Faculty Development Program
on
Machine Learning and Image Processing

Convolutional Neural Network

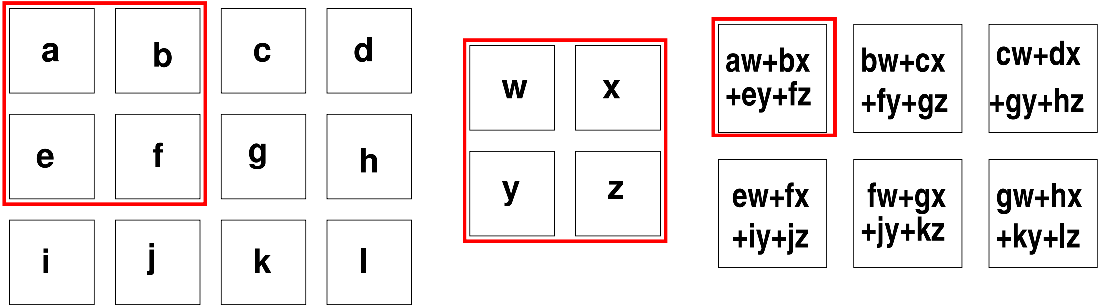
Introduction

- Specialized neural network for processing data that has grid like topology
  - Time series data (one dimensional)
  - Image (two dimensional)
- Found to be reasonably suitable for certain class of problems eg. computer vision
- Instead of matrix multiplication, it uses convolution in at least one of the layers

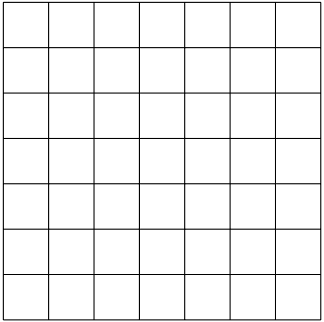
Convolution operation

- Consider the scenario of locating a spaceship with a laser sensor
- Suppose, the sensor is noisy
  - Accurate estimation is not possible
- Weighted average of location can provide a good estimate $s(t) = \int x(a)w(t-a)da$
  - $x(a)$ — Location at age a by the sensor, t — current time, w — weight
  - This is known as convolution
  - Usually denoted as $s(t) = (x * w)(t)$
- In neural network terminology x is input, w is kernel and output is referred as feature map

2D convolution

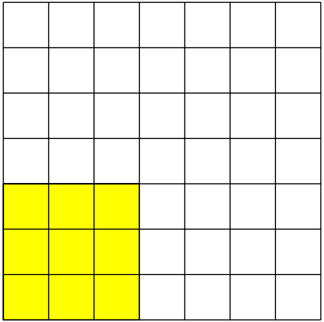


2D Convolution



Grid size: 7×7

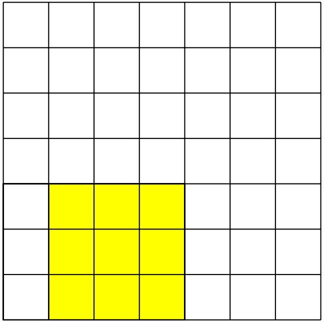
2D Convolution



Grid size: 7×7

Filter size: 3×3
Stride: 1

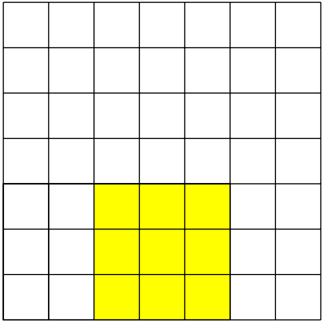
2D Convolution



Grid size: 7×7

Filter size: 3×3
Stride: 1

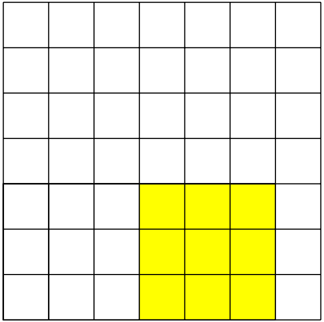
2D Convolution



Grid size: 7×7

Filter size: 3×3
Stride: 1

2D Convolution

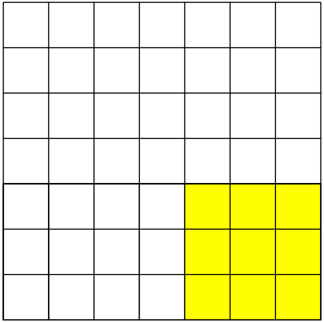


Grid size: 7×7

Filter size: 3×3

Stride: 1

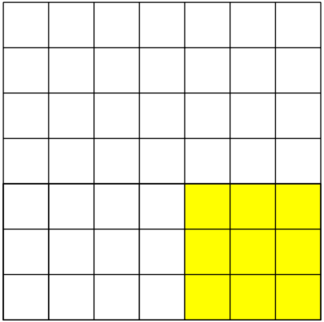
2D Convolution



Grid size: 7×7

Filter size: 3×3
Stride: 1

2D Convolution

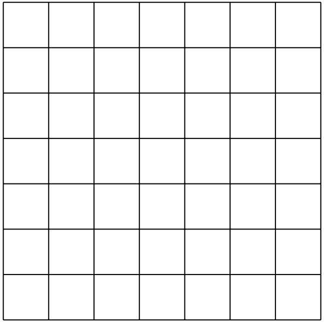


Grid size: 7×7

Filter size: 3×3
Stride: 1

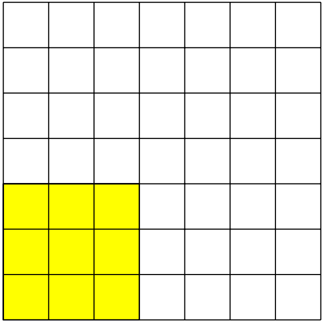
Output size: 5×5

2D convolution with stride



Grid size: 7×7

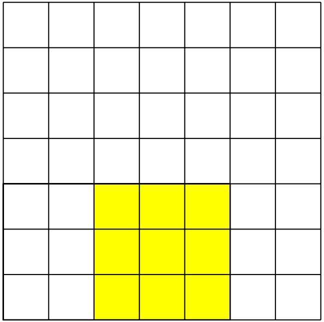
2D convolution with stride



Grid size: 7×7

Filter size: 3×3
Stride: 2

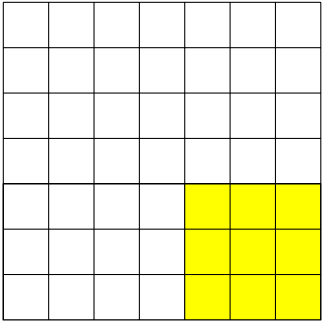
2D convolution with stride



Grid size: 7×7

Filter size: 3×3
Stride: 2

2D convolution with stride

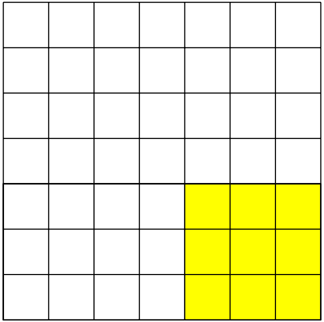


Grid size: 7×7

Filter size: 3×3

Stride: 2

2D convolution with stride

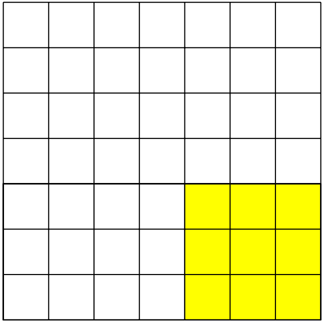


Grid size: 7×7

Filter size: 3×3
Stride: 2

Output size: 3×3

2D convolution with stride



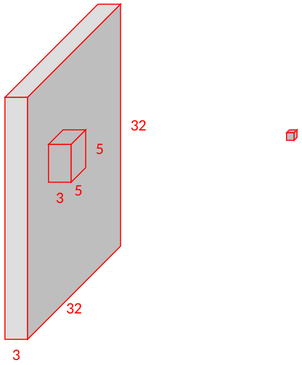
Grid size: 7×7

Filter size: 3×3
Stride: 2

Output size: 3×3

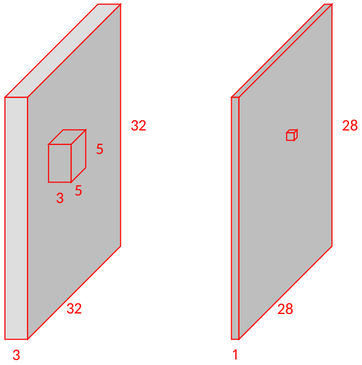
Output size: $(N - F)/S + 1$
N - input size, F - Filter size,
S - Stride

Convolution operation



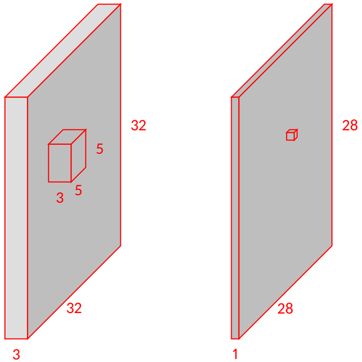
Filters are specified as 5×5. Channel depth is implicit.

Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

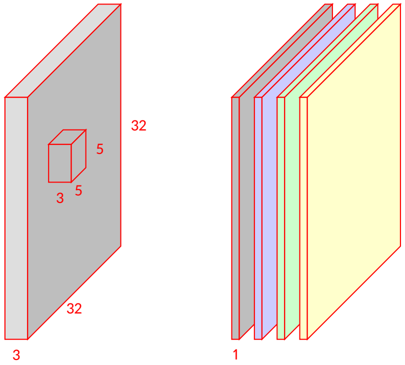
Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

No. of paramters 75 excluding bias. Computation (multiplication) - $28 \times 28 \times 5 \times 5 \times 3$

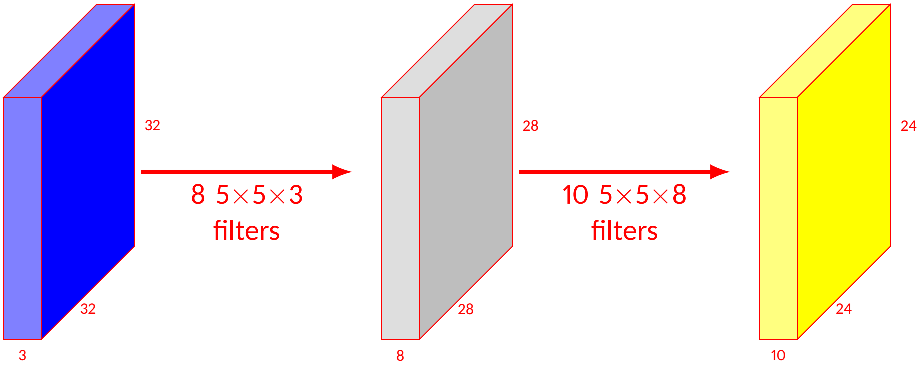
Convolution operation



Filters are specified as 5×5. Channel depth is implicit.

No. of paramters 75 excluding bias. Computation (multiplication) - $28 \times 28 \times 5 \times 5 \times 3$

Convolution example



Edge detection using naive filter

- **Filter** $[-1 \ 1]$

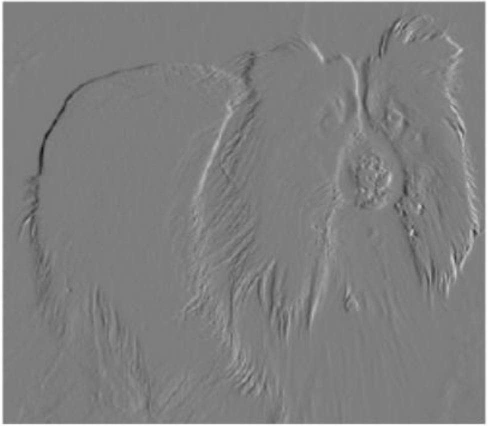


Image source: Deep Learning Book

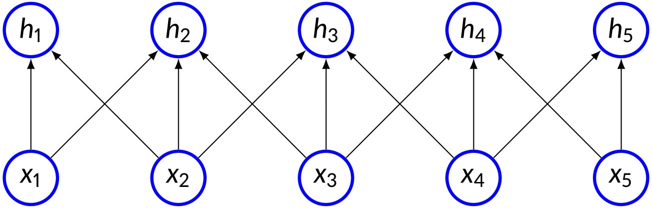
Advantages

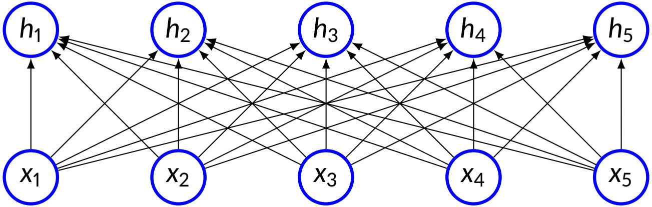
- Convolution can exploit the following properties
  - Sparse interaction (Also known as sparse connectivity or sparse weights)
  - Parameter sharing
  - Equivariant representation

Sparse interaction

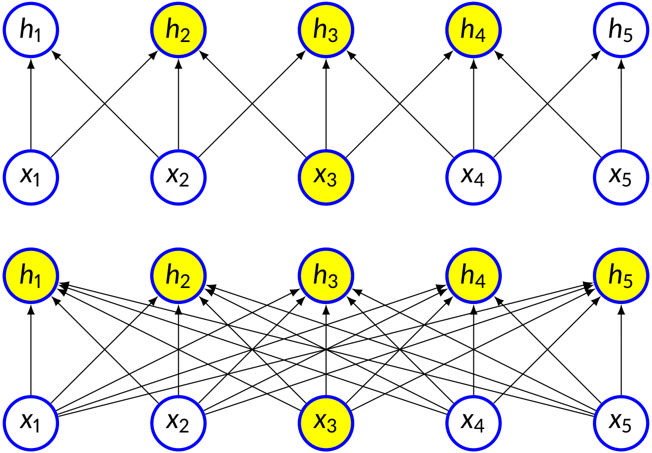
- Traditional neural network layers use matrix multiplication to describe how outputs and inputs are related
- Convolution uses a smaller kernel
  - Significant reduction in number of parameters
  - Computing output require few comparison
- For example, if there is m inputs and n outputs, traditional neural network will require $m \times n$ parameters
- If each of the output is connected to at most k units, the number of parameters will be $k \times n$

Sparse connectivity

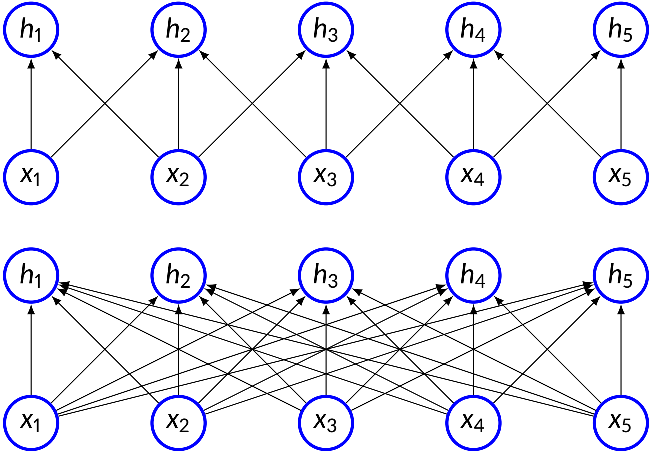




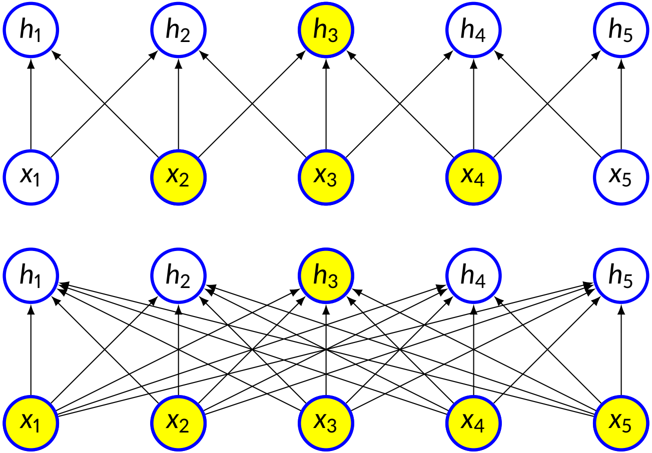
Sparse connectivity



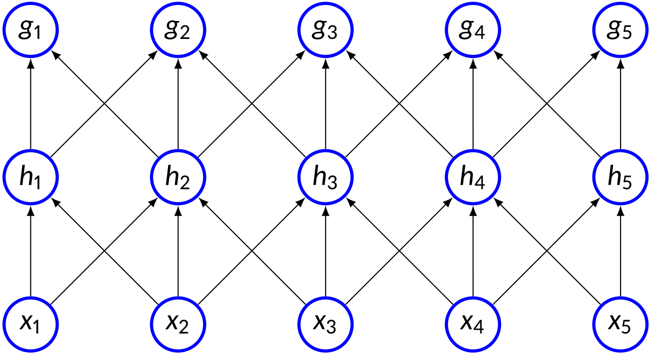
Sparse connectivity



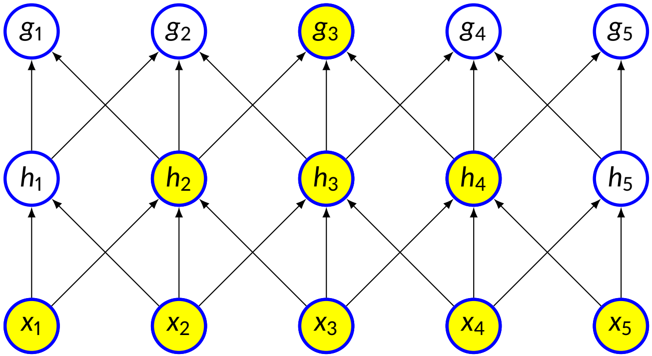
Sparse connectivity



Receptive field



Receptive field



Parameter sharing

- Same parameters are used for more than one function model
- In tradition neural network, weight is used only once
- Each member of kernel is used at every position of the inputs
- As $k \ll m$, the number of parameters will reduced significantly
- Also, require less memory

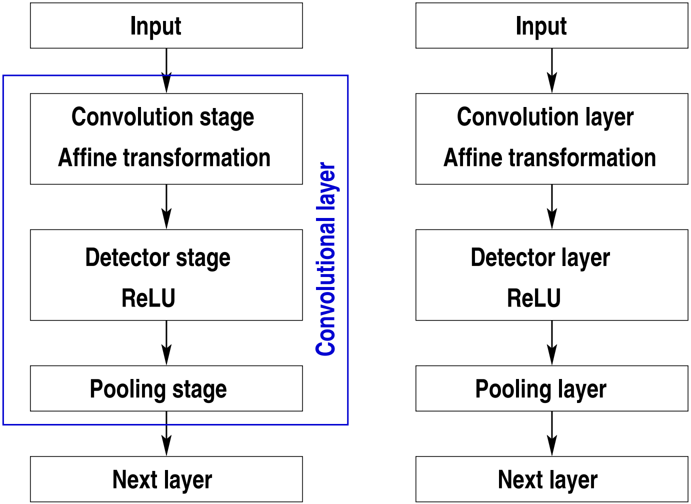
Equivariance

- If the input changes, the output changes in the same way
- Specifically, a function $f(x)$ is equivariant to function g if $f(g(x)) = g(f(x))$
  - Example, g is a linear translation
  - Let B be a function giving image brightness at some integer coordinates and g be a function mapping from one image to another image function such that $I' = g(I)$ with $I'(x, y) = I(x-1, y)$
- There are cases sharing of parameters across the entire image is not a good idea

Pooling

- Typical convolutional network has three stages
  - Convolution — several convolution to produce linear activation
  - Detector stage — linear activation runs through the non-linear unit such as ReLU
  - Pooling — Output is updated with a summary of statistics of nearby inputs
    - Maxpooling reports the maximum output within a rectangular neighbourhood
    - Average of rectangular neighbourhood
    - Weighted average using central pixel
- Pooling helps to make representation invariant to small translation
  - Feature is more important than where it is present
- Pooling helps in case of variable size of inputs

Typical CNN


Input
Convolution stage
Affine transformation
Detector stage
ReLU
Pooling stage
Convolutional layer
Next layer
Input
Convolution layer
Affine transformation
Detector layer
ReLU
Pooling layer
Next layer

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

8	

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

8	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

9	
8	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

9	8
8	5

Max Pool

0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

9	8
8	5

No. of Parameters: 0

Max Pool

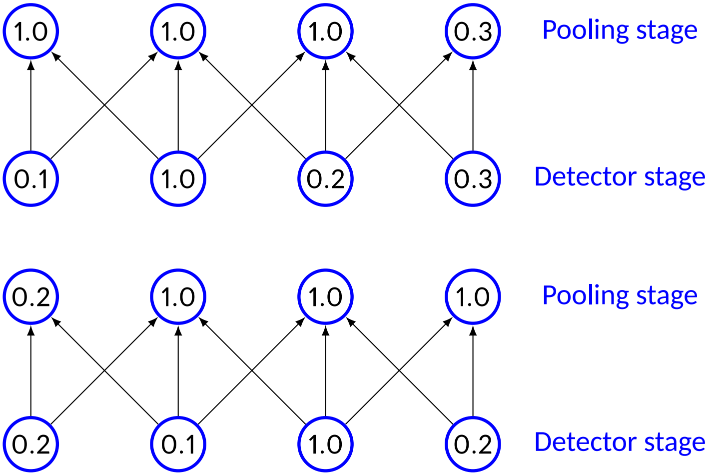
0	4	7	8
9	2	4	5
6	7	3	4
8	2	1	5

9	8
8	5

No. of Parameters: 0

Gradient computation?

Invariance of maxpooling



Learned invariances

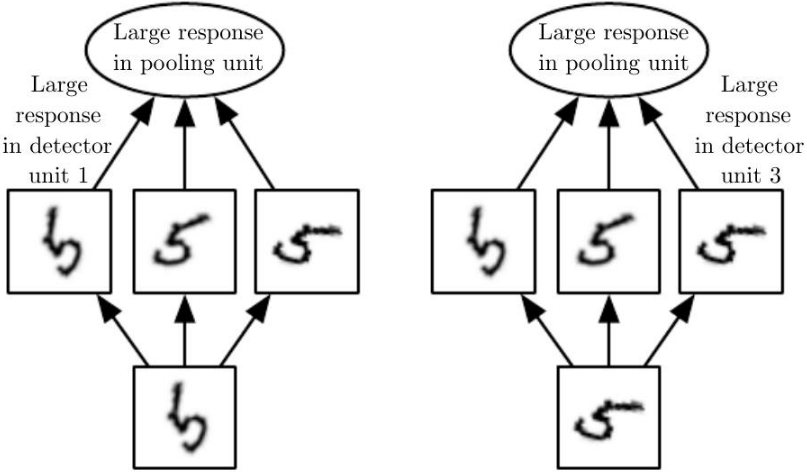
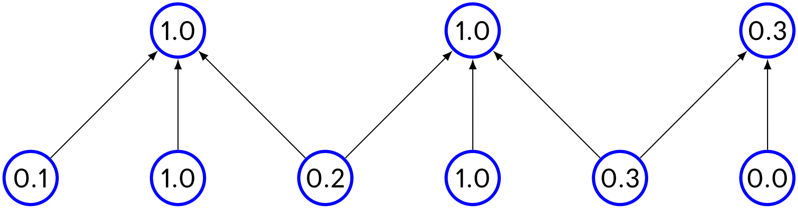
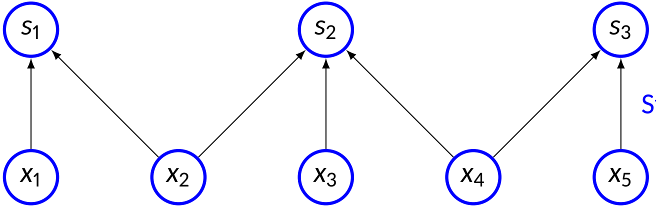


Image source: Deep Learning Book

Pooling with downsampling

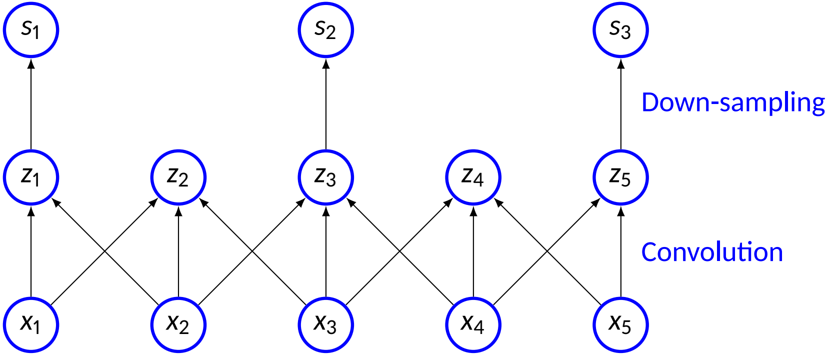


Strided convolution

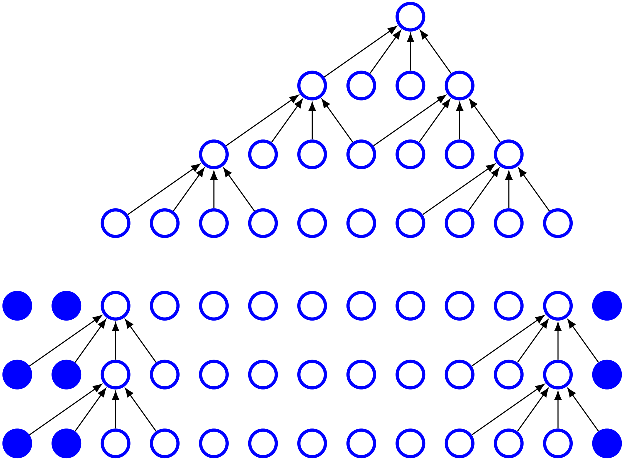


Strided convolution

Strided convolution (contd)



Zero padding



AlexNet

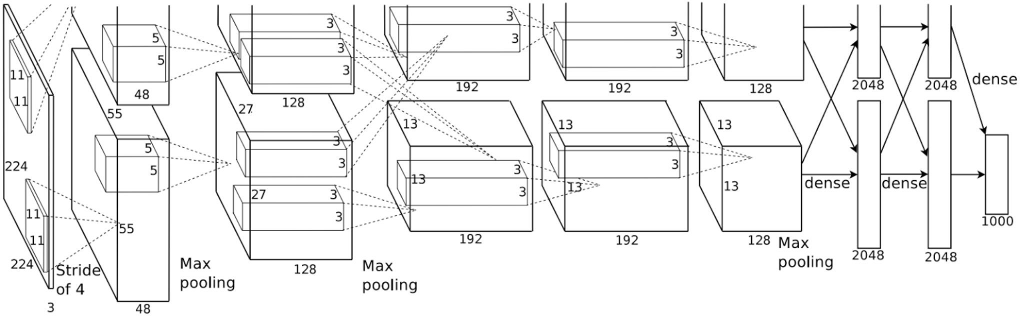
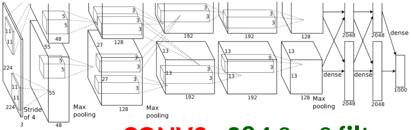


Image source: https://worksheets.codalab.org

AlexNet

- **Architecture**



- **INPUT** - $227 \times 227 \times 3$
- **CONV1** - **96** 11×11 **filters at stride 4, pad 0, Output:** $55 \times 55 \times 96$
- **MAX POOL1** - 3×3 **filter, stride 2 Output:** $27 \times 27 \times 96$
- **NORM1** - **Output:** $27 \times 27 \times 96$
- **CONV2** - **256** 5×5 **filters at stride 1, pad 2, Output:** $27 \times 27 \times 256$
- **MAX POOL2** - 3×3 **filter, stride 2 Output:** $13 \times 13 \times 256$
- **NORM2** - O $13 \times 13 \times 256$
- **CONV3** - **384** 3×3 **filter, stride 1, pad 1, Output:** $13 \times 13 \times 384$
- **CONV4** - **384** 3×3 **filter, stride 1, pad 1, Output:** $13 \times 13 \times 384$
- **CONV5** - **256** 3×3 **filter, stride 1, pad 1, Output:** O $13 \times 13 \times 256$
- **MAX POOL3** - 3×3 **filter, stride 2, Output:** $6 \times 6 \times 256$
- **FC6** - **4096 Neurons**
- **FC7** - **4096 Neurons**
- **FC8** - **1000 Neurons**

Image source: https://worksheets.codalab.org

VggNet

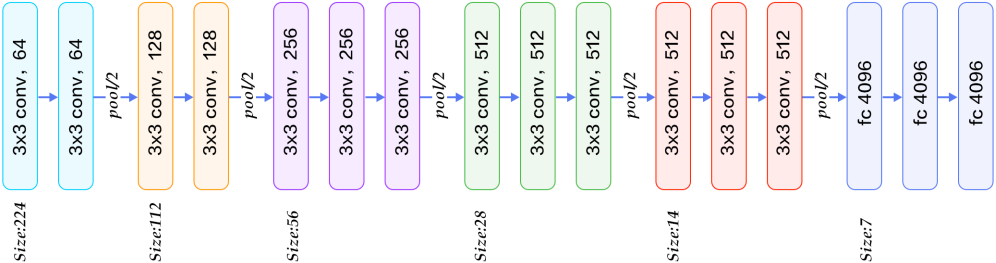


Image source: internet

GoogleNet

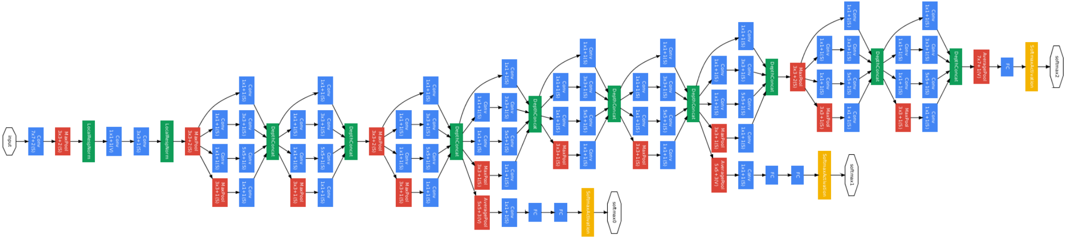
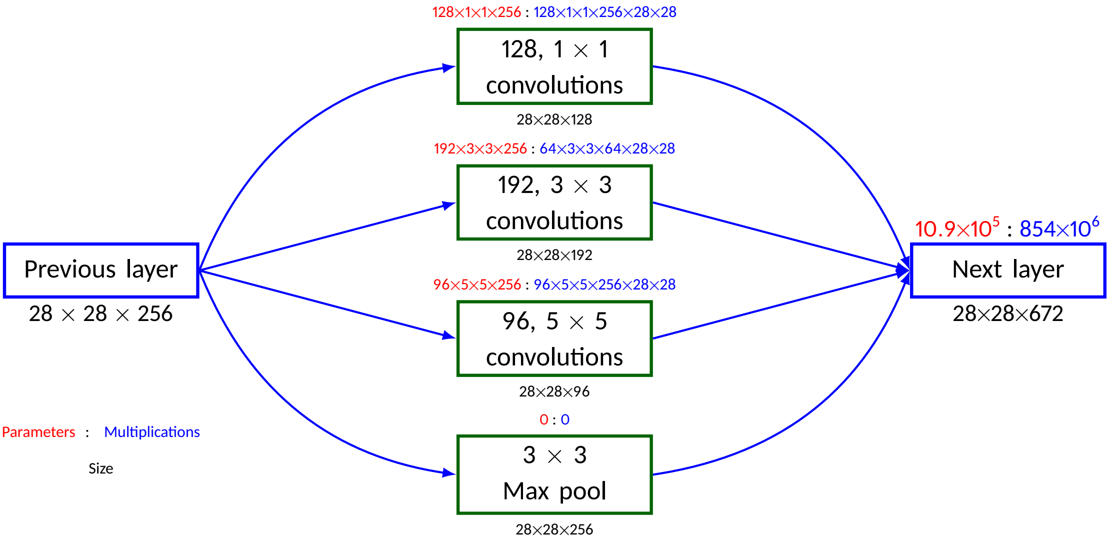
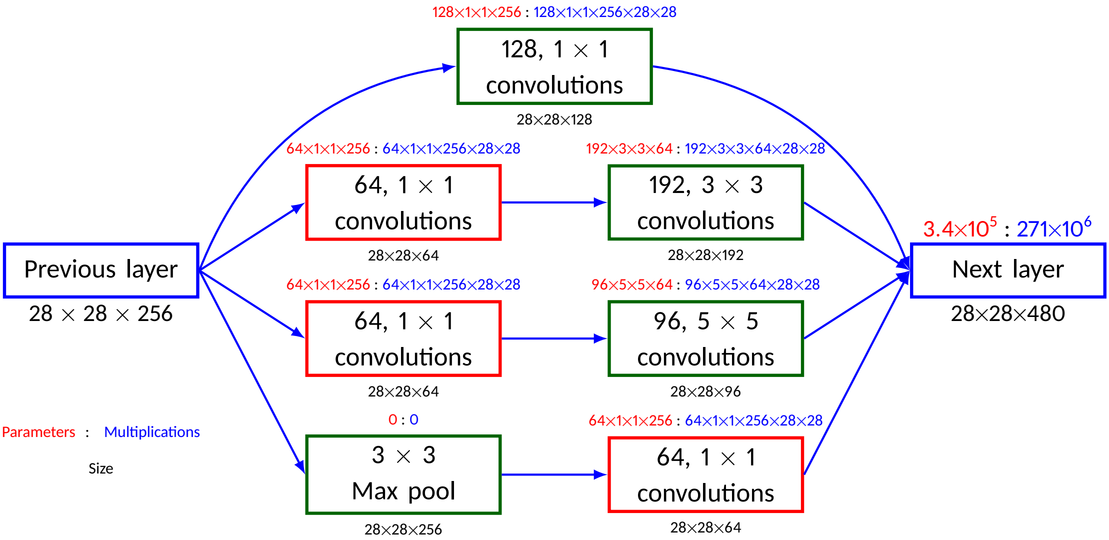


Image source: http://joelouismarino.github.io

Naive inception



Inception



128×1×1×256 : 128×1×1×256×28×28
128, 1 × 1 convolutions
28×28×128
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64
192×3×3×64 : 192×3×3×64×28×28
192, 3 × 3 convolutions
28×28×192
3.4×10^5 : 271×10^6
Previous layer
28 × 28 × 256
Next layer
28×28×480
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64
96×5×5×64 : 96×5×5×64×28×28
96, 5 × 5 convolutions
28×28×96
Parameters : Multiplications
Size
0 : 0
3 × 3 Max pool
28×28×256
64×1×1×256 : 64×1×1×256×28×28
64, 1 × 1 convolutions
28×28×64

ResNet

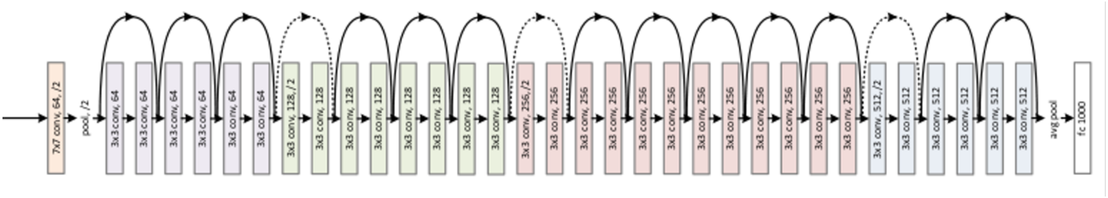


Image source: internet

Comparison of CNN architecture

Model	Size (M)	Top-1/top-5 error (%)	# layers	Model description
AlexNet	238	41.00/18.00	8	5 conv + 3 fc layers
VGG-16	540	28.07/9.33	16	13 conv + 3 fc layers
VGG-19	560	27.30/9.00	19	16 conv + 3 fc layers
GoogleNet	40	29.81/10.04	22	21 conv + 1 fc layers
ResNet-50	100	22.85/6.71	50	49 conv + 1 fc layers
ResNet-152	235	21.43/3.57	152	151 conv + 1 fc layers

Image source: internet

Covnet demo resource

- **URL:** `https://cs.stanford.edu/people/karpathy/convnetjs/`