Measuring diffusion of water with magnetic resonance imaging

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Useful background literature is chapter 1 in Diffusion MRI edited by Heidi Johansen Berg and Tim Behrens, chapter 22 in Diffusion MRI edited by Derek Jones, and chapter 1 in Introduction to Diffusion Tensor Imaging by Susumu Mori. You will have received MRI diffusion data, via a dropbox link or similar.

The data are saved as Matlab workspace, and can be imported into e.g. python (scipy.io.loadmat). Variables are

Name	**Size**	**Description**
DKIdata	480x480x496	Image data, a single slice
bvals	496x1	The b-values of the images (s/mm2)
gradient_dirs ($\hat{g}$)	495x3	Directions of the diffusion gradients. b=0 does not have a direction
mask	480x480	Mask of the brain

Note that each "array-slice" in DKIdata contains a mosaic of all slices recorded at a fixed diffusion weighting. In the following, you need only data corresponding to b-values up to 1000 s/mm2.

1. Produce a map (image) of the mean diffusivity, $\bar{D} = \mathrm{Tr(D)}$. You can determine approximately the voxel wise diffusion tensor by fitting (using e.g. lsqcurvefit in Matlab) the signal expression

$$S(b) = S_0 \exp(-\sum_{i,j} b_{ij} D_{ij}) \tag{1.1}$$

   to the measured signal as function of b-value in each voxel. Here $b_{ij} = b g_i g_j$. Remember that the diffusion tensor must be symmetric and positive definite. You should at least enforce the former constraint in your fitting.

2. Make a map of fractional anisotropy,

$$FA = \sqrt{\frac{3}{2}\frac{(\lambda_1 - \bar{D})^2 + (\lambda_2 - \bar{D})^2 + (\lambda_3 - \bar{D})^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \tag{1.2}$$

Comment and compare the contrast in the mean diffusivity and fractional anisotropy maps.

3. Compute the eigenvalues of the diffusion tensor in each voxel, and overlay the primary eigenvectors on an FA map. You may find the Matlab commands eig and quiver useful.
4. Perform a simple fiber tracking. You should at least to be able to identify the nerve bundle connecting the two hemispheres, the corpus callosum. You should of course show an image of your results. A simple fiber tracking algorithm is based on generating of velocity field $\mathbf{v}(\mathbf{r}) = \mathbf{e}_1(\mathbf{r})$, where $\mathbf{e}_1(\mathbf{r})$ is the primary diffusion eigenvector in the voxel corresponding to the position $\mathbf{r}$. Next, you identify one or more so-called seed points, where you initialize a number of particles, which will subsequently move according to the velocity field above:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}(\mathbf{r}(t)) \tag{1.3}$$

The equation can be solved e.g. by the Euler scheme. The particles should cease their motion when they enter a region in which the FA drops below a threshold value, say 0.2, or if they take a turn which is sharper than a threshold value, say 60°. Play around with the threshold values if you don't get anything useful. Now show each particles trajectory as lines, and you have a simple fiber tractogram. You can find useful brain structures in http://www.med.harvard.edu/aanlib/cases/caseNA/pb9.htm . In the axial view FA image below, you can see two separate parts of the corpus callosum in human.