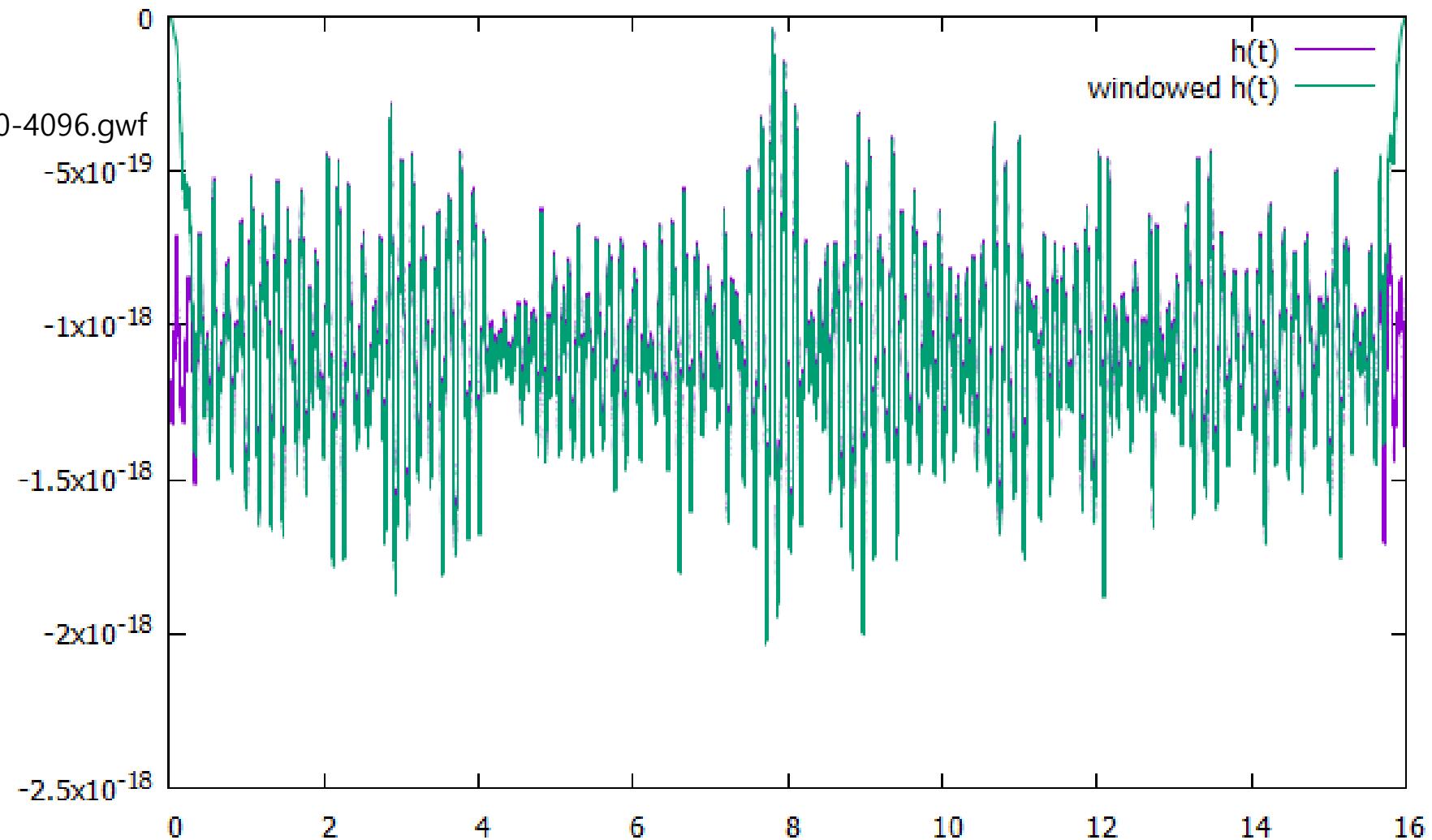
Window function : Tukey with 5% padding

Strain data : L-L1\_LOSC\_16\_V1-1126256640-4096.gwf

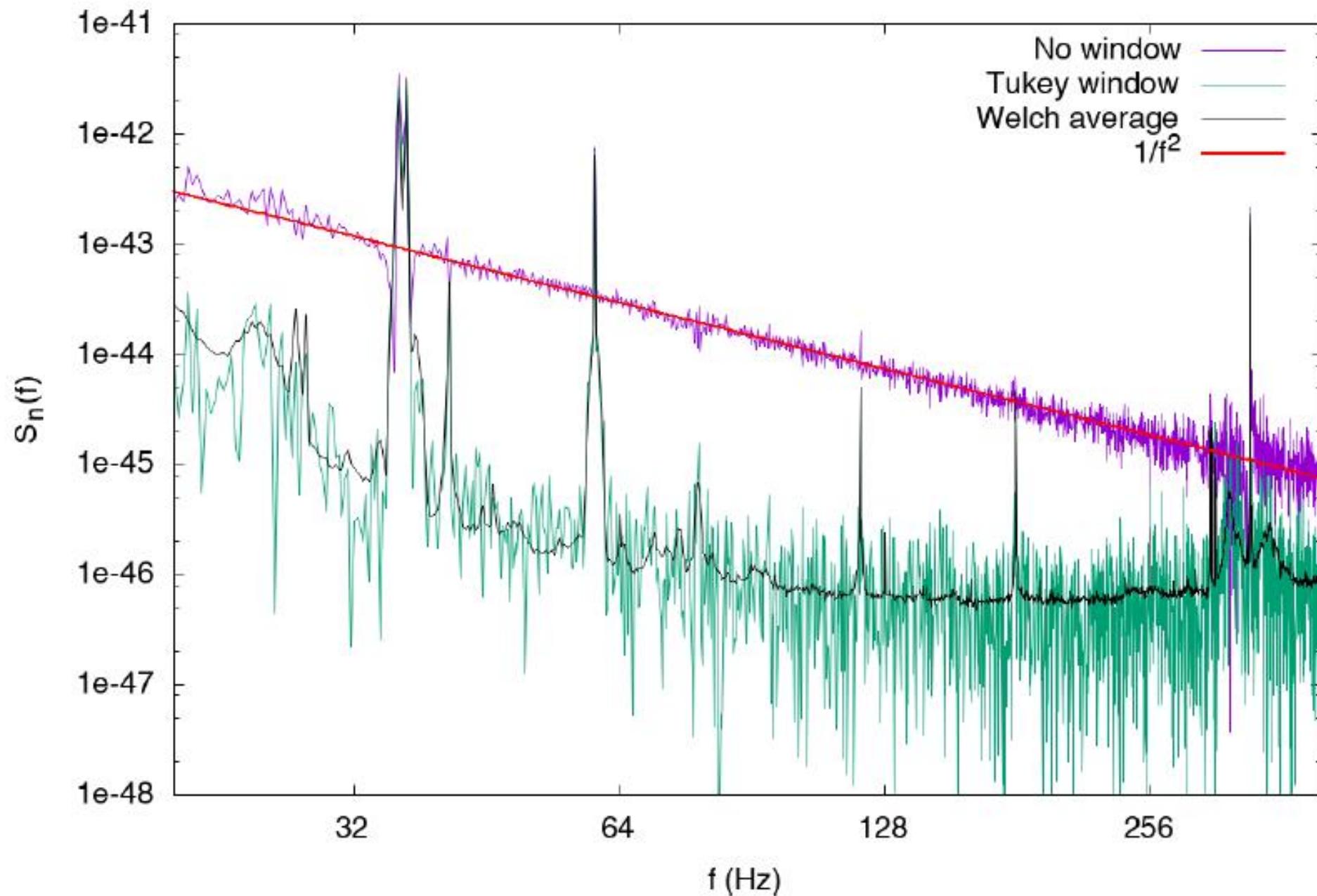
Channel : L1:GWOSC-16KHZ\_R1\_STRAIN

Trigger time : 1126259462

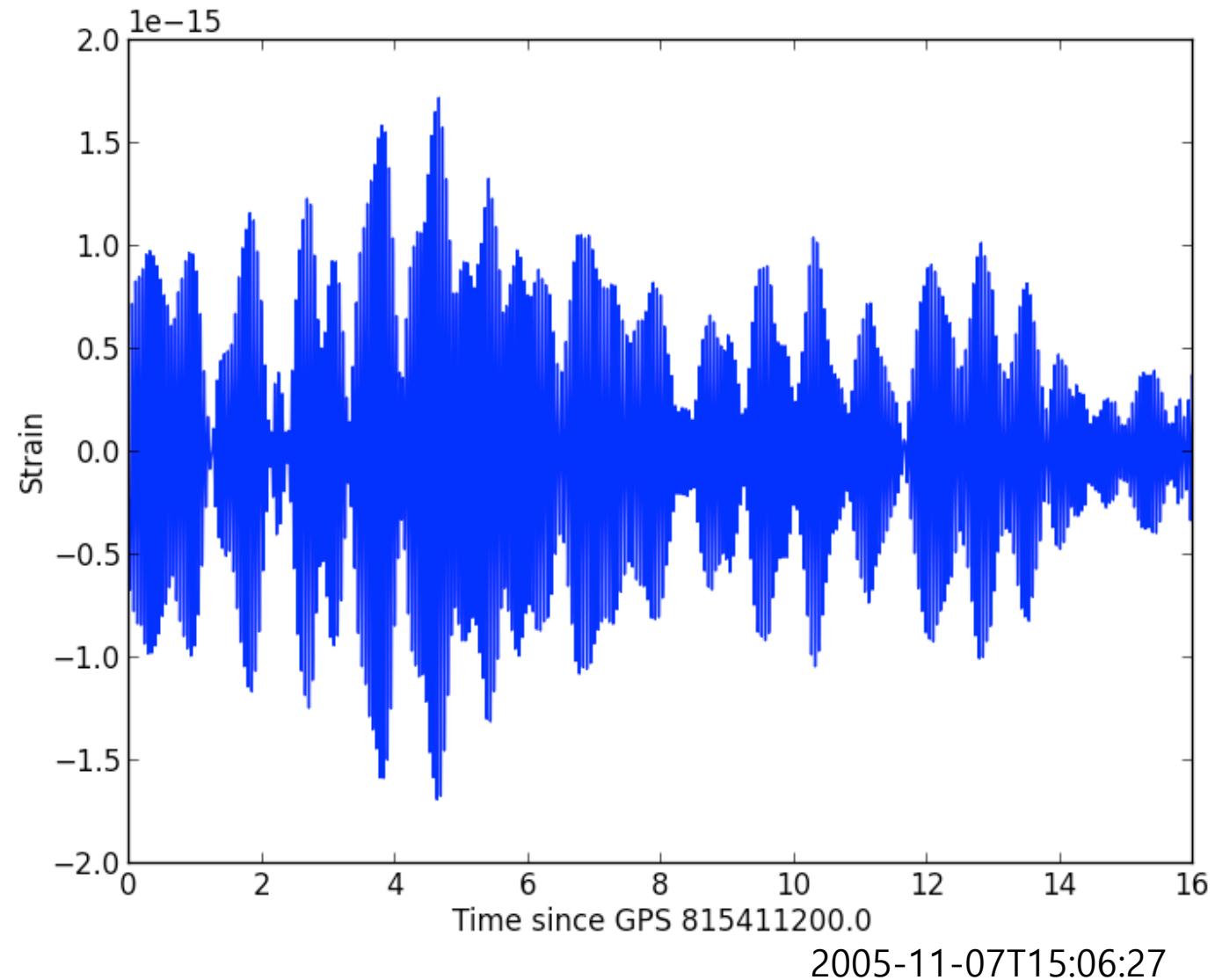
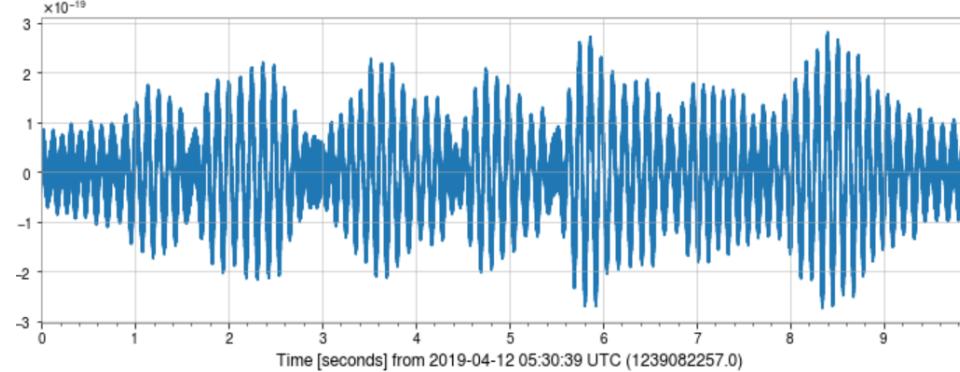
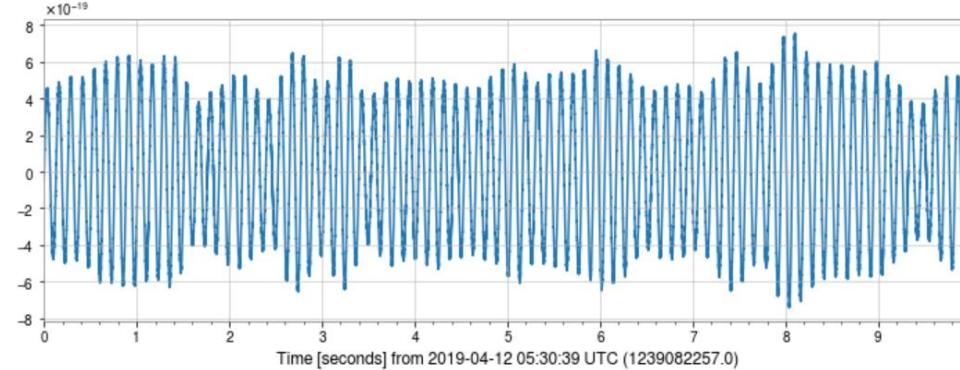
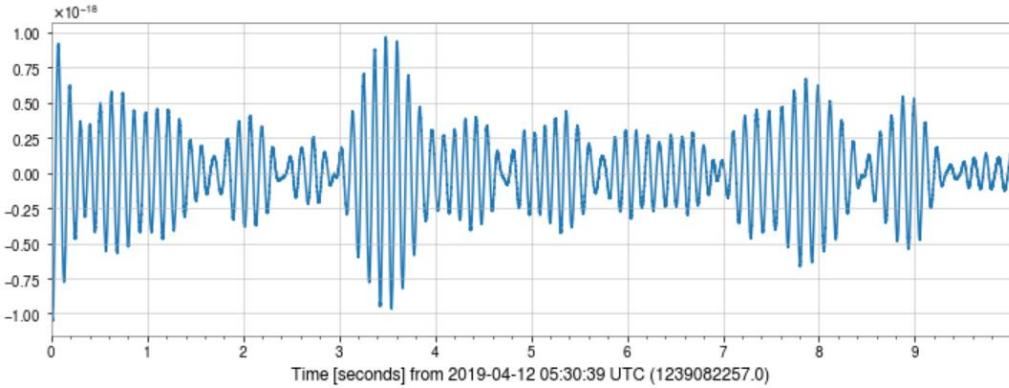
Segment length : 16s



# 스펙트럼



# LIGO 자료 예

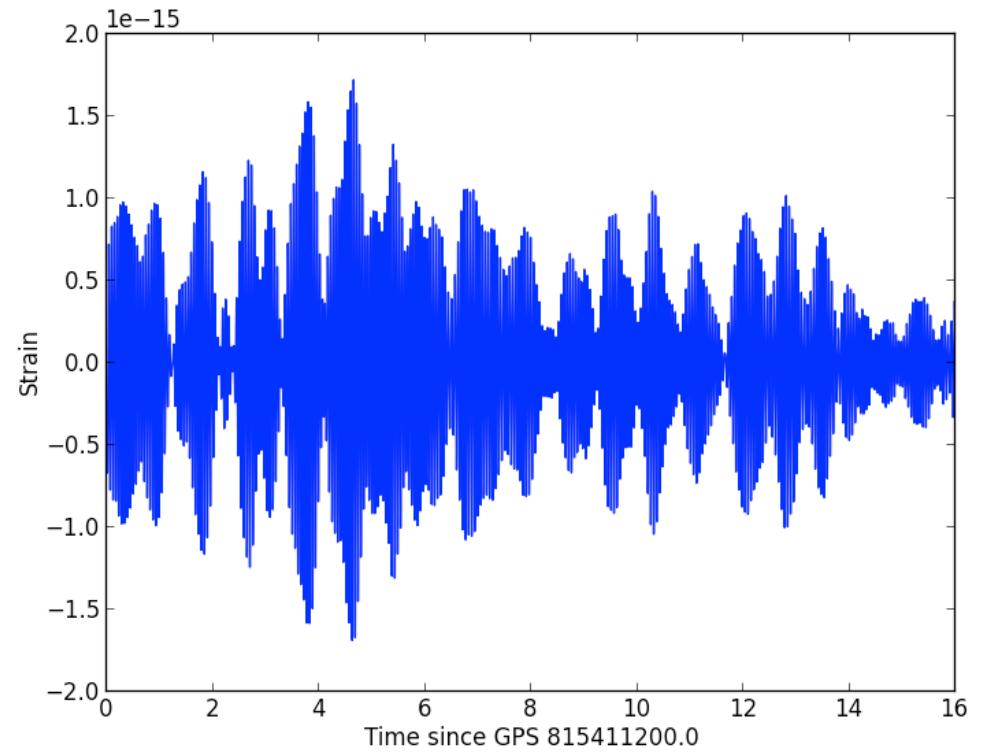


Odw4, GW190412

[GWOSC \(gw-openscience.org\)](http://gw-openscience.org)

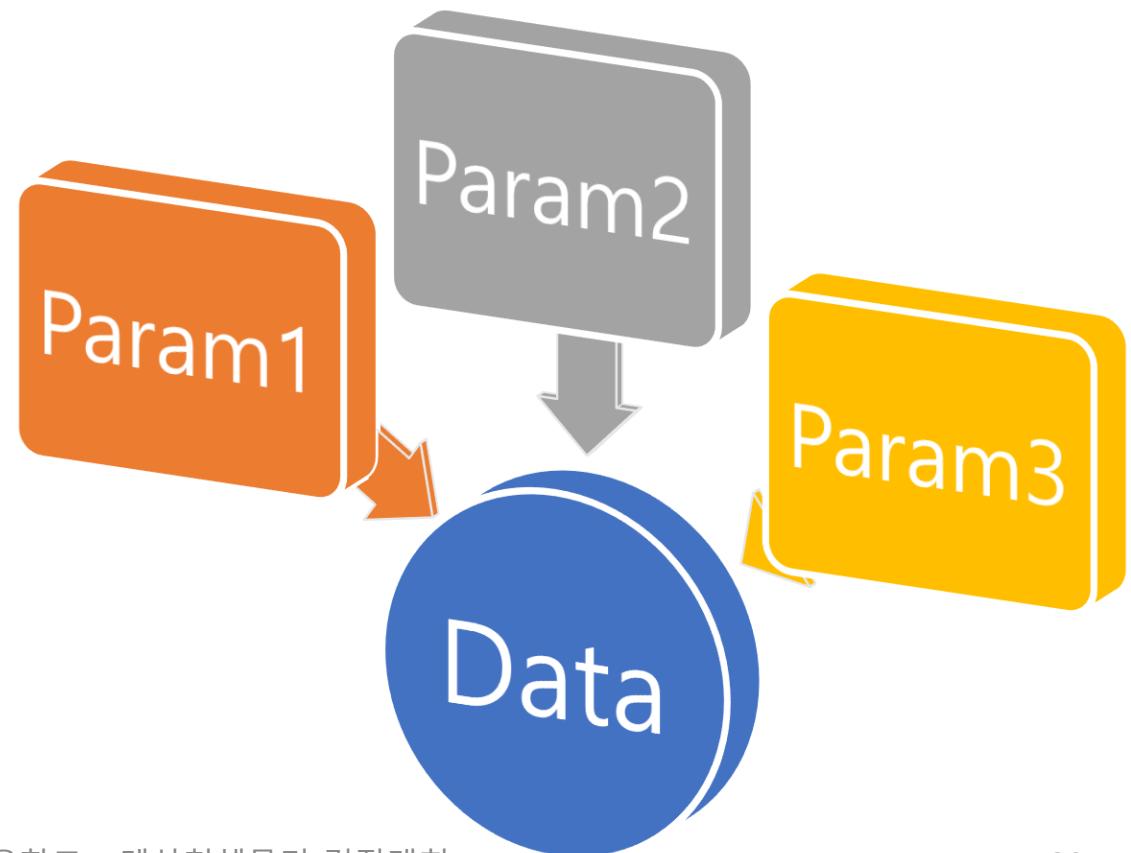
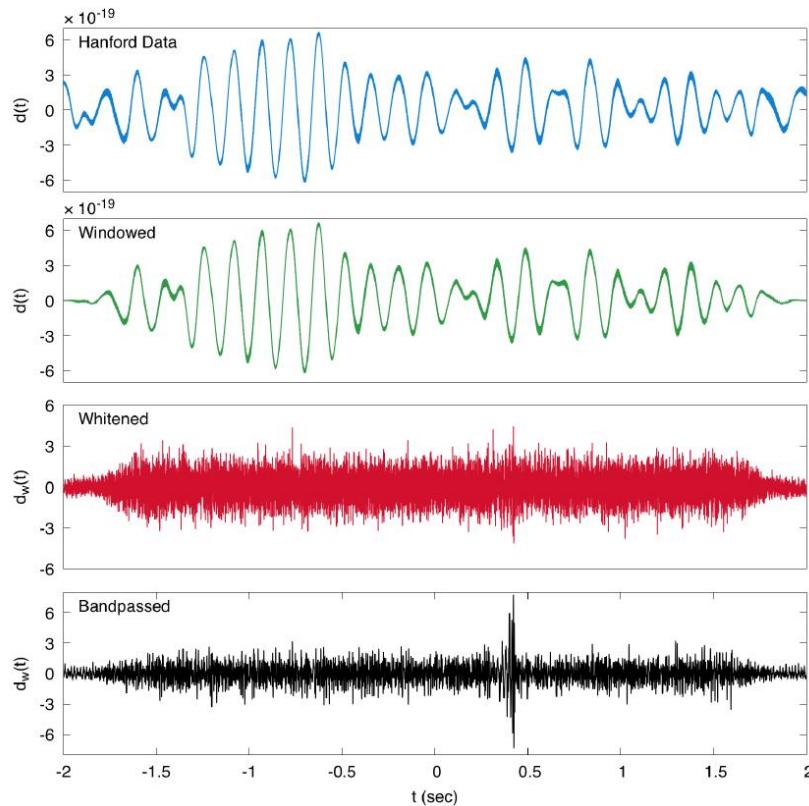
# LIGO 자료

- Discretely sampled time-series data
- Sampling rate (fs)
- $h(t)$  – calibrated strain
  - ALSO: hundreds of “auxiliary” channels
- Recorded at 16384 Hz sample rate
- ~300 MB per hour
- Stored in .gwf “frame” files
  - Also HDF5(Hierarchical Data Format version 5)



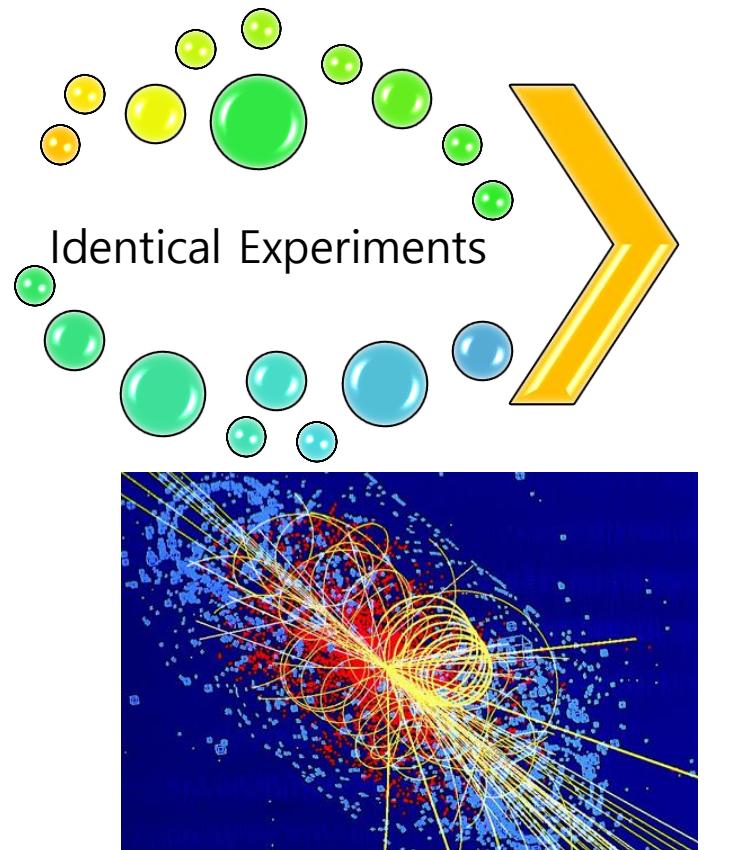
# 베이지언 추론

- 질문: “관측된 자료를 가장 잘 설명하는 중력파원의 물리적 매개변수의 분포는 어떻게 되는가?”

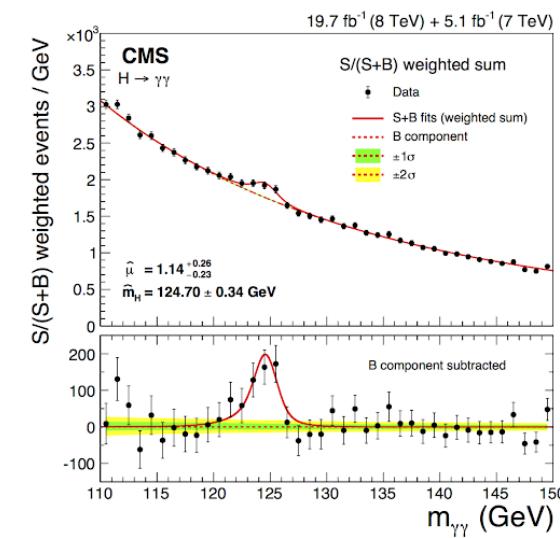
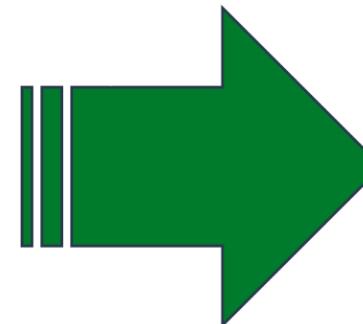
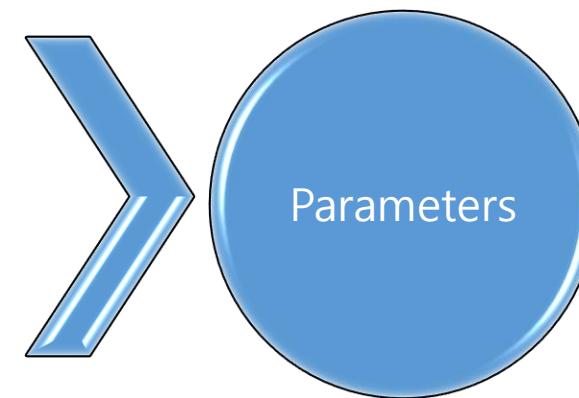


# 실험실 자료 처리

- 질문 : “동일한 조건의 많은 실험을 통하여 획득한 물리적인 매개변수의 분포는 어떻게 되는가?”



Data Process



# 베이지언 추론 기본

- Bayes' rule(theorem)

$$p(M|D) = \frac{p(D|M) p(M)}{p(D)}$$

사후확률 → 가능도 함수 → 사전확률 → 결합확률

$$p(M, D) = p(M|D)p(D) = p(D|M)p(M)$$

- 모델에 대한 개선된 믿음의 정도는 기존 믿음의 정도와 모델이 관측된 자료를 생성할 확률의 곱이다.

$$p(M, \theta|D, I) = \frac{p(D|M, \theta, I) p(M, \theta|I)}{p(D|I)}$$

$$p(M, \theta|I) = p(\theta|M, I) p(M|I)$$

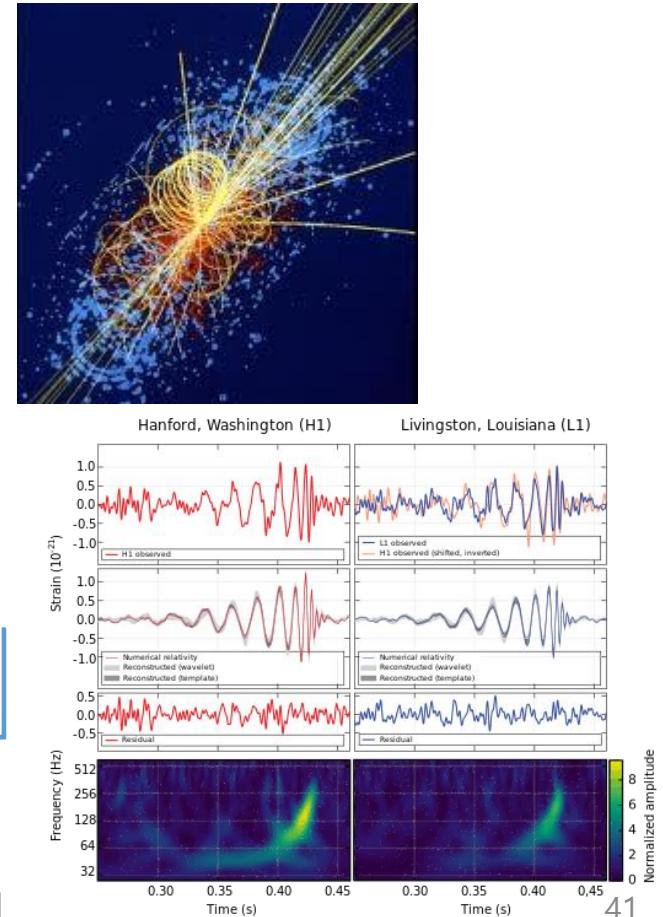
# 가능도 함수(Likelihood Function)

- 관측된 자료에 대한 가능도 함수는 실험장치에 대한 정량적 지표이다.

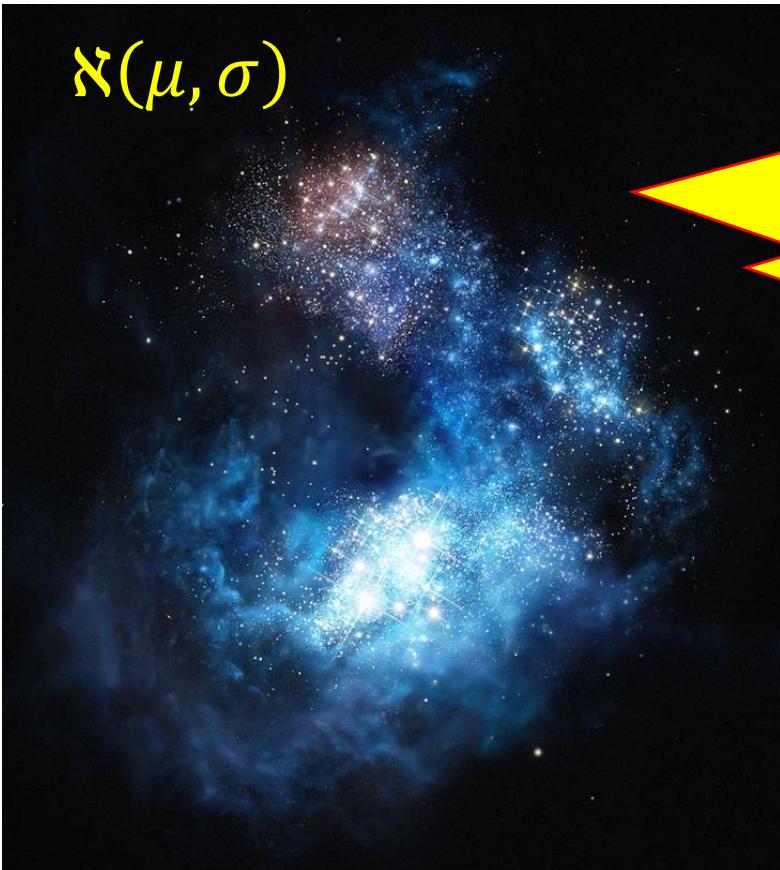


2022-01-15

2022 수치상대론 및 중력파 겨울학교 / 계산천체물리 경진대회

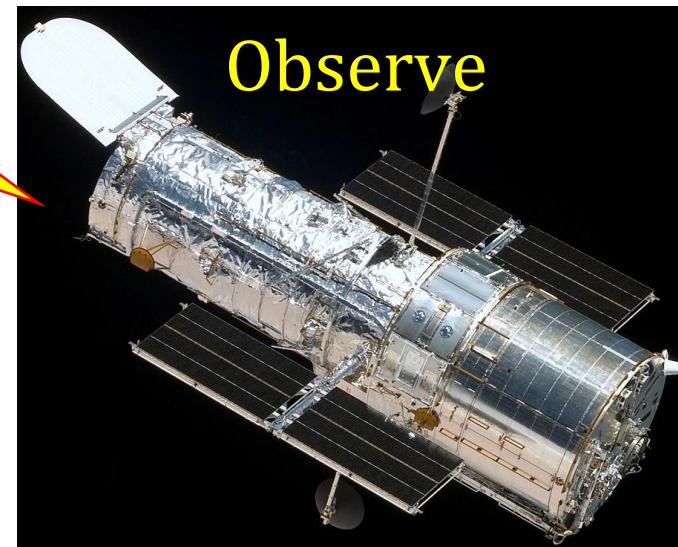


# 가능도 함수



The probability of the data given the model

$$L \equiv p(\{x_i\}|M(\theta)) = \prod_{i=1}^n p(x_i|M(\theta))$$



$$x_i: \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$
$$\prod_{i=1}^N \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right)$$

Likelihood is a function of  $x$  for given model

Likelihood is a function of model parameters for given data

# 내적(Inner Product)

$$K(f) \propto \frac{h(f)}{S_n(f)}$$

- $\langle a|b \rangle \equiv 4\Re \int_0^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S(f)} df$
- $\langle a|b \rangle = 2\Re \int_{-\infty}^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)}{S(|f|)} df = \int_{-\infty}^\infty \frac{\tilde{a}(f)\tilde{b}^*(f)+\tilde{a}^*(f)\tilde{b}(f)}{S(|f|)} df$
- $p_x[x(t)] \propto e^{-\langle x|x \rangle / 2}$

- We can consider each frequency bin distributed as

$$\cdot p_x[\tilde{x}(f)] = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{4\Delta f}{S_x(f)}} e^{-\frac{1|\tilde{x}(f)|^2 \Delta f}{2 S_x(f)/4}}$$

$$\Delta f = \frac{1}{T}$$

$$p_x[x(t)] \propto \exp \left\{ -\frac{1}{2} 4 \int_0^\infty \frac{|\tilde{x}(f)|^2}{S_x} df \right\}$$

# 가능도 함수(Likelihood)

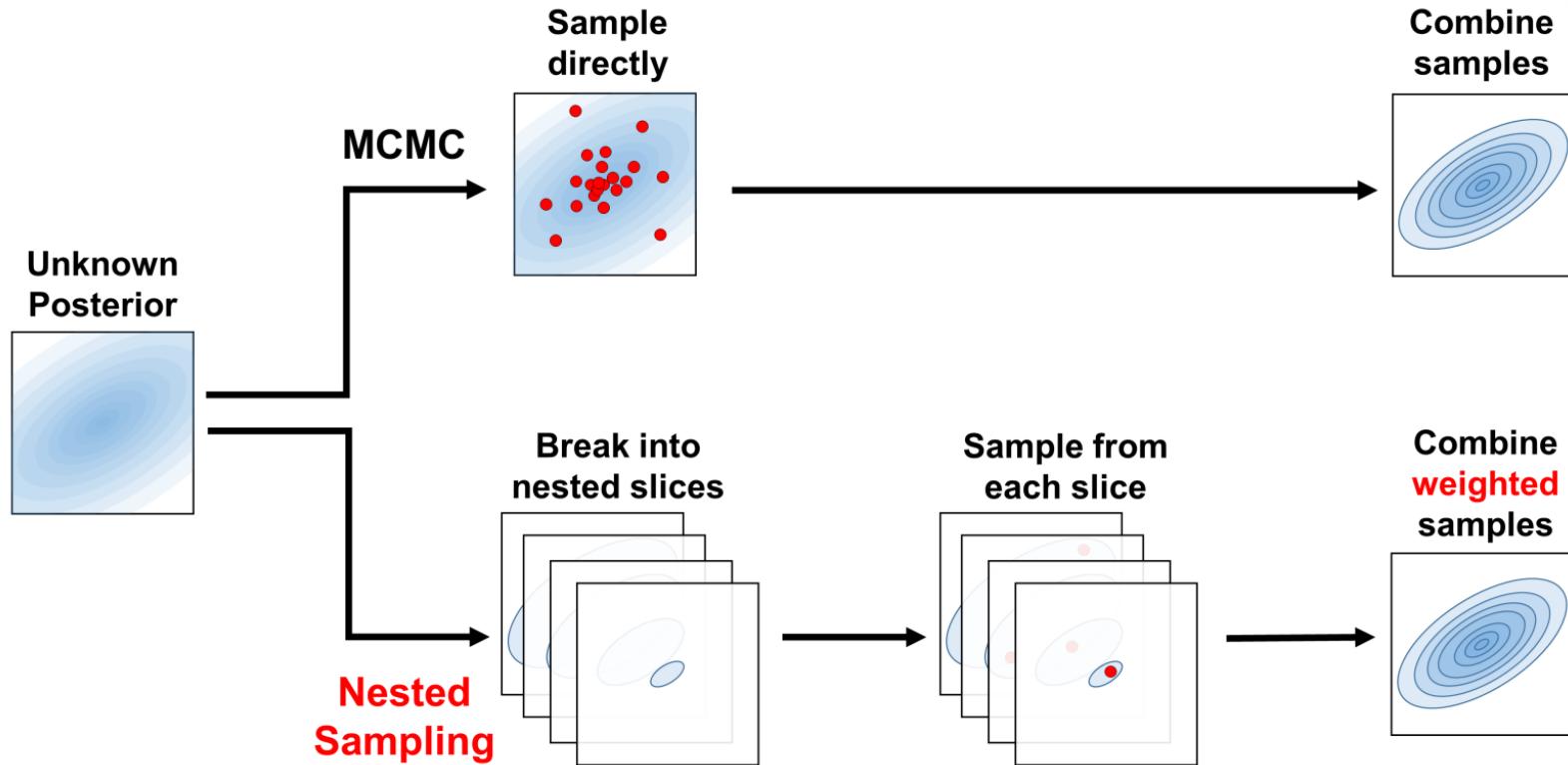
- $p(d|\theta)$  : 모델 매개변수  $\theta$  가 만든 중력파가 신호  $d$ 를 생성할 확률
- 신호  $d$ 가 파형  $h(\theta)$ 를 포함할 확률
  - 만약  $d$ 가 파형  $h(\theta)$ 를 포함한다면,  $d - h$ 는 순 잡음이다.
- $p(d - h|\theta) = p(n|\theta)$
- $p_d[\tilde{d}(f_j) - \tilde{h}(f_j)] = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{4\Delta f}{S_n(f_j)}} e^{-\frac{1}{2} \frac{|\tilde{d}(f_j) - \tilde{h}(f_j)|^2 \Delta f}{S_n(f_j)/4}}$
- $p(d|\theta) = \prod_{j=0}^{N-1} \frac{1}{\sqrt{2\pi}} \sqrt{\frac{4\Delta f}{S_n(f_j)}} e^{-\frac{1}{2} \frac{|\tilde{d}(f_j) - \tilde{h}(f_j)|^2 \Delta f}{S_n(f_j)/4}} \propto e^{-\langle d - h | d - h \rangle / 2}$

# 베이지언 추론 과정

1. 가능성 함수  $p(D|\vec{\theta}, M)$  결정
2. 매개변수에 대한 사전확률 함수  $p(\vec{\theta}|M)$  결정
3. 사후 확률함수밀도  $p(\vec{\theta}|D, M)$  계산
4. 최대 사후확률 값(MAP)  
최대 가능성 함수 값(ML)  
사후확률평균  $\bar{\theta} = \int \theta p(\theta|D, M) d\theta$   
 $p(\theta|D, M) = \int p(\vec{\theta}|D, M) d\vec{\theta}'$ , 다른 매개변수에 대한 적분 marginalization
5. 매개변수에 대한 분산
6. 모델과 매개변수에 대한 가설검증

# 샘플 생성 방법

- Markov Chain Monte Carlo(MCMC)
- Nested Sampling



$$Z = \int p(M|D)dM$$

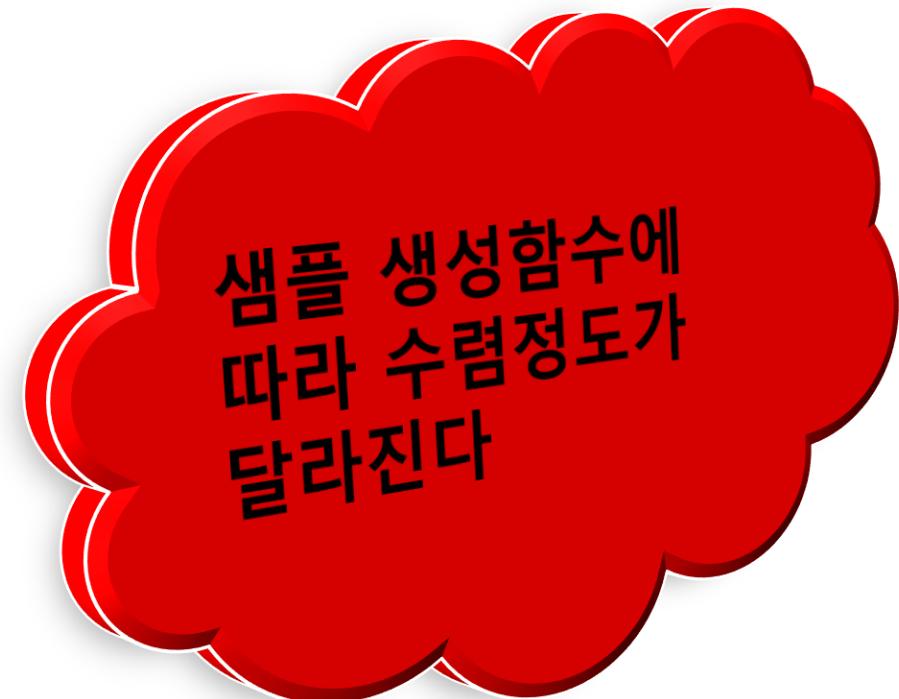
$$p(M|D) = \frac{p(D|M) p(M)}{p(D)}$$

[MNRAS 493, 3132\(2020\)](#)

사후분포를 따르는  
많은 독립적인 샘  
플을 생성한다

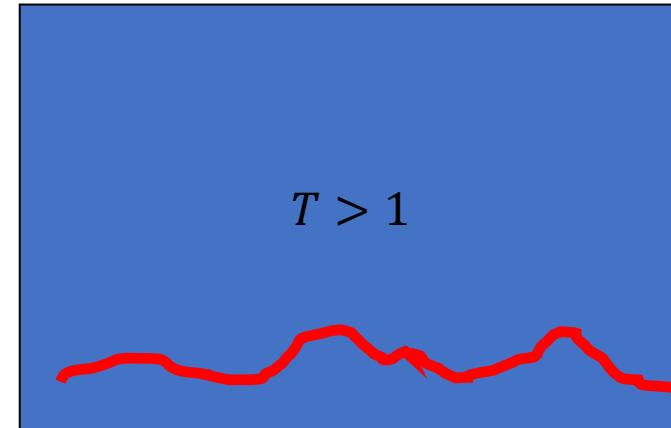
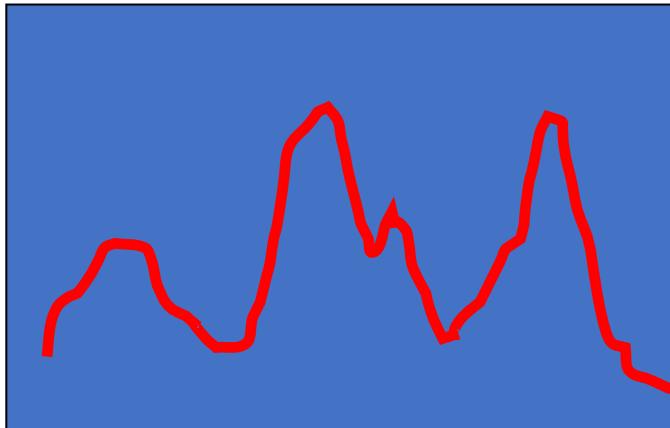
# MCMC 알고리즘

- Metropolis algorithm
  - Metropolis-Hastings algorithm
  - Gibbs sampling algorithm
  - Hamiltonian Monte Carlo
  - ...
- 
- 초기수렴(Burn In)
  - 수렴(Convergence)
  - 혼합(Mixing)

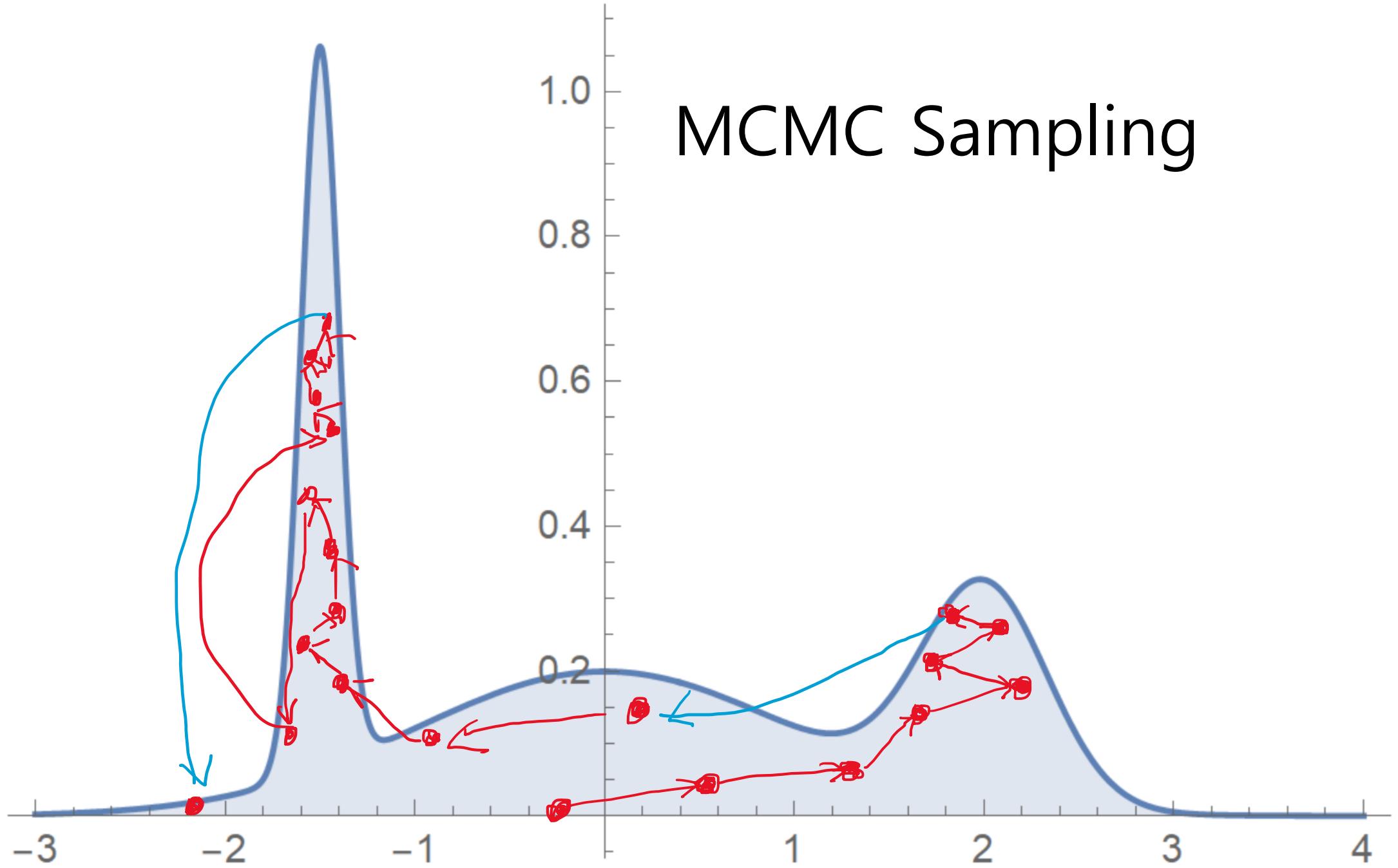


# 온도 병렬계산(parallel tempering)

- 몇 개의 다른 가상의 온도를 사용한다
- 가능도 함수를 온도로 조정한다  $p(d|\theta)^{\frac{1}{T}}, T > 1$
- $T_{max} = \frac{(Network SNR)^2}{n_{par}}$
- 수렴과 혼합을 개선한다



# MCMC Sampling



# Nested Algorithm

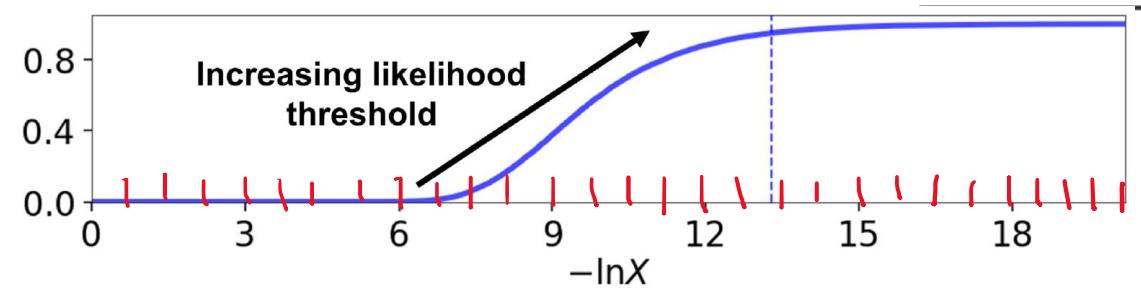
- 믿음(evidence) 계산
- $Z = \int_{\Omega_{\vec{\theta}}} p(D|\vec{\theta}, M)p(\vec{\theta}|M)d\vec{\theta} = \int_0^1 p(D, M, X)dX$
- $X(\lambda) = \int_{\Omega_{\vec{\theta}}:p(D|\vec{\theta}, M) \geq \lambda} p(\vec{\theta}|M)d\vec{\theta}, X(\lambda = 0) = 1, X(\lambda = \infty) = 0$

높은 차원의 적분을 효과적으로 1차원 적분으로 계산

### Algorithm 1: Static Nested Sampling

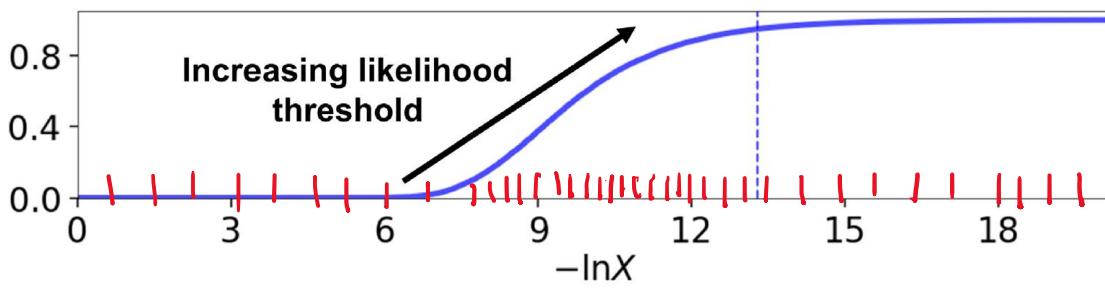
```
// Initialize live points.  
Draw  $K$  “live” points  $\{\Theta_1, \dots, \Theta_K\}$  from the prior  $\pi(\Theta)$ .  
// Main sampling loop.  
while stopping criterion not met do  
    Compute the minimum likelihood  $\mathcal{L}^{\min}$  among the current set of live points.  
    Add the  $k$ th live point  $\Theta_k$  associated with  $\mathcal{L}^{\min}$  to a list of “dead” points.  
    Sample a new point  $\Theta'$  from the prior subject to the constraint  $\mathcal{L}(\Theta') \geq \mathcal{L}^{\min}$ .  
    Replace  $\Theta_k$  with  $\Theta'$ .  
    // Check whether to stop.  
    Evaluate stopping criterion.  
end  
// Add final live points.  
while  $K > 0$  do  
    Compute the minimum likelihood  $\mathcal{L}^{\min}$  among the current set of live points.  
    Add the  $k$ th live point  $\Theta_k$  associated with  $\mathcal{L}^{\min}$  to a list of “dead” points.  
    Remove  $\Theta_k$  from the set of live points.  
    Set  $K = K - 1$ .  
end
```

등간격 적분



MNRAS 493, 3132(2020)

## 반응형 간격 적분



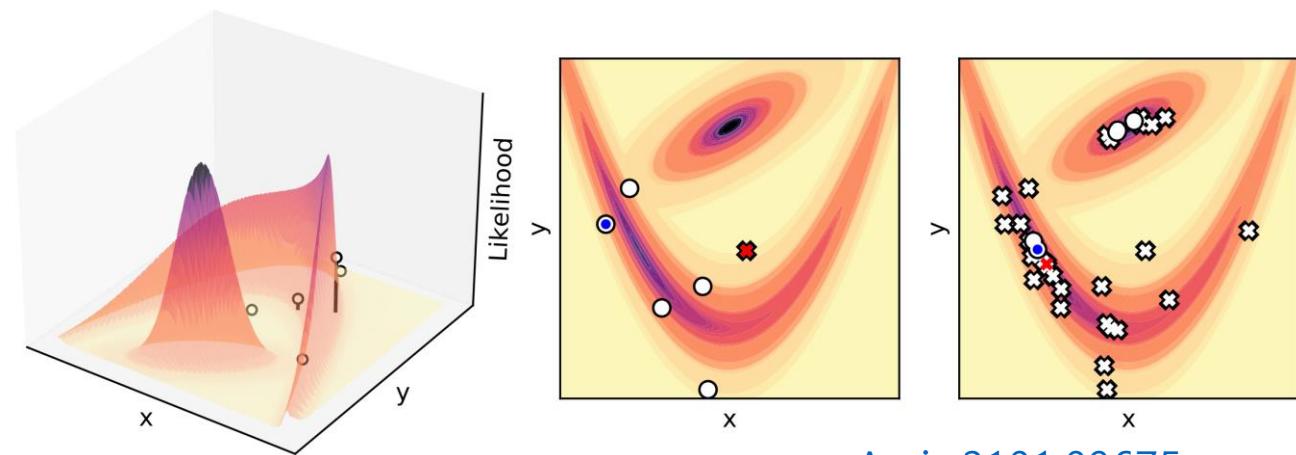
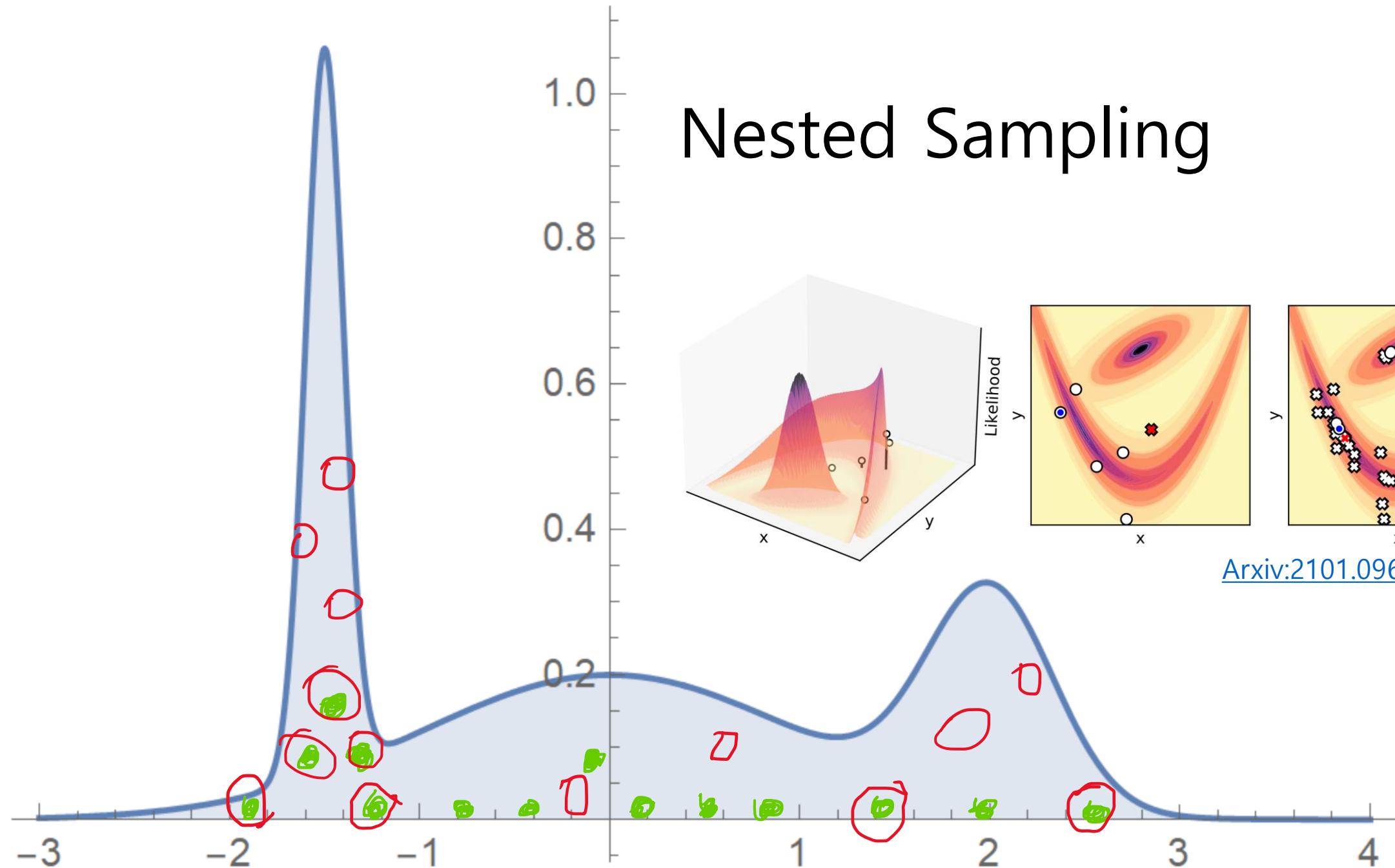
### Algorithm 2: Dynamic Nested Sampling

```

// Initialize first set of live points.
Draw  $K$  “live” points  $\{\Theta_1, \dots, \Theta_K\}$  from the prior  $\pi(\Theta)$ .
// Main sampling loop.
Set  $\mathcal{L}^{\min} = 0$  and  $K_0 = K$ .
while stopping criterion not met do
    // Get current number of live points.
    Compute the previous number of live points  $K$  and the current number of live points  $K'$ .
    if  $K' \geq K$  then
        // Add in new live points.
        while  $K' > K$  do
            Sample a new point  $\Theta'$  from the prior subject to the constraint  $\mathcal{L}(\Theta') \geq \mathcal{L}^{\min}$ .
            Add  $\Theta'$  to the set of live points.
            Set  $K = K + 1$ .
        end
        // Replace worst live point.
        Compute the minimum likelihood  $\mathcal{L}^{\min}$  among the current set of  $K$  live points.
        Add the  $k$ th live point  $\Theta_k$  associated with  $\mathcal{L}^{\min}$  to a list of “dead” points.
        Replace  $\Theta_k$  with  $\Theta'$ .
    else
        // Iteratively remove live points.
        while  $K' < K$  do
            Compute the minimum likelihood  $\mathcal{L}^{\min}$  among the current set of  $K = K'$  live points.
            Add the  $k$ th live point  $\Theta_k$  associated with  $\mathcal{L}^{\min}$  to a list of “dead” points.
            Remove  $\Theta_k$  from the set of live points.
            Set  $K = K - 1$ .
        end
        // Check whether to stop.
        Evaluate stopping criterion.
    end
    // Add final live points.
    while there are live points remaining do
        Compute the minimum likelihood  $\mathcal{L}^{\min}$  among the current set of live points.
        Add the  $k$ th live point  $\Theta_k$  associated with  $\mathcal{L}^{\min}$  to a list of “dead” points.
        Remove  $\Theta_k$  from the set of live points.
    end

```

[MNRAS 493, 3132\(2020\)](#)



[Arxiv:2101.09675](https://arxiv.org/abs/2101.09675)

# 분석도구

# LALSuite

([LALSuite: Main Page \(ligo.org\)](#))

- C based LSC Algorithm Library Suite
- LALInferenceMCMC.c : MCMC Sampler
- LALInferenceNest.c : Nested Sampler
- RIFT(Rapid Iterative Fitting)

# LALSuite from source

## Building LALSuite from source

### Dependencies

### Build tools

The following build tools will be needed to build LALSuite components from source

- a C compiler with support for the C99 standard
- autoconf (building from git only)
- automake (building from git only)
- make
- pkg-config

### Library dependencies

For LAL the library dependencies are:

- [GSL](#) - The GNU Scientific Library.
- [FFTW](#) - The Fastest Fourier Transform in the West.
- [HDF5](#) - The HDF5 library
- [zLib](#) - A Massively Spiffy Yet Delicately Unobtrusive Compression Library

Other subpackages need at least (but not limited to):

- [!FrameL](#) - LIGO/Virgo Frame library (needed for LALFrame)
- [!MetaIO](#) - LIGO\_LW XML library ([LALMetaIO](#))
- [CFITSIO](#) - A FITS File Subroutine Library (needed for LALPulsar)

All Dependencies can be installed using an appropriate Package manager, **you should not need to compile any of these yourself.**

The Python layers for each subpackage will have extra requirements that are not specified here.

# LALSuite from source

## Building from the git repository

The repository is hosted on the LIGO [GitLab](#) instance, please see the following [computing guide](#) page for details on accessing repositories hosted here. The LALSuite repository also utilizes [git-lfs](#) for the management of large file so please ensure that you have configured [git-lfs](#) on your system.

You can then clone the repository using:

```
git clone git@git.ligo.org:lscsoft/lalsuite.git
```

You can also clone using the https interface but the above SSH URL is recommended as this is more robust:

```
git clone https://git.ligo.org/lscsoft/lalsuite.git
```

If you are cloning anonymously then you *must* use the https URL.

You can then install LAL as follows:

```
LAL_INSTALL_PREFIX="${HOME}/opt/lalsuite" # change as appropriate
pushd lal
./00boot
./configure --prefix=${LAL_INSTALL_PREFIX}
make
make install
```

# LALSuite install in conda

## Cloning the Repository

We now utilize [Git LFS](#) for the management of large files and as such `git-lfs` needs to be installed and configured to correctly clone this repository. After installing `git-lfs` it can be configured using:

```
$ git lfs install
```

This only needs to be done once for each machine you access the repository. It can then be cloned using:

```
$ git clone git@git.ligo.org:lscsoft/lalsuite.git
```

## Building from Source

The recommended way to build LALSuite from source is in a `conda` environment. A [recipe file](#) is available with all main dependencies. This can serve as the base for custom [recipes](#), or be used directly via:

```
$ conda env create -f conda/environment.yml
```

Pulling in dependencies may take a while depending on your internet connection. After the environment setup succeeded, you can activate it with:

```
$ conda activate lalsuite-dev
```

You can then build the suite by executing, in order:

1. `./00boot` (once at first time)
2. `./configure` with appropriate options (see `./configure --help`)
3. `make`

After pulling updates or making your own changes, you will usually only need to call `make` again, as reconfiguration and re-running `00boot` should be handled automatically if needed.

# LALInference

([LALInference: Main Page \(ligo.org\)](#))

## Documentation

Here is a list of all modules:

### ▼ General Packages

[Header LALInference.h](#)

Main header file for LALInference common routines and structures

[Header LALInferenceLikelihood.h](#)

Header file for likelihood functions used by LALInference codes

[Header LALInferenceNestedSampler.h](#)

Nested sampler written for LALInference

[Header LALInferencePrior.h](#)

Collection of commonly used Prior functions and utilities

[Header LALInferenceProposal.h](#)

Jump proposals for exploring the GW signal parameter space

[Header LALInferenceReadData.h](#)

Utility functions for handling IFO data

[Header LALInferenceRemoveLines.h](#)

Utility functions for identifying lines in IFO data to be removed in LALInference

[Header LALInferenceTemplate.h](#)

Main header file for LALInference signal template generating functions

[Header LALInferenceVCSInfo.h](#)

VCS and build information for LALInference

### ▼ SWIG Interfaces

[Interface SWIGLALInferenceAlpha.i](#)

SWIG code which must appear *before* the LALInference headers

[Interface SWIGLALInferenceOmega.i](#)

SWIG code which must appear *after* the LALInference headers

### Python Packages

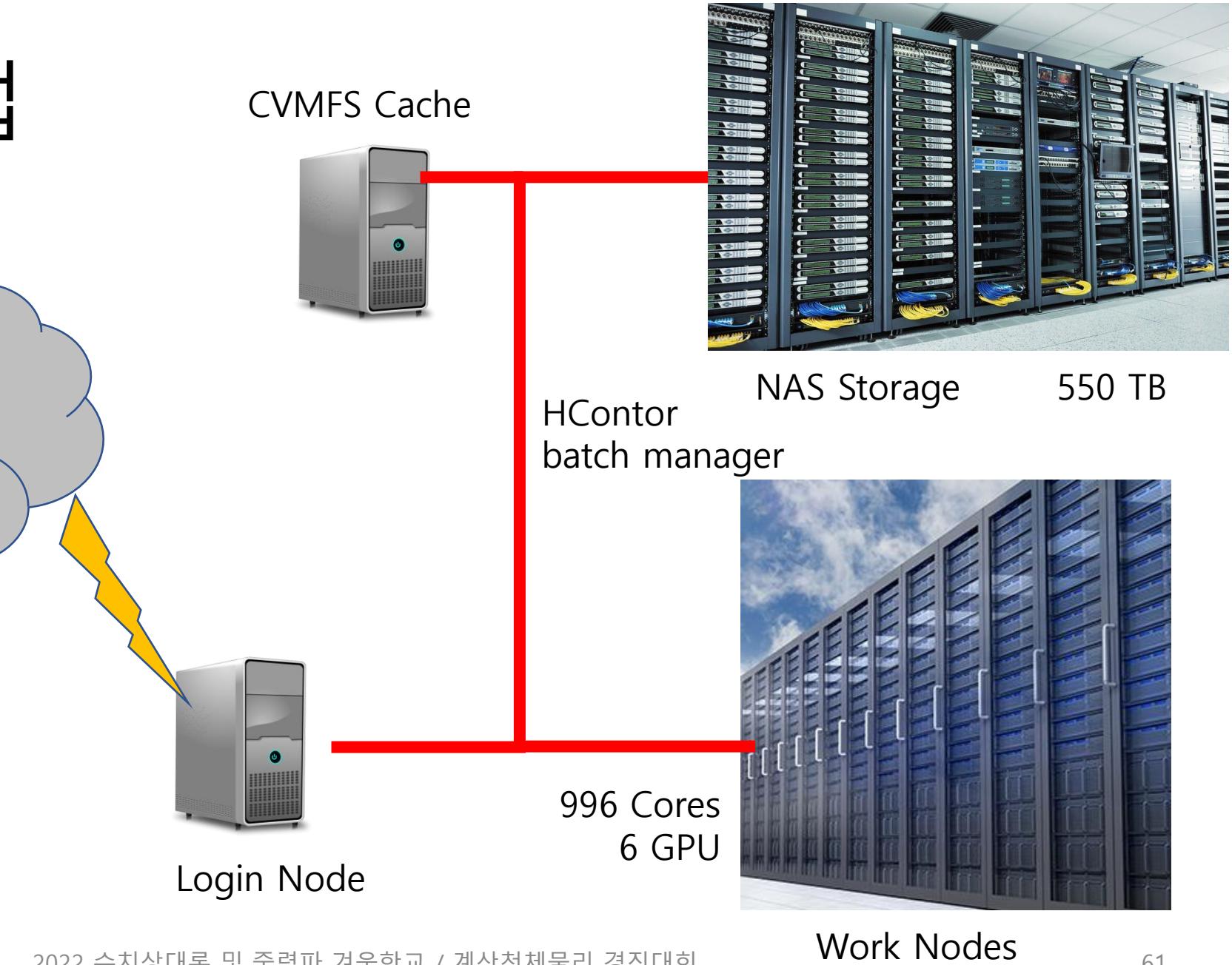
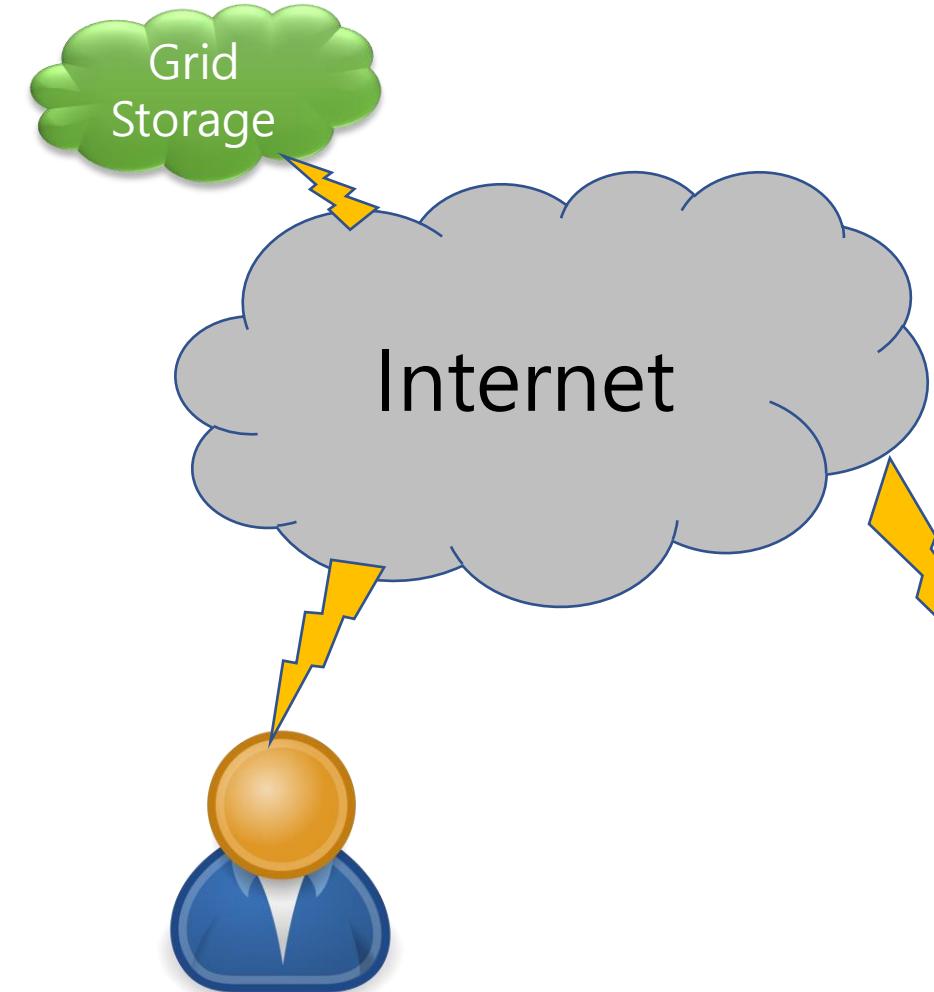
lalinference\_mcmc  
lalinference\_nest

# SWIG(Simplified Wrapper and Interface Generator)

- SWIG is a software development tool that connects programs written in C and C++ with a variety of high-level programming languages. SWIG is used with different types of target languages including common scripting languages such as Javascript, Perl, PHP, Python, Tcl and Ruby. The list of [supported languages](#) also includes non-scripting languages such as C#, D, Go language, Java including Android, Lua, OCaml, Octave, Scilab and R. Also several interpreted and compiled Scheme implementations (Guile, MzScheme/Racket) are supported. SWIG is most commonly used to create high-level interpreted or compiled programming environments, user interfaces, and as a tool for testing and prototyping C/C++ software. SWIG is typically used to parse C/C++ interfaces and generate the 'glue code' required for the above target languages to call into the C/C++ code. SWIG can also export its parse tree in the form of XML. SWIG is free software and the code that SWIG generates is compatible with both commercial and non-commercial projects.



# 키스티 작업



# 작업 수행

```
[screen 0: bash] hwlee@ldg-ui01:/data/ligo/scratch/hwlee/eccwork/gw151226/ecc-ecc-0.04
#!/bin/sh

module load openmpi-x86_64
#source /opt/intel/parallel_studio_xe_2018.1.038/bin/psxevars.sh
#export masterdir=/data/ligo/scratch/pe/LAL/lalinference_o2_eccTides/
export masterdir=/data/ligo/scratch/pe/LAL/eccentricity_160810
source $masterdir/etc/lscsoftrc
export PYTHON=/usr/lib64/python2.7/site-packages

date

cd /data/ligo/scratch/hwlee/eccwork/gw151226/ecc-ecc-0.04
condor_submit Onoiseecc0.sub
condor_submit Onoiseecc0.sub
condor_submit Onoiseecc0.sub
condor_submit Onoiseecc0.sub
condor_submit Onoiseecc0.sub
condor_submit Onoiseecc0.sub

exit
```

```
[screen 0: bash] hwlee@ldg-ui01:/data/ligo/scratch/hwlee/eccwork/gw151226/ecc-ecc-0.04
universe      = vanilla
getenv        = true
executable   = /usr/lib64/openmpi/bin/mpirun
arguments     = -np 11 /data/ligo/scratch/pe/LAL/eccentricity_160810/bin/lalinference_mcmc --outfile PTMCMC.output.$(Cluster)-$(Process)
               .h5 --ifo H1 --H1-cache LALSimAdLIGO --H1-flow 25 --trigtime 894383679.0 --psdstart 894383379.0 --psdlength 1024.0 --seglen 16 --rate
               2048 --inj ./taylorF2EccH1onlyBBHGW151226.xml --event 4 --inj-fref 100 --inj-spinOrder 0 --inj-tidalOrder -1 --approx TaylorF2Ec
               chthreePointFivePN --fref 100 --nsteps 20000000 --skip 100 --neff 10000 --amporder Newtonian --spinOrder 0 --tidalOrder -1 --radiation-frame --margtime --tempLadderBottomUp --differential-buffer-limit 100000 --dataseed 12345 --noise --noSpin --tidalT --quadparam

output       = $(Cluster)-$(Process).f2ecc-ecc.gw151226.Onoise.out
error        = $(Cluster)-$(Process).f2ecc-ecc.gw151226.Onoise.err
log          = $(Cluster)-$(Process).f2ecc-ecc.gw151226.Onoise.log
request_cpus = 11
request_memory = 11*6*1024

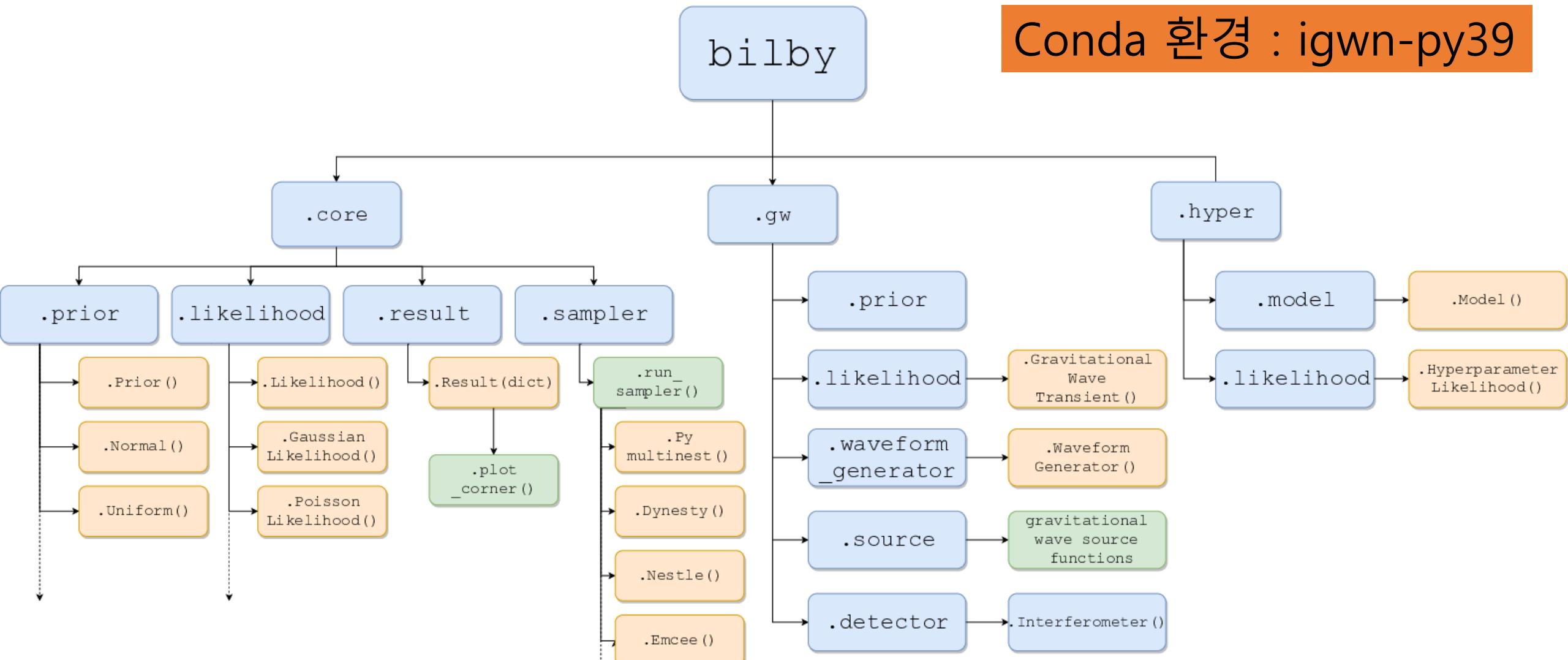
queue 1
```



# Bilby

(Welcome to bilby's documentation! — bilby 1.1 documentation (ligo.org))

Conda 환경 : igwn-py39



# Installation

Conda

Pip

```
$ conda install -c conda-forge bilby
```

Supported python versions: 3.6+.

This will install all requirements for running `bilby` for general inference problems, including our default sampler `dynesty`. Other samplers will need to be installed via pip or the appropriate means.

# Dynesty Guide

The Dynesty sampler is just one of the samplers available in bilby, but it is well-used and found to be fast and accurate. Here, we provide a short guide to its implementation. This will not be a complete guide, additional help can be found in the [Dynesty documentation](#).

All of the options discussed herein can be set in the `bilby.run_sampler()` call. For example, to set the number of live points to 1000

```
>>> bilby.run_sampler(likelihood, priors, sampler="dynesty", nlive=1000)
```

# Bilby MCMC Guide

Bilby MCMC is a native sampler built directly in `bilby` and described in [Ashton & Talbot \(2021\)](#). Here, we describe how to use it.

## Quickstart and output

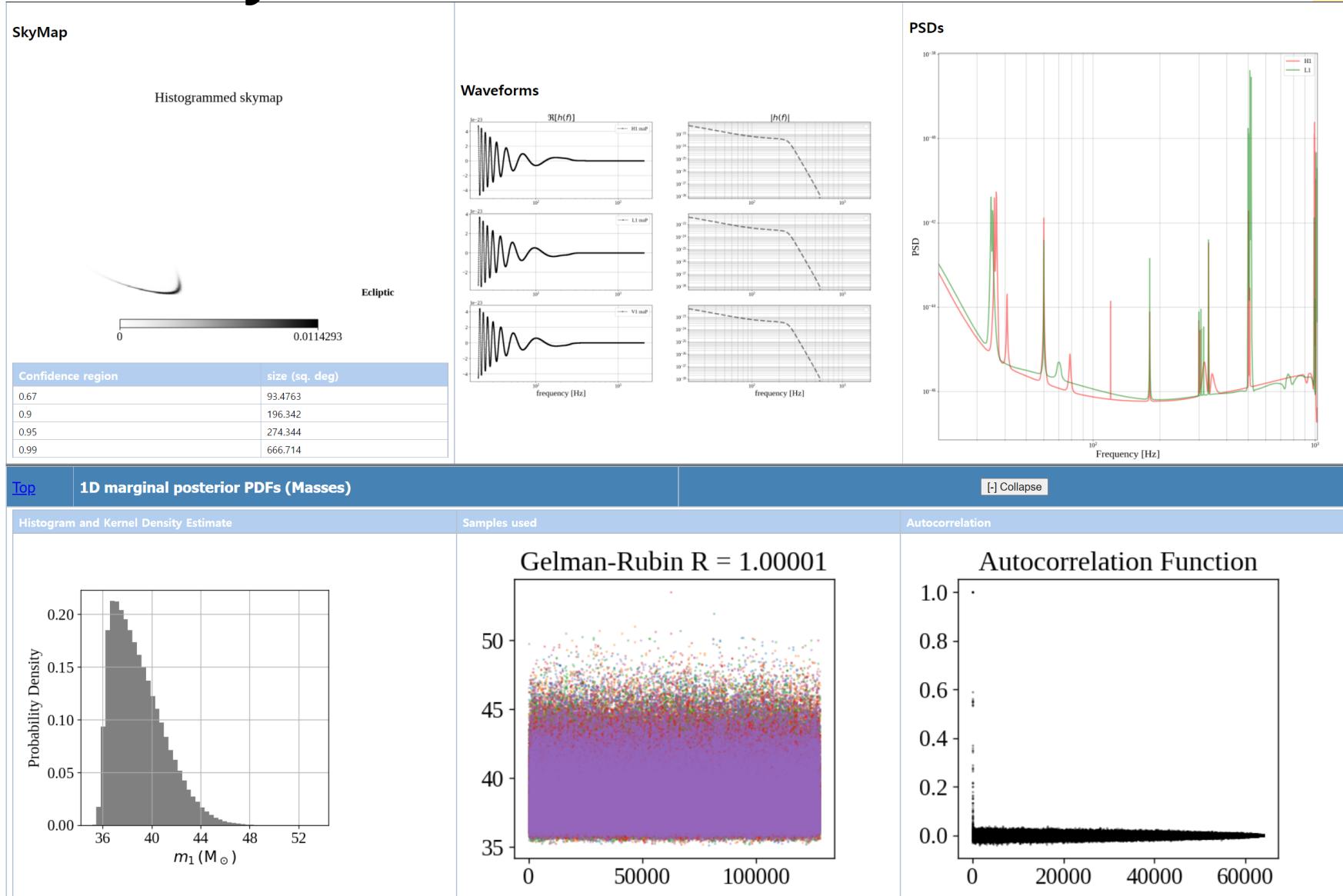
To use the `bilby_mcmc` sampler, we call

```
>>> bilby.run_sampler(likelihood, priors, sampler="bilby_mcmc", nsamples=1000)
```

This will run the MCMC sampler until 1000 independent samples are drawn from the posterior. As the sampler is running, it will print output like this

# 후처리 도구

# CBCBayesPostProc



# Install PESummary

`PESummary` is developed and tested for python 3.5+. We recommend that this code is installed inside a virtual environment using `virtualenv`. This environment can be installed with python 3.5+ using `pyenv`.

For detailed instructions on how to set up your virtual environment, please refer to [setting up a virtual environment](#).

## Installing PESummary using pip

If you choose to install `PESummary` using `pip`, then simply run:

```
$ source ~/virtualenvs/pesummary_py3.6/bin/activate  
$ pip install pesummary
```

## Installing PESummary using conda

If you choose to install `PESummary` using `conda`, then simply run:

```
$ source ~/virtualenvs/pesummary_pyenv3.6/bin/activate  
$ conda install -c conda-forge pesummary
```

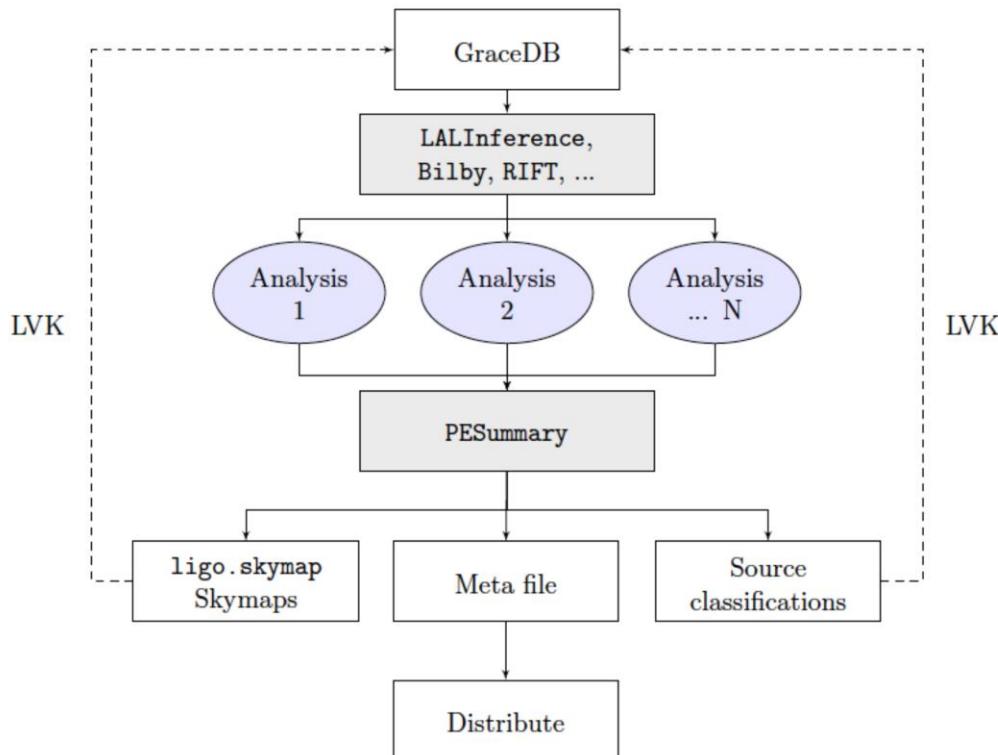
## Pulling the PESummary docker image

If you would like, you are able to pull the `PESummary` docker image. To do this, simply run:

```
$ docker pull 08hoyc/pesummary
```

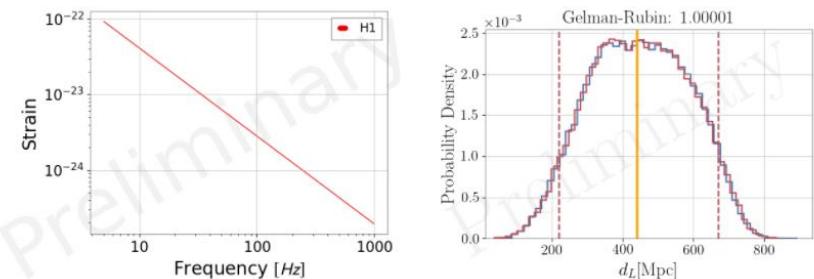
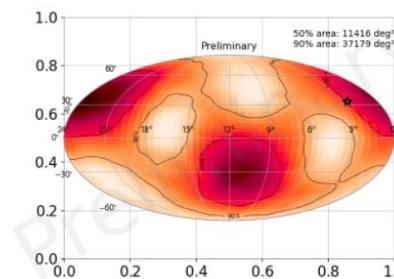
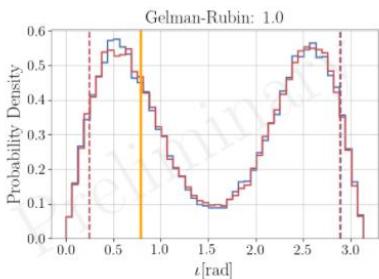
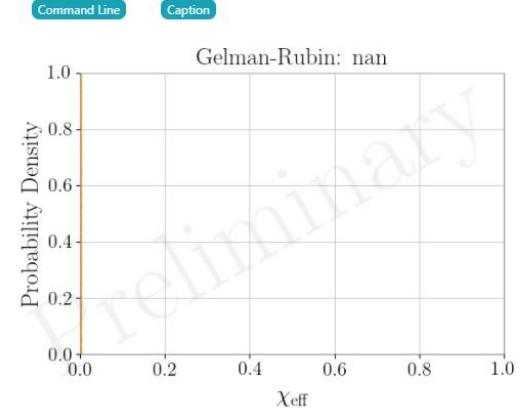
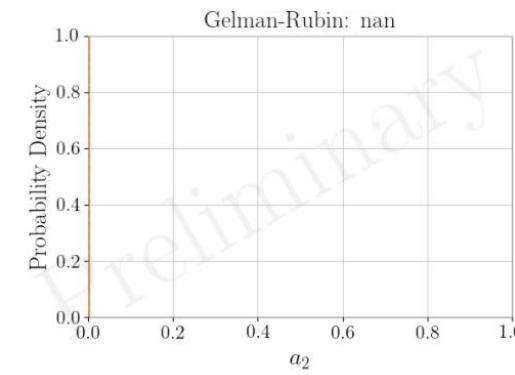
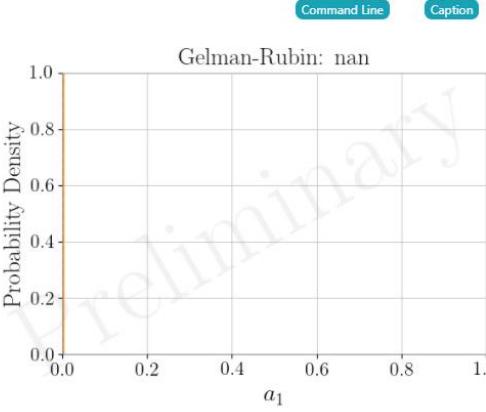
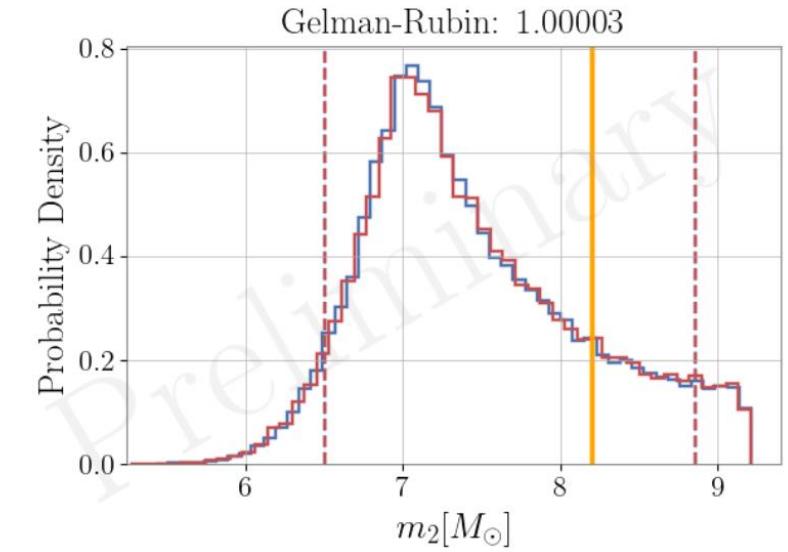
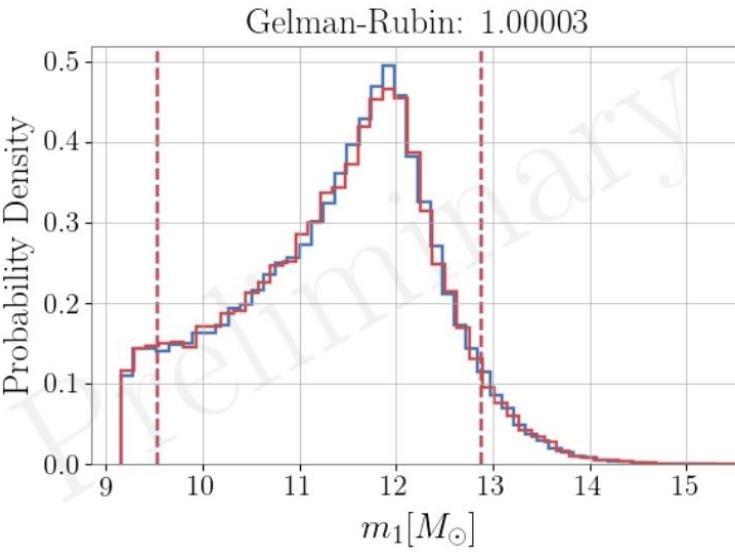
# PESummary ([PESummary \(ligo.org\)](#))

- 모수추정 후처리 결과 생성  
라이브러리



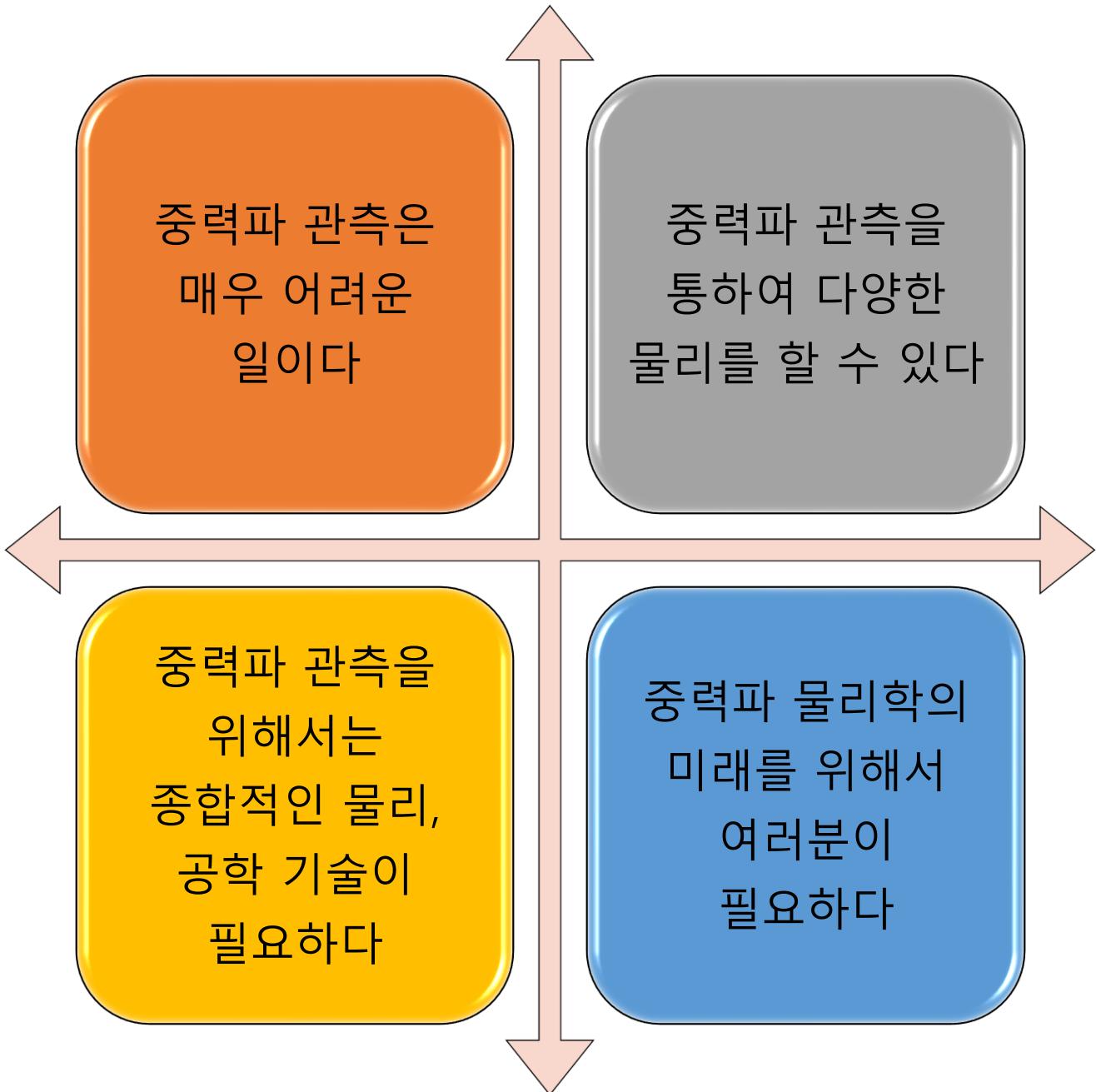
# PESummary

- Bootstrap css
- Interactive



맺음 말

# 결론



# 질문

감사합니다