Crypto Trading Algorithms

nr282

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1 Introduction

Cryptocurrency markets have a large number of speculators. Speculators will often buy due to the increased attention and sentiment brought to cryptocurrency caused by price increases. Likewise, speculators will often sell due to price decreases and reduced sentiment. A cycle is clearly formed where price action induces sentiment, and the sentiment induces further price action. At some point, increased prices grow to unsustainable levels where some speculators begin to profit-take, and there is price deceleration which leads to collapse. Spectral Technologies has developed non-linear dynamical techniques similar to those used in weather forecasting to help develop price signals that capture above dynamics and provide intelligence about cryptocurrency markets, including (1) price trend, (2) price peaks, (3) price busts and (4) forecast limitations.

2 Setup

- Let the number of market participants be M.
- Let the non-public view of trading participant i on the product be denoted by $V_i(t)$
- Let the non-public quantity of market participant i be denoted by $q_i(t)$
- Let $P(t)$ be the price at which trading product trades

3 System Dynamics

A set of axioms are provided below.

3.1 Quantity Accumulation

$$\frac{dq_i}{dt} = \alpha(V_i(t) - P(t)) \tag{1}$$

- If $V_i(t) \geq P(t)$, then $q_i(t)$ increases, indicating market participant i is buying.
- If $V_i(t) \leq P(t)$, then $q_i(t)$ decreases, indicating market participant i is selling.

4 Balancing Equation

What gets sold, must also be bought and hence the following equations hold.

$$\sum_{i=1}^{M} \frac{dq_i}{dt} = 0 \tag{2}$$

5 Development of V$_i$

The key now is to determine how each of the V_i is formulated. This will be determined by the types of different market participants that exist. The question is what market participants exist. Below is listed a set of these market participants.

- Momentum Trader
- Value Trader

5.1 Momentum Trader

Likewise, the momentum trader changes their V_i on the basis of the price action. The formulas for the price action are provided by:

$$V_i(t+1) - V_i = \alpha_i(P(t) - P(t-1)) + \beta_i(P(t) - 2P(t-1) + P(t-2)) \tag{3}$$

The value of α_i and the value β_i control the dependence on the first and second derivatives of the price trend. Velocity of prices is determined by α_i and acceleration in the trend is determined by β_i.

There will be a minimum V_{min} and V_{max} to provide bounds on $V_i(t)$, to represent an upper limit to the momentum traders desire to follow trends. This will lead to price weakness when prices get to high and price strength when prices get too low.

The V_{min} and V_{max} play important roles. The momentum trader with the highest V_{max} will tend to lose as he buys the top, and the player with the lowest V_{min} will tend to lose, since he will sell the dip. This will allow for tulip-like bubble behavior.

5.2 Value Trader

The value trader sets their V_i to a fixed price V.

5.3 Dynamical Systems

We need to look at how the above formulation maps to a set of mathematical equations that can be studied.

As was stated before, let there be M_v value traders and let there be M_m momentum traders, where we have $M = M_v + M_m$.

The dynamical system can be represented with the following equations:

$$V(t+1) = f(V(t), P(t), P(t-1), P(t-2), \beta)$$

where $V \in R^M$, where beta is a set of vectors representing the parameters of the system. Furthermore, prices are created via the following equation:

$$P(t) = \frac{1}{M} \sum_{i=1}^{M} V_i(t)$$

which is based off the balancing equation and (1) substituted into it.

Here we can look at this like a discrete time dynamical system, where we observe a certain average of state variables, where $x \in R^{3M}$

$$x(t+1) = Ax(t)$$

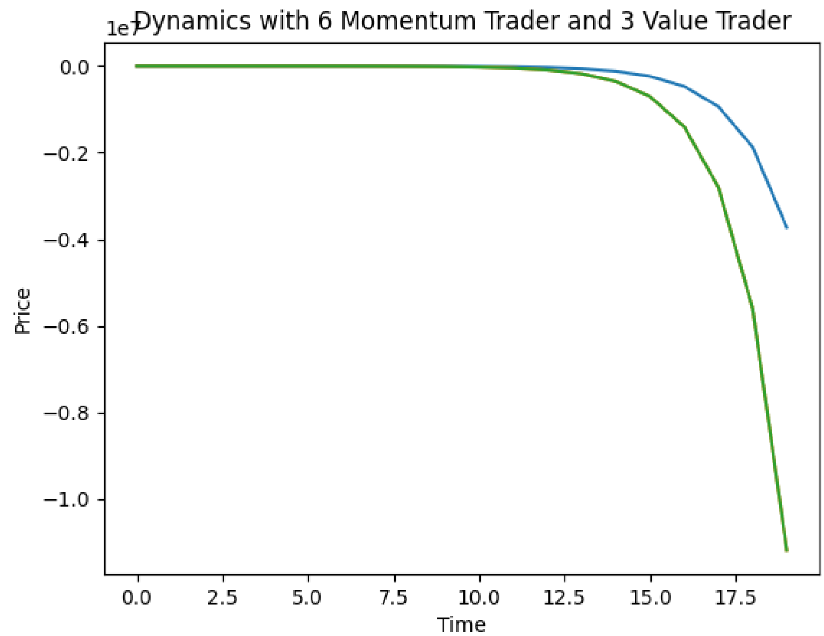
where x has the following form:

$$x(t+1) = \begin{bmatrix} V_1(t+1) \\ ... \\ V_{M_m}(t+1) \\ V_{M_m+1}(t+1) \\ ... \\ V_M(t+1) \\ V_1(t) \\ ... \\ V_{M_m}(t) \\ V_{M_m+1}(t) \\ ... \\ V_M(t) \\ V_1(t-1) \\ ... \\ V_{M_m}(t-1) \\ V_{M_m+1}(t-1) \\ ... \\ V_M(t-1) \end{bmatrix}$$

$$
x(t) = \begin{bmatrix} V_1(t) \\ ... \\ V_{M_m}(t) \\ V_{M_m+1}(t) \\ ... \\ V_M(t) \\ V_1(t-1) \\ ... \\ V_{M_m}(t-1) \\ V_{M_m+1}(t-1) \\ ... \\ V_M(t-1) \\ V_1(t-2) \\ ... \\ V_{M_m}(t-2) \\ V_{M_m+1}(t-2) \\ ... \\ V_M(t-2) \end{bmatrix}
$$

$$
A = \begin{bmatrix}
1+\frac{\alpha_1+\beta_1}{N} & \frac{\alpha_1+\beta_1}{N} & ... & \frac{\alpha_1+\beta_1}{N} & \frac{-(\alpha_1+2\beta_1)}{N} & ... & \frac{-(\alpha_1+2\beta_1)}{N} & \frac{\beta_1}{N} & ... & \frac{\beta_1}{N} \\
\frac{\alpha_2+\beta_2}{N} & 1+\frac{\alpha_2+\beta_2}{N} & ... & \frac{\alpha_2+\beta_2}{N} & \frac{-(\alpha_2+2\beta_2)}{N} & ... & \frac{-(\alpha_2+2\beta_2)}{N} & \frac{\beta_2}{N} & ... & \frac{\beta_2}{N} \\
... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\
\frac{\alpha_N+\beta_N}{N} & \frac{\alpha_N+\beta_N}{N} & ... & 1+\frac{\alpha_N+\beta_N}{N} & \frac{-(\alpha_N+2\beta_N)}{N} & ... & \frac{-(\alpha_N+2\beta_N)}{N} & \frac{\beta_N}{N} & ... & \frac{\beta_N}{N} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\
... & ... & ... & 1 & ... & ... & ... & ... & ... & ... \\
... & ... & ... & ... & 1 & ... & ... & ... & ... & ... \\
... & ... & ... & ... & ... & 1 & ... & ... & ... & ... \\
... & ... & ... & ... & ... & ... & ... & ... & ... & 1
\end{bmatrix}
$$

The dynamics are presented below where we have 6 momentum and 3 value traders, where there are no inclusions of V_{min} and V_{max} into the dynamics.



6 Additional Equations

As was stated above, the dynamics seem to be unbounded if there is not a V_{min} or V_{max}. So there can be proposed a new update for the momentum trader:

$$V_i(t+1) = V_i + \alpha(P(t) - P(t-1)) + \beta(P(t) - 2P_j(t-1) + P(t-2))$$

We need to add appropriate dynamics here that slows down the contribution of the price action as V approaches either V_{min} or V approaches V_{max}. One proposition is: $(V_i - V_{min})(V_{max} - V_i)$.

6.1 Continuous Nonlinear Equations

Hence we have the following:

$V_i(t+1) = V_i(t) + \alpha(P(t) - P(t-1))(V_i(t) - V_{min})(V_{max} - V_i(t)) + \beta(P(t) - 2P(t-1) + P(t-2))(V_i(t) - V_{min})(V_{max} - V_i(t))$

Such an equation is no longer linear in the state vector V unlike the previous equation.

6.2 Discountinous Nonlinear Equations

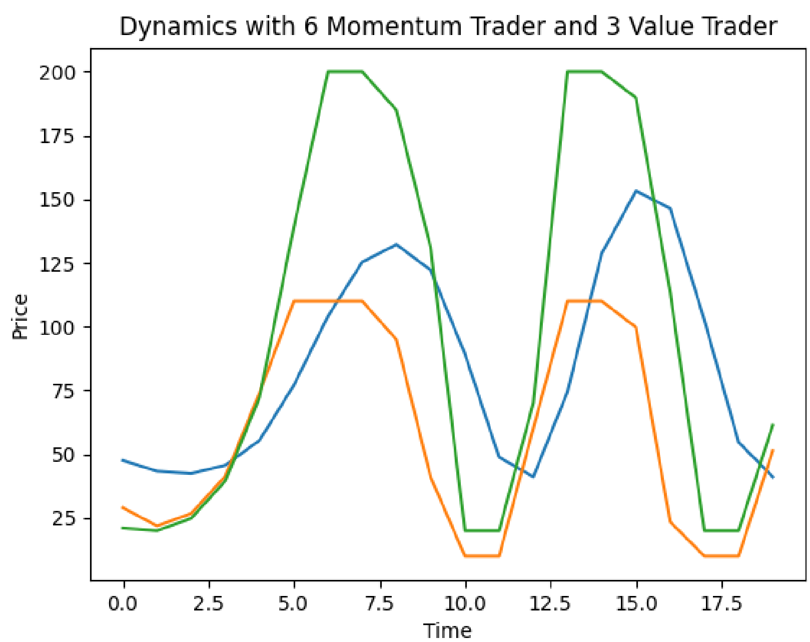
In the case here, we look at adding a discontinuous non-linearity, that probably more accurately models reality, where there are fixed, non-continuous bounds for where momentum traders will move.

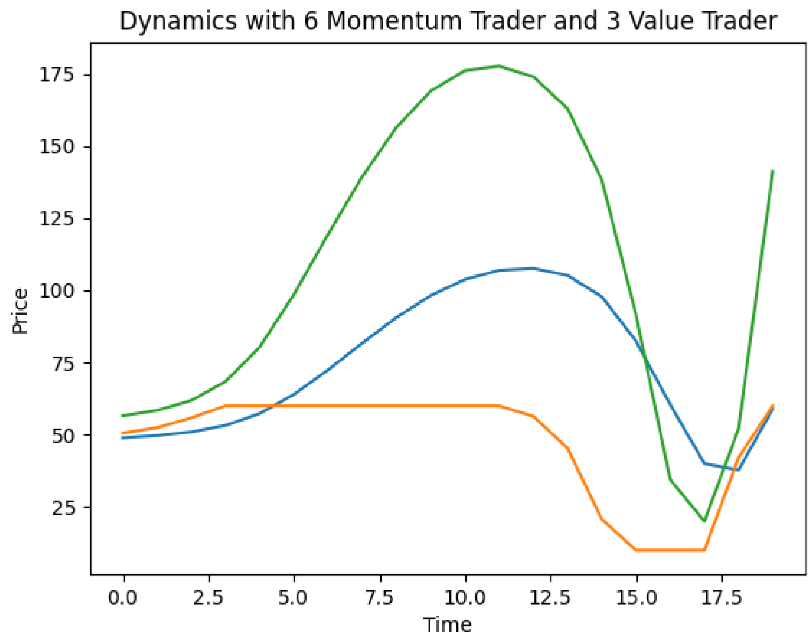
$$\overline{V_i}(t+1) = V_i + \alpha_i(P_j(t) - P_j(t-1)) + \beta_i(P_j(t) - 2P_j(t-1) + P_j(t-2))$$

In the next equation, we have:

$$V_i(t+1) = min(max(\overline{V_i}, V_{min,i}), V_{max,i}) \tag{4}$$

After adding V_{max} and V_{min} to the dynamics, get the following plot. In the below charts, the blue line represents the aggregate price, the orange and green lines represent the values as seen by two of the 6 momentum traders. As one can see the views of the momentum traders oscillate and cause oscillation in the actual prices.





Based off the addition of V_{min} and V_{max} for momentum traders, we can reformulate our non-linear dynamcial system as:

$$x(t+1) = g(Ax(t))$$

where $g : R^{3N} \rightarrow R^{3N}$. g takes on the following form:

$$g(x(t)) = \begin{bmatrix} min(max(V_1(t), V_{1,min}), V_{1,max}) \\ \dots \\ min(max(V_{M_m}(t), V_{M_m,min}), V_{M_m,max}) \\ V_{M_m+1}(t) \\ \dots \\ V_M(t) \\ min(max(V_1(t-1), V_{1,min}), V_{1,max}) \\ \dots \\ min(max(V_{M_m}(t-1), V_{M_m,min}), V_{M_m,max}) \\ V_{M_m+1}(t-1) \\ \dots \\ V_M(t-1) \\ min(max(V_1(t-2), V_{1,min}), V_{1,max}) \\ \dots \\ min(max(V_{M_m}(t-2), V_{M_m,min}), V_{M_m,max}) \\ V_{M_m+1}(t-2) \\ \dots \\ V_M(t-2) \end{bmatrix}$$

where:

$$x(t) = \begin{bmatrix} V_1(t) \\ ... \\ V_{M_m}(t) \\ V_{M_m+1}(t) \\ ... \\ V_M(t) \\ V_1(t-1) \\ ... \\ V_{M_m}(t-1) \\ V_{M_m+1}(t-1) \\ ... \\ V_M(t-1) \\ V_1(t-2) \\ ... \\ V_{M_m}(t-2) \\ V_{M_m+1}(t-2) \\ ... \\ V_M(t-2) \end{bmatrix}$$

6.3 Distribution of $\mathbf{V}_i's$

6.3.1 Characterization of Momentum Traders

The V_i for m_v value traders and the m_m pairs of $V_{min,i}$ and $V_{max,i}$ for momentum traders are critical to the dynamics of the market and the $P(t)$ prices.

For each momentum trader, define the momentum constant as

$$M_i = \frac{V_{max,i} - V_{min,i}}{(V_{max,i} + V_{min,i})/2}$$

The momentum constant measures how much of a momentum trader a specific trader is. As $M \to 0$, the trader essentially becomes a value trader. M_i is scale independent. If

$$\bar{V}_{max,i} = \alpha V_{max,i}$$

and

$$\bar{V}_{min,i} = \alpha V_{min,i}$$

, then:

$$\bar{M}_i = \frac{\bar{V}_{max,i} - \bar{V}_{min,i}}{(\bar{V}_{max,i} + \bar{V}_{min,i})/2}$$

An example can be seen here: if $V_{max,i} = 100$ and $V_{min,i} = 10$, then $M_i \approx 1.8$. Another example can be seen as: $V_{max,i} = 60$ and $V_{min,i} = 40$, then $M_i \approx 0.4$. Furthermore, another example can be seen as: $V_{max,i} = 51$ and

$V_{min,i} = 49$, then $M_i \approx 0.02$. As one can see, as M becomes closer to 0, the trader is no longer momentum, but more of a value trader.

Furthermore, one can look to define $V_i = (V_{max,i} + V_{min,i})/2$, as the mean for the value trader. These two fully define the momentum trader.

6.3.2 Bounds on P(t)

From our discussion above, there are the following equations

$$P(t) \leq \frac{\sum_{i=1}^{m_v} V_i + \sum_{j=m_v+1}^{m_v+m_m} V_{max,j}}{m_v + m_m} = P_{max} \tag{5}$$

and likewise the lower bound is provided by:

$$P(t) \geq \frac{\sum_{i=1}^{m_v} V_i + \sum_{j=m_v+1}^{m_v+m_m} V_{min,j}}{m_v + m_m} = P_{min} \tag{6}$$

This means that there exists i such that $V_{max,i} \geq P_{max}$ and there exists a j such that $V_{min,j} \leq P_{min}$.

6.3.3 Min-to-Max Prices From Bubbles

Looking at ETH, and it seems that the minimum price 537.0 and the maximum price seems to be 4000. This would imply $M_i \geq 1.52$. Now looking at BTC, it seems that the minimum price is 9000 with the top being 60000. This would imply $M_i \geq 1.47$. $CSCO$ during the tech bubble had a minimum price of 8.38 and has a higher price of 60.5, and hence $M_i \geq 1.51$. It can also be seen that the if $V_{max,i} = \alpha V_{min,i}$, where $\alpha \in [7, 8]$.

6.3.4 Distribution of M_i and V_i

As was discussed above, it is important now to examine the distribution of M_i and V_i that is needed for a bubble.

6.3.5 Analysis of Acceleration and Velocity

The eigenvalue decomposition given by $A = V^{-1}\Sigma V$ can help us analyze how the dynamics will evolve, and can help us relate certain properties of the momentum traders to the overall price trend and the price trends frequency.

$$x_{k+1} = A^k x_0$$

Substituting eigenvalue decomposition from above:

$$x_{k+1} = (V^{-1}\Sigma V)^k x_0$$

6.3.6 Analysis Of Price Increase

$$P(t+1) - P(t) = \frac{1}{M} \sum_{i=1}^{M_v+M_m} V_i(t+1) - V_i(t)$$

Since $V_i(t+1) = V(t)$ for $i = 1...M_v$, since these are value traders.

$$P(t+1) - P(t) = \frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} V_i(t+1) - V_i(t)$$

$$= \frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} V_i(t+1) - V_i(t)$$

$$= \frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} \alpha_i(P(t) - P(t-1)) + \beta_i(P(t) - 2P(t-1) + P(t-2))$$

$$= (P(t)-P(t-1))(\frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} \alpha_i) + (P(t)-2P(t-1)+P(t-2))(\frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} \beta_i)$$

Defining the average sensitivity to velocity as $\overline{\alpha}$ and average sensitivity to acceleration as $\overline{\beta}$, which removes dependence on the α_i and β_i.

$$\overline{\alpha} = \frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} \alpha_i$$

$$\overline{\beta} = \frac{1}{M} \sum_{i=M_v+1}^{M_v+M_m} \beta_i$$

$$= (P(t) - P(t-1))\overline{\alpha} + (P(t) - 2P(t-1) + P(t-2))\overline{\beta}$$

As one can see, when not at the top or bottom of the market, the price increase or decrease is mostly determined by $\overline{\alpha}$ and $\overline{\beta}$

If one can calibrate $\overline{\alpha}$ and $\overline{\beta}$ to historical data, one may be able to good forward predictions when price is in the middle of the market (ie V_i

6.3.7 Analysis Of Price Increase with Active Set

$$P(t+1) - P(t) = \frac{1}{M} \sum_{\Omega(t)} V_i(t+1) - V_i(t)$$

6.3.8 Frequency Domain

An analysis can be done in the frequency domain and related to the velocity and acceleration properties of the individual players. The change of $P(t)$ must be a function of α_i and β_i which are the drivers of the growth in $P(t)$ in aggregate. From here, one can see that this will allow us to correlate α_i and β_i with $P(t)$

6.3.9 Notes on Further Investigation

Examine the empirical aspects of bubbles, and more so crypto dynamics in general.

6.4 Analysis Of Volume

Defining $S(t)$ for the volume as the following:

$$S(t) = \frac{1}{2}\sum_{i=1}^{N} |\frac{dq_i}{dt}|$$

$$S(t) = \frac{1}{2}\sum_{i=1}^{N} |\alpha_i(V_i(t) - P(t))|$$

$$P(t) = \frac{1}{N}\sum_{i=1}^{N} V_i(t)$$

$$S(t) = \frac{1}{2}\sum_{i=1}^{N} |\alpha_i(V_i(t) - \frac{1}{N}\sum_{j=1}^{N} V_j(t))|$$

Supposing that α_i is constant across all players, taking $\alpha = \alpha_i$ then

$$S(t) = \frac{1}{2}\alpha\sum_{i=1}^{N} |(V_i(t) - \frac{1}{N}\sum_{j=1}^{N} V_j(t))|$$

We see that the right hand side contains the average absolute deviation of the players views, which we will denote as $AAD(V(t))$.

$$S(t) = \frac{1}{2}\alpha AAD(V(t))$$

B

The quantity that can be controlled is:

$$K = \frac{AAD(V(t))}{\overline{V(t)}}$$

Suppose that

$$\overline{V(t)} \approx \overline{P(t)}$$

Suppose that

$$AAD(V(t)) \approx K\overline{P(t)}$$

I can then calibrate, using $\overline{S(t)}$ as realizied prices:

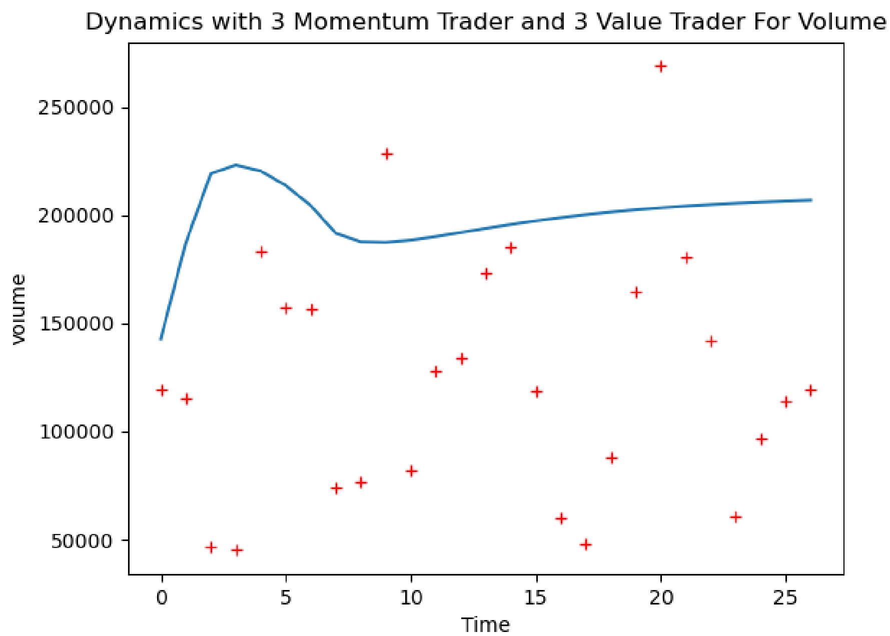
$$\alpha \approx \frac{2}{K\overline{P(t)}} \frac{1}{T} \sum_{t=1}^{T} \overline{S(t)}$$

$$S(t) \approx (\frac{1}{2}\alpha \sum_{i=1}^{N} |(V_i(t) - \frac{1}{N} \sum_{j=1}^{N} V_j(t))|$$

$$S(t) \approx \frac{1}{2}\alpha \sum_{i=1}^{N} |(V_i(t) - \frac{1}{N} \sum_{j=1}^{N} V_j(t))|$$

We can then try to minimize, where we take $\overline{S(t)}$ as the realized volume.

$$\frac{1}{N} \sum_{i=1}^{N} ((\overline{S(t)} - S(t)))^2$$



6.5 Analysis of Equilibrium Points

It is important to consider the set of equilibrium points $x_{eq} = g(A(x_{eq}))$ that will be produced by the system provided above. One can then take steps to analyze the system by doing the following:

1. Find an equilibrium point of the system you are interested in.

2. Calculate the Jacobian matrix of the system at the equilibrium point.

3. Calculate the eigenvalues of the Jacobian matrix.

4. Analyze the eigenvalues to understand dynamics.

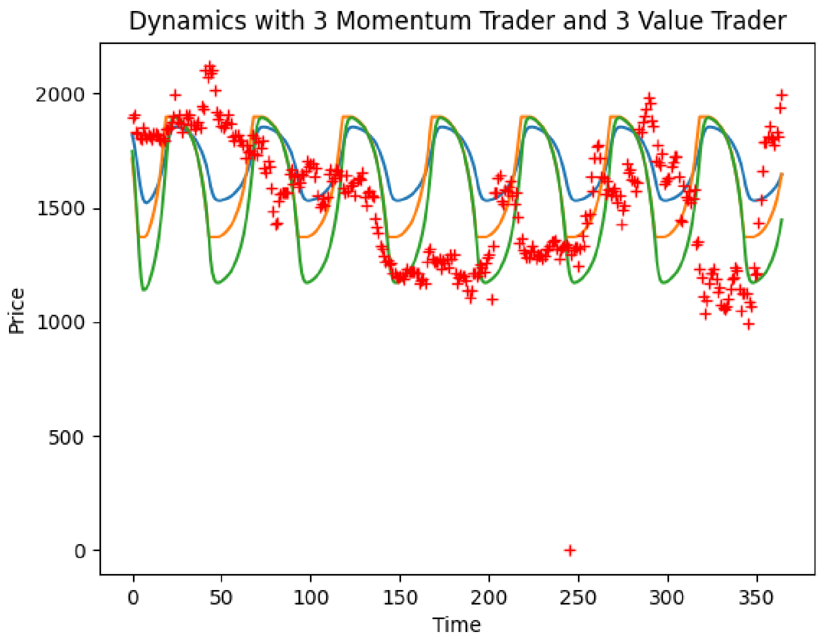
How should we aim to calculate the equilibrium points of the system provided above. In order for this to be in equilibrium x_{eq} must satisfy certain types of conditions. In order to look at equilibrium it suffices to look at the M_m momentum traders. For each of these traders the following conditions must be satisfied, for each $V_i(t)$ for $i = 1...M_m$ in x_{eq}

- if $V_i(t) = V_{max,i}$ then $\overline{V_i}(t+1) \geq V_{max,i}$
- if $V_i(t) = V_{min,i}$ then $\overline{V_i}(t+1) \leq V_{min,i}$
- if $V_{min,i} \leq \overline{V_i}(t) \leq$V$_{max,i}$, then $\overline{V_i}(t+1) = V_i(t)$

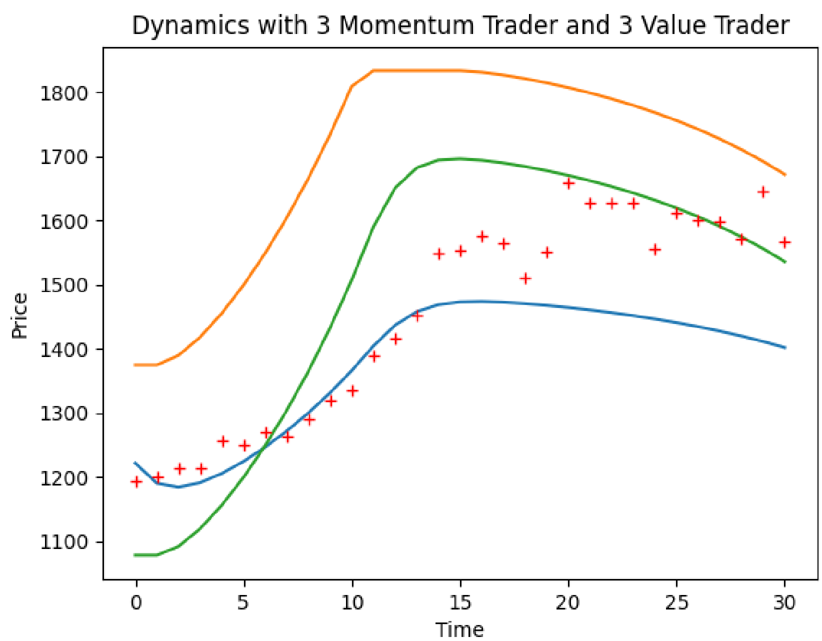
There are 3^{M_m} potential settings for x_{eq} for the M_m players. Each of these equilibrium enforce various setting on the other $V_i(t)$. Suppose that $V_{min,i} \leq V_i(t-2) \leq V_{max,i}$, then we have that $V_i(t-1) = V_i(t-2)$ for all i. Hence $V_{min,i} \leq V_i(t-1) \leq V_{max,i}$ holds, and so $V_i(t) = V_i(t-1)$ for all i. This implies by the recurrence relation for momentum traders $V_i(t+1) = V_i(t)$ for all i. Hence, if we have that if $V_{min,i} \leq V_i(t-2) \leq V_{max,i}$ for all i, then $V_i(t+1) = V_i(t) = V_i(t-1)$ for all, which implies that $P(t+1) = P(t) = P(t-1)$, and the corresponding x_{eq} is a solution.

6.6 Calibration of Dynamical Systems

Given the additional elements that have been added, important calibration topics involve the solution and calibration of nonlinear dynamical systems. This can be done with a global optimization algorithm. In the below picture, the red crosses represents the actual realized ETH prices. Blue represents the projected ETH prices. Orange and Green Curves represent the values as seen by the first and second momentum traders.


Dynamics with 3 Momentum Trader and 3 Value Trader
Price
Time
0
500
1000
1500
2000
0
50
100
150
200
250
300
350

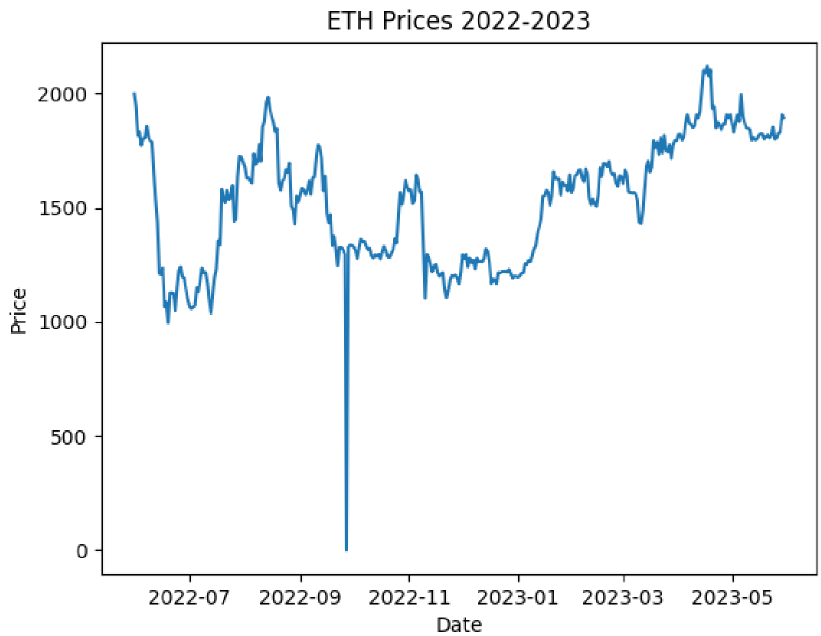
6.7 January 2023 - February 2023



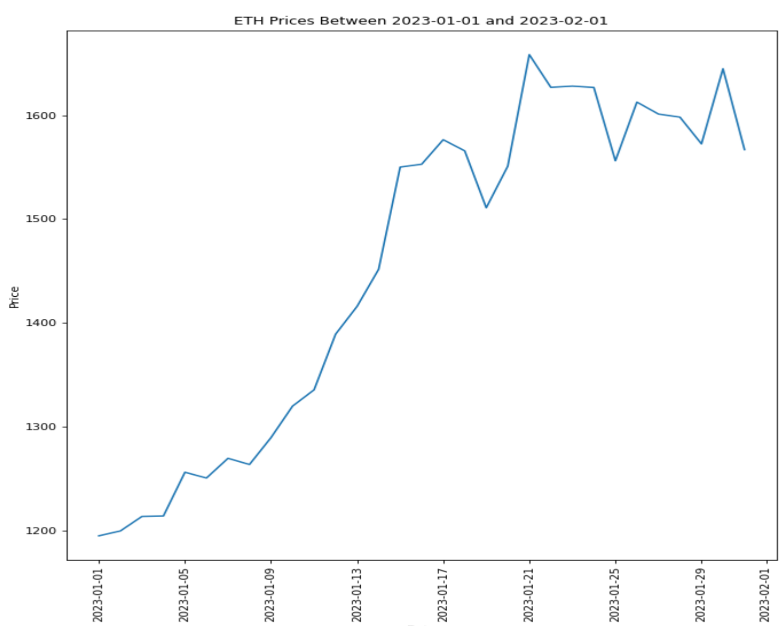
7 ETH Prices, 2022-2023

Prices for ETH, 2022-2023 are plotted below. As one can see during the period of 2023-01 to 2023-02, there is some parabolic growth in the prices of Crypto, that look significantly like the price dynamics created by the model that we have developed. Likewise, there is some significant decline before 2022-07 that could also be expressed by the dynamics.

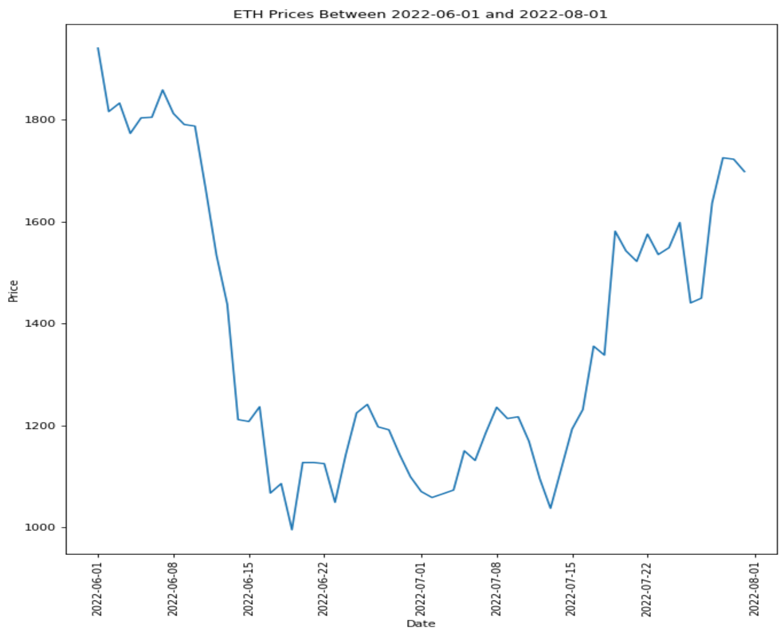
In the below, chart it is clear that there is a series of ups and downs. It seems to me that we can constrain the V's to be distributed over prices ranges seen in the objective function.



The build up from 2023-01 to 2023-02 is shown below. A



One can see something looks like a micro cycle here:


ETH Prices Between 2022-06-01 and 2022-08-01
Price
1800
1600
1400
1200
1000
Date
2022-06-01
2022-06-08
2022-06-15
2022-06-22
2022-07-01
2022-07-08
2022-07-15
2022-07-22
2022-08-01