Project

Code

#Executive Summary

###Data Set

This project takes a look at 6 different Vanguard funds over the course of November 2014 to November 2019.

###Mutual fund description

S&P 500 Index: vfinx The mutual fund employs an indexing investment approach designed to track the performance of the S&P 500 Index, a widely recognized benchmark of U.S. stock market performance that is dominated by the stocks of large U.S. companies.

European Stock Index: veurx The mutual fund is set to track the performance of the MSCI Europe index that contains stocks issued by companies in Europe.

Emerging Markets Fund: veiex The mutual fund invests in the stocks in the MSCI Emerging Markets Index. This index contains the stocks of companies located in emerging markets, or nations with rapid growth and industrialization.

Long-term Bond Fund: vbltx The mutual fund tracks the performance of a market-weighted bond index called the Barclays Capital U.S. Long Government/Credit Float Adjusted Index. This includes government and high-quality corporate bonds that have an average maturity of 15-30 years.

Short-term Bond Fund: vbisx The mutual fund tracks the performance of a market-weighted bond index called the Barclays Capital U.S. 1-5 Year Government/Credit Float Adjusted Index, which includes government and high-quality corporate bonds that have an average maturity of 1-5 years.

Pacific Stock Index: vpacx The mutual fund attempts to replicate the performance of MSCI Pacific Index, which contains the stocks of companies located in Japan, Australia, Hong Kong, Singapore, and New Zealand.

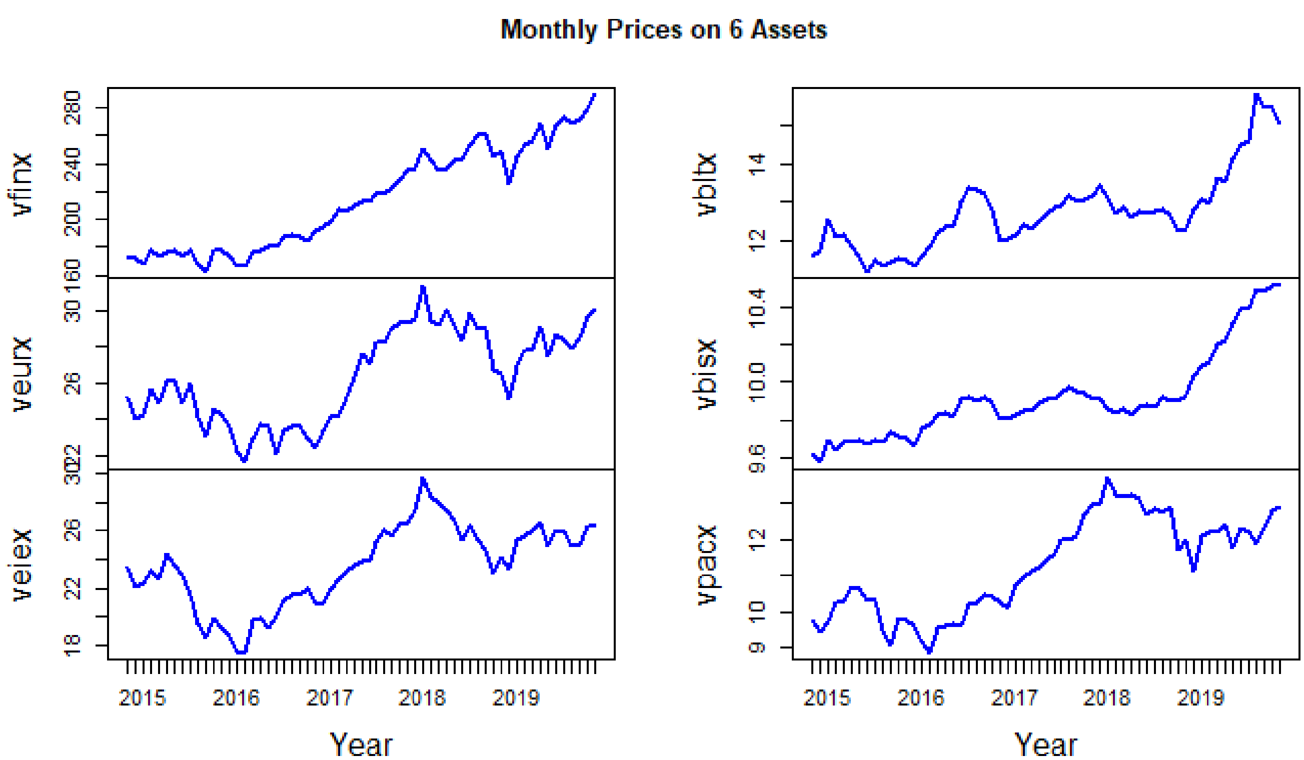
###Main Findings

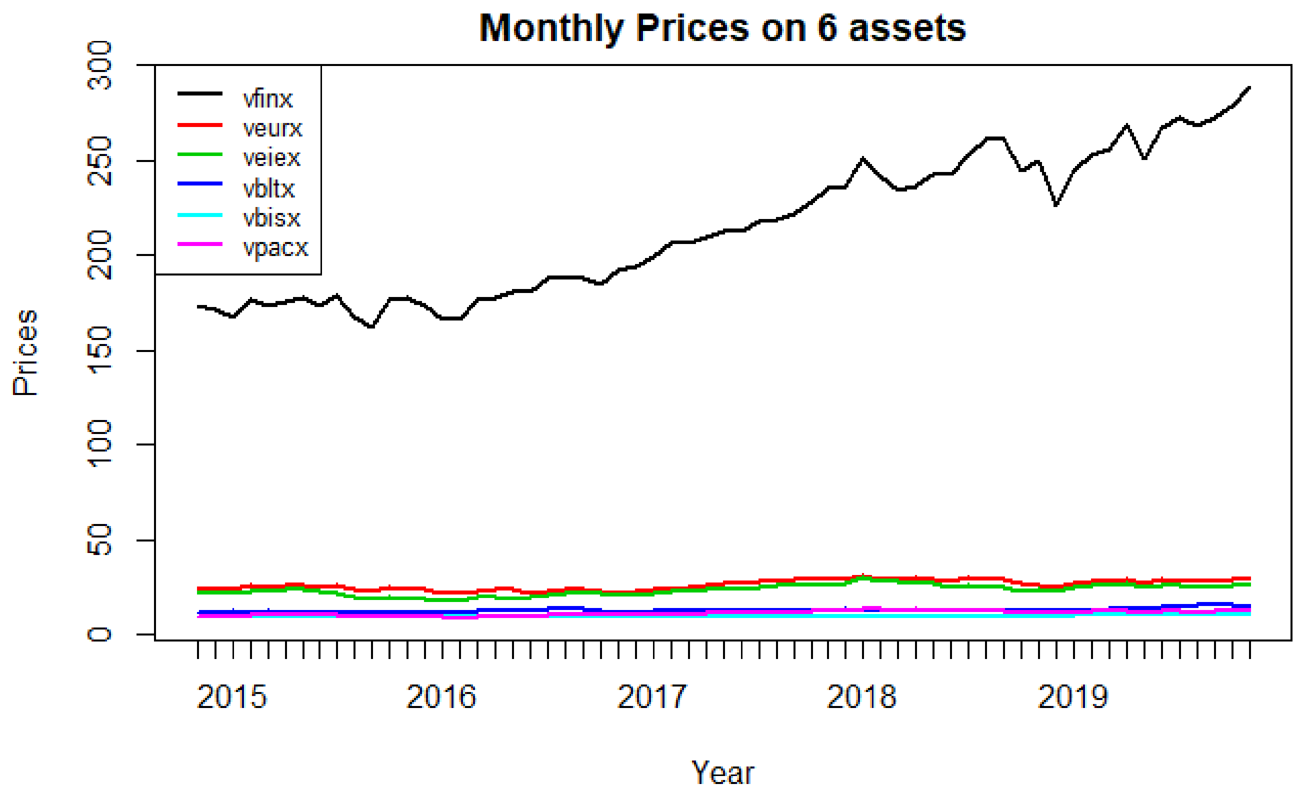
- The three mutual funds: veurx, veiex and vpacx prices all seem to follow the same trend with a drop in 2016, a peak in 2018 and another drop at the end of 2018. The other three indexes seem to follow a continually gradual growth with no severe drops.
- All of the mutual funds seem to have returns that are not normally distributed. The closest ones to normal distribution are the emerging markets index (veiex) and the long-term bonds fund (vbltx), although both have signs that they are not normally distributed.
- The S&P 500 index (vfinx) has the highest average return, while also having the third lowest volatility behind the bond funds (vbltx & vbisx). The short term bond fund (vbisx) has the lowest average return but also coming with the lowest volatility.
- There is usually a trade off of return and risk, which is exhibited with the S&P 500 index (vfinx) and the Pacific fund (vpacx) and the bond funds (vbltx & vbisx). Although this is not true for the other 2 indexes, with both the European stock index (veurx) and the emerging markets fund (veiex) having lower average returns and higher volatility than the long-term bond fund (vbltx).
- The Sharpe Ratio gives us a measure of the excess return per unit of risk. The short-term bond fund (vbisx) has the highest ratio and the emerging markets fund (veiex) has the lowest Sharpe Ratio.

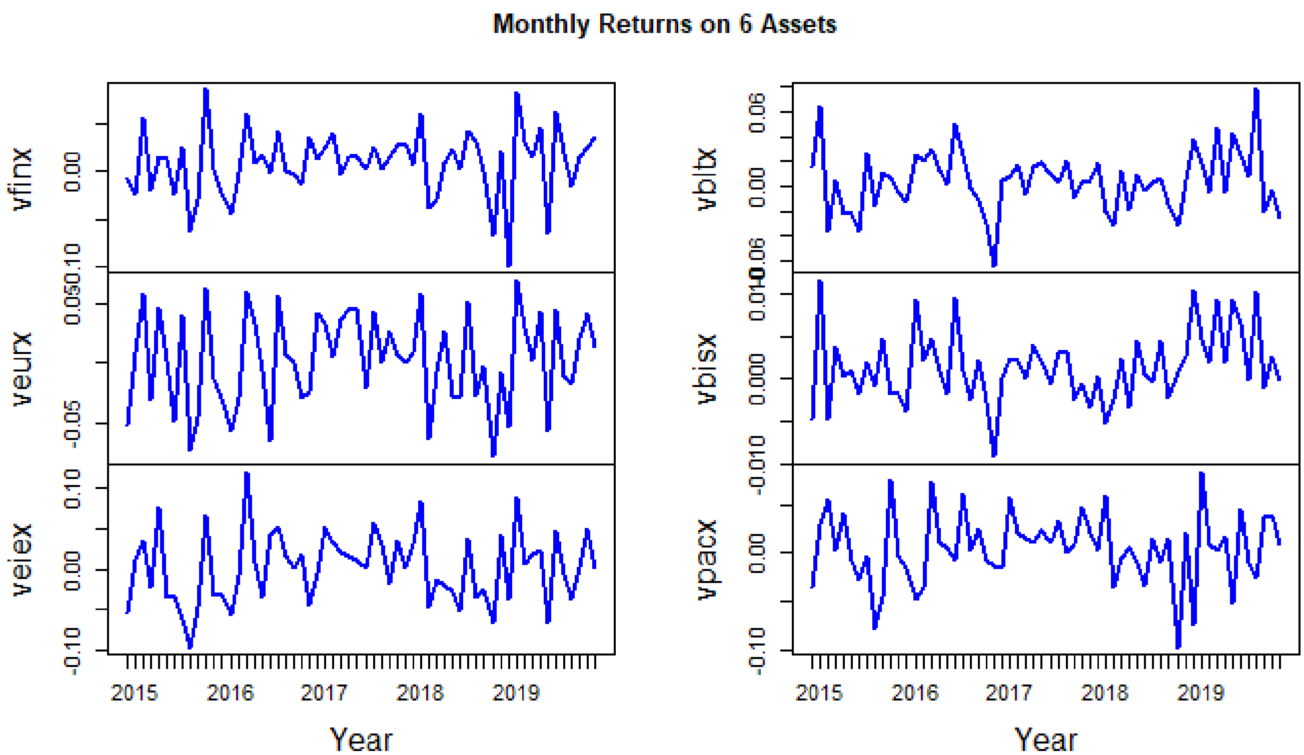
- All of the country stock index funds (vfinx, veurx, veiex, vpacx) have strong positive linear relationship. The country and bond funds have small, mostly slightly negative correlations. The two bond funds (vbltx & vbisx) have strong positive linear correlation.
- The value at risk over a one month horizon is the highest for the emerging markets fund (veiex) and by far the smallest for the short-term bond fund (vbisx). These intuitions are constant for the one year investment horizon too.
- The Sharpe Ratio is higher for the global minimum portfolio that allows short sale, which also has a lower expected return and standard deviation. The value at risk on a month investment horizon is greater for the global minimum portfolio with no short sale.
- The tangency portfolio with short sale has a higher expected return, lower standard deviation and lower Sharpe ratio than the tangency portfolio without shorting.
- Due to the fact that 401k plans do not allow short-selling, an efficient portfolio must rely on portfolio diversification in order to achieve a targeted return and keep the risk at a minimum.

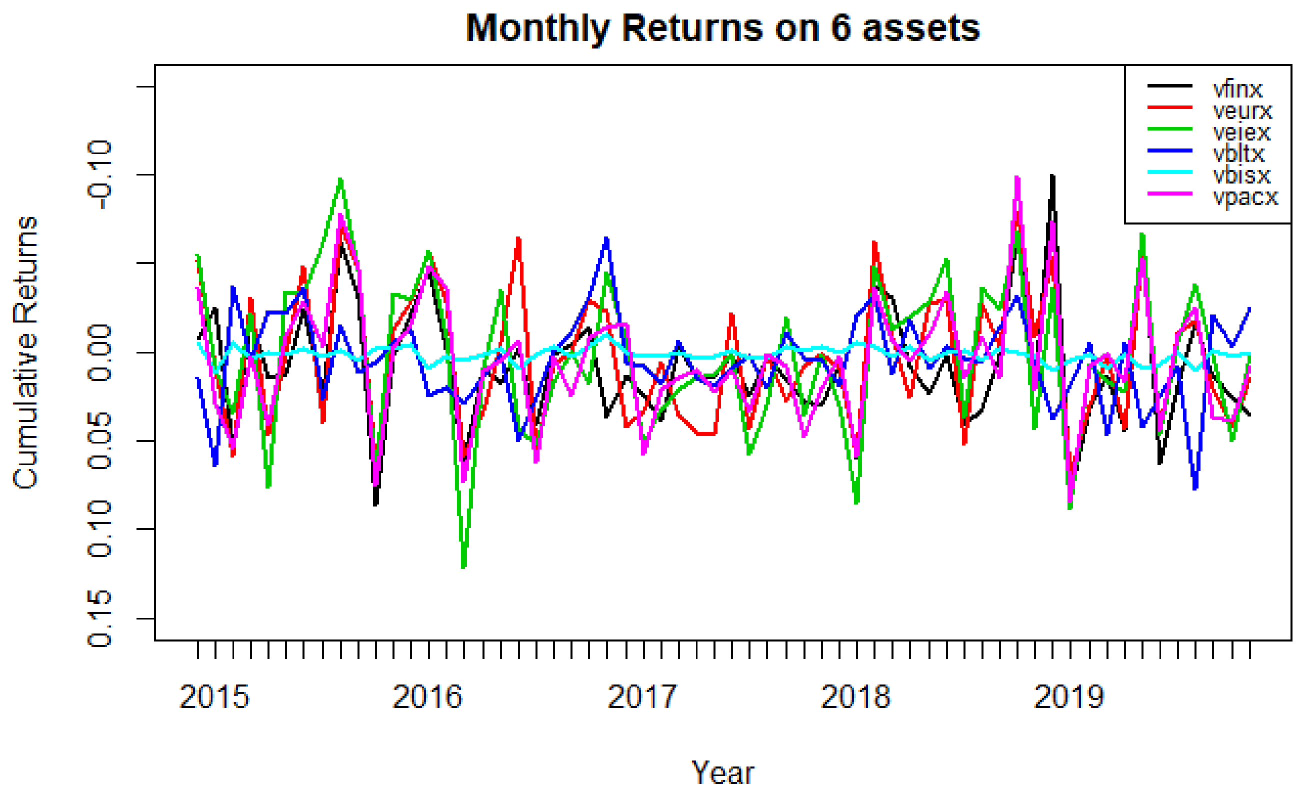
```
time series ends   2019-11-01
time series ends   2019-11-01
time series ends   2019-11-01
time series ends   2019-11-01
time series ends   2019-11-01
time series ends   2019-11-01
```

#Return Calculations and Sample Statistics

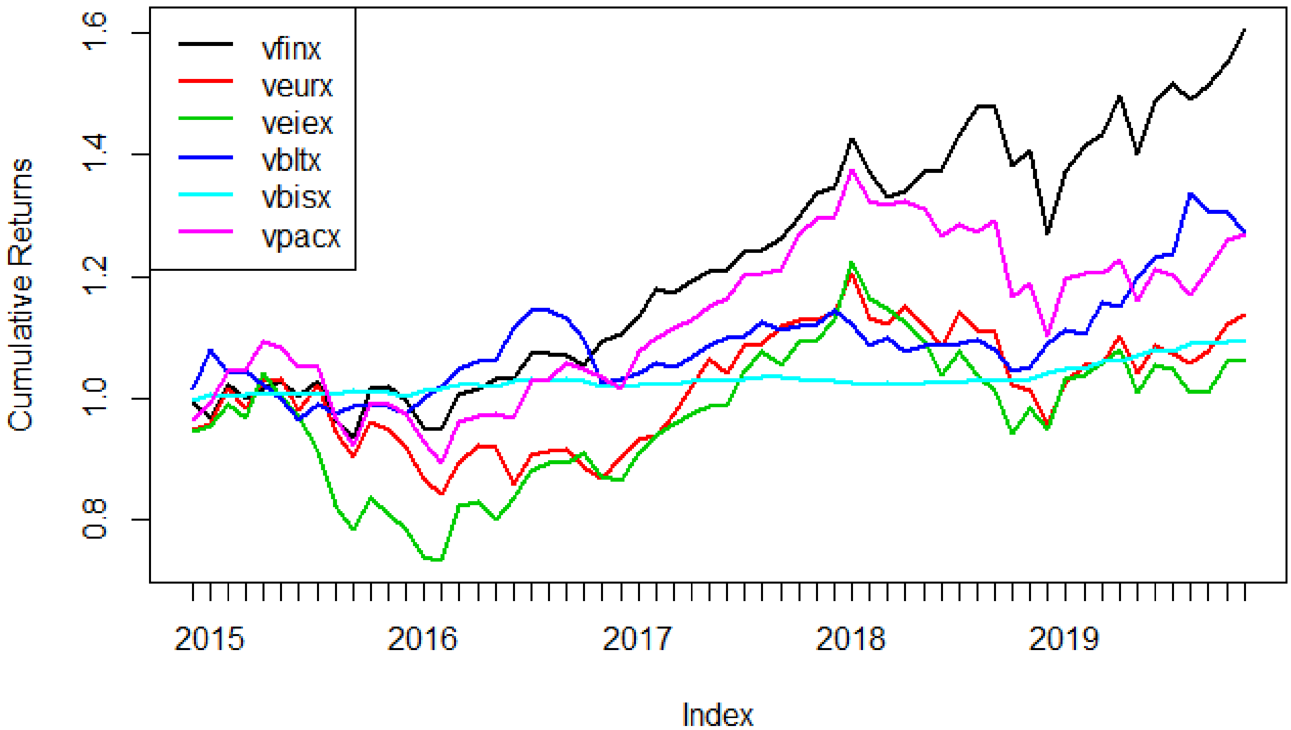



Monthly Prices on 6 assets
vfinx
veurx
veiex
vbltx
vbisx
vpacx
Prices
0
50
100
150
200
250
300
2015
2016
2017
2018
2019
Year


Monthly Returns on 6 Assets
vfinx
veurx
veiex
vbltx
vbisx
vpacx
2015
2016
2017
2018
2019
Year
Year



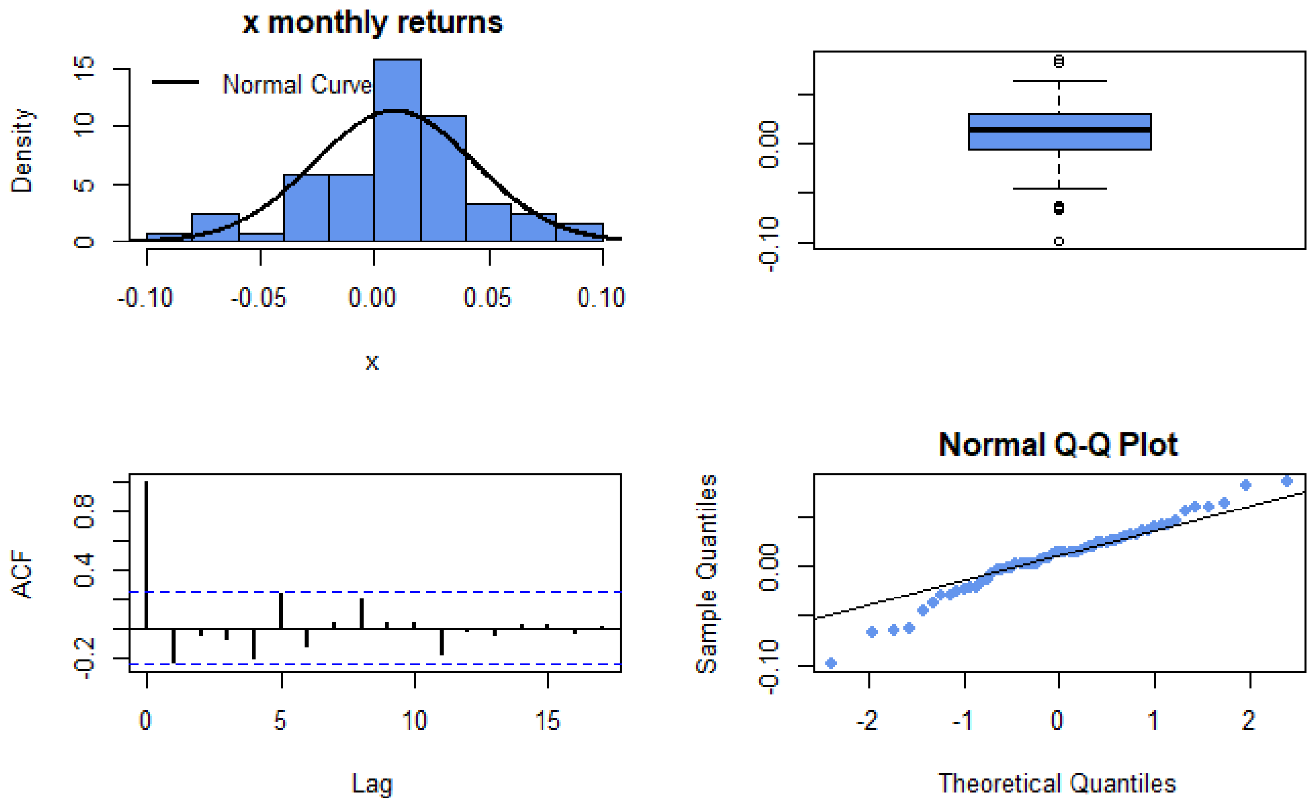
The mutual funds for the European stock index (veurx), the emerging markets fund (veiex) and the Pacific stock index (vpacx) all seem to follow the same trends for their prices and continuously compounded returns. The only country stock index that does not seem to follow the same trend is the S&P 500 index which has more of a positive trend in prices over time. The European stock index (veurx), the emerging markets fund (veiex) and the Pacific stock index (vpacx) all exhibit visible price drops around 2016, while the S&P 500 index (vfinx) does not exude as substantial of a price drop. Later in 2018 the same three funds (veurx, veiex, vpacx) were experienced a sharp decline creating a peak, while the S&P 500 index (vfinx) doesn’t seem to show as steep of a reduction. Looking at the continuously compounded returns, these same three assets (veurx, veiex, vpacx) seem to exhibit larger shifts about the mean than the S&P 500 (vfinx) from mid 2016 to 2018. These differences could be explained by the Italy and European Debt crisis that occurred in mid-2016. For the two bond funds, long-term bond fund (vbltx) and short-term bond fund (vbisx), they seem to move together for both price and returns. In comparison to the country stock indexes, the bond funds have a smoother increase in price except in comparison to the S&P 500 index (vfinx), which seems to have the smoothest increase in price.



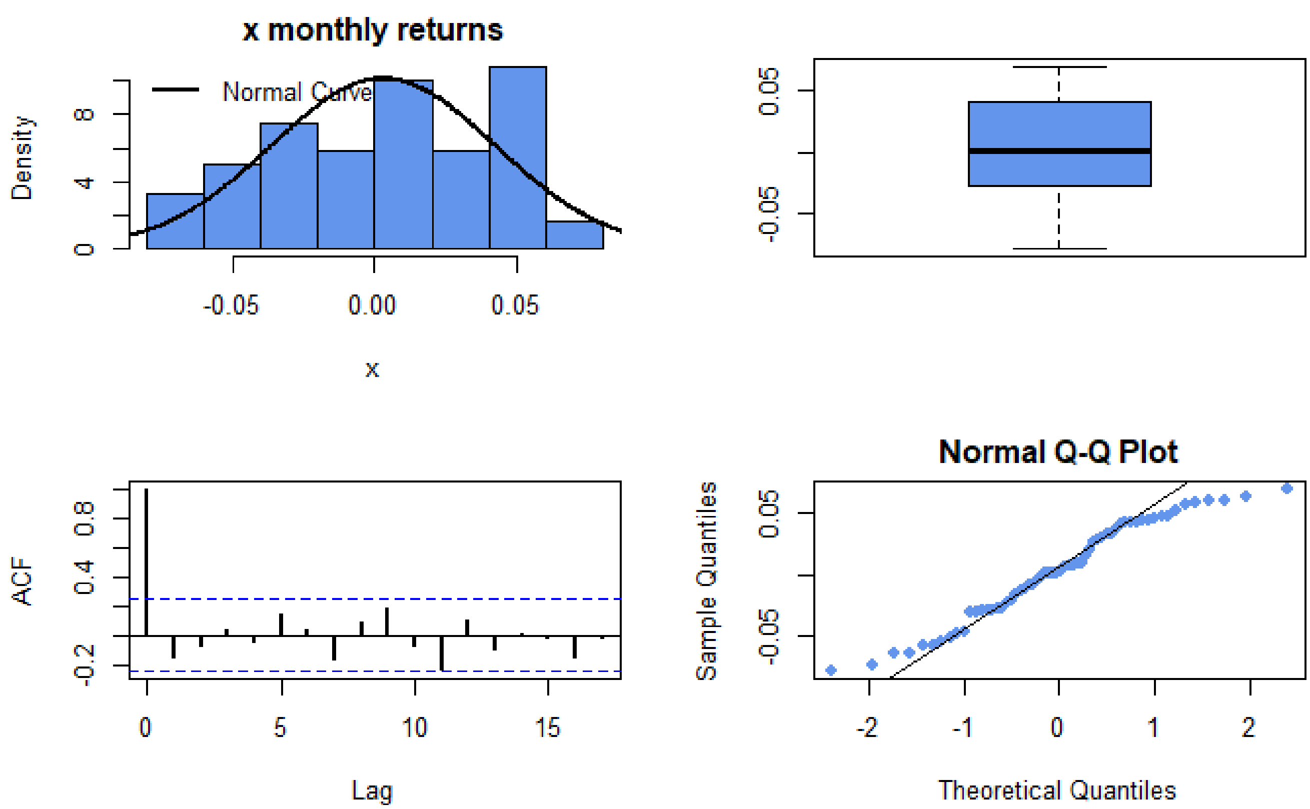
Looking at the equity curve, we can see that the S&P 500 index (vfinx) had a substantially higher future value than the other funds. I am not surprised by this, because looking at the prices and return graphs, it is apparent that the S&P 500 index had a more positive trend than the other assets.

This next section displays four graphs per fund in order to be able to determine if the funds are normally distributed or not. The four graphs are comprised of a histogram, boxplot, QQ-plot and SACF.

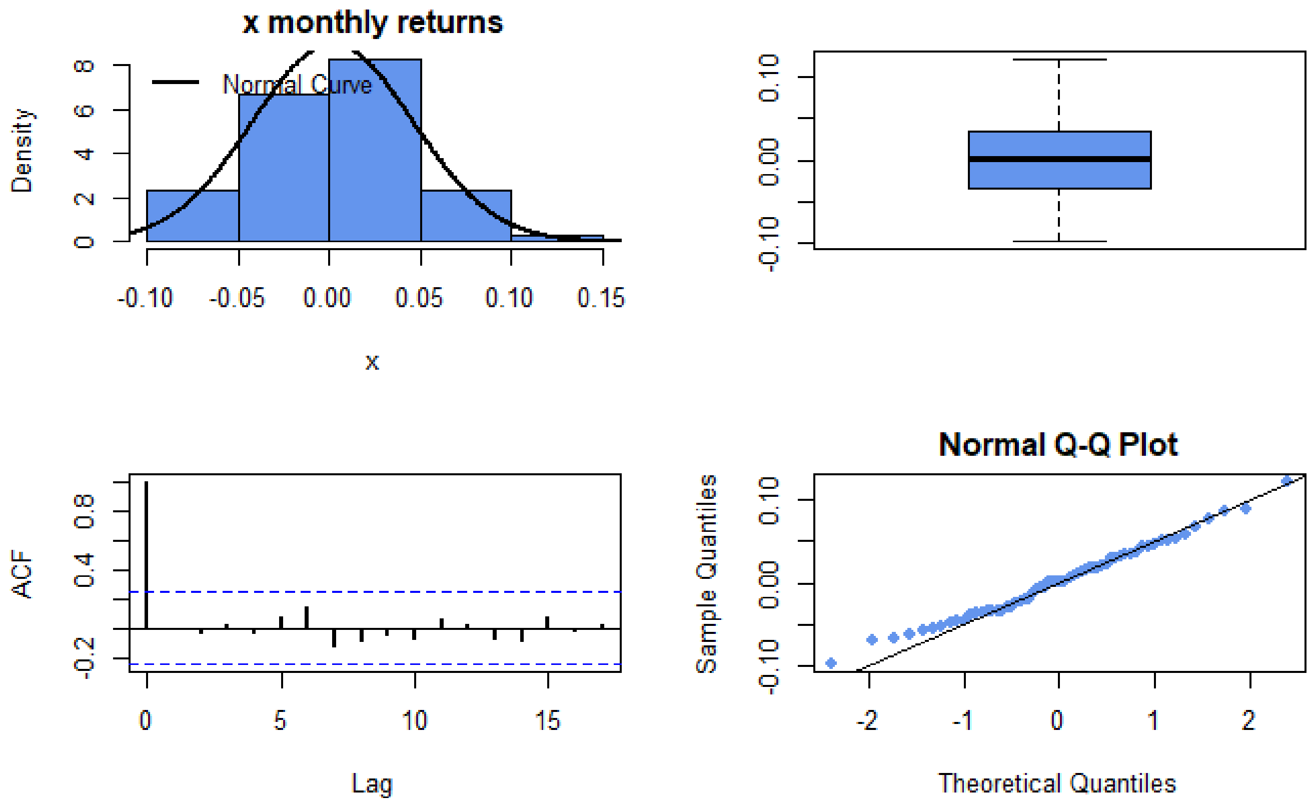
S&P 500 index: vfinx



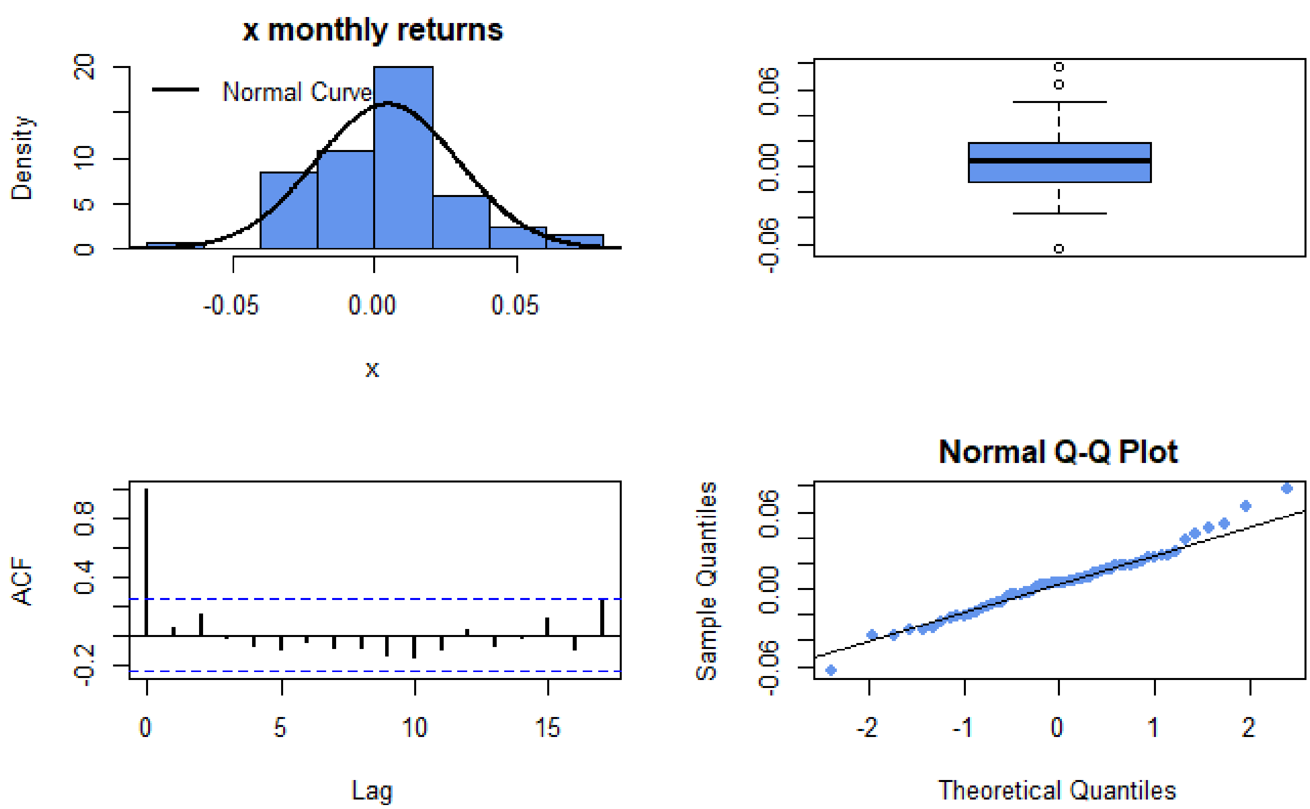
European stock index: veurx



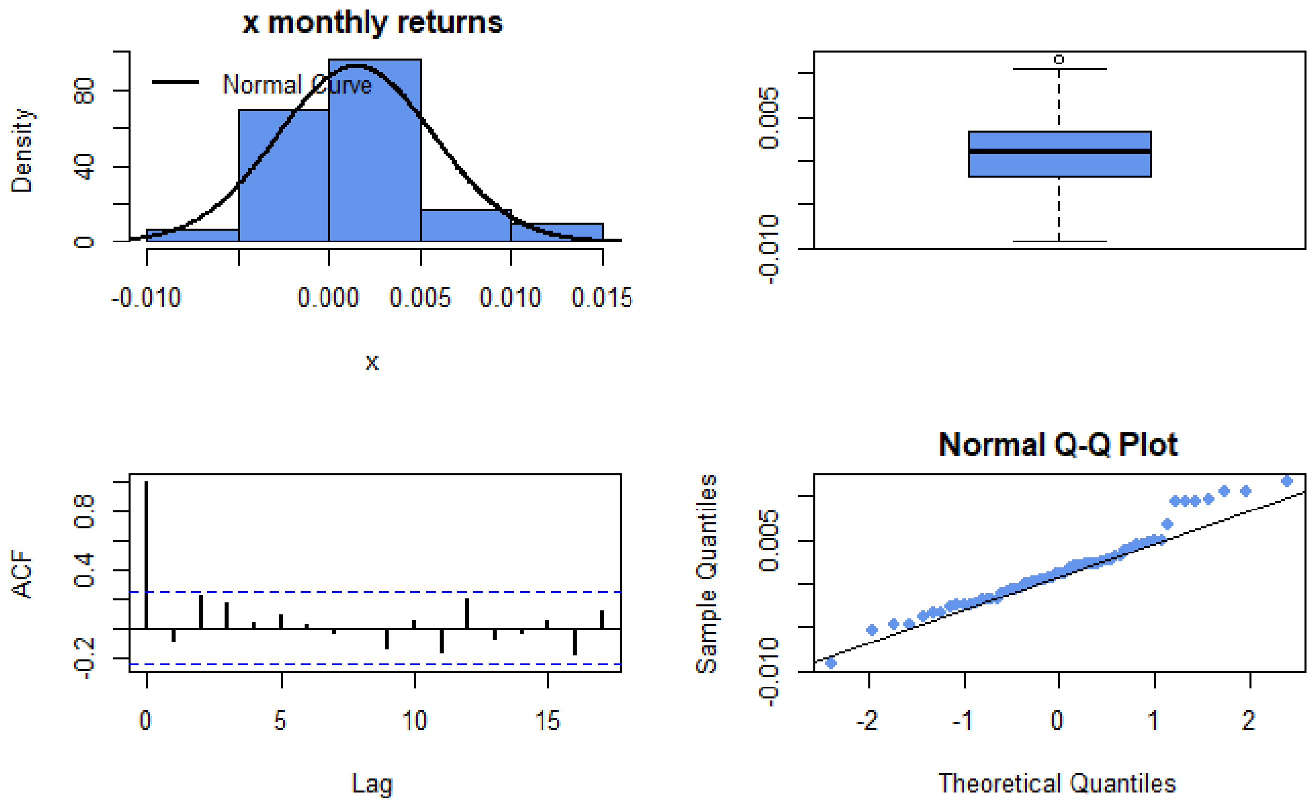
Emerging markets fund: veiex



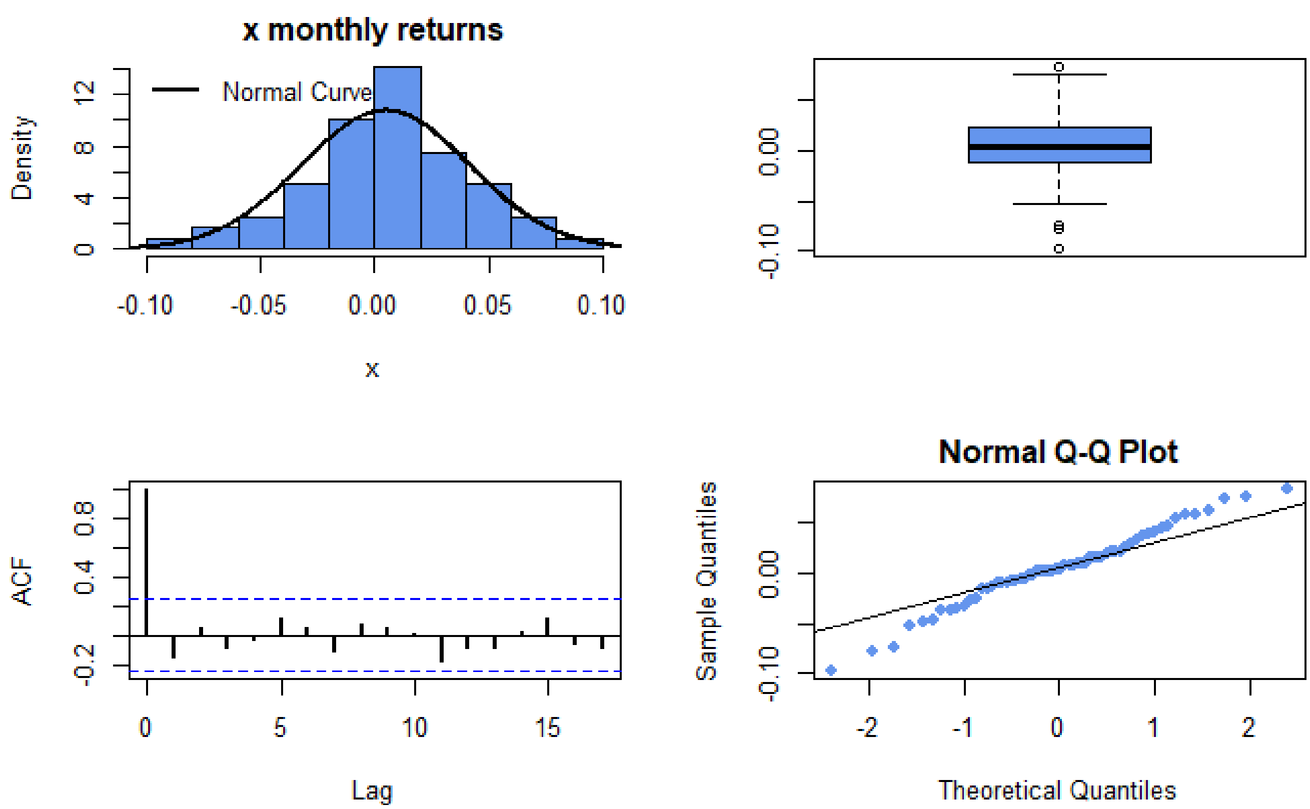
Long-term bond fund: vbltx



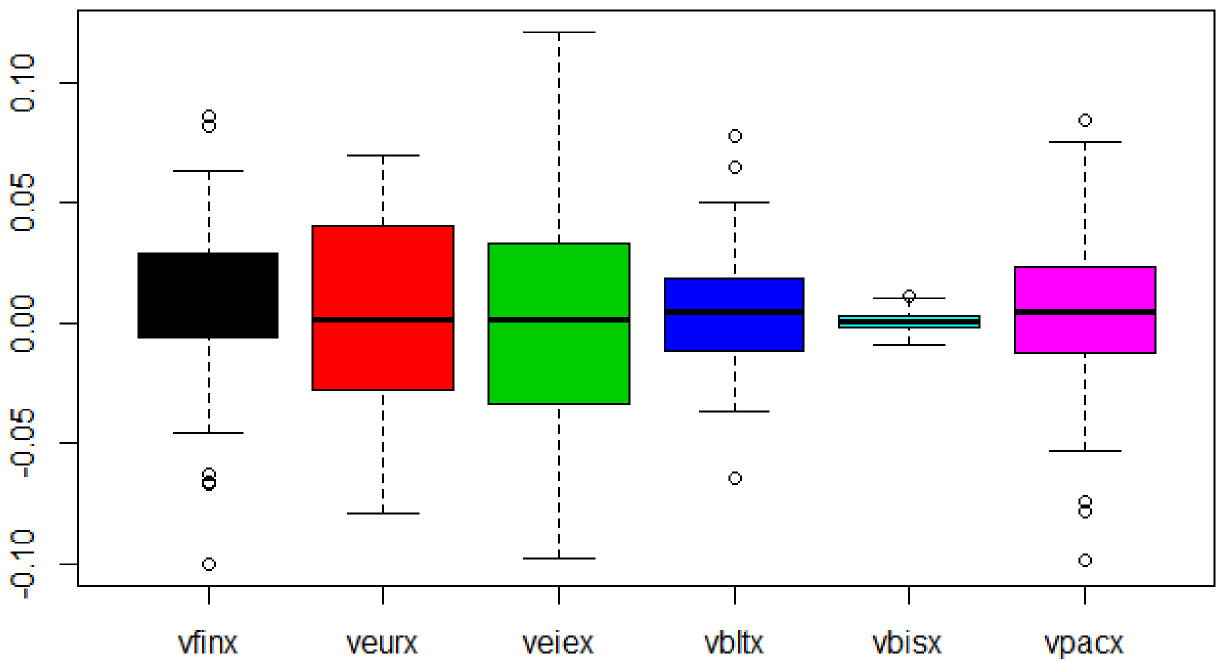
Short-term bond fund: vbisx



Pacific stock index: vpacx



Looking at all of the sets of graphs, it seems that all of the stocks are not normally distributed. The two that are closest to being normally distributed are the Emerging markets index (veiex), although it looks to be skewed which can be seen from the QQ-plot, and the long-term bond fund (vbltx), who's outliers cause for some skew. Looking at the box plots of all the assets we can tell that all of the funds have outliers, except for the European stock index (veurx) and the emerging markets fund (veiex) which had no outliers. Below is a better visualization of all of the funds box plots allowing you to see the outliers in greater detail. Looking at all of the funds' SACF graphs, it does not seem that there is any linear time dependency since none of the bars cross the blue line past the point of 0 lag.



In this section the univariate descriptive statistics are determined so that we can have a more in depth look at the funds and their returns.

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
Mean	0.0085659	0.0029284	0.0019573	0.0043304	0.0014629	0.0046846
Variance	0.0012492	0.0015638	0.0019748	0.0006363	0.0000186	0.0013821
Std Dev	0.0353442	0.0395447	0.0444391	0.0252245	0.0043100	0.0371763
Skewness	-0.5237857	-0.2202063	0.2524267	0.2495723	0.4412992	-0.2808517
Excess Kurtosis	0.8477628	-0.9645830	-0.2448978	0.8716905	0.1935469	0.3538219
1%	-0.0802822	-0.0753218	-0.0800366	-0.0479864	-0.0069300	-0.0864587
5%	-0.0624900	-0.0631494	-0.0619962	-0.0318154	-0.0047407	-0.0541882

The S&P 500 index (vfinx) has the highest average return and the short-term bond fund has the lowest average return. The emerging markets fund (veiex) has the highest volatility, while the short-term bond fund (vbisx) has the smallest volatility. Looking at normality, the European stock index (veurx) has the least amount of skewness in either direction, with the S&P 500 index has the greatest amount of skewness, giving it the longest tail. The European stock index (veurx) has the greatest amount of excess kurtosis meaning it has the thinner tail than a regular normal distribution and the short-term bond fund's (vbisx) kurtosis is closest to 0 meaning its tail size is the closest to being normally distributed. Looking at the kurtosis and skewness of all of these funds, we can tell that none of them are in fact normally distributed with the short-term bond fund being the closest to normal distribution.

```
     vfinx      veurx      veiex      vbltx      vbisx      vpacx
0.23056693 0.06351545 0.03466794 0.15515587 0.24274869 0.11480251
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 1]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)


Bootstrap Statistics :
     original     bias    std. error
t1* 0.2305669 0.01298427   0.1434047

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 2]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)


Bootstrap Statistics :
      original      bias    std. error
t1* 0.06351545 0.002885945   0.1320836

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 3]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)


Bootstrap Statistics :
      original       bias    std. error
t1* 0.03466794 -0.003221104   0.1292655

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 4]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)


Bootstrap Statistics :
     original      bias    std. error
t1* 0.1551559 0.002038827   0.1307697

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 5]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)
```

```
Bootstrap Statistics :
     original       bias    std. error
t1* 0.2427487 -0.004425816   0.1276533

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = as.numeric(projectReturns.z[, 6]), statistic = sharpeRatio.boot,
    R = 999, rf = r.f)


Bootstrap Statistics :
     original      bias    std. error
t1* 0.1148025 0.006923306   0.1323182
```

	vfinx	veurx	veiex	vbltx	vbisx	vpacx
sharpe.vals	0.2305669	0.0635155	0.0346679	0.1551559	0.2427487	0.1148025
sharpe.se.vals	0.1480000	0.1320000	0.1320000	0.1300000	0.1350000	0.1320000

The asset with the highest slope is the short-term bond fund (vbisx), telling us that this asset has the best return per unit of risk. Looking at the standard error values in the table you can see that all of the error values are relatively close to the estimated values, telling us that these are most likely not very good estimates.

	mu	SE for mu	Upper CI	Lower CI	Width	SD	SE for SD	Upper CI	Lower CI	Width
vfinx	0.008566	0.004563	0.017692	-0.000560	0.018252	0.035344	0.003226	0.041797	0.028891	0.012906
veurx	0.002928	0.005105	0.013139	-0.007282	0.020421	0.039545	0.003610	0.046765	0.032325	0.014440
veiex	0.001957	0.005737	0.013431	-0.009517	0.022948	0.044439	0.004057	0.052553	0.036326	0.016227
vbltx	0.004330	0.003256	0.010843	-0.002183	0.013026	0.025225	0.002303	0.029830	0.020619	0.009211
vbisx	0.001463	0.000556	0.002576	0.000350	0.002226	0.004310	0.000393	0.005097	0.003523	0.001574
vpacx	0.004685	0.004799	0.014284	-0.004914	0.019198	0.037176	0.003394	0.043964	0.030389	0.013575

From this table you can see that the mean values are not calculated very accurately. The standard error values are large relative to the estimates and the confidence interval widths are also large in comparison. The SD values are estimated with greater accuracy shown by the widths of the CI's and the SE values being low relative to the SD estimates.

	Annualized mu values	Annualized SD values
vfinx	0.1027908	0.1224358
veurx	0.0351408	0.1369869
veiex	0.0234878	0.1539417

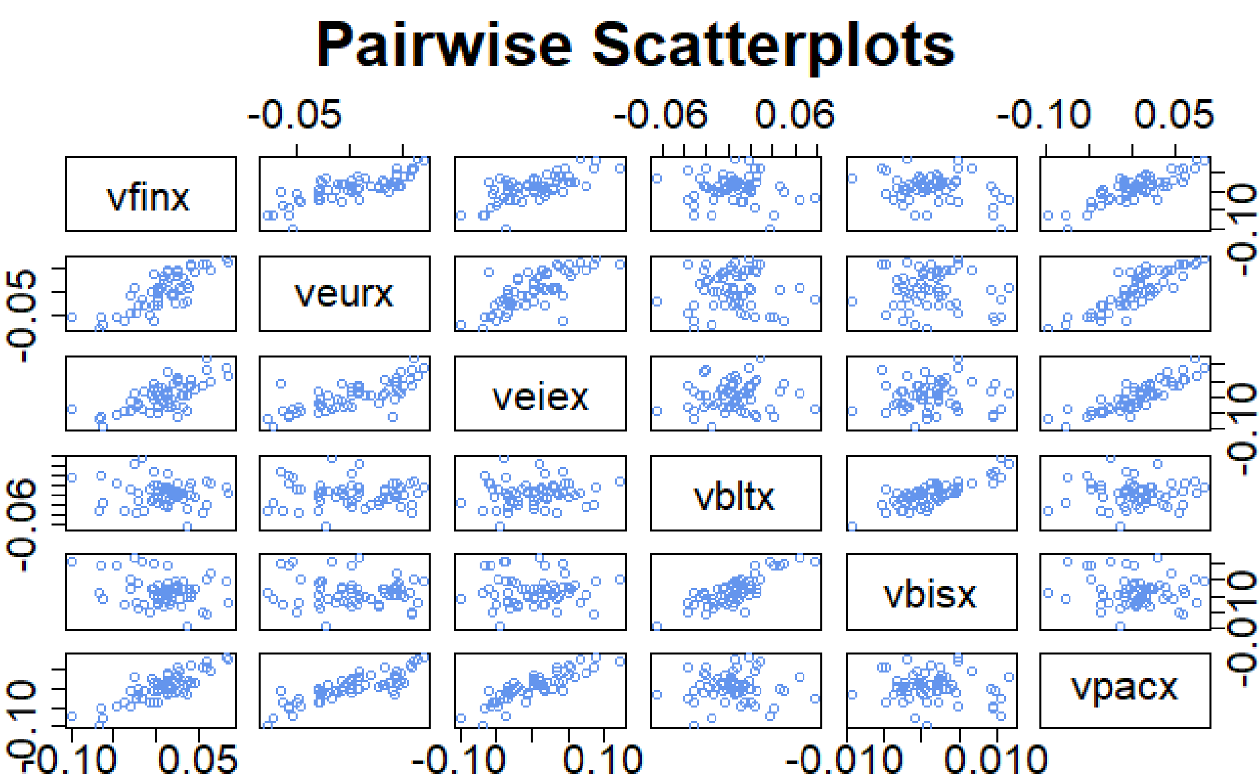
	Annualized mu values	Annualized SD values
vbltx	0.0519652	0.0873804
vbisx	0.0175553	0.0149302
vpacx	0.0562156	0.1287825

	Monthly Sharpe Ratio	Yearly Sharpe Ratio
vfinx	0.2305669	0.7987073
veurx	0.0635155	0.2200240
veiex	0.0346679	0.1200933
vbltx	0.1551559	0.5374757
vbisx	0.2427487	0.8409061
vpacx	0.1148025	0.3976875

Looking at the table above of the monthly Sharpe ratios and the annualized Sharpe ratios, you can see that the stocks maintained their ranking of Sharpe ratio values.

```
   vfinx    veurx    veiex    vbltx    vbisx    vpacx
1.671888 1.192085 1.124613 1.296705 1.091744 1.324557
```

The table above displays the estimate return that you would get from investing $1 over 5 years. The S&P 500 index grants us the greatest return based on these estimates.



Within the scatter plots, you can see that there is an obvious positive correlation between the country stock index funds (vfinx, veurx, veiex, vpacx). There seems to be no real correlation between the bond funds (vbltx, vbisx) and the country stock index funds. Within the bond funds themselves, there seems to be strongly positive correlations.

As observed in the scatter plot above, we can easily see that these country stock index funds (vfinx, veurx, veiex, vpacx) are heavily correlated. With the correlation plot, we can now see that the bond funds (vbltx, vbisx) and the country stock index funds are mostly near zero with the correlation of the S&P 500 index (vfinx) and the short-term bond fund having a slightly more negative correlation than the rest, although still quite small. Finally, we are also reassured on our past intuition that the bond funds are positively correlated is true. Looking at the correlation values, diversification may be slightly difficult since none of these assets have substantial negative correlation, but between the bond funds and the country stock index there is some slight negative correlation. So, some diversification could come from investing in both the bond funds and country stock indexes.

#Value at Risk

	5% VaR	1% VaR
vfinx	-4836.1542	-7100.9657
veurx	-6022.6933	-8521.5114
veiex	-6866.7160	-9644.9766
vbltx	-3647.8265	-5290.0013
vbisx	-561.0545	-852.7007
vpacx	-5490.0395	-7854.4133

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vfinx"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -4836.154 108.6619    838.8185

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vfinx"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -7100.966 151.3301     1063.98
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "veurx"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -6022.693 66.45261    679.1282

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "veurx"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -8521.511 92.49259    816.7027
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "veiex"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -6866.716 78.22741    668.1976

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "veiex"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -9644.977 109.6352     891.271
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vbltx"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -3647.827 58.76634    496.5683

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vbltx"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -5290.001 60.86567    652.5415
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vbisx"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -561.0545 8.965312    74.78027

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vbisx"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original  bias    std. error
t1* -852.7007 9.64348    97.17815
```

```
ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vpacx"]), statistic = ValueAtRisk.boot.5,
    R = 999)


Bootstrap Statistics :
     original   bias    std. error
t1* -5490.039 100.3438     777.976

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = coredata(projectReturns.z[, "vpacx"]), statistic = ValueAtRisk.boot.1,
    R = 999)


Bootstrap Statistics :
     original  bias    std. error
t1* -7854.413 113.442    1000.003
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = vfinx.boot.VaR.5, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-6589, -3301 )   (-6397, -3093 )
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = vfinx.boot.VaR.1, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-9338, -5167 )   (-9190, -4961 )
Calculations and Intervals on Original Scale
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = veurx.boot.VaR.5, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-7420, -4758 )   (-7251, -4579 )
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = veurx.boot.VaR.1, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-10215,  -7013 )   ( -9986,  -6786 )
Calculations and Intervals on Original Scale
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = veiex.boot.VaR.5, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-8255, -5635 )   (-8066, -5461 )
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = veiex.boot.VaR.1, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-11501,  -8008 )   (-11279,  -7782 )
Calculations and Intervals on Original Scale
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = vbltx.boot.VaR.5, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-4680, -2733 )   (-4530, -2582 )
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL :
boot.ci(boot.out = vbltx.boot.VaR.1, conf = 0.95, type = c("norm",
    "perc"))

Intervals :
Level      Normal             Percentile
95%   (-6630, -4072 )   (-6626, -4015 )
Calculations and Intervals on Original Scale
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL : 
boot.ci(boot.out = vbisx.boot.VaR.5, conf = 0.95, type = c("norm", 
    "perc"))

Intervals : 
Level      Normal             Percentile     
95%   (-716.6, -423.5 )   (-696.5, -405.0 )  
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL : 
boot.ci(boot.out = vbisx.boot.VaR.1, conf = 0.95, type = c("norm", 
    "perc"))

Intervals : 
Level      Normal             Percentile     
95%   (-1052.8,  -671.9 )   (-1038.5,  -664.2 )  
Calculations and Intervals on Original Scale
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL : 
boot.ci(boot.out = vpacx.boot.VaR.5, conf = 0.95, type = c("norm", 
    "perc"))

Intervals : 
Level      Normal             Percentile     
95%   (-7115, -4066 )   (-6889, -3829 )  
Calculations and Intervals on Original Scale
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 999 bootstrap replicates

CALL : 
boot.ci(boot.out = vpacx.boot.VaR.1, conf = 0.95, type = c("norm", 
    "perc"))

Intervals : 
Level      Normal             Percentile     
95%   (-9928, -6008 )   (-9814, -5776 )  
Calculations and Intervals on Original Scale
```

	Estimates_0.05 <dbl>	SE_0.… <dbl>	CI_0.05_lower <dbl>	CI_0.05_upper <dbl>	Estimates_0.01 <dbl>	SE_0.… <dbl>	CI_0.0
vfinx	-4836.1542	876.0	-6597	-3165	-7100.9657	1016.0	
veurx	-6022.6933	710.0	-7456	-4672	-8521.5114	781.0	
veiex	-6866.7160	726.0	-8358	-5512	-9644.9766	809.0	

	Estimates_0.05	SE_0.…	CI_0.05_lower	CI_0.05_upper	Estimates_0.01	SE_0.…	CI_0.0
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	
vbltx	-3647.8265	505.0	-4695	-2715	-5290.0013	620.0	
vbisx	-561.0545	74.5	-718	-426	-852.7007	93.8	
vpacx	-5490.0395	810.0	-7136	-3959	-7854.4133	946.0	

6 rows | 1-8 of 8 columns

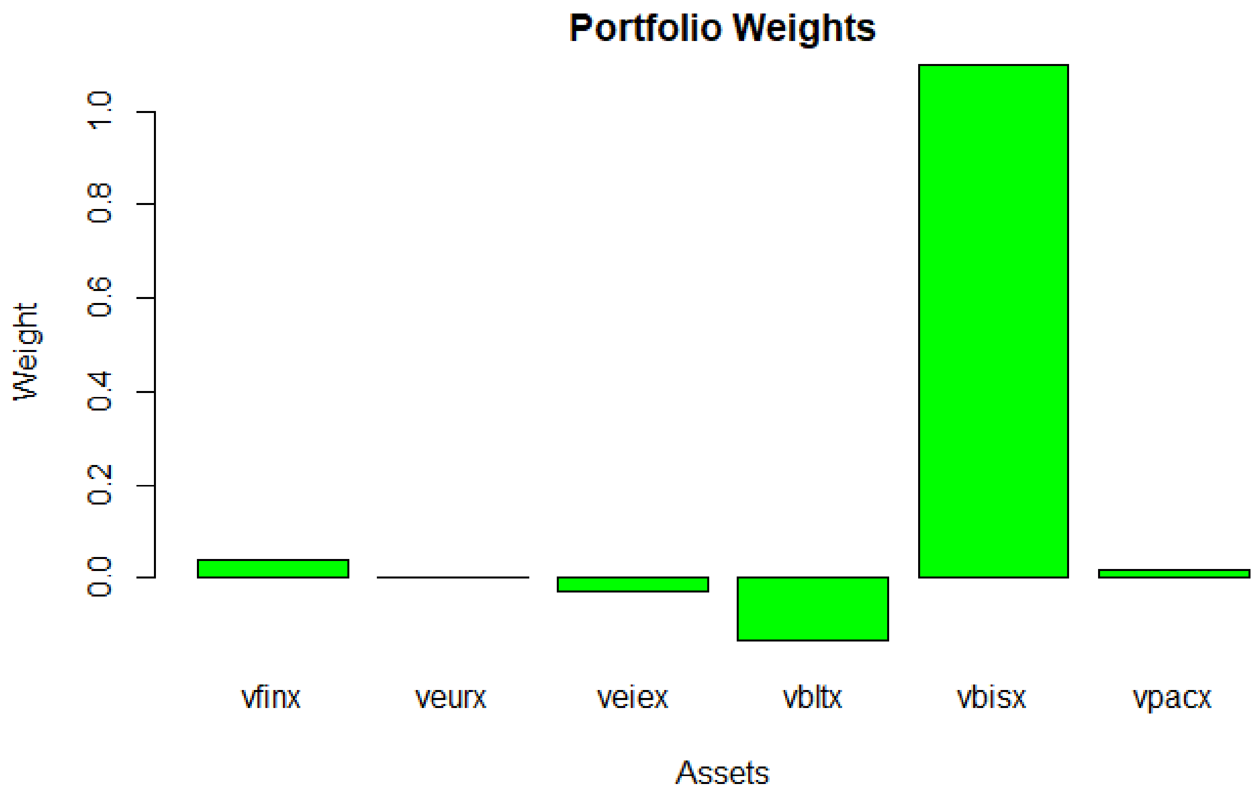
The standard error values for the value at risks estimates seem to be a little bit high, but do not seem high enough to call these bad estimates. The standard error values also seem to be relatively smaller for the 1% estimates in comparison to the 5% estimates. The short-term bond fund (vbisx) has by far the lowest VaR for both the 5% and the 1% estimates. The emerging markets fund (veiex) has the highest VaR for both the 5% and 1%, having over ten times the value at risk than the short-term bond fund (vbisx).

	5% VaR annually	1% VaR annually
vfinx	-9389.3288	-16642.991
veurx	-17319.1822	-24688.574
veiex	-20524.6890	-28440.015
vbltx	-8767.8358	-14042.043
vbisx	-697.8241	-1703.086
vpacx	-14410.9460	-21602.436

```
             5%        1%
vfinx -6057.7532 -7714.4157
veurx -6119.6776 -7255.5032
veiex -6011.3562 -7691.7404
vbltx -3131.4662 -4685.3249
vbisx  -472.9456  -690.6057
vpacx -5274.6190 -8282.6592
```

These numbers are slightly different than the previously estimated values based on a normal distribution. They do relate in that the bond funds (vbltx, vbisx) are still the assets with the lowest value at risk. Although now there are some changes within ranking from the 5% estimates to the 1% estimates. For example, at 5% VaR the Pacific stock index (vpacx) is ranked 4th out of the 6 funds, but at the 1% VaR it is ranked 1st with the most VaR.

#Portfolio Theory




```
      vfinx        veurx        veiex        vbltx        vbisx        vpacx 
0.038427616  0.003089439 -0.027551580 -0.131229223  1.100144743  0.017119005
```

Looking at the global minimum variance portfolio's (that allows short-sales) weights, we get both the emerging markets fund (veiex) and the long-term bond fund (vbltx) having negative weights in this portfolio. With shorts being allowed, this has made it so the short-term bond fund has a weight greater than 1 by itself.

Now we will be annualizing the portfolio's expected return and standard deviation so that we can compare it to the previously calculated annual Sharpe ratios for the funds.

	Asset annual Sharpe Ratio	Annual Gmin Sharpe Ratio
vfinx	0.7987073	1.330746
veurx	0.2200240	1.330746
veiex	0.1200933	1.330746
vbltx	0.5374757	1.330746
vbisx	0.8409061	1.330746
vpacx	0.3976875	1.330746

Looking at the Sharpe ratio of this hypothetical portfolio in comparison to all of the funds' ratios, we can see that this portfolio has a much higher ratio telling us that the risk to return rate would be a lot better for you to invest in these weights rather than in any of these individual stocks.

Here we will be looking to compare the global minimum variance portfolio's VaR against the funds that we have been assessing.

	5%	1%
GMin	-282.4808	-457.2734
vfinx	-4836.1542	-7100.9657
veurx	-6022.6933	-8521.5114
veiex	-6866.7160	-9644.9766
vbltx	-3647.8265	-5290.0013
vbisx	-561.0545	-852.7007
vpacx	-5490.0395	-7854.4133

This global minimum variance portfolio has a much lower value at risk than any of the individual funds could offer. This portfolio has half of the value at risk of the short-term bond fund (vbisx) at both levels.

```
Call:
globalMin.portfolio(er = muhat.vals, cov.mat = sigma.mat, shorts = F)

Portfolio expected return:     0.001781076
Portfolio standard deviation:  0.003982994
Portfolio weights:
 vfinx  veurx  veiex  vbltx  vbisx  vpacx
0.0448 0.0000 0.0000 0.0000 0.9552 0.0000
```

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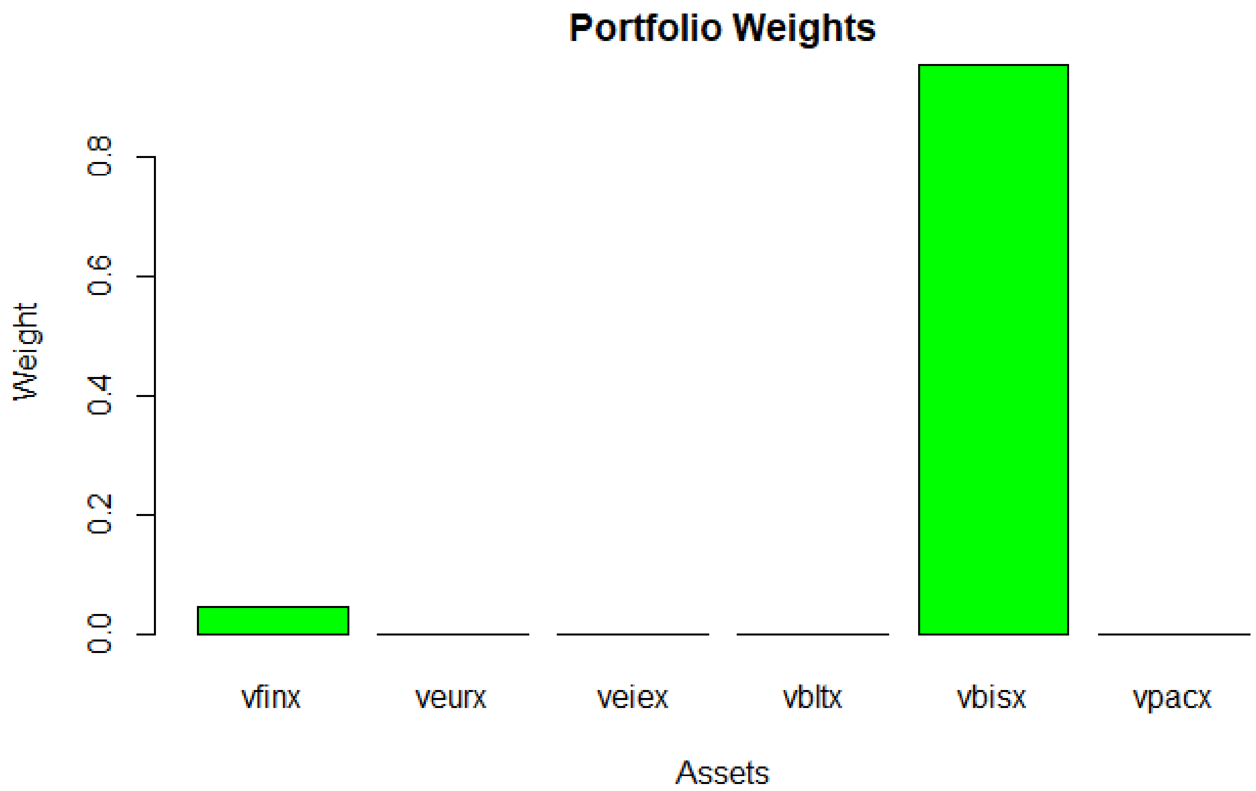
```
#Plot of gmin portfolio weights w/ no short-sale
plot(gmin.port.ns, col = "green")
```



	ER	SD	Sharpe
W/ Short	0.0168678	0.0089179	1.330746
No Short	0.0213729	0.0137975	1.186629

Here, we have calculated the annualized expected return, standard deviation and Sharpe ratio for both the global minimum portfolio with and without short sales. This table allows us to easily tell the differences between the two portfolios. The new portfolio that we have calculated (global minimum portfolio with out short sale) has a greater expected return, although it also comes with a higher standard deviation. Looking at the Sharpe ratios, we can see that the global minimum portfolio with short sales has a decently higher ratio, telling us that the portfolio with short-sales would be the better bet.

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```
#Calculating VaR values with no short sale for gmin and comparing against VaR values with short sa
les
gmin.ns.qhat.05 = exp(gmin.port.ns$er + gmin.port.ns$sd * qnorm(.05))-1
gmin.ns.qhat.01 = exp(gmin.port.ns$er + gmin.port.ns$sd * qnorm(.01))-1
gmin.ns.VaR.05 = gmin.ns.qhat.05 * W0
gmin.ns.VaR.01 = gmin.ns.qhat.01 * W0
gmin.VaR.05.vals = cbind(gmin.VaR.05, gmin.ns.VaR.05)
gmin.VaR.01.vals = cbind(gmin.VaR.01, gmin.ns.VaR.01)
VaR.gmin.analysis = rbind(gmin.VaR.05.vals, gmin.VaR.01.vals)
row.names(VaR.gmin.analysis) = c("5%", "1%")
colnames(VaR.gmin.analysis) = c("W/ short", "No short")
VaR.gmin.analysis
```

```
    W/ short  No short
5% -282.4808 -475.9006
1% -457.2734 -745.6813
```

Comparing these two portfolios further by looking at their differences in value at risk, we can see that there is a substantial change. The value at risk for the portfolio with out short-sales has almost double the value at risk than the portfolio allowing short sales.

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```
#Markowitz bullet
ef = efficient.frontier(muhat.vals, sigma.mat, alpha.min = -1, alpha.max = 1, shorts = T)
#Efficient portfolio w/ ER = to vfinx
ef.port = efficient.portfolio(muhat.vals, sigma.mat, target.return = muhat.vals["vfinx"], shorts =
T)
ef.port
```

```
Call:
efficient.portfolio(er = muhat.vals, cov.mat = sigma.mat, target.return = muhat.vals["vfinx"],
    shorts = T)

Portfolio expected return:     0.008565897
Portfolio standard deviation:  0.02013112
Portfolio weights:
  vfinx   veurx   veiex   vbltx   vbisx   vpacx
 0.8838 -0.3756 -0.3162  0.4234  0.2859  0.0988
```

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```
#Graphing Markowitz bullet and portfolios
plot(ef, pch = 16)
points(ef.port$sd, ef.port$er, col = "blue", pch = 16, cex = 2)
```

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```
text(ef.port$sd, ef.port$er, labels = "EP", pos = 4)
points(gmin.port$sd, gmin.port$er, col = "yellow", pch = 16, cex = 2)
```

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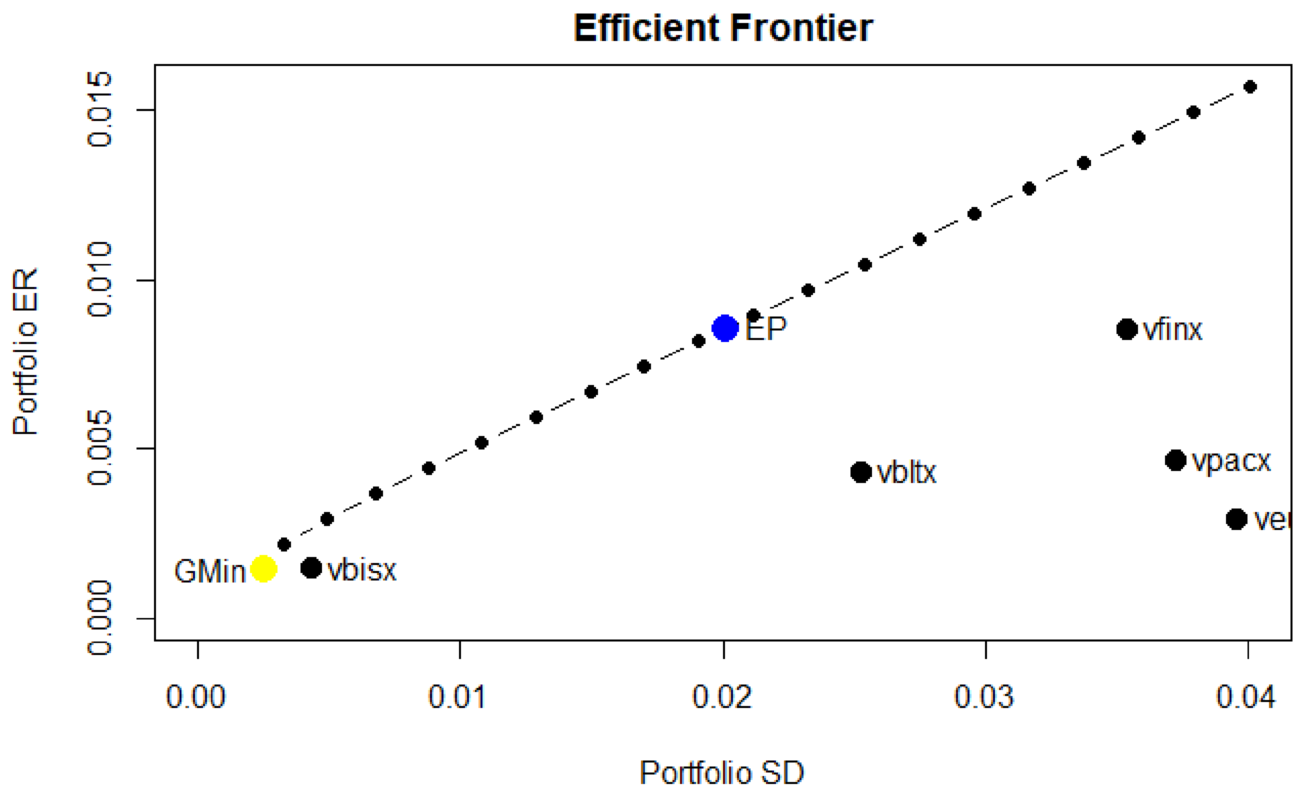
```
text(gmin.port$sd, gmin.port$er, labels = "GMin", pos = 2)
points(sd.vals, muhat.vals, pch = 16, cex = 1.5)
```

Hide

```
text(sd.vals, muhat.vals, labels = c("vfinx", "veurx", "veiex", "vbltx", "vbisx", "vpacx"), pos =
4)
```




```
Call:
tangency.portfolio(er = muhat.vals, cov.mat = sigma.mat, risk.free = r.f,
    shorts = T)

Portfolio expected return:     0.002267531
Portfolio standard deviation:  0.003521813
Portfolio Sharpe Ratio:        0.5255336
Portfolio weights:
  vfinx   veurx   veiex   vbltx   vbisx   vpacx
 0.1402 -0.0425 -0.0623 -0.0645  1.0021  0.0269
```

With these above calculations we have now created the tangency portfolio that allows short sales for these funds. This tangency portfolio is the point along the efficient frontier that has the highest Sharpe slope.

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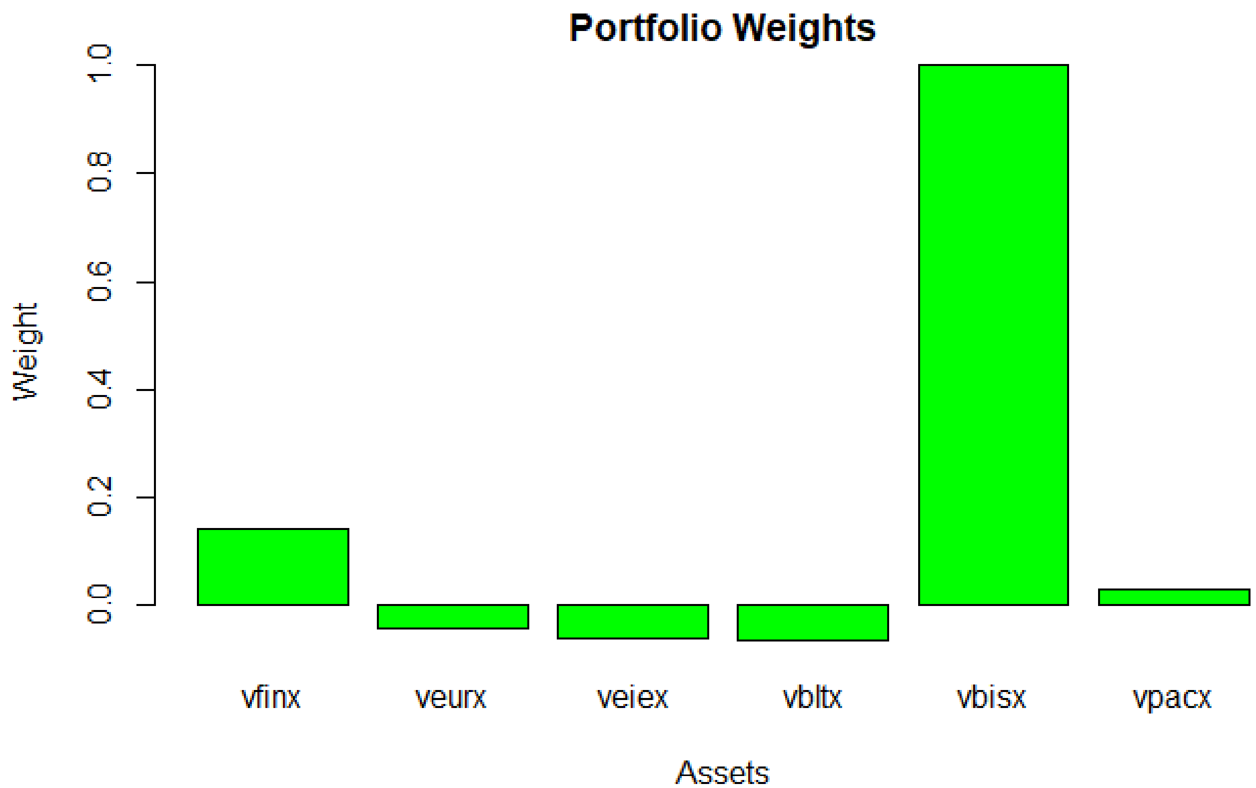
```
#Graphing tangency portfolio
plot(tan.port, col = "green")
```



Looking at the distribution of weights for the weights of the funds, we can see that this portfolio has assigned three of the six funds a negative weight. These three stocks being: the European stock index (veurx), the emerging market fund (veiex) and the long-term bond fund (vbltx).

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```
#ER, Var & SD of tangency portfolio
tan.port.er = tan.port$er
tan.port.var = tan.port$sd^2
tan.port.sd = tan.port$sd
tan.port.vals = cbind(tan.port.er, tan.port.var, tan.port.sd)
kable(tan.port.vals, col.names = c("ER", "Variance", "SD"), digits = 6)
```

ER	Variance	SD
0.002268	1.2e-05	0.003522

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```
#Calculating the Sharpe ratio for the tangency portfolio and comparing it to the individual funds'
ratios
sharpe.tan.port = as.data.frame((tan.port.er - r.f) / tan.port.sd)
colnames(sharpe.tan.port) = c("Tangency Portfolio Sharpe")
cbind(as.data.frame(sharpe.vals), sharpe.tan.port)
```

	sharpe.vals <dbl>	Tangency Portfolio Sharpe <dbl>
vfinx	0.23056693	0.5255336

	sharpe.vals	Tangency Portfolio Sharpe
	<dbl>	<dbl>
veurx	0.06351545	0.5255336
veiex	0.03466794	0.5255336
vbltx	0.15515587	0.5255336
vbisx	0.24274869	0.5255336
vpacx	0.11480251	0.5255336

6 rows

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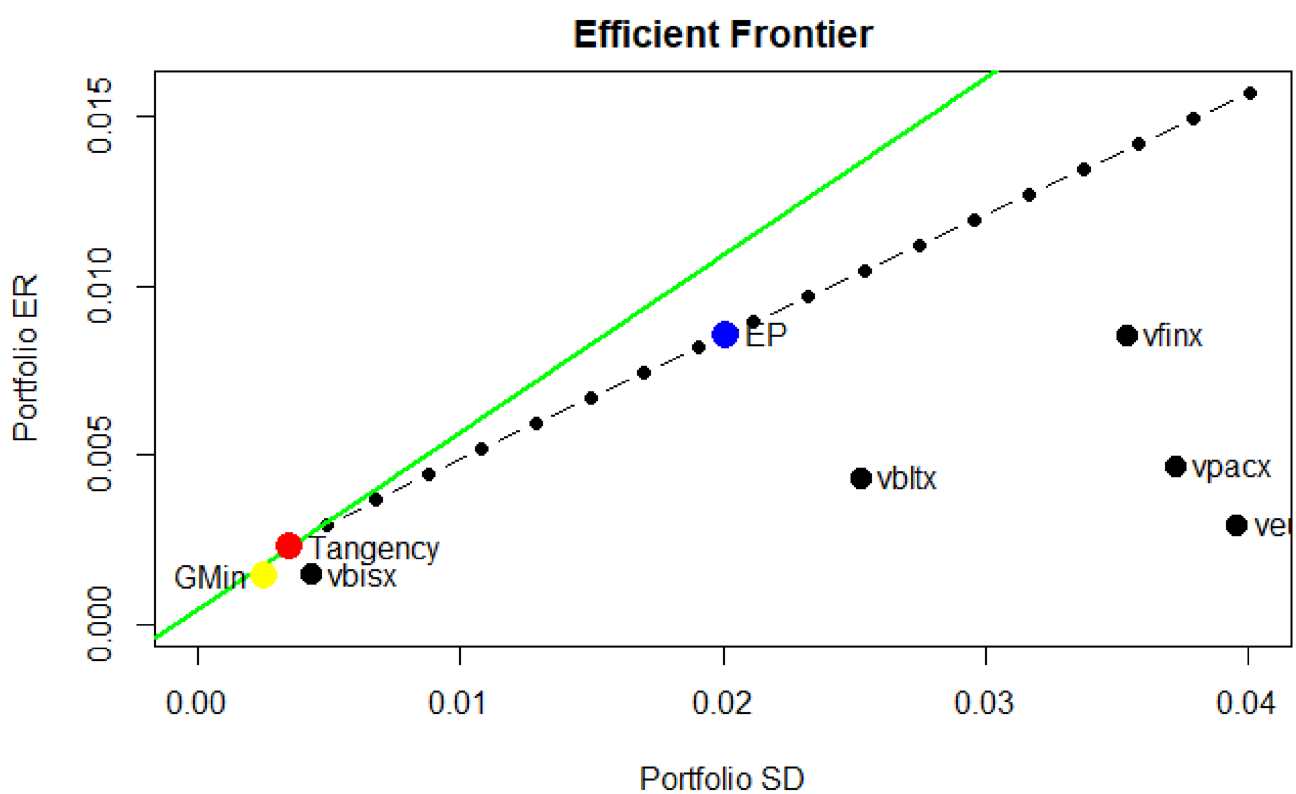
```
NA
```

Looking at the table, we have calculated the tangency portfolio's Sharpe ratio with the expected returns and standard deviation of the portfolio. We can see that this tangency portfolio has a a Sharpe ratio of nearly double that of any of the individual funds.



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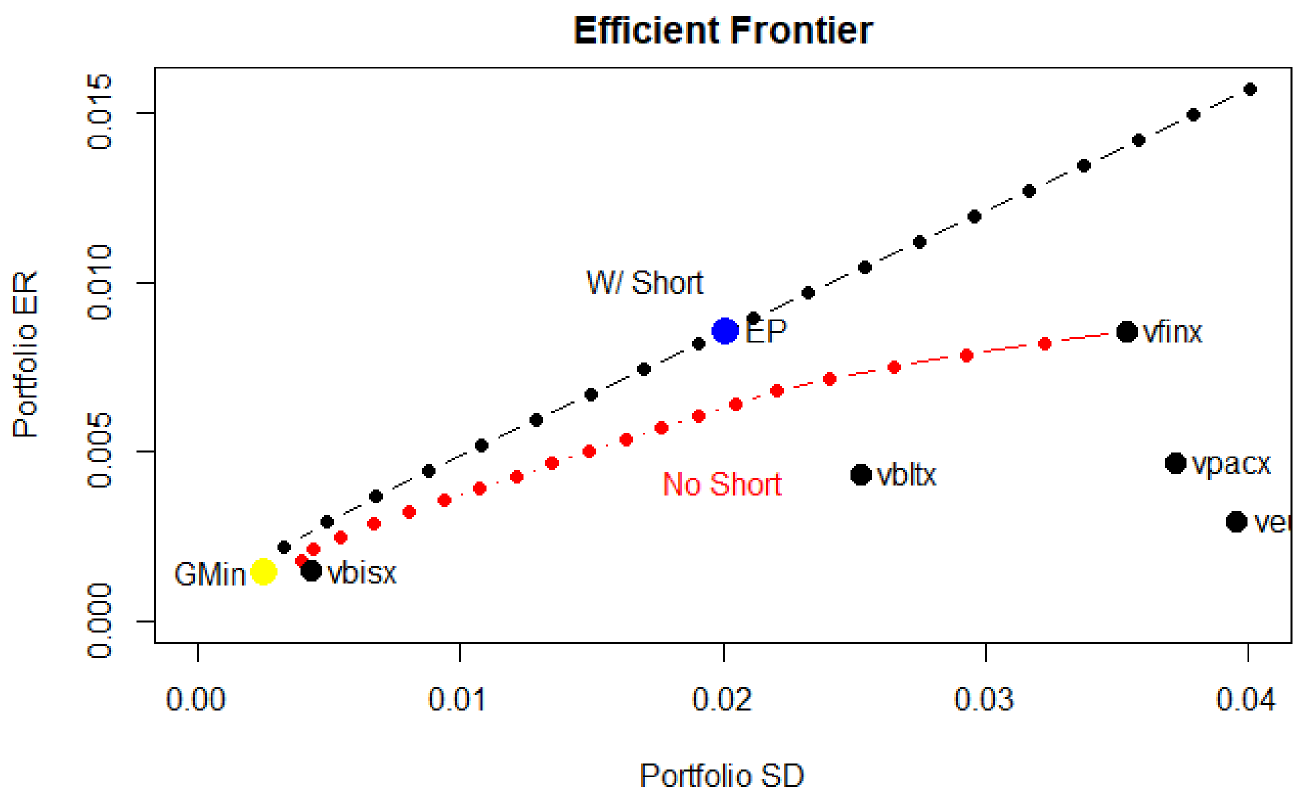
```
#Annualizing the tangency portfolio and computing the Sharpe ratio
annualized.tan.port.er = tan.port.er * 12
annualized.tan.port.sd = tan.port.sd * sqrt(12)
annualized.tan.port.sr = (annualized.tan.port.er - r.f.a) / annualized.tan.port.sd
annualized.tan.port.sr
```

```
[1] 1.820502
```

This tangency portfolio has a Sharpe ratio of 1.82, which is much higher than any of the individual funds' annualized Sharpe ratio. Referring back to the previously calculated annualized Sharpe ratios, the highest for the funds was a ratio of 0.841.



Using this graph of the different efficiency frontiers as an aid, you can easily see the difference between the frontier that allows short-sells and the one that does not. The frontier that does not allow short-sales maxes its expected returns out at a level equivalent to that of the S&P 500 index (vfinx). With no short-sales, the frontier line is less linear and more concave than the frontier allowing short-sales.

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```
#Loss in ER for efficient portfolio with short and no short @ .02 variance
ef.port.ns = efficient.portfolio(muhat.vals, sigma.mat, target.return = 0.0063, shorts = F)
ef.port$er - ef.port.ns$er
```

```
[1] 0.002265894
```

To determine the amount of expected returns lost from an efficient portfolio allowing short-sales vs one that doesn't I needed to figure out the distance from the efficiency frontier without short sales to the efficiency frontier with short sales. The efficient portfolio that was previously calculated that had the same expected return as the S&P 500 index (vfinx) had a standard deviation of about 0.02. Then I found that a target return of 0.0063 had a standard deviation of 0.02 along the efficiency frontier without short sale. Subtracting the first efficiency portfolio from this new found efficiency portfolio I was able to determine that the loss in expected returns was about 0.00227.

```
Call:
tangency.portfolio(er = muhat.vals, cov.mat = sigma.mat, risk.free = r.f,
    shorts = F)

Portfolio expected return:     0.002215067
Portfolio standard deviation:  0.004572792
Portfolio Sharpe Ratio:        0.3932754
Portfolio weights:
 vfinx  veurx  veiex  vbltx  vbisx  vpacx
0.1059 0.0000 0.0000 0.0000 0.8941 0.0000
```

ER	Variance	SD
0.002215	2.1e-05	0.004573

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```
#Calculating the Sharpe ratio of the tangency portfolio without short sales
tan.port.ns.SR = (tan.port.ns$er - r.f) / tan.port.ns$sd
tan.port.ns.SR
```

```
[1] 0.3932754
```

	ER	SD	Sharpe Ratio
W/ Short	0.027210	0.012200	1.820502
No Short	0.026581	0.015841	1.362346

The table above gives us an easy comparison of the annualized tangency portfolios' expected return, standard deviation and Sharpe ratio for with and without short-sales. For these two portfolios, the tangency portfolio with shorts has a higher expected return and a lower standard deviation. This yields the portfolio a higher Sharpe ratio, showing how limiting it is to not be able to short-sell when it comes to balancing out your portfolio.

#Asset Allocation

```
Call:
efficient.portfolio(er = muhat.vals, cov.mat = sigma.mat, target.return = 0.005,
    shorts = F)

Portfolio expected return:     0.005000002
Portfolio standard deviation:  0.01490122
Portfolio weights:
 vfinx  veurx  veiex  vbltx  vbisx  vpacx
0.3772 0.0000 0.0000 0.2992 0.3236 0.0000
```

The efficient portfolio that is able to achieve this 6% expected yearly return with no short sales is comprised of only 3 of the 6 funds. This portfolio includes the S&P500 index (vfinx), the long-term bond fund (vbltx) and the short-term bond fund (vbisx), having weights of 0.377, 0.299 and 0.324 respectively.

	5%	1%

	5%	1%
VaR	-1932.123	-2922.973

Compared to the previously calculated VaRs this stands as the most risky portfolio. This exhibits the trade off of risk and return. Trying to achieve a yearly return of 12% with no short sales is impossible. This can be seen visually on the graph containing the efficiency frontier without short sales. In order to achieve this level of return, you would need a monthly return of 1%, yet the highest expected return is the expected return equal to that of the S&P500 index (vfinx) which is 0.857% monthly return.