Econ 424 Lab 7, Winter 2020

Introduction

In this lab you will

- Compute portfolios consisting of Amazon and Boeing, T-bills and Amazon, T-bills and Boeing and T-bills and combinations of Amazon and Boeing.
- Use R functions to compute the global minimum variance portfolio and the tangency portfolio
- Do simple asset allocation with efficient portfolios

This notebook walks you through all of the computations for the lab. You will use the following R packages

- **IntroCompFinR**
- **PerformanceAnalytics**.
- **zoo**
- **xts**

Make sure to install these packages before you load them into R. As in the previous labs, use this notebook to answer all questions. Insert R chunks where needed. I will provide code hints below.

Reading

- EZ chapter 11 (Introduction to Portfolio Theory)
- Ruppert and Matteson, Chapter 16 (Portfolio Selection)

Load packages and set options

```
library(IntroCompFinR)
```

```
## Loading required package: xts
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
library(PerformanceAnalytics)
```

```
##
## Attaching package: 'PerformanceAnalytics'
```

```
## The following object is masked from 'package:graphics':
##
##     legend
```

```
library(xts)
options(digits = 3)
Sys.setenv(TZ="UTC")
```

Data

For this lab you will use annualized estimates of the CER model for Amazon and Boeing based on monthly simple returns over the period January 2010 through December 2014. First construct monthly returns from daily prices:

```
data(amznDailyPrices, baDailyPrices)
amznMonthlyPrices = to.monthly(amznDailyPrices, OHLC=FALSE)
baMonthlyPrices = to.monthly(baDailyPrices, OHLC=FALSE)
amznMonthlyRetS = na.omit(Return.calculate(amznMonthlyPrices, method="simple"))
baMonthlyRetS = na.omit(Return.calculate(baMonthlyPrices, method="simple"))
amznBaRetS = merge(amznMonthlyRetS, baMonthlyRetS)
smpl = "2010::2014"
amznBaRetS = amznBaRetS[smpl]
head(amznBaRetS, n=3)
```

```
##             AMZN     BA
## Jan 2010 -0.0677 0.1195
## Feb 2010 -0.0559 0.0494
## Mar 2010  0.1467 0.1495
```

Next, estimate the CER model parameters for Amazon and Boeing and annualize using the square root of time rule:

```
muhat = apply(amznBaRetS, 2, mean)*12
sig2hat = apply(amznBaRetS, 2, var)*12
sighat = apply(amznBaRetS, 2, sd)*sqrt(12)
covhat = cov(amznBaRetS)[1,2]*12
rhohat = cor(amznBaRetS)[1,2]
```

The annualized expected returns are:

```
muhat
```

```
##  AMZN    BA
## 0.205 0.222
```

The annualized variances are

```
sig2hat
```

```
##   AMZN     BA
## 0.0771 0.0453
```

The annualized volatilities are

```
sighat
```

```
##  AMZN    BA
## 0.278 0.213
```

The annualized covariance is

```
covhat
```

```
## [1] 0.02
```

The correlation, which is invariant to annualization, is

```
rhohat
```

```
## [1] 0.338
```

Assume a risk-free T-bill rate of 1% per year.

```
rf = 0.01
```

Exercises

1. Create the following portfolios:
    - Combinations of Amazon and Boeing with $x_A = -1, -0.9, \cdots, 2$ and $x_B = 1 - x_A$.
    - Combinations of Amazon and T-Bills with $x_A = 0, 0.1 \cdots, 2$ and $x_f = 1 - x_A$.
    - Combinations of Boeing and T-Bills with $x_B = 0, 0.1 \cdots, 2$ and $x_f = 1 - x_B$.

For each set of portfolios compute $E[R_p]$, $var(R_p)$, $SD(R_p)$ using the appropriate formulas. For each set of portfolios plot $E[R_p]$ vs $SD(R_p)$ and put these all on the same risk-return graph. Compute Sharpe's slope for Amazon and Boeing. Which asset has the highest slope value? Are you surprised?

```
amznRet=amznBaRetS[,1]
Amuhat = apply(amznRet, 2, mean)*12
baRet=amznBaRetS[,2]
Bmuhat=apply(baRet, 2, mean)*12
asigmahat=apply(amznRet, 2, sd)*sqrt(12)
bsigmahat=apply(baRet, 2, sd)*sqrt(12)
x.A=seq(from=-1,to=2,by=.1)
x.B=1-x.A
mu.p1=x.A*Amuhat + x.B*Bmuhat
mu.p1
```

```
##  [1] 0.239 0.237 0.235 0.234 0.232 0.230 0.229 0.227 0.225 0.224 0.222 0.220
## [13] 0.219 0.217 0.215 0.214 0.212 0.210 0.209 0.207 0.205 0.204 0.202 0.200
## [25] 0.199 0.197 0.195 0.194 0.192 0.190 0.188
```

```
var1=(x.A)^2*(asigmahat)^2 + (x.B)^2*(bsigmahat)^2 +2*x.A*x.B*rhohat*asigmahat*bsigmahat
var1
```

```
##  [1] 0.1786 0.1578 0.1387 0.1213 0.1055 0.0913 0.0788 0.0680 0.0588 0.0512
## [11] 0.0453 0.0411 0.0385 0.0375 0.0383 0.0406 0.0446 0.0503 0.0576 0.0665
## [21] 0.0771 0.0894 0.1033 0.1189 0.1361 0.1549 0.1755 0.1976 0.2214 0.2469
## [31] 0.2740
```

```
sd1=sqrt(var1)
sd1
```

```
##  [1] 0.423 0.397 0.372 0.348 0.325 0.302 0.281 0.261 0.242 0.226 0.213 0.203
## [13] 0.196 0.194 0.196 0.202 0.211 0.224 0.240 0.258 0.278 0.299 0.321 0.345
## [25] 0.369 0.394 0.419 0.445 0.471 0.497 0.523
```

```
x.AT = seq(from=0, to=2, by=0.1)
 mu.p.A = rf + x.AT*(Amuhat - rf)
sig.p.A = x.AT*asigmahat
var.p.A=x.AT^2*asigmahat^2
sharpe.A = (Amuhat - rf)/asigmahat
sharpe.A
```

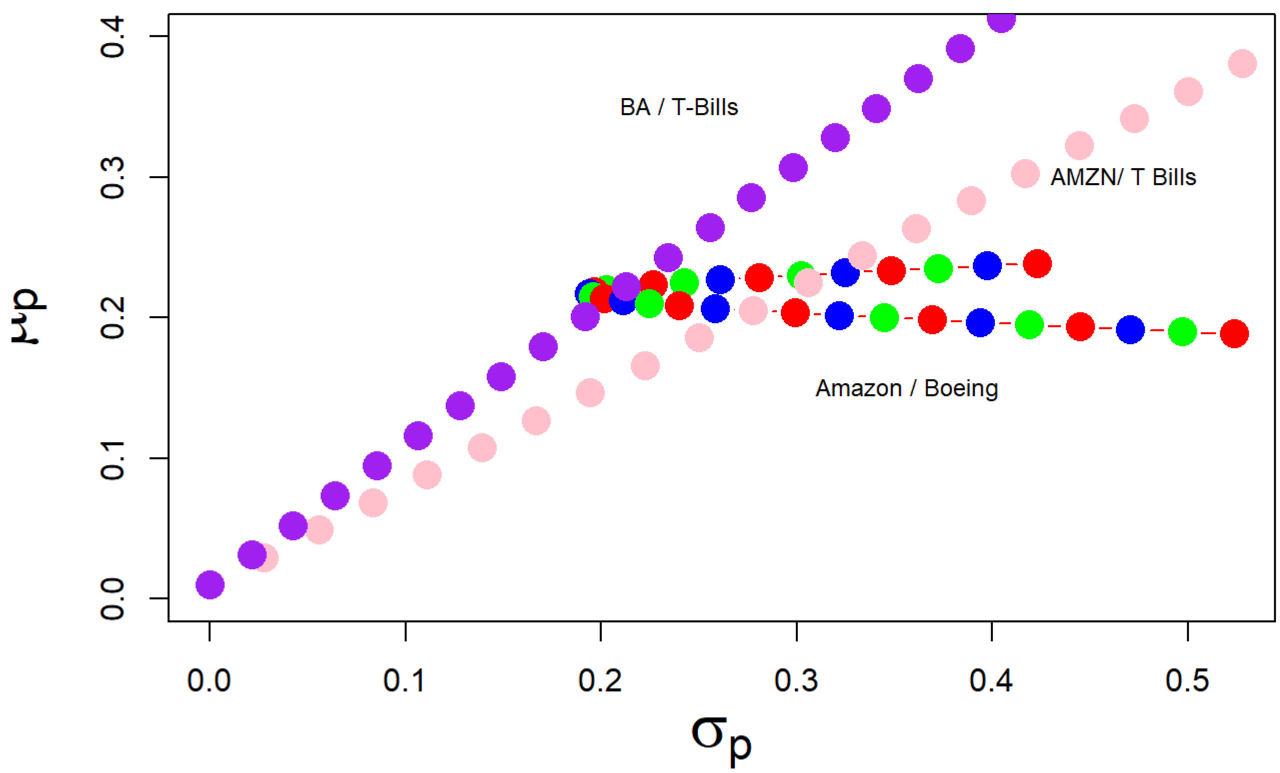
```
##  AMZN
## 0.703
```

```
x.BT = seq(from=0, to=2, by=0.1)
mu.p.B = rf + x.BT*(Bmuhat - rf)
sig.p.B = x.BT*bsigmahat
var.p.B = x.BT^2*bsigmahat^2
sharpe.B = (Bmuhat - rf)/bsigmahat
sharpe.B
```

```
##    BA
## 0.996
```

Boeing has the higher Sharpe Ratio telling us that it has higher excess return per unit of risk. This isn’t surprising, because it has a higher annualized mean and a lower annualized standard deviation than Amazon.

```
cex.val=2
plot(sd1, mu.p1, type="b", pch=16, cex = cex.val,
     ylim=c(0,0.4), xlim=c(0, max(sd1)),
     xlab=expression(sigma[p]), ylab=expression(mu[p]), cex.lab=cex.val,
     col=c(rep("red", 1), "blue", rep("green", 1)))
points(sig.p.A,mu.p.A, col="pink", pch=16, cex = cex.val)
points(sig.p.B,mu.p.B,col="purple",pch=16, cex = cex.val)
text(x=.2, y=.35, labels="BA / T-Bills", pos=4, cex = .75)
text(x=.42, y=.3, labels="AMZN/ T Bills", pos=4, cex = .75)
text(x=.3, y=0.15, labels="Amazon / Boeing", pos=4, cex = .75)
```



2. Compute the global minimum variance portfolio using the analytical formula presented in class and check with the **IntroCompFinR** function `globalMin.portfolio()` .
   - Make a bar chart showing the weight of Amazon and Boeing in global minimum variance portfolio.
   - Compute $E[R_m]$, $var(R_m)$, and $SD(R_m)$
   - Compute Sharpe's slope for the global minimum variance portfolio
   - Indicate the location of the global minimum variance portfolio on the graph you created previously in question 1

```
X.b = as.numeric(sig2hat[2])
X.a = as.numeric(sig2hat[1])
sig_ab = as.numeric(rhohat * sighat[1]*sighat[2])
X.a_min = (X.b - covhat)/(X.a+ X.b -2*covhat)
X.b_min = 1-X.a_min
gmin_mu = as.numeric(X.a_min*muhat[1] + X.b_min*muhat[2])
gmin_mu
```

```
## [1] 0.217
```

```
globalMin.portfolio(((X.a_min*muhat[1] + X.b_min*muhat[2])), sig_ab)
```

```
## Call:
## globalMin.portfolio(er = ((X.a_min * muhat[1] + X.b_min * muhat[2])),
##     cov.mat = sig_ab)
##
## Portfolio expected return:     0.217
## Portfolio standard deviation:  0.141
## Portfolio weights:
## AMZN
##    1
```

```
gmin_var = as.numeric((((X.a_min^2) * sig2hat[1]) + ((X.b_min^2) * sig2hat[2]) + (2*X.a_min*X.b_m
in*covhat))
gmin_var
```

```
## [1] 0.0375
```

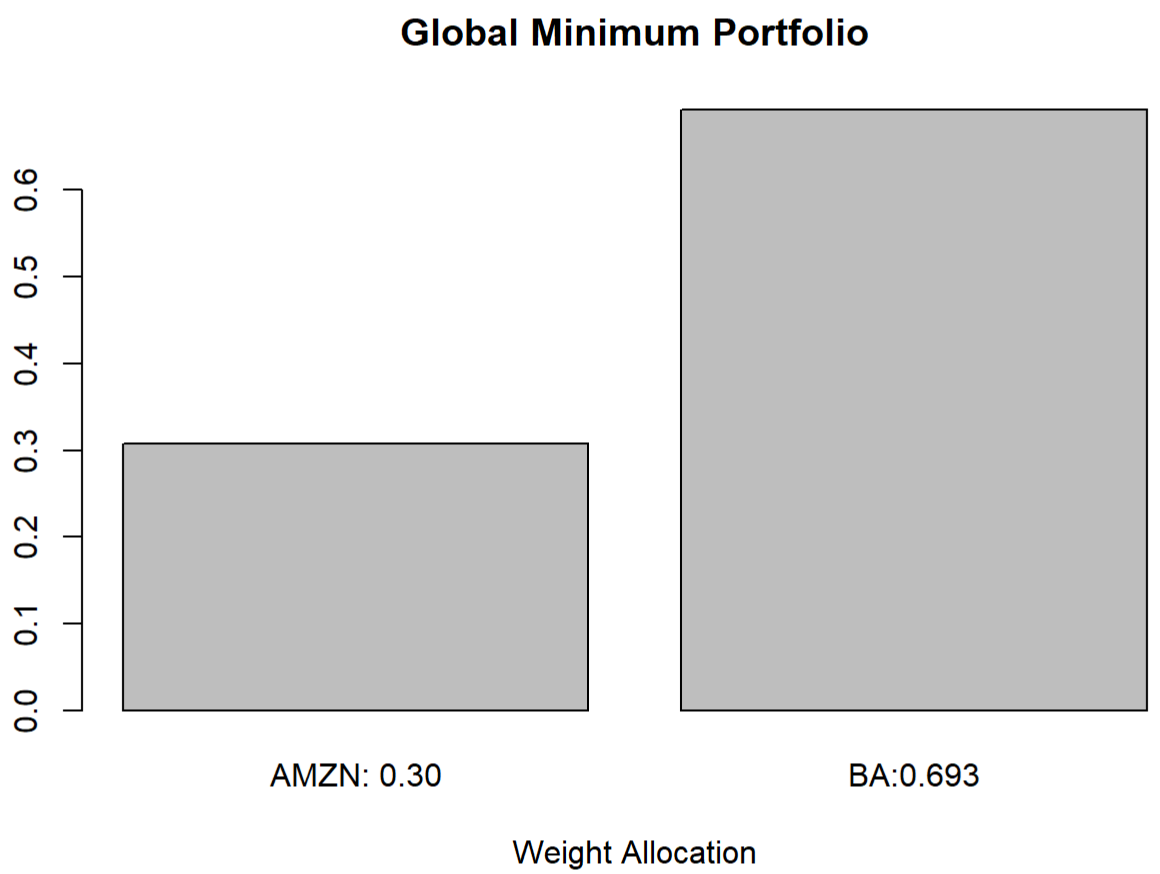
```
gmin_sd = as.numeric(sqrt(gmin_var))
gmin_sd
```

```
## [1] 0.194
```

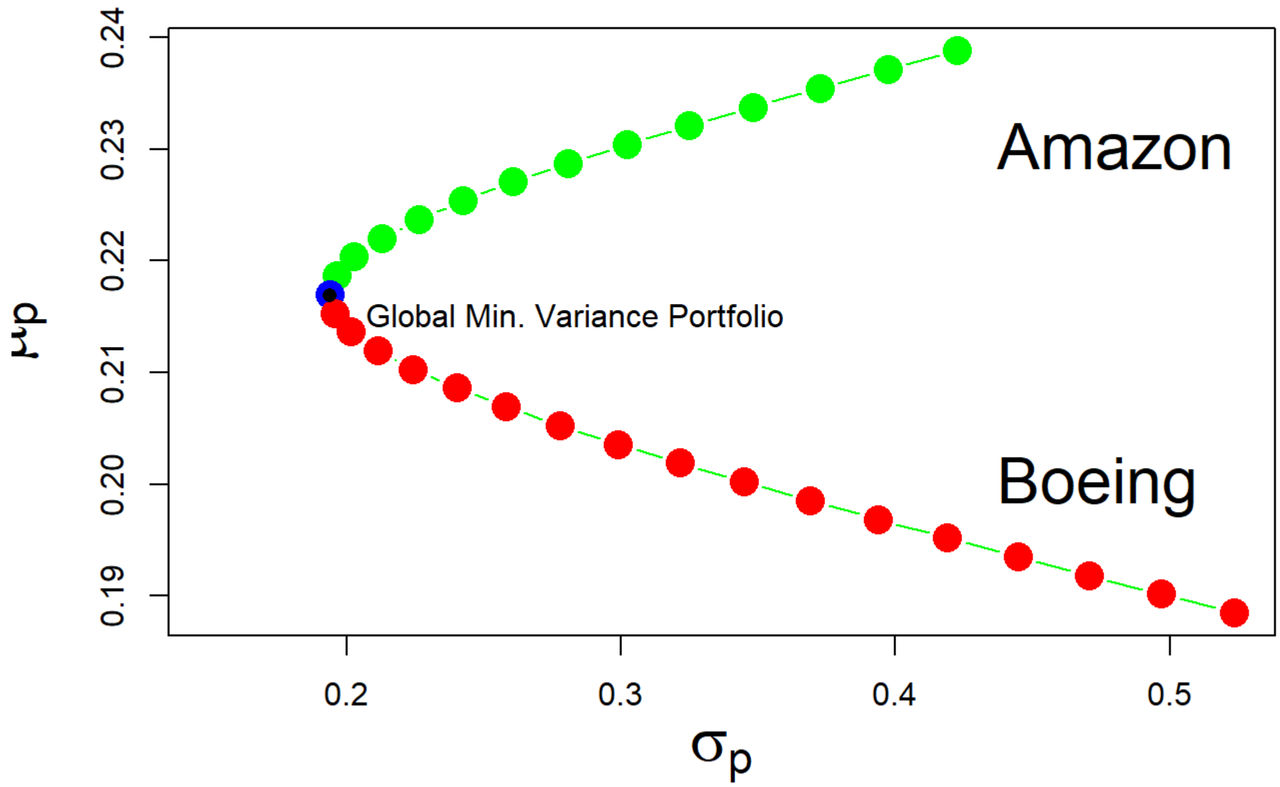
```
gmin_sharpe = as.numeric((gmin_mu - rf)/gmin_sd)
gmin_sharpe
```

```
## [1] 1.07
```

```
gmin_weights = c(X.a_min, X.b_min)
barplot(gmin_weights, main='Global Minimum Portfolio',
        xlab='Weight Allocation', names.arg=c('AMZN: 0.30', 'BA:0.693'))
```




```
cex.val = 2
plot(sd1, mu.p1, type="b", pch=16, cex = cex.val,
ylim=c(min(mu.p1), max(mu.p1)), xlim=c(0.15, max(sd1)),
xlab=expression(sigma[p]), ylab=expression(mu[p]), cex.lab=cex.val,
col=c(rep("green", 13), "blue", rep("red", 30)))
points(gmin_sd, gmin_mu, type='b', pch=16)
text(x=.43, y=.23, labels="Amazon", pos=4, cex = cex.val)
text(x=.43, y=.20, labels="Boeing", pos=4, cex = cex.val)
text(x=.2, y=.215, labels="Global Min. Variance Portfolio", pos=4, cex = 1)
```



3. Using a risk-free rate of 3% per year for the T-bill, compute the tangency portfolio using the analytical formula presented in class and check using the **IntroCompFinR** function `tangency.portfolio()` .
    - Make a pie chart showing the weight of Amazon and Boeing in the tangency portfolio.
    - Compute $E[R_t]$, $var(R_t)$, and $SD(R_t)$
    - Compute Sharpe's slope for the tangency portfolio.
    - Indicate the location of the tangency portfolio on the graph you created previously in question 1.

```
rf = 0.03
top = (muhat[1] - rf)*sig2hat[2] - (muhat[2] - rf)*sig_ab
bot = (muhat[1] - rf)*sig2hat[2] + (muhat[2] - rf)*sig2hat[1] - (muhat[1] - rf + muhat[2] - rf)*
sig_ab
x.A.tan = as.numeric(top/bot)
x.B.tan = as.numeric(1 - x.A.tan)
mu.p.tan = as.numeric(x.A.tan*muhat[1] + x.B.tan*muhat[2])
mu.p.tan
```

```
## [1] 0.218
```

```
tangency.portfolio(mu.p.tan, sig_ab, risk.free = 0.03)
```

```
## Call:
## tangency.portfolio(er = mu.p.tan, cov.mat = sig_ab, risk.free = 0.03)
##
## Portfolio expected return:     0.218
## Portfolio standard deviation:  0.141
## Portfolio weights:
## [1] 1
```

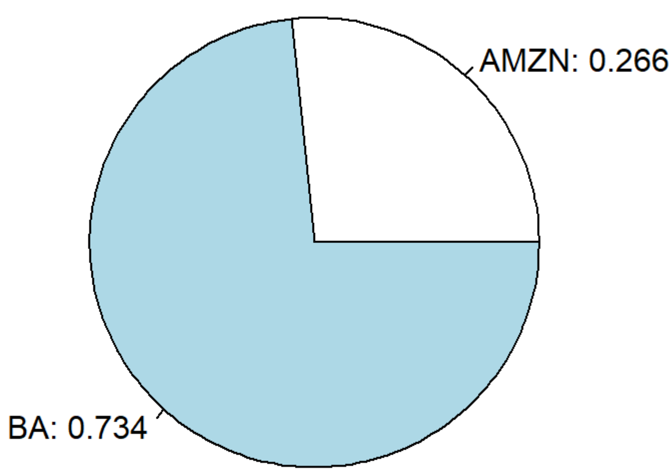
```
sig2.p.tan = as.numeric(x.A.tan^2 * sig2hat[1] + x.B.tan^2 * sig2hat[2] + 2*x.A.tan*x.B.tan*sig_
ab)
sig2.p.tan
```

```
## [1] 0.0377
```

```
sig.p.tan = as.numeric(sqrt(sig2.p.tan))
sig.p.tan
```

```
## [1] 0.194
```

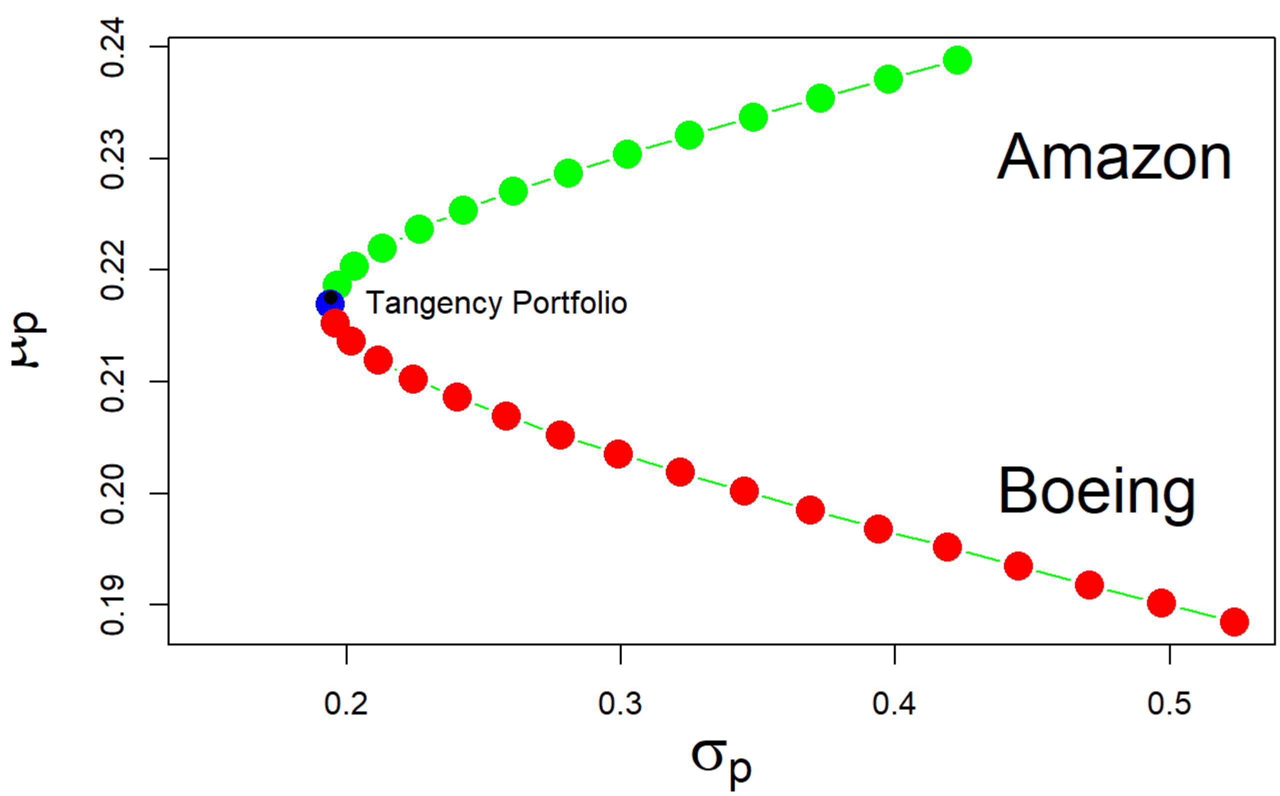
```
tan_weights = c(x.A.tan,x.B.tan)
lbls = c('AMZN: 0.266','BA: 0.734')
pie(tan_weights,lbls)
```



```
sharpeR = (mu.p.tan - rf)/sig.p.tan
sharpeR
```

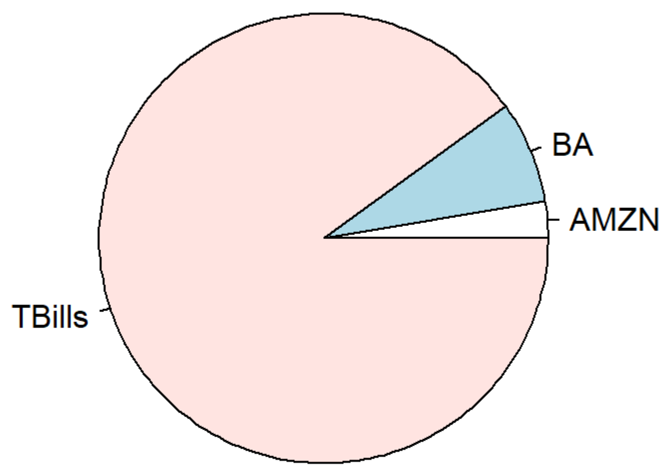
```
## [1] 0.966
```

```
cex.val = 2
plot(sd1, mu.p1, type="b", pch=16, cex = cex.val,
ylim=c(min(mu.p1), max(mu.p1)), xlim=c(0.15, max(sd1)),
xlab=expression(sigma[p]), ylab=expression(mu[p]), cex.lab=cex.val,
col=c(rep("green", 13), "blue", rep("red", 30)))
points(sig.p.tan, mu.p.tan, type='b', pch=16)
text(x=.43, y=.23, labels="Amazon", pos=4, cex = cex.val)
text(x=.43, y=.20, labels="Boeing", pos=4, cex = cex.val)
text(x=.2, y=.217, labels="Tangency Portfolio", pos=4, cex = 1)
```



4. Consider a portfolio that has 10% in the tangency portfolio and 90% in T-bills.
    - In this portfolio, what is the percent invested in Amazon and what is the percent invested in Boeing? Give a pie chart showing the percent invested in T-bills, Amazon and Boeing.
    - Compute $E[R_p]$, $var(R_p)$, $SD(R_p)$
    - Compute Sharpe's slope for this portfolio
    - Indicate the location of this portfolio on the graph you created previously in question 1.

```
x.A.tan.rf = x.A.tan/10
x.B.tan.rf = x.B.tan/10
new.tan =(x.A.tan.rf + x.B.tan.rf)
x.TBill.tan.rf = 1 - (x.A.tan.rf + x.B.tan.rf)
tan_weights = c(x.A.tan.rf,x.B.tan.rf,x.TBill.tan.rf)
lbls = c('AMZN','BA','TBills')
pie(tan_weights,lbls)
```




```
safe.port.mu= rf + new.tan*(mu.p.tan - rf)
safe.port.mu
```

```
## [1] 0.0488
```

```
safe.port.sd = .3*(sig.p.tan)
safe.port.sd
```

```
## [1] 0.0582
```

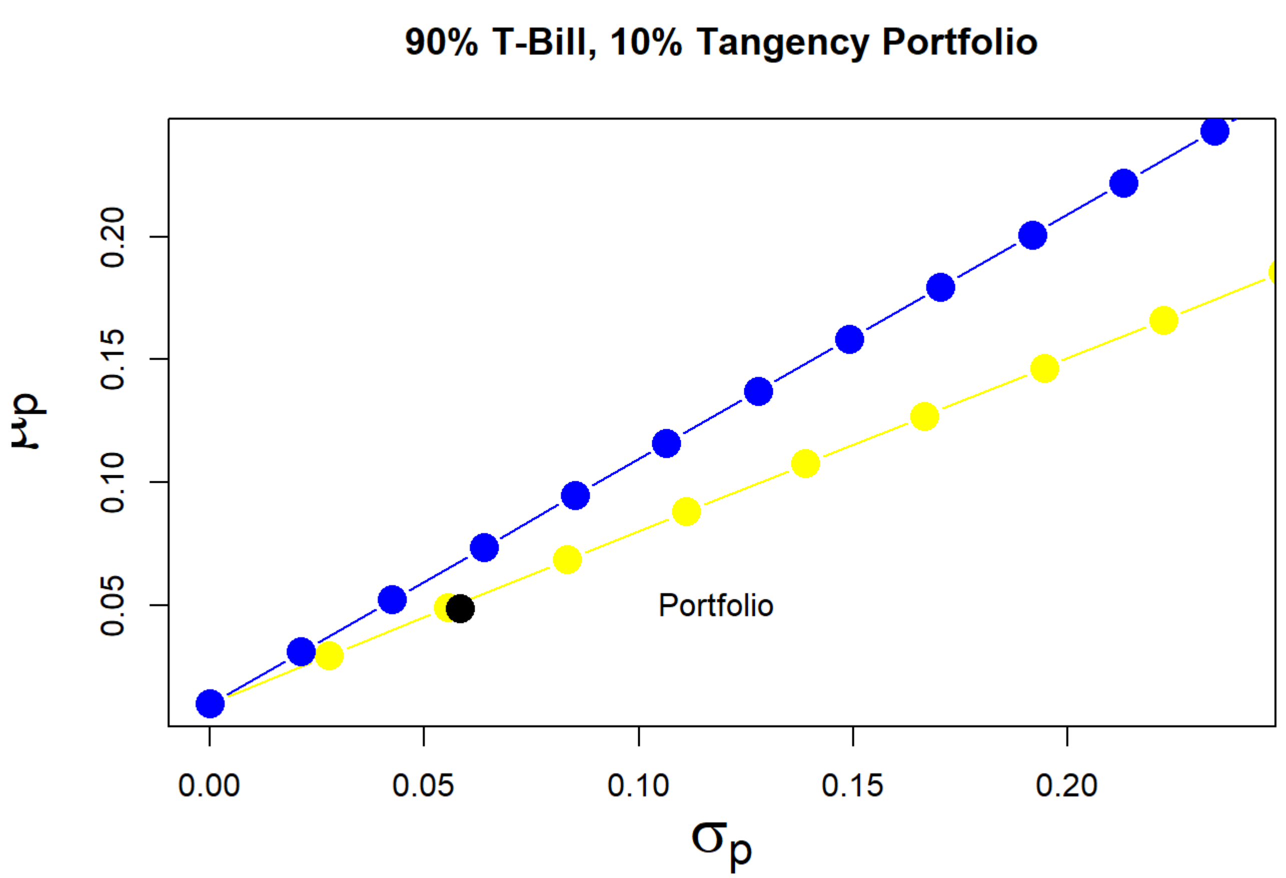
```
safe.port.var = (safe.port.sd)^2
safe.port.var
```

```
## [1] 0.00339
```

```
safe.port.sharpe = as.numeric((safe.port.mu - rf)/safe.port.sd)
safe.port.sharpe
```

```
## [1] 0.322
```

```
cex.val = 2
plot(sig.p.A, mu.p.A, type="b", pch=16, cex = cex.val, main='90% T-Bill, 10% Tangency Portfolio'
, ylim=c(min(mu.p.A), max(mu.p1)), xlim=c(min(sig.p.A), max(mu.p1)),
xlab=expression(sigma[p]), ylab=expression(mu[p]), cex.lab=cex.val,
col=c(rep("yellow", 1), "yellow", rep("yellow", 30)))
points(sig.p.B, mu.p.B, type="b", col="blue", pch=16, cex = cex.val)
points(safe.port.sd, safe.port.mu, type="b", pch=16, cex = cex.val)
text(x=.2, y=.4, labels="BA / T-Bills", pos=4, cex = 1)
text(x=.42, y=.3, labels="AMZN/ T Bills", pos=4, cex = 1)
text(x=.1, y=0.05, labels="Portfolio", pos=4, cex = 1)
```



5. Find the efficient portfolio (combination of T-bills and tangency portfolio) that has the same risk (SD) as Amazon.
    - In this portfolio, what is the percent invested in Amazon and what is the percent invested in Boeing? Give a pie chart showing the percent invested in T-bills, Amazon and Boeing.
    - Compute $E[R_p], var(R_p), SD(R_p)$

- Compute Sharpe's slope for this portfolio
- Indicate the location of this portfolio on the graph you created previously in question 1.

```
xtan = as.numeric(sighat[1] / sig.p.tan)
xf = 1-xtan
xtan
```

```
## [1] 1.43
```

```
xf
```

```
## [1] -0.431
```

```
x.mu = rf + xtan*(mu.p.tan -rf)
x.mu
```

```
## [1] 0.298
```

```
x.sd = xtan*sig.p.tan
x.sd
```

```
## [1] 0.278
```

```
x.var = as.numeric((x.sd)^2)
x.var
```

```
## [1] 0.0771
```

```
(x.mu - rf)/x.sd
```

```
## [1] 0.966
```

```
x.sd / x.A.tan
```

```
## [1] 1.04
```

```
x.sd / x.B.tan
```

```
## [1] 0.379
```

```
plot(sig.p.A, mu.p.A, type="b", pch=16, cex = cex.val, main='Portfolio w/ Risk = AMZN',
xlab=expression(sigma[p]), ylab=expression(mu[p]), cex.lab=cex.val,
col=c(rep("yellow", 1), "yellow", rep("yellow", 30)))
points(sig.p.B, mu.p.B, type="b", col="blue", pch=16, cex = cex.val)
points(x.sd, x.mu, type="b", pch=16, cex = cex.val)
text(x=.2, y=.4, labels="BA / T-Bills", pos=4, cex = 1)
text(x=.42, y=.3, labels="AMZN/ T Bills", pos=4, cex = 1)
text(x=.2, y=0.3, labels="Portfolio", pos=4, cex = 1)
```