Robust Information Design

V Bhaskar (UT Austin)

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Robustness = Purification

- How robust are the insights of information design to the agent having some (small) private payoff information (Harsanyi, 1973).
- Principal does not exact preferences of the agent over action-state pairs.
- Is the principal's value, $V(\pi)$, the limit of a sequence of equilibrium values as payoff uncertainty vanishes?
  - If the principal is uninformed, answer is YES.
  - If the principal is informed, there are multiple equilibria in the unperturbed problem (even with quasi-cooperative equilibrium selection).
  - Main result: unique purifiable (robust) equilibrium outcome in a class of informed principal problems, using a purely non-cooperative approach.
  - Private information of the principal effectively becomes public.

Some literature

- Information design: Kamenica-Gentzkow (2011) etc.
- ID with agent private information – Kolotilin, Mylovanov, Zapechelnyuk, Li (2017).
- Informed principal – Myerson (1983).
- Informed information design: Koessler-Skreta (2023).
- Purification as refinement: Bhaskar, Mailath & Morris (2013), Diehl-Kuzmics (2021), Jiang (2023).
- Unravelling: Milgrom (1981).

Model

- States: $\Omega = \{\omega_1, .., \omega_m\}$. Common prior π.
- Agent's action set $A = \{a_1, a_2, ..., a_n\}$.
- Agent's utility function $u : A \times \Omega \to \mathbb{R}$.
- Principal's payoff function is $\tilde{v} : A \times \Omega \to \mathbb{R}$.
- $u(a, \mu)$ denotes the agent's payoff from a at belief μ.
- Agent chooses optimal action at μ, breaking any ties in favor of the principal's preferred action.
- Defines the principal's payoff at any final belief, $v(\mu)$.
- Principal's value at prior π, $V(\pi)$, is given by the concavification of $v(\mu)$.

The perturbed game, Γ^k

- Sequence of random variables, with distribution F^k.
- Each F^k has support $Z^k \subset \mathbb{R}^n$, with typical element $z = (z_i)_{i=1}^n$.
- If agent takes action a_i, then his payoff is $u(a_i, \omega) + z_i$.
- Support of Z^k contains $[0, \bar{z}]^n$ (for every k).
- $\bar{z}$ is large enough that for any ω_j any a_i is strictly optimal for some z-value.
  - Lebesgue measure on the interval $[0, \bar{z}]^n$ is absolutely continuous with respect to F^k.
  - F^k is absolutely continuous with respect to Lebesgue measure.
  - F^k converges to the point mass on 0 as $k \to \infty$.

Let $V^k(\pi, F^k)$ denote the principal's value function in the perturbed information design problem Γ^k.

Definition

$V(\pi)$ is a robust value function if for each π and every sequence $< F^k >$ satisfying the above conditions, $\lim_{k\to\infty} V^k(\pi; F^k) = V(\pi)$.

Definition

The agent's (unperturbed) decision problem is *generic* if:

- For each $\omega \in \Omega$, there is a unique optimal action.
- If action a is optimal at some belief $\mu \in \Delta(\Omega)$, there exists a nearby belief such that a is strictly optimal.

Proposition

If the agent's decision problem is generic and the principal is uninformed, then the value function $V(\pi)$ is robust.

Informed principal

- Principal privately observes the outcome of a private experiment $\hat{\xi}$.
- Induces belief-types $\mathcal{M} = \{\mu_1, \mu_2.,, \mu_L\}$, $\mu_i \in \Delta(\Omega)$.
- Informed principal conducts an public experiment ξ.
- We allow experiment ξ to be:
    - arbitrarily correlated with the state and with the private information of the principal.
    - An experiment ξ is a Lebesgue measurable mapping from $\Omega \times [0, 1]$ to finite signal space S (Green-Stokey, 1978).
    - $\hat{\xi}$, the private experiment of the principal, belongs to the same class.
    - Principal may choose a compound experiment (ξ_1, ξ_2), where ξ_2 depends on the signal realized in ξ_1.
- The agent knows that the principal's private experiment is $\hat{\xi}$.

Informed principal game, $\Gamma_{\mathcal{I}}$

- Nature chooses $\omega \in \Omega$ according to π.
- Principal privately observes outcome of experiment $\hat{\xi}$. Agent only knows that $\hat{\xi}$ has been conducted.
- The principal chooses a public experiment ξ.
- Principal and agent observe the outcome of ξ and agent chooses an action in A.

Note: We do not need to assume any commitment on the part of the principal.

Economic question: How does the expected ex ante value of the informed principal, $W(\pi)$, compare with $V(\pi)$, her value in the absence of private information?

Assumptions: Supermodularity

- Order states and actions so that $\omega_{i+1} > \omega_i$ for $i < m$ and $a_{j+1} > a_j$ if $j < n$.
- Agent's utility function $u : A \times \Omega \to \mathbb{R}$ is strictly supermodular.
- Principal's payoff function is $\tilde{v} : A \times \Omega \to \mathbb{R}$, satisfies ordinal state independence, and is strictly increasing in the agent's action.
- for any $\omega \in \Omega$, $\tilde{v}(a_{i+1}, \omega) > \tilde{v}(a_i, \omega)$.

Since Ω is a totally ordered set, we can partially order beliefs in $\Delta(\Omega)$ by first order stochastic dominance.

Equilibrium Concept: Purifiable PBE

- Outcome: a distribution over $\mathcal{M} \times \Delta(\Omega \times A)$.
- Focus on PBE of Γ that are strongly purifiable.
- A PBE is (strongly) purifiable if its outcomes are limits of a sequence of PBEs of Γ^k's for *any sequence* of shock distributions F^k as $k \to \infty$.
- We assume that if the principal conducts experiment $\hat{\xi}$, then she perfectly discloses her private information, i.e. principal and agent's beliefs coincide.

Theorem

Suppose that the principal types are ordered by first order stochastic dominance, and the agent's decision problem is generic.

- *A purifiable PBE exists.*
- *In any purifiable PBE, the value of each principal type $= V(\mu_i)$, her value in the uninformed principal game with public belief μ_i.*
- *The ex ante expected value of the informed principal, $W(\pi)$, $\leq$ the value of uninformed principal at the prior, $V(\pi)$.*
- *$W(\pi) < V(\pi)$ if $\hat{\xi}$ is sufficiently informative.*

The *outcome*, in any purifiable PBE, is the same as when the experiment $\hat{\xi}$ is publicly conducted.

However, the agent may or may not learn the principal's type.

Existence

A robust equilibrium that achieves $V(\mu_i)$ for each principal i.

- All principal types conduct compound experiment ξ^*:
- She conducts $\hat{\xi}$, resulting in some public belief $\mu_i \in \mathcal{M}$.
- She follows up with the optimal experiment corresponding to public belief μ_i.
- If any principal type deviates to a different experiment ξ, the agent attributes this deviation to the worst principal type consistent with the outcome of ξ.

Uniqueness

- Strategy ξ^* implies lower bound on a each type's payoff, $V(\mu_i)$.
- Robustness requirement is key showing that this is also an upper bound.
- Focus on the conditional distribution of agent belief (ν) given principal type.
- Key step: $\mathbb{E}(\nu|\mu_i) = \mu_i$ in any robust equilibrium.
- If $\mathbb{E}(\nu|\mu_i) > \mu_i$ for some i, Bayes plausibility implies $\mathbb{E}(\nu|\mu_j) < \mu_j$ for some $\mu_j \in \mathcal{M}$.

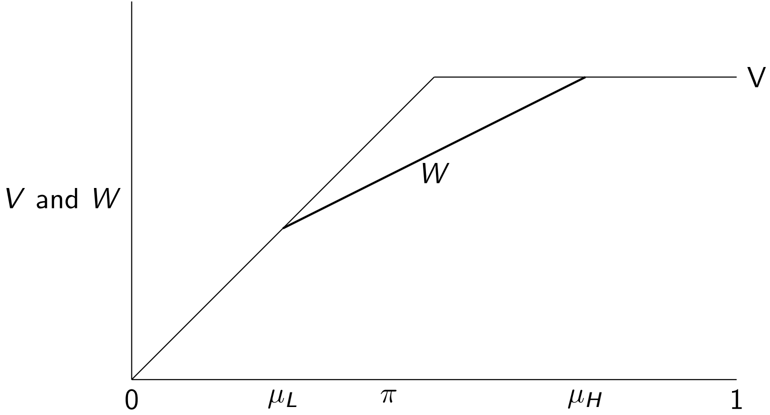
- In any perturbed game, if $\mathbb{E}(\nu|\mu_j) < \mu_j$ in a candidate equilibrium, then type μ_j has a profitable deviation.
- This type can conduct experiment $\hat{\xi}$ and conduct an experiment τ that first order stochastic dominates the conditional distribution $\nu|\mu_i$.
- Yields a strictly higher payoff in any perturbed game, since the distribution of agent actions, $\alpha|\mu_i$, FOSD that in candidate equilibrium.
- Thus, $\mathbb{E}(\nu|\mu_i) = \mu_i$ in any robust equilibrium.

Investment example

	ω_B	ω_G
Y	-3,1	2,1
N	0,0	0,0

- agent wants to invest (Y) only at ω_G, principal always wants Y.
- prior belief is $\pi = 0.4$. Threshold belief is 0.6.
- Principal gets private information via binary signals.
- Less informative $\hat{\xi}$: $\mu_H = 0.5$, $\mu_L = 0.3$, induced beliefs are 0.6 and 0, $W(\pi) = V(\pi)$.
- More informative $\hat{\xi}$: $\mu_H = 0.7$, $\mu_L = 0.1$, induced beliefs are $0,7$, 0.6 and 0, $W(\pi) < V(\pi)$.

Informative $\hat{\xi}$



$\mu_H = 0.7$, $\mu_L = 0.1$,
induced beliefs are $0, 7$, 0.6 and 0, $W(\pi) < V(\pi)$.

When principal belief types are not ordered by FOSD

- Our analysis so far does not need to assume commitment by the principal.
- Simple two stage game: principal chooses public experiment; agent observes signal realization and takes an action.
- When types are not ordered by FOSD, private information may remain private.
- However, once the principal observes the outcome of the public experiment ξ, she may learn differently depending on her private information.
- This may induce her to conduct further experiments (absent commitment).

Example

	ω_1	ω_2	ω_3	$\tilde{v}$
L	3	2	0	0
M	2	3	2	4
H	0	2	3	5

- Prior is uniform $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.
- Private information binary: types are ω_2 and $\neg\omega_2$.
- If private information $\hat{\xi}$ is revealed, then :
  - type ω_2 has nothing further to reveal, so agent chooses M.
  - type $\neg\omega_2$ induces beliefs $(\frac{2}{3}, 0, \frac{1}{3})$, inducing H with prob. $\frac{1}{2}$ and $(\frac{1}{3}, 0, \frac{2}{3})$, inducing M with prob. $\frac{1}{2}$.

Ex ante optimal experiment ξ^* has binary signals:

- Recommend H after ω_3; with prob. $\frac{1}{3}$ after both ω_2 and ω_3.
- Recommend M with prob. $\frac{2}{3}$ after both ω_2 and ω_3.
- Both principal types are better off than by revealing ξ^*.
- However, after recommendation M is made, type ω_2 has an incentive to reveal her type in the perturbed game.
- Whether principals can commit not to conduct further experiments or not is crucial.

Remark on Myerson, 1983

- Myerson observed different principal types may benefit from being inscrutable.
- Incentive and participation constraints of the agent need only hold on average, across different principal types.
- Inscrutable contract must be weakly better for any principal type τ, compared to deviation.
- However, if the contract is only weakly better for some τ, then this may turn out to be inferior when there is slight uncertainty about agent preferences.
- Important in environments without transfers since bets between different principal types, via the agent, are not feasible.

Conclusions

- Uninformed principal: information design is generically robust.
- Informed principal + belief types ordered by FOSD: principal's private information becomes effectively public. Reduces ex ante payoff of principal.

Extensions

- Many agents playing a supermodular Bayesian game.
- Types not ordered by FOSD.
- Informed principal problems without transfers.