ECON0072: Empirical Industrial Organisation

Lecture 2: Standard Oligopoly Models

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Introduction

- Reading
  1. Tirole (1993): Introduction to Section 2 and Chapter 5, pages 205–210, 218–224. Skip sections 5.2 and 5.3.
  2. Davis and Garcés (2010) Chapter 1.
- If there is more than one firm with market power, then we must take into account firms' strategic interactions.
- Precise nature of competition is important.

Nature of competition

- Key features of the nature of competition include:
  1. **What is the market?** Is the softdrink market the market for sugary carbonated drinks only? Or does it also include bottled water?
  2. The **number of firms** in the market.
  3. The **strategy space** of firms. Do firms compete by setting prices or quantities? Or, do they compete through product or quality differentiation or advertising and promotion?
  4. Is competition **static or dynamic**?
  5. What **information** does each firm have about its competitors and about market demand?
  6. Constraints on competition due to **regulations or legal institutions.**
- In this lecture we will **first discuss market definition** and then begin to discuss the basic models of firm competition: **Bertrand price competition and Cournot quantity competition**.

What is a market? Economic definition

1. **Each good is a market?** Only goods that are perfect substitutes are in the same market? This definition is too narrow. In reality, no two goods are ever identical and rarely are goods perfect substitutes.
2. **The entire economy is a market?**. This is too broad a definition because it is intractable. We want to be able to do partial equilibrium analysis and capture the most important interactions amongst firms and goods.
3. The "right" economic definition depends on the **research or policy question** to be addressed.

Market definition is problem specific

- In this course, a market will be defined in a problem. For example,
  1. **A group of differentiated products** that are fairly good substitutes (or complements).
  2. **A group of goods** for which there are limited interactions with the rest of the economy so that we can ignore prices and quantities of other goods.
- **Demand side definition:**
  - All goods that are **reasonably close complements or substitutes** are in the same market. Goods that enter utility in an additively separable way or approximately in an additively separable way can be considered to be in a different market.
- **Supply side definition:**
  - All goods that **directly impact the cost of production** (not mediated by price) are in the same market.

What is a market? Legal definition

- Legal definition is often based on a **SSNIP** test
- What is a **SSNIP** test?
  A test seeks to identify the **smallest** set of products on which a (hypothetical) monopolist could impose a profitable small price increase.
  1. Start with a set of products.
  2. Would a hypothetical monopolist producing this set of goods be able to increase profits using a **"small but significant and non-transitory increase in price"** (SSNIP)?
     1. Usually taken to mean a 5% increase in price.
     2. The idea of the test is to check if consumers could substitute the items in the set with something outside the set following the price increase. If they can, one needs to include more products into the set.
  3. If no, add the next closest substitute product to the set and repeat.
  4. Repeat until the answer is yes. That defines a relevant market.

SSNIP test: Comments

- Results in a "relevant market" where a merger potentially could have anti-competitive effect.
- Multiple markets can be defined. Often interested in smallest one.
- Must be careful to avoid the "cellophane" fallacy.
    1. Suppose baseline price of initial set of products is a monopoly price (due to collusion for example). Then, a 5% price increase will not be profitable. As a result, the market definition will be too wide and result may be failure to identify anti-competitive harm.
    2. Implementing the test starting from baseline monopoly price is known as the "cellophane" fallacy (see Stocking and Mueller, 1955).
    3. Must ensure that baseline price is a "competitive" price.
- We will come back to these topics when we talk more about market definition after talking about demand estimation. For more on competition policy and anti-trust policy, see Whinston (2007), *2010 US DOJ Horizontal Merger Guidelines*, or *2010 UK CC Merger Guidelines*. These can be found on the course moodle page.

Bertrand model

A key determinant of market outcomes is the strategy space in which firms compete. The Bertrand model of market competition assumes that firms compete by setting prices. This section outlines the simplest Bertrand competition model. The model assumes that there are two firms who compete by setting prices. The setting is static and both firms know the market demand and the costs of their rival. Key assumptions of the model include:

1. Homogenous products (perfect substitutes). Both firms produce exactly the same good (no product differentiation). Consumers buy a product that has the lowest price.
2. No fixed costs, constant marginal costs and no capacity constraints.

Nash equilibrium

- Bertrand model computes equilibrium prices as a Nash Equilibrium in a game where both firms set prices.

Definition

Nash Equilibrium. Given a set of players, a set of feasible actions for each player, and a payoff function for each player, a set of actions is a Nash Equilibrium if, given the actions of rivals, no player can increase their payoff by altering their own action.

- In the Bertrand pricing game, the players are the firms, the strategies are prices, and the payoffs are the profits.

Nash equilibrium: example

Example

Suppose there are two firms and let (a_1, a_2) be the actions of firms 1 and 2 respectively. Let $\pi_1(a_1, a_2)$ and $\pi_2(a_1, a_2)$ be the profits of firms 1 and 2 respectively. Let A_1 be the set of feasible actions of firm 1. Let A_2 be the set of feasible actions of firm 2. Then a pair $\left(a_1^{NE}, a_2^{NE}\right) \in A_1 \times A_2$ is a Nash Equilibrium if

$$\begin{aligned} \pi_1\left(a_1, a_2^{NE}\right) &\leq \pi_1\left(a_1^{NE}, a_2^{NE}\right) \text{ for all } a_1 \in A_1 \\ \pi_2\left(a_1^{NE}, a_2\right) &\leq \pi_2\left(a_1^{NE}, a_2^{NE}\right) \text{ for all } a_2 \in A_2 \end{aligned}$$

- **Optional reading: Tirole (1993) Chapter 11., Mascollel, Whinston and Green (1995) Chapters 7–9, Fudenberg and Tirole (1993).**

Nash equilibrium: comments (1)

- A game can have no equilibria, one equilibrium or many. Many applications in empirical IO involve games with multiple equilibria. However, it is often difficult to draw conclusive inferences in such games so other applications restrict their analysis to games with a unique equilibrium. In this course, we will largely stick to games with a unique equilibrium.
- A game may have an equilibrium in which players' equilibrium strategies are mixed strategies. In a mixed strategy, a player doesn't choose a single strategy or action with probability one but rather mixes over several strategies each with some probability. For example, a mixed strategy in the Bertrand pricing game could involve one firm setting the price equal to p_L with probability 0.5 and p_H with probability 0.5.

Nash equilibrium: comments (2)

- For some games, it is virtually impossible to compute an equilibrium because it is too computationally costly. Chess is one such game. There are too many possible strategies to evaluate the payoffs to all. A supermarket entry game is another example. Suppose there are 10 grocery firms and 10,000 possible store locations. Suppose each firm simultaneously chooses to open 10 - 1000 stores The set of possible strategies is too big to evaluate. In such games, it may be possible to prove that an equilibrium exists but may be impossible to compute an equilibrium. In such cases, some form of approximate equilibrium is required to make progress.
- The empirical predictions of an oligopoly model are contingent on the model (assumptions about costs, demands, payoffs, number of players), the type of equilibrium (the strategy space), and also on information.

Bertrand pricing model (continued)

- Key Assumptions
  1. Both firms have constant marginal cost c (constant returns to scale with no capacity constraint).
  2. Both firms produce exactly the same good.
  3. Firms choose price simultaneously in a "one shot" game.

Bertrand demand

The residual demand of firm j depends on the prices of both firms. Let $D(p)$ be the market demand curve. The residual demand for firm j is

$$q_j(p_j, p_k) = \left\{ \begin{array}{cc} D(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}D(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{array} \right\}.$$

Bertrand equilibrium

In this economy, it is a Nash Equilibrium for both firms to choose $p_j = p_k = c$. If firm j expects firm k to choose $p_k = c$, then profits of firm j as a function of p_j are

$$\pi_j(p_j, p_k) = \left\{ \begin{array}{cc} 0 & \text{if } p_j > c \\ 0 & \text{if } p_j = c \\ (p_j - c)\, d(p_j) & \text{if } p_j < c \end{array} \right\}.$$

But, the last row is negative because $p_j < c$. Hence, firm j maximises profits by choosing $p_j \geq c$. Every $p_j \geq c$ yields zero profits. Since the firm is indifferent, it is an equilibrium for them to choose $p_j = c$. This is the unique Nash Equilibrium. You can check this by conjecturing that there is one with $p_j = p_k > c$ or with $p_j > p_k \geq c$, and then checking that none of these conjectured equilibria is an equilbrium.

Different marginal costs

If $c_1 < c_2 < p^m$, then the Nash Equilibrium outcome is $p = c_2$ and only firm 1 sells to the consumers.

Several ways to get to this conclusion:

- assume tie-breaking in favour of low cost firm
- assume that money are discreet: in this case $p = c_2 - 0$
- look at mixed strategy equilibria: firm 2 chooses the price uniformly on $[c_2, c_2 + x]$, where $x > 0$ (see Blume, 2003)

Bertrand discussion

The Bertrand model predictions are stark. They are not robust to changes in the key assumptions:

1. If firms have capacity constraints or decreasing returns to scale, the predictions change.
2. If the firm's products are not perfect substitutes the predictions change.
3. If the game is repeated, the predictions change.

Bertrand model with fixed costs

Suppose firm 1 incurs a fixed cost of F_1 and firm 2 incurs a fixed cost of F_2. Then profits of firm $i \in \{1, 2\}$ are

$$\pi_i = (p_i - c)\, D_i\, (p_1, p_2) - F_i \cdot \mathbb{I}\{D_i\, (p_1, p_2) > 0\}.$$

- Timing is important: fixed costs are not sunk when the firms are setting prices.
- $p_1 = p_2 = c$ is not an equilibrium. Due to the fixed costs, profits are negative if price equals marginal cost.
- Let $F_1 < F_2$ Set $p_1 = p_2 - 0$ (or use mixed strategies) so that

$$(p_2 - c)\, D\, (p_2, p_2) = F_2.$$

- This is an equilibrium. At these prices, the high-cost firm is driven out of the market. Price is higher than marginal cost.

Bertrand model with differentiated products

In the previous models, the goods produced by the two firms were perfect substitutes. Suppose instead that they are imperfect substitutes and that demand for q_1 and q_2 is given by

$$
\begin{aligned}
q_1(p_1, p_2) &= a - b_{11}p_1 + b_{12}p_2 \\
q_2(p_1, p_2) &= a + b_{21}p_1 - b_{22}p_2
\end{aligned}
$$

as long as $q_1 \geq 0$ and $q_2 \geq 0$. Assume that $b_{11} > 0$, $b_{22} > 0$ and that $b_{12} > 0$ and $b_{21} > 0$. The demand functions slope down and the goods are substitutes. In this model we can work out a Bertrand pricing equilibrium in which both firms choose prices that are higher than marginal cost.

Best response correspondence for Firm 1

Firm 1 maximizes

$$\pi_1 = (p_1 - c_1)(a - b_{11}p_1 + b_{12}p_2).$$

The first order condition for this problem is

$$\frac{\partial \pi_1}{\partial p_1} = (a - b_{11}p_1 + b_{12}p_2) - (p_1 - c_1)\, b_{11} = 0.$$

Solving for $p_1(p_2)$:

$$p_1(p_2) = \frac{a_1 + b_{12}p_2}{2b_{11}} + \frac{c_1}{2} \tag{1}$$

Best response (discussion)

- Equation (1) is the best response correspondence (BRC) for firm 1.
- It describes the optimal price of firm 1 as a function of the price of firm 2. Or in other words, it describes how firm 1 should react to firm 2 when setting its price.
- The slope of the BRC depend on the ratio b_{12}/b_{11} (this ratio determines an isoquant).
- This ratio is positive and the BRC is increasing. In this case, the game is one in which prices are "strategic complements": Higher prices set by firm 2 incentivize firm 1 to increase its price and vice versa.

(Linear) Bertrand Equilibrium

Nash equilibrium:

$$p_1^* = \frac{a_1 + b_{12}\left(\frac{a_2 + b_{21} p_1^*}{2b_{22}} + \frac{c_2}{2}\right)}{2b_{11}} + \frac{c_1}{2}$$

We can then solve for p_1^* and obtain

$$p_1^* = \frac{2b_{22}a_1 + b_{12}a_2 + 2b_{11}b_{22}c_1 + b_{12}b_{22}c_2}{4b_{11}b_{22} - b_{12}b_{21}}.$$

Validity

- $4b_{11}b_{22} - b_{12}b_{21} > 0$ this condition ensures no “divergence”
- checking for corners: $q_i \geq 0$:
  - $a - b_{11}p_1^* + b_{12}p_2^* \geq 0$
  - $a - b_{21}p_1^* + b_{22}p_2^* \geq 0$

Cournot model

Cournot model

- Two firms; Static setting
- Firms set their outputs (quantities) rather than prices. Formally, firm i chooses $q_i \geq 0$.
- Market "sets" one price given the total output of firms: $p = D^{-1}(q_1 + q_w)$.
- Profits of firm i:

$$\pi_i(q_1, q_2) = D^{-1}(q_1 + q_2)\, q_i - c_i q_i. \tag{2}$$

- Assume π_i is strictly concave in q_i.

Cournot best reply correspondences

Firm 1 chooses q_1 to maximize (2). The first order conditions for this problem are

$$\frac{\partial D^{-1}\left(q_1+q_2\right)}{\partial q_1} q_1 + D^{-1}\left(q_1+q_2\right) - c_1 = 0. \tag{3}$$

Let $Q = q_1 + q_2$. This can be rewritten as

$$-\frac{\partial \ln p}{\partial \ln Q}\frac{q_1}{Q} = \frac{p - c_1}{p}. \tag{4}$$

Lerner condition for Cournot

This equation is version of the Lerner condition for the Cournot model:

$$-\frac{s_1}{\varepsilon} = \frac{p - c_1}{p} \tag{5}$$

where

- $\varepsilon = \frac{\partial \ln Q}{\partial \ln p}$ is the elasticity of demand,
- $s_1 = \frac{q_1}{Q}$ is the market share of firm 1, and
- $\frac{p-c_1}{p}$ Lerner index (markup)

Cournot discussion

Several points are worth noting:

1. In contrast to monopoly, firm 1 takes into account impact of q_1 on revenue of firm 1 only. It does not take into account impact on total industry revenue (externality).
2. An increase in q_1 increases revenue from increase in volume (2nd term) but reduces price (1st term). Relative to monopoly outcome, firms jointly produce more.
3. Cournot prices are below monopoly price and above competitive price.
4. Firm 1 faces the residual demand curve (that is endogenous to firm 2's output) and acts as a monopolist.
5. The markup of each firm is tied to the elasticity and to its market share.

Best response correspondences

The best response of firm 1 is the function $q_1(q_2)$ that satisfies (3). The best response of firm 2 is analogous.

- In general, $q_1(q_2)$ and $q_2(q_1)$ could be either downward or upward sloping
  - If q_1 and q_2 are strategic substitutes $\left(\frac{\partial^2 \pi_i}{\partial q_1 \partial q_2} < 0\right)$, then they will be downward sloping. To see this, differentiate BRC:

$$0 = \frac{d}{dq_2}\left(\frac{\partial \pi_1(q_1(q_2), q_2)}{\partial q_1}\right) = q_1'(q_2)\frac{\partial^2 \pi_1(q_1(q_2), q_2)}{\partial q_1^2} + \frac{\partial^2 \pi_1(q_1(q_2), q_2)}{\partial q_1 \partial q_2}$$

  - This will be true if the inverse demand is concave:

$$\frac{\partial^2 \pi_1}{\partial q_1 \partial q_2} = \frac{dD^{-1}(Q)}{dQ} + q_1 \frac{d^2 D^{-1}(Q)}{dQ^2} < 0$$

Cournot equilibrium

A pair $\left(q_1^*, q_2^*\right)$ is a Nash-Cournot equilibrium if and only if

$$\begin{aligned} q_1\left(q_2^*\right) &= q_1^* \\ q_2\left(q_1^*\right) &= q_2^*. \end{aligned}$$

In general, it is possible that there is no equilibrium, one equilibrium or many equilibria.

Cournot linear example

Example

Linear demand. Suppose market inverse demand is

$$p = a - bQ$$

where $Q = q_1 + q_2$ is the market demand. Firm 1 solves

$$\max_{q_1} \left\{ q_1 \left(a - b \left(q_1 + q_2 \right) \right) - c_1 q_1 \right\}$$

with first order condition

$$\left(a - bq_1 - bq_2 \right) - bq_1 - c_1 = 0.$$

Cournot linear example (continued)

Example

Linear demand (continued). Therefore

$$q_1 = \frac{a - bq_2 - c_1}{2b}.$$

and by symmetry, firm 2's best response is

$$q_2 = \frac{a - bq_1 - c_2}{2b}$$

implying that equilibrium is

$$\begin{aligned} q_1 &= \frac{a + c_2 - 2c_1}{3b} \\ q_2 &= \frac{a + c_1 - 2c_2}{3b}. \end{aligned}$$

Cournot linear example: comments (1)

- q_1 increases with c_2 and decreases with c_1
- With n firms, in Nash equilibrium, the market price is above the competitive price c and below the monopoly price p^m.
- Total output is lower than the competitive output and larger than the monopoly output.
- The Bertrand model predicts that duopoly prices and quantities will equal the competitive prices and quantities.
- The Cournot model predicts that the duopoly prices and quantities will be somewhere between the monopoly levels and the competitive levels.

Cournot linear example: comments (2)

- Suppose there are $n > 2$ firms. With n firms,

$$q_i = \frac{a - b \sum_{j \neq i} q_j - c}{2b}.$$

$$2bq_i = a - b \sum_{j \neq i} q_j - c.$$

- Guess that $q_i = q_j$ for all i, j. (using symmetry.). Then this becomes

$$\begin{aligned} q_i &= \frac{a-c}{b\left(n+1\right)}. \\ p &= \frac{\frac{1}{n}a + c}{1 + \frac{1}{n}} \end{aligned}$$

- When n increases price falls. Price falls to competitive price when n grows large.

Cournot linear example: comments (3)

In this example, one can work out the impact of merger:

1. If $c_1 > c_2$ shut down plant 1.
   - if marginal costs are not constant, reallocate production to minimize total costs
2. Benefit to a merger: costs go down (synergies result in efficiency gains)
3. Cost to merger: market power goes up

Relation between market “parameters” and outcomes

- In both Bertrand and Cournot models, outcomes depend on
  1. cost structure of firms
  2. number of firms
  3. type of competition (strategy space)
  4. demand function
- Bertrand model predicts that 2 firms are sufficient to restore the competitive outcome.
- Cournot model predicts that prices fall to the competitive level as more firms enter but that prices fall more slowly than the Bertrand model predictions.

Measures of "competitiveness"

- IO economists use several indicators to "measure" the competitiveness of a market.
- Some measures of competition include:
  1. Profits: can be hard to measure. Need to be clear about variable vs. fixed.
  2. Price/cost margins. Marginal cost data are often unavailable.
  3. Market shares
     - Often easy to estimate.
     - Depend on market definition.
     - Not conclusive measures of market power
     - Connection to efficiency is not straightforward (e.g. returns to scale)
  4. Market concentration: based on market share data, how concentrated is the market? In general, market concentration can be indicative. However, it is not in general conclusive.

Market concentration index

The $m-$ *firm* or $K-$ *firm* concentration index is defined as

$$I_m = \sum_{j=1}^{m} s_j.$$

It is the sum of the market shares of the biggest m firms.

- If m is small and I_m is close to 1 (or 100) then market is dominated by a small number of firms.
- When I_m is close to 1 there might be potential for exercise of unilateral market power.
- When I_m is close to 1, it might be easier to sustain collusion.

HHI (1)

Another measure of market concentration is the Herfindahl-Hirschman index (HHI) . It is defined as

$$HHI = \sum_j s_j^2.$$

For a monopoly

$$HHI = (100)^2 = 10,000.$$

HHI (2)

For n firms with equal market shares

$$
\begin{aligned}
HHI(n) &= \sum_{j=1}^{n} s_j^2 \\
&= n\left(\frac{100}{n}\right)^2 \\
&= \frac{10,000}{n}
\end{aligned}
$$

so that $HHI(n)$ decreases with n.
If shares are **asymmetric**, HHI tends to go up. For example, if 3 firms have market share of 5% each, 1 firm has market share of 45% and 1 firm has market share of 40%, the HHI is

$$
\begin{aligned}
H &= 3 * 5^2 + 45^2 + 40^2 \\
&= 3,750
\end{aligned}
$$

HHI (3)

- Many competition authorities use a rule of thumb based on the HHI to pre-screen merger cases. For example, in EU, UK, US
  - if $HHI < 1,000$, there is nothing to worry about; there is no need to investigate the merger further.
  - if $HHI \in [1,000, 2,000]$ after a merger and if the merger increases HHI by less than 250, then there is no need to investigate the merger further.
  - If $HHI > 2,000$ after the merger, then there is no need to ivnestigate if the increase is less than 150.
- One needs data on all firms in market to calculate HHI.
- One should never judge a merger to be anti-competitive based on HHI alone.

Theoretical justification for HHI

- In general (without assuming specific models of competition, cost and demand), these measures are not good measures of competition.
- However, in the Cournot model from the previous section industry profits are

$$\begin{aligned}
\Pi &= \sum_{j=1}^{n} \left(p - c_j\right) q_j \\
&= \frac{pq}{\varepsilon} \sum_{j=1}^{n} s_j \frac{q_j}{q} \\
&= \frac{pq}{\varepsilon} \sum_{j=1}^{n} s_j^2
\end{aligned}$$

- Industry profits are directly proportional to *HHI*. Even in this case, a high *HHI* is insufficient evidence that a market is non-competitive; profits also depend on the demand elasticity.

Why are market concentration indexes useful?

Market concentration indexes are useful because:

1. they are easy to compute
2. they are potentially correlated with industry profitability, competition, demand
3. they are also potentially correlated with the potential for firms to exercise unilateral or collusive market power.
4. when used with caution, they can be good screening devices to start an investigation of a market.

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