DISCRETE STRUCTURE & APPLICATIONS BUM1433

BASIC COUNTINGS

1.5 The Pigeonhole Principle

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Objectives

Find the least number of pigeons if given a number of pigeonholes.

Introduction

There are many problems in which an object with certain properties need to be determined. For example:

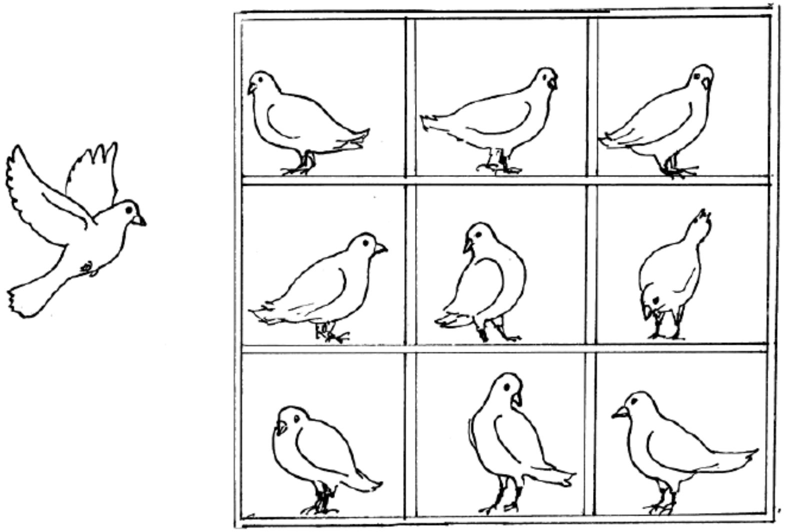
- *If 101 people are booked for a trip and the plane has only 100 seats, then at least 2 people must be assigned the same seat.*

- *If there are 13 people in a room, at least 2 people must be born in the same month.*

This problem can be solved by applying the principle known as the **pigeonhole principle**. The pigeonhole principle is also known as the **Dirichlet drawer principle**. This principle was first formally stated by Peter Gustav Lejeune Dirichlet (1805-1859).

The Pigeonhole Principle

If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.



In order to apply the **pigeonhole principle**, we need to identify which objects are pigeons and which objects are pigeonholes.

Another examples of **pigeonhole principle**:

- Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

- If eight people are chosen in any way from some group, at least two of them will have been born on the same day of the week.

- In any group of 27 English words, there must be at least two that begin with the same letter, since there are 26 letters in the English alphabet.

Example 1:

How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Solution:

$0 - 100 = 101$

atleast 102 #

Example 2:

Suppose the positive integers in the set A are ≤ 32. Determine the least number of integers that must be chosen so that at least one of them is divisible by 3 or 5.

Solution:

The Generalized Pigeonhole Principle

If N objects are placed into k boxes, then there is at least one box containing at least objects. Mathematically,

$$\left\lceil \frac{N}{k} \right\rceil = m$$

If $\left\lceil \frac{N}{k} \right\rceil = m$, then $N = [(m-1).k] + 1$

where $\lceil \cdot \rceil$ represents the ceiling function, which assigns to the real number by the smallest integer that is greater than or equal to real number. For example:

$$\lceil 1.3 \rceil = 2; \quad \lceil 0.9 \rceil = 1; \quad \lceil 25.56 \rceil = 26; \quad \lceil -3.65 \rceil = -3$$

Example 3:

In a group of 100 people, what is the least number of people born in the same month for any given year?

Solution:

100 people → N

1 year = 12 months → k

$$\left\lceil \frac{N}{k} \right\rceil = \left\lceil \frac{100}{12} \right\rceil = \lceil 8.3333 \rceil = 9$$

Example 4:

$N = 11$

$k = 2 < {}^{F}_{m}$

$\left\lceil \frac{11}{2} \right\rceil = \lceil 5.5 \rceil = 6$

1 In a group of 11 people, there are at least 6 men or 6 women.

$N = 30$

$k = 7$

$\left\lceil \frac{30}{7} \right\rceil = \lceil 4.2857 \rceil$

$= 5$

If any 30 people are selected, then 5 were born on the same day of the week. 2

3 In a group of 1099 people, at least four share a birthday.

$N = 1099$

$k = 365$

$\left\lceil \frac{1099}{365} \right\rceil = \lceil 3.011 \rceil = 4$

If 30 dictionaries in a library contain a total of 61327 pages, 4
then one of the dictionaries must have at least 2045 pages.

Example 5:

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D, and F?

Solution:

A, B, C, D, F → boxes (k)

N = ?

$\left\lceil \frac{N}{k} \right\rceil = \left\lceil \frac{N}{5} \right\rceil = 6 \to m$

$N = [(m-1)\cdot k] + 1$

$= [(6-1)\cdot 5] + 1$

$= 26$

Example 6:

A box contains eight green balls and six red balls. How many balls one must take out from the box to be sure that at least two balls have the same colour?

Solution:

Exercises

1. Let X be a set of 100 distinct positive integers. If these positive integers are divided by 75, show that at least two of the remainders must be the same.

2. Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

3. A box contain 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
   (a) How many balls must she select to be sure of having at least three balls of the same color.
   (b) How many balls must she select to be sure of having at least three red balls?

4. A basket contains fruits such as nine apples and nine bananas. Maryam takes the fruit at random without looking at them.
   (a) How many fruits must she select to be sure of having at least three of the same types of fruits?
   (b) How many fruits must she select to be sure of having at least three bananas?


thank you