Propagating state distributions: `prop`

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Abstract

This is a short documentation of `prop.m`.

At time t the state distribution consists of two copies of the representation, the actual state, $s(t)$, and the forward predicted belief state, $r(t)$. These are jointly Gaussian

$$\begin{bmatrix} s(t) \\ r(t) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_s \\ \mu_r \end{bmatrix}, \begin{bmatrix} \Sigma_s & \Sigma_c \\ \Sigma_c^\top & \Sigma_r \end{bmatrix} \right).$$

The belief state, $b(t)$, is computed by combining the noisy actual state $\mathcal{N}(\mu_s, \Sigma_y = \Sigma_s + \Sigma_n)$ with $r(t)$ using a convex combination where each mean is weighted by its precision. Note, that this only corresponds to the Bayesian posterior if $s(t)$ and $r(t)$ are uncorrelated.

$$\begin{aligned} \mu_b &= \left(\Sigma_s^{-1} + \Sigma_r^{-1}\right)^{-1} \left(\Sigma_s^{-1}\mu_s + \Sigma_r^{-1}\mu_r\right) \;=\; Z_r\mu_s + Z_n\mu_r, \\ \Sigma_b &= Z_r(\Sigma_s + \Sigma_n)Z_r^\top + Z_n\Sigma_r Z_n^\top + Z_r\Sigma_c Z_n^\top + Z_n\Sigma_c^\top Z_r^\top, \end{aligned}$$

where we have defined $Z_r = \Sigma_r(\Sigma_s + \Sigma_r)^{-1}$ and $Z_n = \Sigma_n(\Sigma_s + \Sigma_r)^{-1}$.

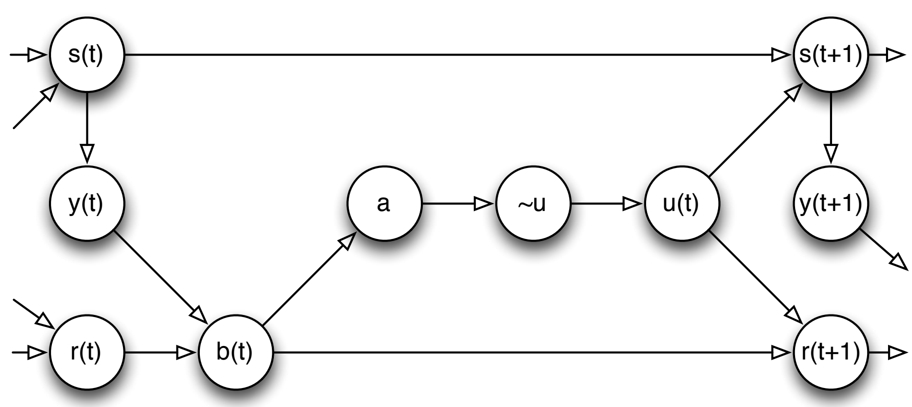

Figure 1: Graphical model representing the time evolution of the state s and belief-state b.

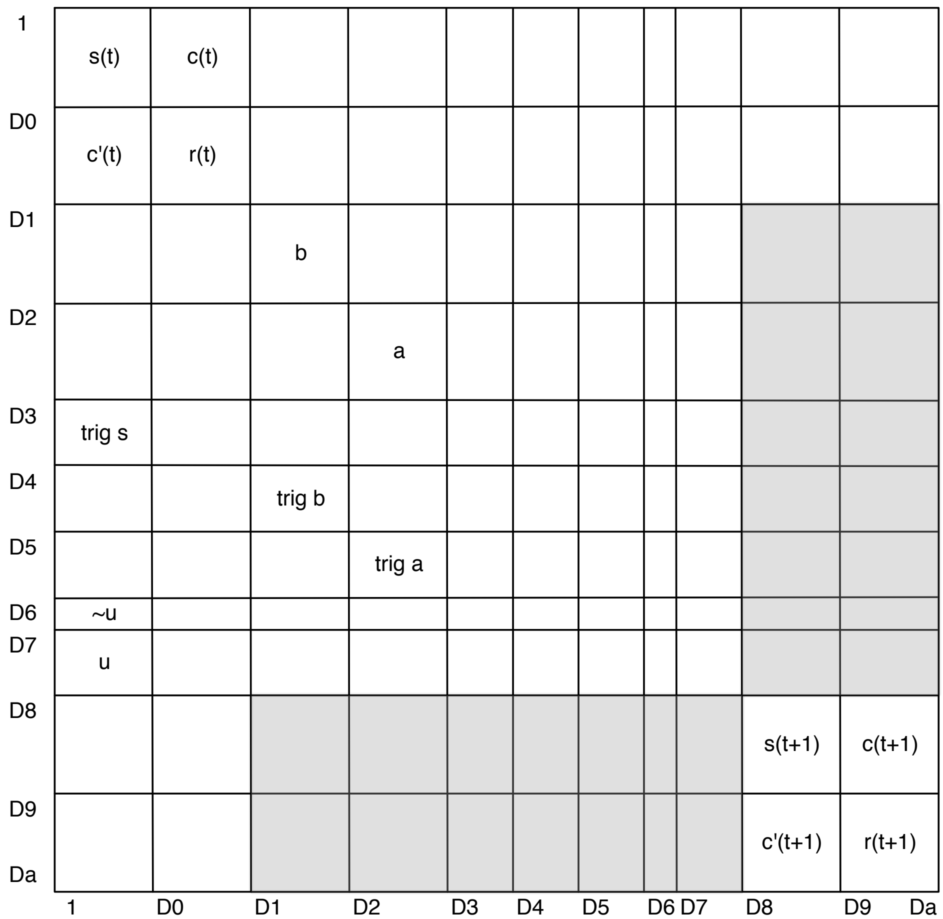

Figure 2: Diagram showing the memory layout for the covariance matrix associated with a single forward propagation. The mean vector uses a similar ordering. Shaded variables are never computed.