## Supplementary Results and Figures

Family	#	Avg. time	Speedup
miter-unsat	14	34.86	0.30
profitable-robust-production-sat	5	7.19	0.35
heule-nol-sat	7	18.50	0.43
social-golfer-sat	6	8.23	0.52
grs-fp-comm-unsat	8	75.60	0.62
scheduling-unsat	9	26.66	0.80
software-verification-unsat	10	56.29	0.97
cryptography-simon-sat	8	0.13	0.98
random-circuits-sat	5	24.89	1.01
hamiltonian-unsat	11	7.11	1.05
cryptography-ascon-unsat	6	8.58	1.05
argumentation-unsat	18	7.45	1.06
satcoin-unsat	5	16.36	1.28
brent-equations-sat	7	0.96	1.29
hamiltonian-sat	12	2.14	1.35
maxsat-optimum-sat	5	5.78	1.36
scheduling-sat	9	40.00	1.59
heule-folkman-sat	5	81.82	1.82
school-timetabling-sat	8	21.01	2.00
cryptography-sat	6	12.77	2.05
set-covering-sat	14	11.07	2.09
mutilated-chessboard-unsat	6	31.48	2.27

Table 2 Geometric mean speedup of "search-only" configuration over default Kissat-only configuration, at 768 cores, separated by GBD benchmark family and result, showing all such categories where the number of considered instances ("#") is at least five. "Avg. time" denotes the average running time of the default configuration on the respective instances.

Family	#	Avg. time	Speedup
set-covering-sat	6	0.21	0.04
heule-folkman-sat	6	5.82	0.08
register-allocation-unsat	11	0.12	0.10
random-circuits-sat	10	23.27	0.60
rbsat-sat	5	13.93	0.68
maxsat-optimum-unsat	5	19.11	0.70
hamiltonian-unsat	9	5.19	0.78
brent-equations-sat	9	0.63	0.81
argumentation-unsat	16	12.00	0.84
profitable-robust-production-sat	8	70.91	0.84
cryptography-ascon-unsat	10	8.12	0.88
quantum-kochen-specker-unsat	6	9.92	0.90
hamiltonian-sat	8	1.46	1.03
scheduling-unsat	6	28.64	1.04
scheduling-sat	17	24.12	1.08
satcoin-unsat	10	13.73	1.16
cryptography-sat	8	24.14	1.18
school-timetabling-sat	9	11.98	1.25
miter-sat	9	87.61	1.25
miter-unsat	23	29.96	1.26
coloring-unsat	5	24.48	1.26
software-verification-unsat	5 7	34.61	1.38
hashtable-safety-unsat	7	100.41	1.48
cryptography-ascon-sat	8	3.00	1.57
grs-fp-comm-unsat	6	65.93	1.63

**Table 3** Geometric mean speedup of our new setup over KCL, at 3072 cores, split as in Table 2. "Avg. time" denotes the average running time of KCL on the respective instances.

Procedure	Kissat	$768 \times d$	768×s-o
backbone	0.12	0.18	_
congruence	0.46	1.85	_
eliminate	1.01	1.33	_
extend	0.00	0.00	_
factor	0.55		_
fastel	0.30	2.96	_
focused	43.39	44.65	49.97
lucky	0.08	1.34	1.59
parse	0.15	_	_
preprocess	0.83	4.77	_
probe	11.15	13.52	_
reduce	1.37	2.68	3.24
search	86.32	79.15	95.41
simplify	12.61	13.86	1.77
stable	42.93	34.50	45.44
substitute	0.66	1.39	_
subsume	0.39	0.36	_
sweep	2.18	1.54	_
transitive	0.17	0.20	_
vivify	7.03	8.35	_
walking	0.96	0.65	1.77

**Table 4** Percentages of total ("CPU") time spent in different procedures of Kissat's SAT solving, for sequential Kissat, 768-core default setup, and 768-core search-only setup. Note that some categories subsume each another (e.g., focused and stable are disjoint sub-categories of search).

Here we report two additional parameter studies. They were done with the setting denoted "Ours", with one difference: original LBDs were still used instead of the Deactivated-LBDs setting. Each run consisted of 396 instances with 300 s timeout on 8 nodes (384 cores).

The first study explores clause database reduction. Kissat offers two parameters, reduce-low (default 500) and reduce-high (default 900) which control the aggressiveness of database reductions. The default parameters correspond to reductions by 50% early in the run and by up to 90% later in the run. We tested four parameter settings, described briefly.

Reduce setting	#	+	_	PAR2
Default	323	153	170	137.9
Point	323	154	169	139.2
Gaussian	321	151	170	140.8
Range	318	149	169	144.2

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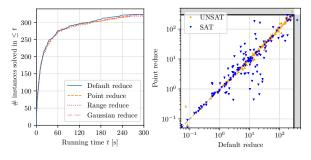


Figure 12 Effects of diversifying Kissat's reduce-low and reduce-high parameters.

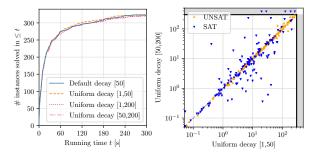
**Default reduce**: Left the reduce parameters at their default. **Point reduce**: A value  $r \in [0, ..., 1000]$  is uniformly sampled per solver and both reduce-low and reduce-high are set to it. This creates some extreme solvers that keep all clauses forever (r = 0) and others that delete every clause almost immediately (r = 1000). **Range reduce**: A value  $r \in [-200, 1200]$  is uniformly sampled per thread; then we set reduce-low=max(0,r-200) and reduce-high=min(1000,r+200). Intuitively this slides the default interval [500,900]

randomly to higher or lower values and leaves per solver the flexibility of shifting from low to high reductions. **Gaussian reduce**: A value r is sampled from a Gaussian distribution with mean 700 and standard deviation 150, then r is clipped to be within [300,980] and both reduce-low and reduce-high are set to it. This sampling specifically avoids the extremes from the other two settings.

The results of the four reduce settings are shown in Fig. 12. Default reduce performs overall best, whereas Point reduce performs strongly on some SAT instances, which might be due to some aggressive solvers being allowed to eliminate almost all learned clauses.

The second study focuses on the decay of (E)VSIDS scores controlled by the decay parameter. Its default value is 50 (per mille), corresponding to an update of variables activity scores to 95% of their former value. Higher decay results in more aggressive updates.

Decay setting	#	+	_	PAR2
Uniform [1,50] Default [50]				135.5 137.9
Uniform [1,200] Uniform [50,200]	322	152	170	140.1
Omnorm [50,200]	321	191	170	141.1



**Figure 13** Effects of diversifying Kissat's decay parameter.

Kissat accepts values in the range of [1,...,200]. We explore this full range by testing three settings: **Uniform**[1,50], **Uniform**[1,200] and **Uniform**[50,200]. In each setting every solver thread samples its decay value uniformly from the given interval. The third setting is thus much more eager than the default, while the first is more conservative.

The results of the different decay settings are shown in Fig. 13. Regarding PAR2 scores, the conservative updating with decay at or below 50 performs better than the more aggressive choices. However, similar to the database reductions, the more eager approaches perform better on some SAT instances, observable in the direct comparison plot.

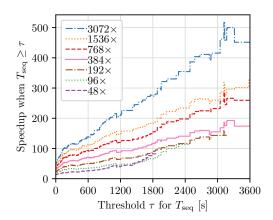
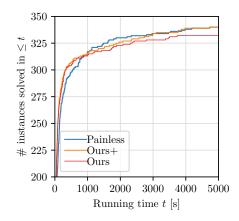


Figure 14 Weak Scaling of KCL configuration (as in Fig. 11).



	#	+	-	PAR2
Ours	332	148	184	1857.0
Ours+	<b>340</b>	154	186	1705.3
Painless	<b>340</b>	159	181	1721.4

Figure 15 Performance of our approach from the paper ("Ours"), an enhanced version ("Ours+") with additional Lingeling-based preprocessing and each 20th thread running YalSAT instead of CDCL, and the state-of-the-art shared-memory solver PL-PRS-BVA-KISSAT, at a single node (48 cores) and, notably, for up to 5000 s of running time as in the SAT Competition Parallel tracks.