Homework 3

Part I

Problem 1

```
library(mvtnorm)
```

```
## Warning: package 'mvtnorm' was built under R version 3.5.3
```

```
mu.x = 0.05
sig.x = 0.10
mu.y = 0.025
sig.y = 0.05

rho.xy = 0.9
sig.xy = rho.xy * sig.x * sig.y
sigma.xy = matrix(c(sig.x^2, sig.xy, sig.xy, sig.y^2), 2, 2, byrow = TRUE)
sigma.xy
```

```
##        [,1]   [,2]
## [1,] 0.0100 0.0045
## [2,] 0.0045 0.0025
```

```
n = 100
set.seed(123)
xy.vals = rmvnorm(n, mean = c(mu.x, mu.y), sigma = sigma.xy)
head(xy.vals)
```

```
##             [,1]         [,2]
## [1,] -0.01055124 -0.002720223
## [2,]  0.19865393  0.081562113
## [3,]  0.12156743  0.091291081
## [4,]  0.04939103 -0.004618093
## [5,] -0.02987387 -0.014866126
## [6,]  0.17729184  0.080391233
```

```
plot(xy.vals[,1], xy.vals[,2], pch = 16, cex = 2, col = "blue", xlab = "x", ylab = "y")
title("Bivariate normal: rho=0.9")
abline(h=mu.y, v=mu.x)
segments(x0=0, y0=min(xy.vals[,2]), x1=0, y1=0, col="red")
segments(x0=min(xy.vals[,1]), y0=0, x1=0, y1=0, col="red")
```

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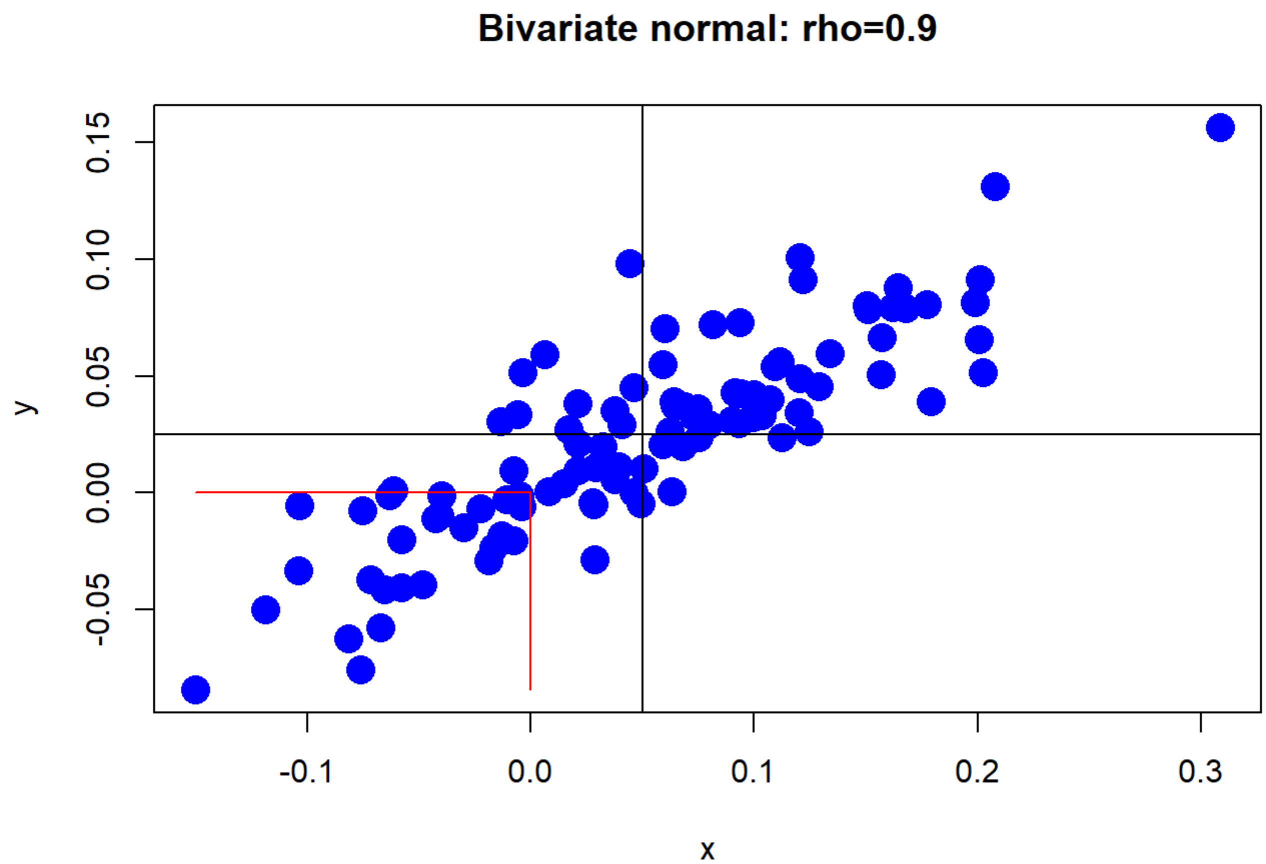


```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu.x, mu.y), sigma=sigma.xy)
```

```
## [1] 0.2453259
## attr(,"error")
## [1] 1e-15
## attr(,"msg")
## [1] "Normal Completion"
```

Promblem 2

```
rho.xy = -0.9
sig.xy = rho.xy * sig.x * sig.y
sigma.xy = matrix(c(sig.x^2, sig.xy, sig.xy, sig.y^2), 2, 2, byrow = TRUE)
n = 100
set.seed(123)
xy.vals = rmvnorm(n, mean = c(mu.x, mu.y), sigma = sigma.xy)
head(xy.vals)
```

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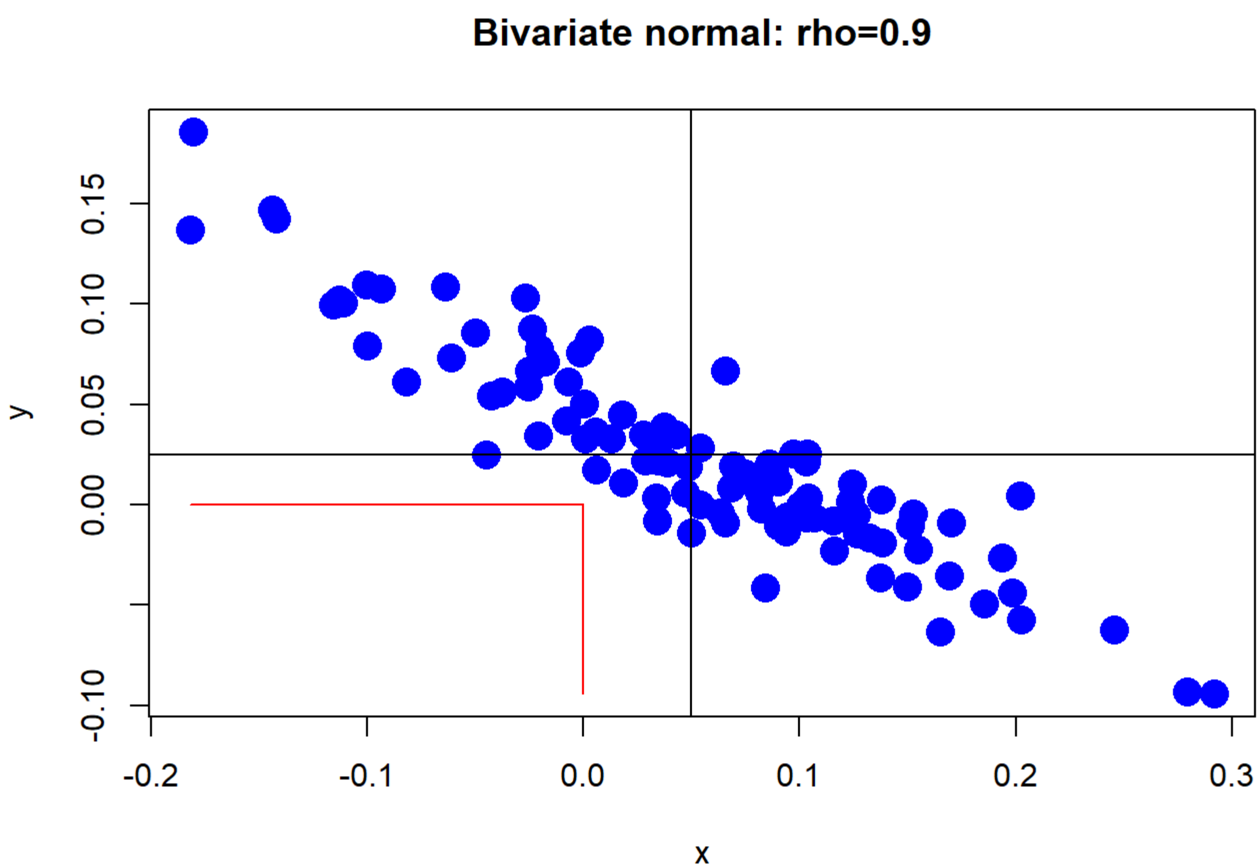
```
##             [,1]         [,2]
## [1,] 0.005403536  0.036129207
## [2,] 0.193766631 -0.026479922
## [3,] 0.002687517  0.082329487
## [4,] 0.137078885 -0.036566550
## [5,] 0.001017241  0.032743153
## [6,] 0.152351306 -0.004456129
```

```
plot(xy.vals[,1], xy.vals[,2], pch = 16, cex = 2, col = "blue", xlab = "x", ylab = "y")
title("Bivariate normal: rho=0.9")
abline(h=mu.y, v=mu.x)
segments(x0=0, y0=min(xy.vals[,2]), x1=0, y1=0, col="red")
segments(x0=min(xy.vals[,1]), y0=0, x1=0, y1=0, col="red")
```




```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu.x, mu.y), sigma=sigma.xy)
```

```
## [1] 0.0008028802
## attr(,"error")
## [1] 1e-15
## attr(,"msg")
## [1] "Normal Completion"
```

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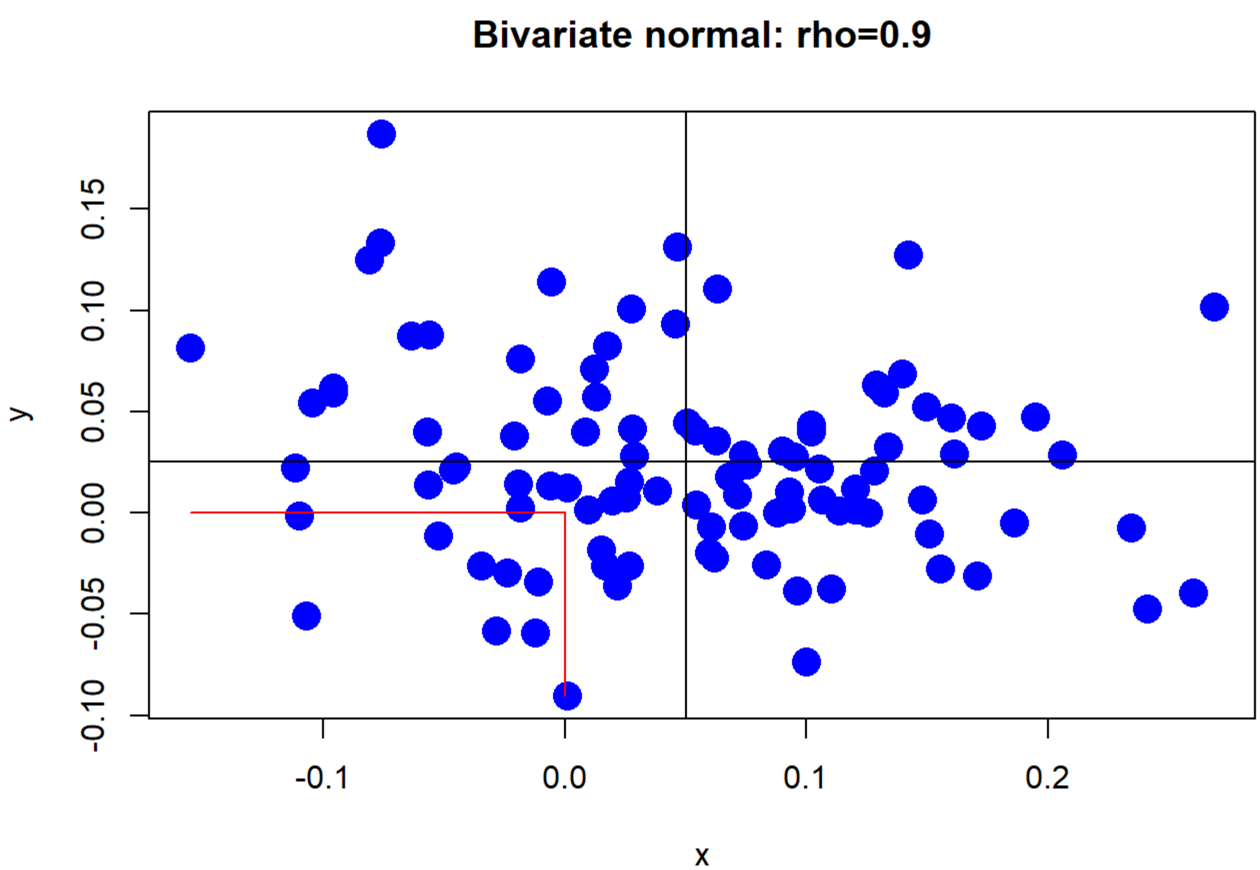
Problem 3

```
rho.xy = 0
sig.xy = rho.xy * sig.x * sig.y
sigma.xy = matrix(c(sig.x^2, sig.xy, sig.xy, sig.y^2), 2, 2, byrow = TRUE)
n = 100
set.seed(123)
xy.vals = rmvnorm(n, mean = c(mu.x, mu.y), sigma = sigma.xy)
head(xy.vals)
```

```
##              [,1]         [,2]
## [1,] -0.006047565  0.013491126
## [2,]  0.205870831  0.028525420
## [3,]  0.062928774  0.110753249
## [4,]  0.096091621 -0.038253062
## [5,] -0.018685285  0.002716901
## [6,]  0.172408180  0.042990691
```

```
plot(xy.vals[,1], xy.vals[,2], pch = 16, cex = 2, col = "blue", xlab = "x", ylab = "y")
title("Bivariate normal: rho=0.9")
abline(h=mu.y, v=mu.x)
segments(x0=0, y0=min(xy.vals[,2]), x1=0, y1=0, col="red")
segments(x0=min(xy.vals[,1]), y0=0, x1=0, y1=0, col="red")
```



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```
pmvnorm(lower=c(-Inf, -Inf), upper=c(0, 0), mean=c(mu.x, mu.y), sigma=sigma.xy)
```

```
## [1] 0.09519541
## attr(,"error")
## [1] 1e-15
## attr(,"msg")
## [1] "Normal Completion"
```

Part II

Problem 1

```
MatA = matrix(c(1, 4, 7, 2, 4, 8, 6, 1, 3), nrow = 3, ncol = 3, byrow = TRUE)
MatB = matrix(c(4, 4, 0, 5, 9, 1, 2, 2, 5), nrow = 3, ncol = 3, byrow = TRUE)
VecX = as.matrix(c(1, 2, 3))
VecY = as.matrix(c(5, 2, 7))
```

Problem 2

```
t(MatA)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    6
## [2,]    4    4    1
## [3,]    7    8    3
```

```
t(MatB)
```

```
##      [,1] [,2] [,3]
## [1,]    4    5    2
## [2,]    4    9    2
## [3,]    0    1    5
```

```
t(VecX)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    3
```

```
t(VecY)
```

```
##      [,1] [,2] [,3]
## [1,]    5    2    7
```

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Problem 3

```
MatA + MatB
```

```
##      [,1] [,2] [,3]
## [1,]    5    8    7
## [2,]    7   13    9
## [3,]    8    3    8
```

```
MatA - MatB
```

```
##      [,1] [,2] [,3]
## [1,]   -3    0    7
## [2,]   -3   -5    7
## [3,]    4   -1   -2
```

```
2 * MatA
```

```
##      [,1] [,2] [,3]
## [1,]    2    8   14
## [2,]    4    8   16
## [3,]   12    2    6
```

```
MatA %*% VecX
```

```
##      [,1]
## [1,]   30
## [2,]   34
## [3,]   17
```

```
t(VecX) %*% MatA %*% VecY
```

```
##      [,1]
## [1,]  369
```

```
t(VecY) %*% MatA %*% VecX
```

```
##      [,1]
## [1,]  337
```

Prolem 4

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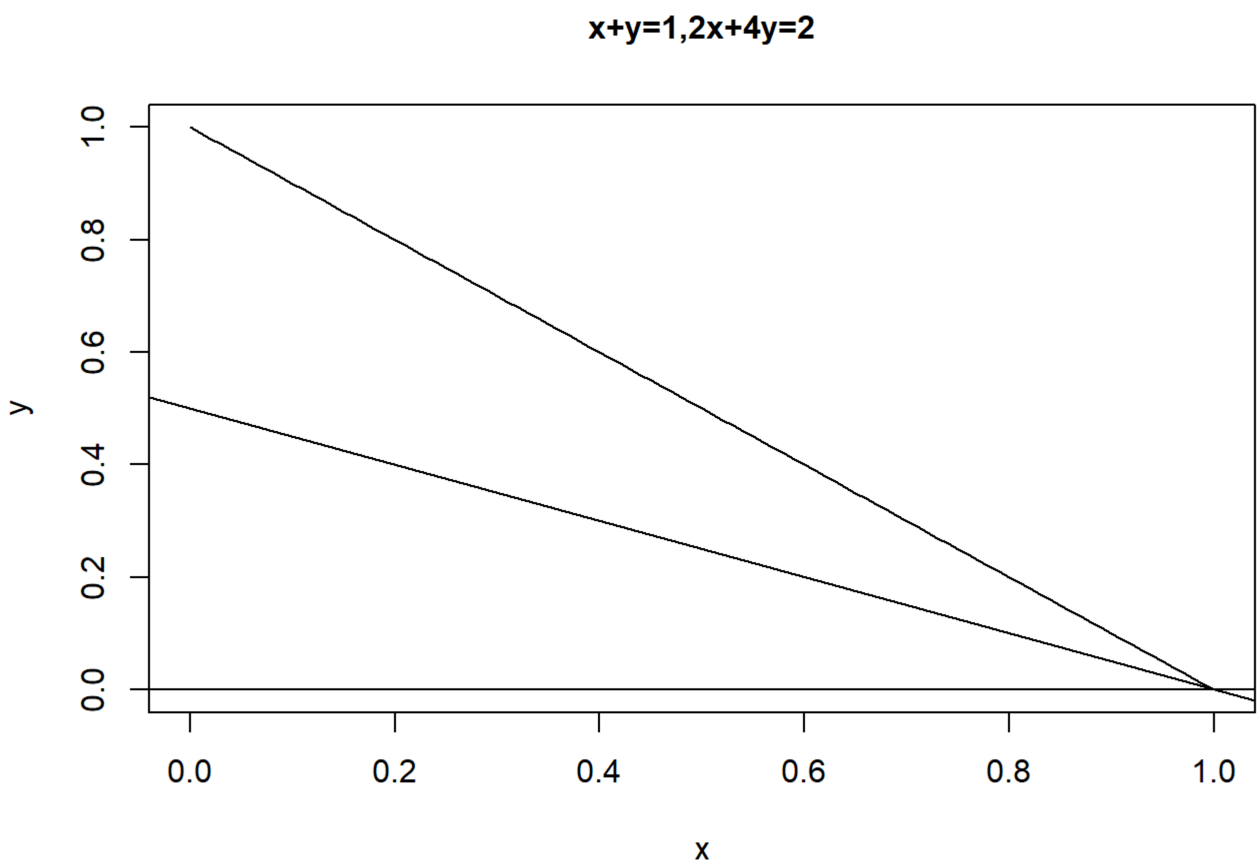
```
curve(1-x,0,1,lwd=1,ylab="y",xlim=c(0,1),ylim=c(0,1))
abline(a=.5,b=-.5,lwd=1,ylim=c(-1,1),xlim=c(0,1.5))
abline(h=0)
title("x+y=1,2x+4y=2",cex.main=1)
```




```
matA=matrix(c(1,1,2,4),2,2,byrow=TRUE)
vecB=c(1,2)
matA.inv=solve(matA)
matA.inv
```

```
##      [,1] [,2]
## [1,]    2 -0.5
## [2,]   -1  0.5
```

```
matA.inv%*%matA
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

```
z=matA.inv%*%vecB
z
```

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```
##      [,1]
## [1,]    1
## [2,]    0
```

Problem 5

```
MatMu = matrix(c(0.01, 0.04, 0.02))
MatSig = matrix(c(.1, .3, .1, .3, .15, -.2, .1, -.2, .08), 3, 3, byrow = TRUE)
Weight = matrix(c(1/3, 1/3, 1/3))
crossprod(MatMu, Weight)
```

```
##            [,1]
## [1,] 0.02333333
```

```
t(Weight)%*% MatSig %*% Weight
```

```
##            [,1]
## [1,] 0.08111111
```

Problem 6

The portolio returns for x is $R_{p,x}$=R'x The portolio returns for y is $R_{p,y}$=R'y The Portfolio expected returns for x is $\mu_{p,x}$=x'μ The Portfolio expected returns for y is $\mu_{p,y}$=y'μ

Problem 7

The matrix algebra expression for the constraint that the portfolio weights sum to one is x'1

Problem 8

The portfolio variance for x is $\sigma^2_{p,x}$=x'Σx The portfolio variance for y is $\sigma^2_{p,y}$=y'Σy The covariance between $R_{p,x}$ and $R_{p,y}$ is x'Σy

Time Series Concepts

Problem 1

Covariance is stationary if the mean and standarad deviation are finite and do not depend on t and the covariance has a jth order of autovariance, is finite, and depends only on j but not on t for j=0,1,2,…

Problem 2

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Process 1 seems to be covariance stationary with a central mean throughout the time series. Process 2, 3, 4 all do not seem to be covariance stationary. Process 2's mean increases over time in a linear manner. Process 3 seems to be on a random walk and does not have a centralized mean. Finally, Process 4 has an increasing variance over time.