# DISCRETE STRUCTURE & APPLICATIONS BUM1433

# CHAPTER 1 : BASIC OF COUNTING

Ref : Rosen K.H., *Discrete Mathematics & Its Applications*, (Sixth Edition), McGraw-Hill, 2007

# 1.8: BINOMIAL COEFFICIENTS

*Lesson outcome:*

- *Find the binomial coefficients of a term from a binomial series.*

# Binomial Theorem

- The binomial theorem states that a binomial expansion $(x + y)^n$ can be expanded as follows:

$$(x + y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \ldots + \binom{n}{r} x^{n-r} y^r \ldots + y^n$$

$$= \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

- Where $\displaystyle {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ is called a binomial coefficient with $k! = k(k-1)(k-2)\cdots(2)(1)$ and $0! = 1$

# Example 1.8.1

- Find the expansion of $(x+y)^3$
  -

    – using the combinatorial reasoning

  **_Solution_:**
  $$(x+y)^3 = (x+y)(x+y)(x+y)$$
  $$=$$

    – using the binomial theorem

  **_Solution_:**
  $$(x+y)^3 = \sum_{r=0}^{3}\binom{n}{r}x^{n-r}y^r$$
  $$= \binom{3}{0}x^3 + \binom{3}{1}x^2y^1 + \binom{3}{2}x^1y^2 + \binom{3}{3}y^3$$
  $$=$$

# EXERCISE 1.8

1. Expand the following by using Binomial theorem:

A $\left(1+x\right)^4$

B $\left(3-x\right)^5$

C $\left(2+3x\right)^3$

D $\left(2x+3y^2\right)^4$

E $\left(\dfrac{1}{x}-2\sqrt{x}\right)^5$

F $\left(\dfrac{1}{2}c+d^3\right)^4$

# Example 1.8.2

1. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(x+y)^{25}$?

*Solution*:

$$\binom{25}{13} =$$

2. What is the coefficient of $x^{12}y^{13}$ in the expansion of $(2x-3y)^{25}$

*Solution*:

$$(2x-3y)^{25} = \sum_{r=0}^{25}\binom{25}{r}(2x)^{25-r}(-3y)^r$$

# EXERCISE 1.8 (cont.)

2. Without expanding completely, find the indicated term(s) in the expansion of the expression.
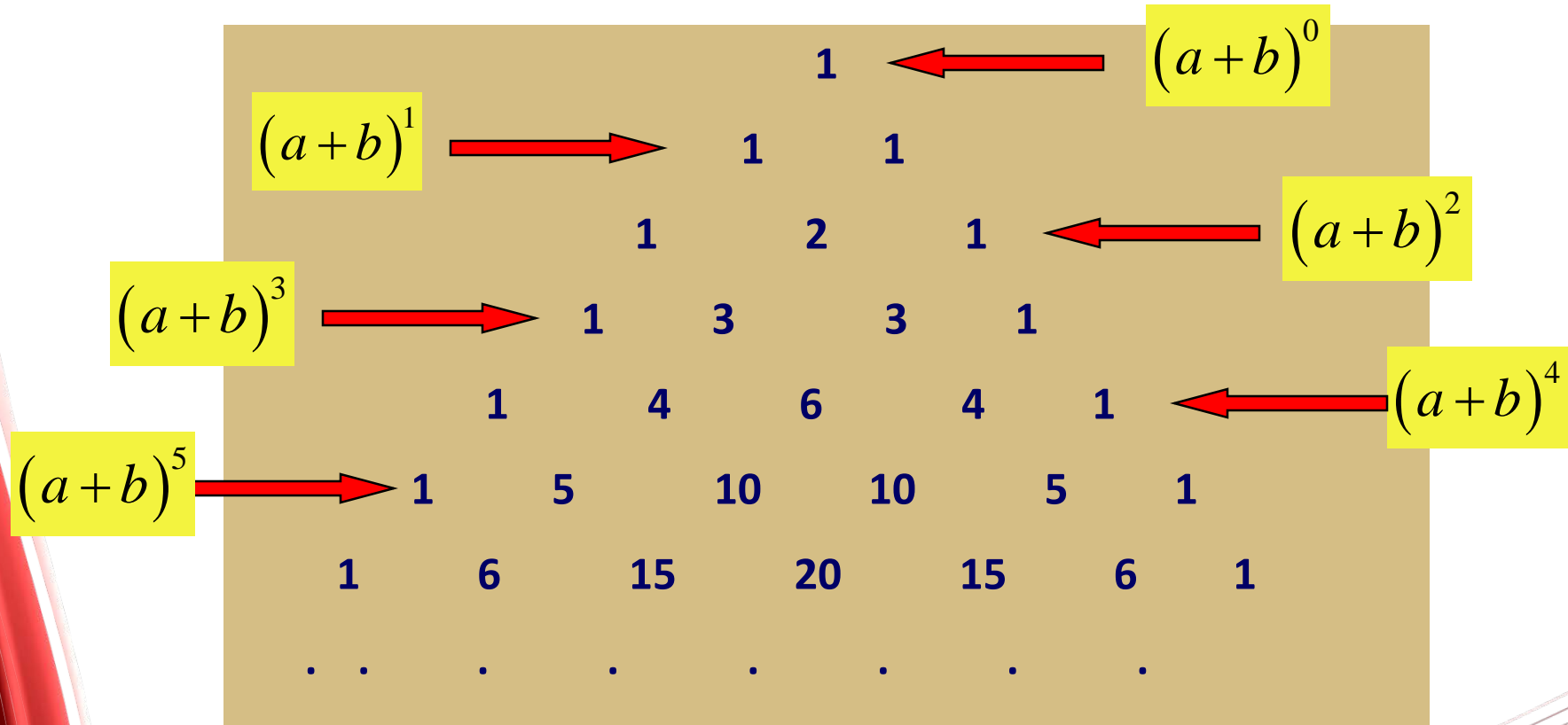
**A** $\left( x^3 + \sqrt{y} \right)^{13}$ , fifth term

**B** $\left( \dfrac{1}{3} p + q^2 \right)^{12}$ , term involving $q^{10}$

**C** $\left( xy - 3y^{-1} \right)^8$ , term that does not contain $y$

# PASCAL'S TRIANGLE

- Used to obtain binomial coefficients

$$1 \quad \longleftarrow \quad (a+b)^0$$

$$(a+b)^1 \quad \longrightarrow \quad 1 \quad 1$$

$$1 \quad 2 \quad 1 \quad \longleftarrow \quad (a+b)^2$$

$$(a+b)^3 \quad \longrightarrow \quad 1 \quad 3 \quad 3 \quad 1$$

$$1 \quad 4 \quad 6 \quad 4 \quad 1 \quad \longleftarrow \quad (a+b)^4$$

$$(a+b)^5 \quad \longrightarrow \quad 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$

$$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$

. . . . . . . .

# EXERCISE 1.8 (cont.)

3. Expand the following by using Pascal Triangle

A
$$(a+b)^7$$

B
$$(3-x)^5$$

C
$$(2+3x)^3$$

# Thank You