Circles of Trust: Social Networks and Rival Information

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Abstract

We analyze the diffusion of rival information in a social network. In our model, rational agents can share information sequentially, unconstrained by an exogenous protocol or timing. We show how to compute the set of eventually informed agents for any network, and show that it is essentially unique under altruistic preferences. The relationship between network structure and information diffusion is complex because the former shapes both the charity and confidentiality of potential senders and receivers.

1 Introduction

Relationships and communication are basic social traits that interact: information is often shared among people with close ties. This would seem particularly true for rival information. Since rival information or knowledge becomes less valuable when shared, people ought to be more (or only) willing to share it with those they care for.

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At first sight, this appears to imply that a network with many altruistic relationships, for instance, encourages the diffusion of rival information.[1] Such a conjecture neglects, however, that people often have multiple relations that can create conflicting loyalties. While P is friends with S and N, the latter may have other friends whom P is not friends with and who in turn have friends and friends-of-friends that are even further removed from P.

Such constellations create complex trust problems. Suppose M is N's friend, but not P's. When P contemplates sharing a "secret" with N, she assesses whether N will pass it on to M. This may depend on whether N trusts M to keep the secret, which in turn may depend on how much M trusts *her* other friends, and so on—which results in non-trivial interdependencies in trust. For instance, P may trust N if N distrusts M, and conversely, distrust N if N trusts M.

In general, therefore, the equilibrium constellation of trust is a function of the entire network structure. Predicting which path, or how far, rival information travels through a network is thus not as straightforward as it may seem.

This paper presents a theory of rival information diffusion in a social network. One agent, the originator, is endowed with a piece of hard information. The private value of this information to any informed agent decreases when the set of agents that possess it expands. Communication is strategic and sequential: At any point, any informed agent can freely communicate the information to any uninformed agents in the network, even those she is not (altruistically) connected to. Everyone is rational and forward-looking. The information diffusion process ends endogenously when no informed agent conveys the information to any of the remaining uninformed agents.

The advantage (and limitation) of our approach is twofold: The diffusion process is a sequential-move model, but no sequences or communication channels are exogenously imposed. The model hence accounts for every possible sequence of communication, but at the same time, allows the use of backward induction to identify what we call *forecasts*—the sets of ultimately informed agents that are compatible with information-

[1] It has been shown that important social relations involve altruism (Becker, 1974), that such relations facilitate the exchange of rival goods (Foster and Rosenzweig, 2001), and that the strength of altruism correlates with measures of distance in social networks (Leider et al. (2009); Goeree et al. (2010)).

sharing incentives in the network—for any arbitrary preference profile. For a large class of preferences, the forecast is essentially unique.

We then impose a particular structure on the preferences that represents a setting in which (i) information is rival and (ii) links in the social network are mutually altruistic relations. Our model shows that, in such a setting, the mapping from network structure to information diffusion is complex: Adding (subtracting) links in a network can have non-monotonic effects on information diffusion, and small changes to the structure can cause large changes in who becomes informed. Relatedly, it is not obvious where within a network information should be seeded to maximize its diffusion. For example, it need not be better to seed it with more agents, or with those who have the highest centrality measures.

One complication arises in evaluating the (yet) uninformed agents' trustworthiness: can an agent be trusted not to share the information further, or at least not too far? In a network, such trust is not a pairwise property, but as stressed earlier, depends on the disposition of trust across all links; trust relations are interdependent. Small changes, like removing or adding a link, can alter trust relations elsewhere in the network, even far away from the location of the change.

Another complication is that informed agents' secrecy may be mutually conditional. For example, information may be contained between two agents by each one's threat of informing friends that the other agent is not friends with. The credibility or strength of such threats, and therefore the balance of power in this "mutual hostage situation," depends on the web of relations the informed agents are embedded in. Changes to the network—again, be it removing or adding links—can tilt the balance in either direction and thereby support or undermine secrecy.

These strategic issues also complicate incentives for network formation (presuming an environment where access to rival knowledge is of such concern that it affects relationship choices). Having many links need not be advantageous, as it could undermine one's trustworthiness. Nor is tying oneself to a single cluster, as keeping links outside a specific circle of trust may preserve the balance of power within it.

There is a wealth of evidence that social ties play a significant role in the diffusion of rival information.[2] Our theory resonates most with studies in which subjects (seem to) take social networks into account to strategically steer or contain information diffusion. The most compelling evidence of this kind comes from randomized controlled trials in which information about scarce opportunities to participate in paid experiments or in aid programs is "seeded" with individuals to then be shared by word of mouth (Banerjee et al., 2012; Bandiera et al., 2018; Vilela, 2019). Another pertinent set of studies traces out networks and information sharing among fishermen, a context in which social ties may even be endogenous to (concerns about) information flows (Palmer, 1991; Turner et al., 2014; Alexander et al., 2020). These too find information flows to be embedded in social relationships, and the role of network structure to be complex.

Although information diffusion is a central topic in the social network literature, the latter features few theoretical analyses of *rival* information. In Immorlica et al. (2014), agents select a time-invariant probability of passing information to friends. Once these choices are set, information travels through the network according to a Poisson process. In Kleinberg and Ligett (2013), agents want all their friends but none of their enemies to become informed. Stable information sharing structures are characterized in a static model with myopic agents. Kushnir and Nichifor (2014) take a similar approach.[3] One distinctive aspect of our model, which is crucial for our results, is that agent selectively choose *which* of their friends to inform based on considerations of trustworthiness and mutual secrecy.

At a conceptual level, our theory is closely related to Barbera et al. (2001)'s analysis of club formation by invitations and to the literature on the resale of information (Muto, 1986; Admati and Pfleiderer, 1986, 1990; Nakayama and Quintas, 1991; Nakayama et al., 1991; Polanski, 2007, 2019; Ali et al., 2020). Last, the notion of *forecast* used in our

[2]See, e.g., Cohen et al. (2008) in the context of fund managers and corporate managers, Iyer and Puri (2012) and Kelly and O Grada (2000) in the context of bank runs and financial panics, and the large literature documenting the role of social networks in disseminating information about job opportunities, which is rival when shared between job seekers (reviewed, e.g., by Ioannides and Datcher Loury (2004), Jackson (2010), Munshi (2011) and Topa (2011)).

[3]These are, of course, not the only papers that analyze strategic behavior with respect to information in a social network. But other papers in this literature focus on different issues such as, e.g., information acquisition and network formation when information is a local public good that "spills over" to neighbors (Bramoullé and Kranton, 2007; Galeotti and Goyal, 2010) or communication of soft information (cheap talk) to influence neighbors' actions or cooperation (Galeotti et al., 2013; Lippert and Spagnolo, 2011).

analysis can be seen as an adaptation of the notion of *expectation* in Jordan (2006) and is akin to solution concepts used in Ray and Vohra (1997) and Acemoglu et al. (2012).

2 General framework

There is a group of agents N with one of them—$o \in N$—possessing a valuable piece of information. We refer to o as the originator. The originator and, later, any other informed agent, can pass the information to uninformed agents in N. We assume that the agents share information sequentially. At any instance, at most one agent passes on the information. However, several agents can receive the information simultaneously.

The act of sharing the information, but not information itself, is publicly observed.[4] If agent i shares the information with some set of agents $R \subset N$, everyone observes that agents in R are now informed and that agent i was the source. This process constitutes a diffusion of information.

A snapshot of the diffusion process is represented by a directed tree $T = (N_T, E_T)$ where $N_T \subset N$ is the set of informed agents and $E_T \subset N_T \times N_T$ records communications (e.g., $(i, j) \in E_T$ means that i informed j). In any such tree, the originator is informed: $o \in N_T$. And no one passes the information to the originator: $(i, o) \notin E_T \ \forall i \in N$. Apart from these two conditions, we assume that agents can share information freely without any restrictions. We refer to such trees as *outcomes*, and denote the collection of feasible outcomes by $\mathscr{I}$.[5] In addition, by $T_o := (\{o\}, \emptyset)$, we denote an empty tree and interpret it as an outcome in which only the originator is informed.

Everyone takes interest in how the information diffuses. We assume that each agent i has complete and transitive preferences $\succeq_i$ over $\mathscr{I}$, and denote strict preferences by $\succ_i$. We abstract from the time dimension in agents' preferences—agents are insensitive to the speed of the information diffusion. We also assume that any uninformed agent always wants to become informed.

[4]This assumption is stricter than needed. For our results, it would suffice to assume that, at any stage, the set of then-informed agents learns whether any one of them passes on the information and to whom it is passed on. See also our concluding discussion in Section 4.

[5]There are several ways to introduce constraints on how agents share information. One possibility is to restrict set $\mathscr{I}$. Another possibility is use large payoff penalties for some acts of sharing.

2.1 Information diffusion

We study a dynamic process in which agents who receive information can pass it on to anyone at any subsequent point in time. As in dynamic programming problems, we examine this process for any initial condition represented by some outcome T. That is, assuming that the information diffused according to T up to a certain point in time, we ask: what happens next?

To answer this question, we must consider the information-sharing incentive of each agent who is informed in outcome T. We do so under the assumption that all agents are forward-looking and that their expectations are common and consistent. We construct a function, $\mu : \mathscr{I} \to \mathscr{I}$, that we call a *forecast*. The forecast returns an ultimate, long-term stable result of information diffusion $\mu(T)$ for each initial condition T. It classifies outcomes as *terminal* or *transitory*. An outcome is transitory if it is followed by further diffusion, and it is terminal otherwise.

Consider agent i who is informed in $T : i \in N_T$. Let

$$\Theta(T,i) = \{\tilde{T} \in \mathscr{I} \mid E_T \subset E_{\tilde{T}} \text{ and } E_{\tilde{T}} \setminus E_T \subset i \times (N \setminus N_T)\}$$

be the set of all outcomes that can result from agent i passing the information to some uninformed agents. This set consists of all trees that can be obtained from T by adding a star with a center at i to it.

Definition 1. *A function $\mu : \mathscr{I} \to \mathscr{I}$ is a forecast if for any $T \in \mathscr{I}$*

(i) $T \subset \mu(T)$*;*

(ii) $\mu(\mu(T)) = \mu(T)$*;*

(iii) if outcome T is terminal—i.e., if $T = \mu(T)$, then for any informed agent $i \in N_T$ and for any $\tilde{T} \in \Theta(T,i)$:

$$\mu(T) \succeq_i \mu(\tilde{T});$$

(iv) if outcome T is transitory—i.e., if $T \neq \mu(T)$ then there is an informed agent $i \in N_T$

such that

$$\mu(T) \in \max_{\tilde{T} \in \Theta(T,i)}{}_{\succeq_i} \left\{\mu(\tilde{T})\right\} \succ_i T.$$

Condition *(i)* and the definition of $\Theta(\cdot,\cdot)$ state that a diffusion is irreversible: once informed, always informed. By condition *(ii)*, a forecast is dynamically consistent. Conditions *(iii)* and *(iv)* require that all agents are rational and forward-looking and agree on their predictions about the future. Moreover, uninformed agents receive information only if some informed agent benefits from it *strictly*; if all informed agents are indifferent between sharing the information further and not, we assume that they stop sharing it.

A forecast is defined as a fixed point. However, we can exploit the irreversibility of information diffusion to characterize a forecast using a backward induction algorithm. Let

$$Q(\mu, T) = \{i \in N_T \mid \forall \tilde{T} \in \Theta(T,i) : T \succeq_i \mu(\tilde{T})\}$$

be a set of agents who have the information in T and prefer not to give it to uninformed agents. The algorithm is as follows:

(i) Let $\mu(T) := T$ for every $T : N_T = N$. This is a starting point of the induction.

(ii) Take an outcome T such that for any $\widehat{T} \supset T$, $\mu(\widehat{T})$ is already defined.[6]

   (a) If $Q(\mu, T) = N_T$, then set $\mu(T) := T$;

   (b) otherwise, pick any agent $i \in N_T \setminus Q(\mu, T)$ and set $\mu(T) := T' \in \max_{\tilde{T} \in \Theta(T,i)}{}_{\succeq_i} \left\{\mu(\tilde{T})\right\}$.

(iii) Repeat (ii) until μ is defined everywhere.

Theorem 2. *For any set of agents N and any preference profile $\{\succeq_i\}_{i \in N}$,*

(a) there exists at least one forecast;

(b) any forecast can be obtained by the backward induction algorithm.

[6] Note that for such a tree T, $Q(\mu, T)$ is well defined because μ is defined for all $\tilde{T} \supset T$ at the previous rounds of the induction.

Proof. The theorem follows from the definition of a forecast and the description of the backward induction algorithm. □

Forecasts are not necessarily unique. As we show next, placing certain restrictions on agent preferences guarantees that running the algorithm once characterizes *all* forecasts, while still allowing for interesting applications of the model.

2.2 Uniqueness

If agents do not care about the entire diffusion *path* but only about who is eventually informed and who is not, finding one forecast is sufficient to characterize every terminal outcome under all forecasts. More formally, we assume the following:

Assumption 3. *For any agent i and for any two outcomes $T, \tilde{T}$ such that $N_T = N_{\tilde{T}}$, we have $T \sim_i \tilde{T}$.*

Under Assumption 3, the solution is *essentially unique*.

Theorem 4. *If preferences satisfy Assumption 3 then:*

(i) for any forecast μ and outcomes $T_1, T_2 \in \mathscr{I}$ such that $N_{T_1} = N_{T_2}$:

$$\text{if } \mu(T_1) = T_1, \text{ then } \mu(T_2) = T_2; \tag{1}$$

(ii) for any two forecasts μ_1, μ_2 :

$$\text{if } \mu_1(T) = T, \text{ then } \mu_2(T) = T. \tag{2}$$

Proof. We prove the first part of this theorem by setting up an induction:

1. If $N_{T_1} = N_{T_2} = N$, then $\mu(T_1) = T_1$ implies $\mu(T_2) = T_2$ because there are no uninformed agents in T_2.

2. Let (1) hold for all $T_1, T_2 : N_{T_1} = N_{T_2}, |N_{T_1}| > k$.

We show that (1) holds for all $T_1, T_2 : N_{T_1} = N_{T_2}, |N_{T_1}| = k$. By contradiction assume that for some $T_1, T_2 : \mu(T_1) = T_1, \mu(T_2) = T_2'$ and $T_2' \neq T_2$. There exists an agent $i \in N_{T_2} : T_2' \succ_i T_2$ and $T_2' \sim_i T_1' \in \Theta(T_1, i)$. Since $|N_{T_2'}| > k$, $\mu(T_2') = T_2'$ implies $\mu(T_1') = T_1'$, hence $\mu(T_1) \neq T_1$ which is a desired contradiction.

We prove the second part of this theorem by setting up an induction:

1. If $N_T = N$, then $\mu(T) = T$ for any μ because there are no uninformed agents in T.

2. Let (2) hold for all $\mu_1, \mu_2, T : |N_T| > k$.

We show that (2) holds for all $\mu_1, \mu_2, T : |N_T| = k$. Assume, by contradiction, that $\mu_1(T) = T$ and $\mu_2(T) = T' : T' \neq T$. There exists an agent $i \in N_T : T' \succ_i T$. Also there exists $T'' \in \Theta(T, i)$ such that $N_{T'} = N_{T''}$. Since $|N_{T'}| > k$, $\mu_2(T') = T'$ implies $\mu_1(T') = T'$. Also, by the first part of this theorem, $\mu_1(T') = T'$ implies $\mu_1(T'') = T''$, hence $\mu_1(T) \neq T$ which is a desired contradiction. $\square$

Let $\Omega = \{T \in \mathscr{I} \mid T = \mu(T)\}$ be the set of all terminal outcomes. Under Assumption 3 we can omit the dependence of this set on the forecast.

The terminal outcome that results from an initial condition with a single informed agent—the originator—is unique up to indifferences in the originator's preferences. It is as if the originator selects her favorite outcome from the set of all terminal ones:

$$\mu(T_o) \in \max_{\succeq_o} \Omega.$$

Suppose the originator wants to achieve outcome T. Given communication is unconstrained, the originator can inform every agent in N_T by herself and thereby arrive at some outcome $\tilde{T} : N_{\tilde{T}} = N_T$. This is as if her choice is restricted to the set Ω by incentives of others, but she is essentially free to choose any element of that set.

When there is more than one originator, a similar principle applies: every forecast is some originator's favorite outcome among the set of the terminal ones.

For an arbitrary initial condition T, the terminal outcome that results is always the "closest", or put differently, it features the smallest (in the set-inclusion sense) group of informed agents that can occur in any terminal outcome.

Theorem 5. *If preferences satisfy Assumption 3, for any $T_1 = \mu(T)$ there exists no $T_2 \neq \mu(T)$ such that $T \subset T_2 \subset T_1$ and $T_2 = \mu(T_2)$.*

Proof. Consider agent i that initiates the transition from T to $\mu(T)$. Note that $T_1 \in \max_{\substack{\succeq_i \\ X \in \Theta(T,i)}} \{\mu(X)\}$. By contradiction, suppose there exists $T_2 : T \subset T_2 \subset T_1$ and $\mu(T_2) = T_2$. There exists $T_2^* \in \Theta(T,i) : N_{T_2^*} = N_{T_2}$ and, therefore, $\mu(T_2^*) = T_2^*$. This implies that $T_1 \succ_i T_2^* \sim_i T_2$. Also, there exists $T_1^* \in \Theta(T_2^*, i)$ such that $N_{T_1} = N_{T_1^*}$ and, therefore, $\mu(T_1^*) = T_1^*$. Since $T_1^* \succ_i T_2^*$, we arrive at the desired contradiction. □

Of course, Theorem 5 also holds for T_o as the initial condition. The group of agents that will be informed is minimal in size (in the set-inclusion sense) across all terminal outcomes, and of all minimal ones, is the originator's favorite group. Theorems 4 and 5 demonstrate that the absence of an exogenous communication protocol, together with Assumption 3 on preferences, greatly simplifies finding the set of terminal outcomes.[7]

3 Directed altruism and rival information

We now study a specific application of our framework by putting more structure on agents' preferences in two ways. First, every informed agent incurs a utility penalty that increases in the number of informed agents. This captures the idea that the information is rival such that purely selfish agents would not share it. Second, we impose the notion of a social network where connected agents are mutually altruistic "friends" who derive additional utility if the other is informed.

Formally, such a network is represented by a symmetric adjacency matrix $\Phi \in \mathbb{R}_+^{N \times N}$. Element $\Phi_{i,j}$ represents the strength of altruism between agents i and j. Generally, the utility of agent i in outcome T is $U_i(N_T, \Phi)$. These preferences satisfy Assumption 3. We use this model to show that strategic sharing of rival information in a social network is a complex problem and that readily generalizable patterns of information diffusion and

[7] If agents care about the exact paths of information diffusion—e.g., if they experience "warm glow" from the *act* of sharing information—both parts of Theorem 4 break down, and non-trivial multiplicity arises. Theorem 2, however, holds for arbitrary preferences, any numbers of originators, and irrespective of constraints on communication.

network formation do *not* emerge without additional restrictions. We will pick different specifications of Φ to illustrate a few of the complexities. An especially tractable class of preferences that we use is linear: every agent gets an additive utility bonus of b for each informed friend and a utility penalty of 1 for each informed person. Formally,

Definition 6. *Agent i's preferences are linear if they are characterized by a utility function of the form*

$$U_i(N_T, \Phi) = b_i \sum_{j \in N_T} \Phi_{i,j} - |N_T|,$$

where $b_i > 0$.[8]

Note that under this specification, agents without any friends are never informed by anyone, so we can ignore them in the analysis.

3.1 Endogenous (dis)trust: If I told her, would she tell anyone?

Since rival information would not be shared in the absence of altruistic preferences in our model, one might conjecture that the degree of information diffusion increases in the number of altruistic links or the strength of the altruism. However, this is not true for reasons that are best exposed in a very simple setting. Suppose all agents have linear preferences with a common parameter $b > 0$, and form a *chain* of friends of length m with the originator at one end. In this case, a sufficient statistic to characterize terminal outcomes is the total number of uninformed agents in the chain.

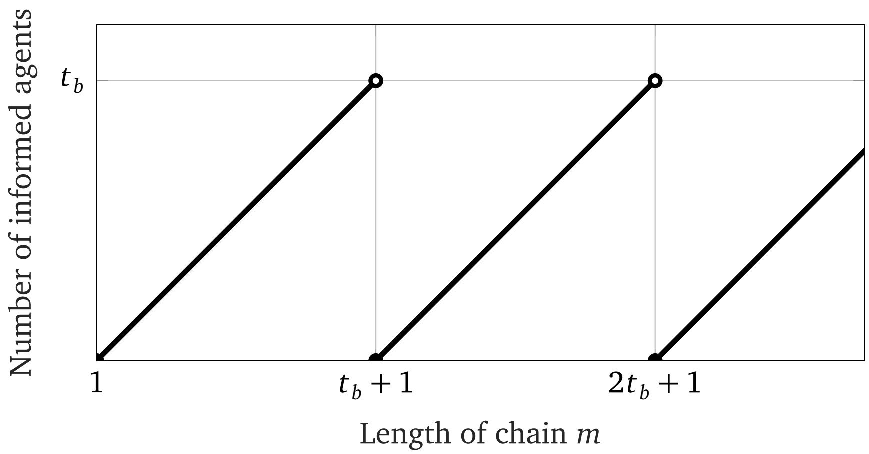
Proposition 7. *Consider a chain of friends where* $\Phi_{i,j} = 1$ *for all* $i, j \leq m$ *and* $|i - j| = 1$ *and* $\Phi_{i,j} = 0$ *otherwise. Let* $t_b \in \mathbb{N}$ *be a number that satisfies* $(t_b - 1) < b < t_b$. *For any forecast* μ, *outcome* T *is terminal if and only if the number of uninformed agents is either zero or a multiple of* t_b.

Proof. Suppose there are t_b uninformed agents. An informed agent who has an uninformed friend will not share the information because once she does, everyone will

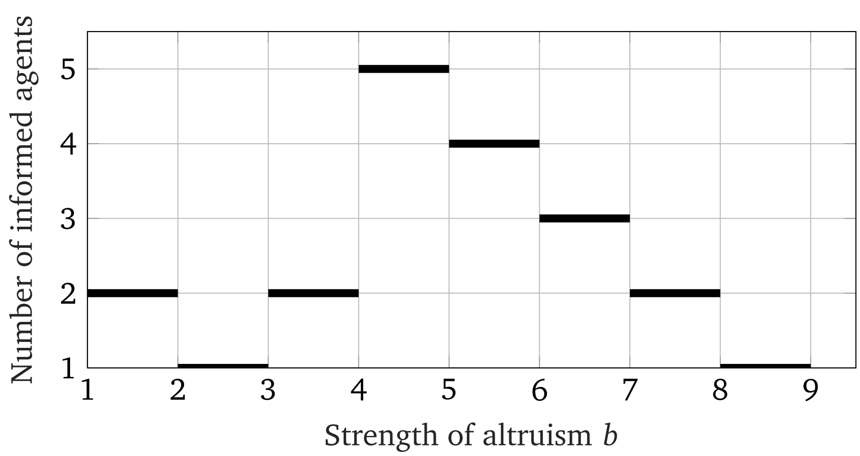
[8] Recall, that if an agent is not informed in outcome T, her utility is assumed to be low, so that agents never refuse an information that is given to them.

be informed. If the number of uninformed agents is a multiple of t_b, informing one of them will result in t_b of them getting the information eventually. $\square$

Two types of comparative statics follow from Proposition 7: one with respect to the length of the chain m, and the other with respect to the strength of altruism b. In either case, the relationship to the number of informed agents is *non-monotonic*, as depicted in Figure 1.



(a) Number of informed agents as a function of m.



(b) Number of informed agents as a function of b $(m = 10)$;

Figure 1: Comparative statics for a chain of friends.

The intuition underlying both comparative statics is that, because the information is rival, an informed agent shares it with her friend only if she is certain that it will not spread too far. Let us elaborate on this for the comparative statics with respect to the chain length (Figure 1a). When the remaining chain of uninformed agents is short, it is guaranteed that the information does not travel far. However, when the chain becomes longer, informed agents must become strategically secretive—taking into account the endogenous trustworthiness of the uninformed agents, which itself is a function of the chain length.

For instance, when the chain is of length $t_b + 1$, none of the agents can be trusted to keep the secret: the originator knows that passing the secret to her friend ultimately result in everyone being informed. When the length of the chain is $t_b + 2$, the originator entrusts her friend with the secret because her friend finds himself in the same situation as the originator with the chain of $t_b + 1$ agents. Namely, the originator's friend will not trust the remaining uninformed agents, which makes her endogenously trustworthy—she is in the originator's endogenous "circle of trust."

Like the sequential information-sharing problem, the problem of trust is recursive: Agent o's trust in agent 1 depends on the latter's trust in agent 2, which in turn depends on agent 2's trust in agent 3, and so forth. For example, agent o may trust agent 1 only if the latter *mis*trusts agent 2, which may be the case only if agent 2 trusts agent 3. More generally, in structures more complex than chains, whether an agent is inside another's circle of trust hence depends on the surrounding network in complex ways.

Figure 1a also illustrates that small changes in the network structure can have large effects on who becomes informed. For instance, adding one link to a chain of length t_b causes a discrete drop in the number of informed agents from t_b to 1, which means that the structural change at the t_bth position in the chain alters the originator's information sharing decision (i.e., $t_b - 1$ nodes away from the change).

3.2 Mutually conditional secrecy: I won't tell, if you don't tell

Another source of complexity is that one informed agent's secrecy may not only take into account the trustworthiness of those yet uninformed but also be *conditional* on the secrecy of others already informed. To see this, consider a chain of four agents in which the originator is one of the interior nodes: 1-o-2-3. Let $b = 1.4$, so an agent informs a friend only if she is sure that it will not trigger further information diffusion.

One forecast is that the information only reaches agent 2 and travels no further. It is not obvious why this outcome is terminal; agent 2 would seem to get 0.4 units of utility from informing agent 3, as would agent o from informing agent 1. What contains the information to agents o and 2 is the mutual threat of sharing the information further. If agent 2 passes the information to agent 3, agent o will pass it to agent 1, and vice versa. Once triggered, this sequence of events would result in a net loss of 0.6 each for agents o and 2. This "mutually assured diffusion" contains the diffusion.

Although no information is shared between o and 1 or between 2 and 3, these links are crucial to the forecast. For example, eliminating the link between 2 and 3 *increases* information diffusion: the information would then reach agents 2 *and* 1. Hence, having the extra link to agent 3 gives agent 2 more "power" over the information.[9]

3.3 Seeding and centrality

The previous arguments highlight that adding or subtracting links in a network can have complex, even counterintuitive, effects on information diffusion. As a result, there is no straightforward relationship between information diffusion and standard network measures in our model. Similarly, there is no obvious relationship between information diffusion and the location of the originator within a network.

This latter point matters for strategies that "seed" information with a small number of recipients in order for it to spread by word-of-mouth. It is sometimes suggested that a very effective seeding strategy is to select agents with high levels of network centrality as initial recipients. One can construct centrality indices through theoretical arguments (degree, betweenness, closeness, etc.) or empirically, using historical diffusion patterns (Banerjee et al., 2013, 2014).

[9] Note that this notion of "power" as a result of network links deviates from the standard emphasis on the degree to which an agent has *access* to resources and the degree to which others *depend* on an agent for resources. Here, it concerns the ability to deter dilutive sharing by others through a credible threat to dilute the value of the resource herself—as in a mutual hostage game.

When information is rival, a very central agent may not be the best seeding point. To give an example, consider the network shown in Figure 2. Common centrality indices—such as degree centrality, betweenness-centrality and eigenvector centrality—indicate 2 and 3 as the most central agents. Agents 1, 7, 8, 9 and 10 form a cluster around agent 3, and agents 1, 4, 5, 6 and 10 form a similar cluster around agent 2.

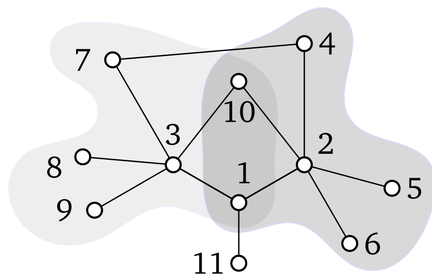


Figure 2: A friendship network with two clusters

Suppose all agents have linear preferences with a common parameter $b = 11/2$. We use the backward induction algorithm to compute the forecast for different originators. If agent 2 is the originator, agents 4, 5, 6 and 10 will be informed. Agents 4 and 10 will not pass the information into the cluster around agent 3—the resulting diffusion would outweigh the benefit of having an additional informed friend.

In comparison, if agent 1 is the originator, *everyone* in the network will eventually be informed. What distinguishes agent 1 from agent 2 or 3 is that she cares equally about the clusters of agents on the left and the right, which makes her a more effective seeding point than either 2 or 3.

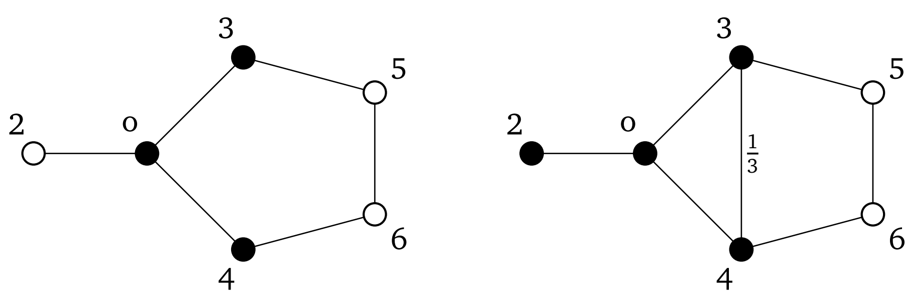
One can also construct examples in which seeding information with *fewer* agents is more effective. Revisit the example from Section 3.2 and suppose o and 2, while connected, are the centers of two equal-sized star structures, which are otherwise disjoint (and include 1 and 3, respectively). In this modified example, for some values of b, it is better to seed information with either o or 2 than with both of them.

3.4 Incentives to form or sever links

The arguments so far have treated the network structure as given since, in practice, the formation of close relationships may result from forces outside of our framework. In environments, however, where receiving and sharing rival information is a key concern, the information diffusion problem analyzed in our model might have some influence on network formation.

A natural conjecture is that agents gravitate toward close-knit clusters within which rival information is shared and contained. Indeed, one might expect that, starting from any given structure, all agents that the information reaches gain from being each other's friends ("inside links") but not from being friends of those that will remain uninformed ("outside links"). Such a logic would strongly favor clusters.

However, while such examples can easily be constructed, the above argument does not generally hold in our model. Take the network shown in Figure 3a with preferences assumed to be such that agents o, 3, and 4 will be informed, and think about pairwise incentives to form links and unilateral incentives to sever links. Notice that this thought experiment presumes that links require (only) bilateral consent.



(a) Network of friendships Φ (every link has a unit weight);

(b) Network of friendships $\hat{\Phi}$ obtained from network Φ by adding a link of weight $1/3$ between agents 3 and 4.

Figure 3: Friendship networks and sets of informed agents for nonlinear preferences.

Inside links. Consider whether agents 3 and 4 benefit from connecting, conditional on staying informed. Among agents o, 3, and 4, such a link would lead to *triadic closure*, a concept associated with *trust* in social network theory at least since Simmel (1908).[10]

It turns out that agents 3 and 4 do not necessarily gain from forming an inside link, for reasons that have to do with trust. To explain this, we first show that they do benefit from such a link if preferences are sufficiently separable, and then analyze an example where this is not the case because the separability condition is violated.

Assumption 8. *For any agent i in a social network Φ, preferences are represented by*

$$U_i(N_T, \Phi) = \sum_{j \in N_T} \Phi_{i,j} - g_i(N_T).$$

Since the altruistic utility bonuses $\Phi_{i,j}$ from informing friends are fixed and additive, an agent's net gain from forming an additional link is independent of which (informed) agents she is already friends with. For example, in Figure 3a, this means that agent 3's incentive to inform agent 5 does not depend on the relationship between agents 3 and 4.

For the class of preferences that satisfy Assumption 8, which nests the class of linear preferences used in Sections 3.1 and 3.3, adding links between informed agents always makes them better off. For a social network Φ, let μ_Φ be a forecast given a preference profile that satisfies Assumption 8.

Proposition 9. *Suppose that the agents' preferences satisfy Assumption 8 for any social network. Consider two agents, i and j who are informed given a social network Φ, namely $\{i, j\} \subset N_{\mu_\Phi(T_o)}$. Let the strength of their friendship increases in the new social network $\hat{\Phi}$ ($\hat{\Phi}_{i,j} > \Phi_{i,j}$) and the rest of the social network remained the same ($\forall \{k,l\} \neq \{i, j\} : \hat{\Phi}_{k,l} > \Phi_{k,l}$). If i and j remain informed under $\hat{\Phi}$—i.e., $\{i, j\} \subset N_{\mu_{\hat{\Phi}}(T_o)}$—then both are better off:*

$$U_i(N_{\mu_{\hat{\Phi}}(T_o)}, \hat{\Phi}) > U_i(N_{\mu_\Phi(T_o)}, \Phi)$$
$$U_j(N_{\mu_{\hat{\Phi}}(T_o)}, \hat{\Phi}) > U_j(N_{\mu_\Phi(T_o)}, \Phi).$$

Proof. Consider any outcome $T : \{i, j\} \subset N_T$. Note that any agent ranks the outcomes

[10] Triadic closure also features prominently in Granovetter (1973).

that follow T under Φ in the same way as under $\hat{\Phi}$. More formally, for any $T_1, T_2 \supset T$, and for any $k \in N$: $U_k(N_{T_1}, \Phi) \geq U_k(N_{T_2}, \Phi)$ iff $U_k(N_{T_1}, \hat{\Phi}) \geq U_j(N_{T_2}, \hat{\Phi})$. This means that when it comes to terminal outcomes, μ_Φ agrees with $\mu_{\hat{\Phi}}$ for all outcomes in which both i and j are informed. Since the originator can essentially select her favorite terminal outcome, under the premise of the proposition, $\mu_\Phi(T_o) = \mu_{\hat{\Phi}}(T_o)$. □

Now consider a case in which Assumption 8 does not hold. Let the preferences of an agent be nonlinear in the number of informed friends:

$$U_i(N_T, \Phi) = b \log\left(\sum_{j \in N_T} \Phi_{i,j}\right) - |N_T|. \tag{3}$$

Under these preferences, an agent's net gain from informing another friend depends on how many informed friends she already has. We will provide an intuitive interpretation further below.

Assume these preferences and set $b = 3$. Then, in network Φ in Figure 3a, agents o, 3, and 4 will be informed. In network $\hat{\Phi}$, which is obtained from network Φ by creating an additional link between agents 3 and 4 with $\Phi_{3,4} = 1/3$, the same agents *plus agent* 2 will be informed. Agents 3 and 4 prefer network Φ, in which they are not friends but fewer agents are informed:

$$U_3(N_{\mu_\Phi(T_o)}, \Phi) = 3 \log 1 - 3 > 3 \log \frac{4}{3} - 4 = U_3(N_{\mu_{\hat{\Phi}}(T_o)}, \hat{\Phi}).$$

What is the intuition? The key is to understand why the information remains contained in network Φ, among agents o, 3, and 4. None of them diffuses the information further because their mutual threat of doing so is credible. The added link in network $\hat{\Phi}$ renders agents 3 and 4 more reluctant to share information with others. This undermines their threat vis-à-vis agent o, who consequently becomes less secretive. This showcases both of the aspects highlighted in Sections 3.1 and 3.2: a friendship between agents 3 and 4 alters their endogenous trustworthiness, which critically tilts the balance in the "mutual hostage" situation in favor of agent o.

Preferences as represented by (3) are by no means pathological, but quite natural.

The same preferences can be represented by the utility function

$$\hat{U}_i(N_T, \Phi) = \left(1 + \sum_{j \in N_T} \Phi_{i,j}\right) e^{-\frac{1}{b}|N_T|}.$$

One can see $e^{-\frac{1}{b}|N_T|}$ as the objective value of information, whose rival nature is reflected in the fact that the value is decreasing in $|N_T|$. In addition to enjoying this value, agent i internalizes the share $\Phi_{i,j}$ of the value enjoyed by each of her informed friends $j \neq i$. So, having more informed friends reduces agent i's incentive to dilute the information, that is, to diffuse it further.

Outside links. Taking again network Φ in Figure 3a, consider whether agents have anything to lose from dropping links to uninformed friends. They do. Suppose agents 3 and 4 drop their links to agents 5 and 6. This harms them because, without the credible threat of informing 5 and 6, they cannot keep agent o from informing agent 2. Notice that this argument also indicates why an isolated cluster between agents o, 3, and 4 is an unstable outcome. For instance, agent o would gain from befriending and informing agent 2. In general, the implications of rival information sharing for network formation seem far from straightforward.

4 Concluding remarks

Our model makes the strong assumption that whether information is shared, that is, whether it diffuses, is observable. This is arguably more plausible for some types of rival knowledge (e.g., trade craft) than for others (e.g., stock tip). Moreover, we make heroic assumptions about agents' comprehension of the network and the information-sharing incentives within it, which may be more appropriate for overseeable networks than for far-reaching ones. In the latter case, introducing elements of bounded rationality in the analysis would be a reasonable extension.

The caveats regarding observability and rationality are potentially less relevant if, in contexts where rival information diffusion is a first-order concern, agents naturally tend to group into small, close-knit clusters. However, in our model, agents' incentives to gravitate toward clusters is ambiguous, at least based on notions of pairwise stability. They may eschew links that reinforce clustering and want links that “keep things open” to support secrecy and trust, or better, to limit information diffusion. As a result, agents' attitudes toward forming links defy simple descriptions in our model.

These complications highlight the difficulty of balancing out *multilateral* incentives to share information (or not), if everyone is free to form and drop links. This points to a possibly important real-world element left out of our analysis: in close-knit groups—such as, e.g., families, cliques, gangs—the members sometimes restrict whom else each of them may connect with, especially when it comes to “outsiders.” In our setting, such power may rest with the originator. Being the source of rival information may empower her to restrict others' actions, including their freedom to enter into other relationships. Developing and analyzing such a model, perhaps with multiple potential originators, is a promising avenue for follow-on work.

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