

Chemical Manufacturing Analysis Report

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Team Mini Project 3

Problem Addressing and Approach

The problem that we are addressing for this chemical manufacturing company is finding a combination of production levels of three product lines and a special waste treatment line that maximizes profit for the company. However, we are constrained to the amount of raw material of the company has and the fact that the EPA requires the proper disposal of the liquid waste, which is a byproduct of the primary product. To solve this problem, we used Microsoft Excel and its solver tool.

In solving this problem, we were given the fixed costs, variable cost, selling prices, and available resources. The fixed costs are the monthly administrative costs and monthly refinery costs, whose values are \$13,000 and \$4,500 respectfully. The variable costs are the costs of raw materials x and y, and the direct labor costs for the primary product, secondary product k, secondary product m, and the special treatment of the liquid waste. For the variable costs of the raw material x, raw material y, and the direct labor costs of the primary product, we were given these variable costs in terms of the previous months overall production level of the primary product. So, we had to get the individual variable costs. In doing so, we get the cost of \$1.33 per pound of raw material x, \$0.73 per pound of raw material x, and direct labor costs \$0.42 per pound of primary product. For the direct labor costs per pound of secondary product k, secondary product m, and the special treatment of the liquid waste we are given estimates of what we expect the costs to be given the different products to follow the EPA regulations in terms of liquid waste disposal. The selling price per pound for the primary product has been \$5.70, and is estimated to be \$0.80 and \$0.65 for secondary products K and M. Furthermore, given the past data, estimated data, and selling prices for the various products we calculated the unit profit of each product. The unit profit values were found by the difference between the selling prices and the various variable cost that go into the production/treatment. Therefore, the unit profit considers all the variable costs per pound of product/treatment. The per pound unit profits for the products are shown below:

Product/Treatment	Unit Profit (per pound of product/treatment)
Primary Product	\$2.49
Secondary Product K	- \$0.07
Secondary Product M	\$0.19
Special Treatment of Liquid Waste	- \$0.25

Linear Programming Model

Given the unit profits, fixed costs, available resources (7500 lbs. of raw material x and 9000 lbs. of raw material y), and the EPA requirements that all liquid waste produced must be properly disposed through the production of secondary product k, secondary product m, and special treatment of liquid waste, we can create the following linear programming problem.

$$\text{Maximize } Z = -17,500 + 2.49x_1 - 0.07x_2 + 0.19x_3 - 0.25x_4$$

Subject to:

$$x_1 + 0.5x_2 \leq 7500 \quad [\text{Raw Material X Constraint}]$$

$$2x_1 + 0.5x_3 \leq 9000 \quad [\text{Raw Material Y Constraint}]$$

$$x_1 - 0.5x_2 - 0.5x_3 - x_4 = 0 \quad [\text{Liquid Waste Disposal Constraint}]$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 0$$

Variable	Product/Treatment
x_1	Primary Product Production Level
x_2	Secondary Product K Production Level
x_3	Secondary Product M Production Level
x_4	Special Treatment of Liquid Waste

For this model, we assume that the monthly administrative and refinery costs are going to remain the same for this upcoming period. Furthermore, the model requires that the product and treatments production levels are nonnegative since it would not make sense to produce a negative amount of something. We thought of putting in an assumption that the production levels had to be integer values, but we decided to leave that requirement since there was no notice that the company couldn't produce and sell a fractional amount of a product.

The constraints for the model include constraints on the amount of raw material x and raw material y that the company uses. The company cannot use more raw materials than the company has access to. Therefore, the first two constraints consider the amount of raw material x and y that the company can use. The coefficients on the variables in these constraints are the number of pounds that the raw material that is needed to produce one pound of product. For example, in the raw material x constraint, 1 pound of raw material x is needed to produce one pound of primary product and 0.5 pounds of primary product are needed to produce 1 pound of primary product k. The coefficients for the rest of the constraints are set up like this.

The final constraint is the Liquid Waste Disposal Constraint. This constraint requires that all the liquid waste be properly disposed of. Therefore, the constraint is equal to 0 since all the liquid waste produced must be

used up by the production of a combination of the secondary products or the special treatment. Since the liquid waste is a byproduct of the primary product on a 1 to 1 ration, the amount of liquid waste used in the secondary products and special treatment is subtracted from the amount of primary product produced. This is why the coefficient for the primary product production level variable is 1, with the rest of the variable coefficients are negative values that correspond to the number of pounds of liquid waste needed to produce one pound of the secondary product and special treatment.

Solution

To solve this problem, we used Excel and the Solver tool. We created a worksheet named 'Model Worksheet' that broke down the problem in terms of the known variables, calculated variables, decision variables, production level overview, constraints, and outputs. The known variables table breakdown of the variables that were given ranging to the costs, selling prices, and available resources. The calculated variables table include the production cost for the products/treatments and the calculated unit profits. These variables were calculated based on the data that was given. The decision variables table include the variables in the linear programming problem: the production levels for the three products and the special treatment. These variables are changed in hopes of finding the highest profit. The constraints table is simply the constraints that the model is confined to the raw material use and liquid waste disposal. Since the model is constrained to these values, the production level overview provides a detail breakdown on how the raw materials and liquid wastes are being used in the various production levels. Finally, the output table provides a breakdown of the revenue, costs, and profit based on the production level.

In using the solver tool, the objective was set to finding the maximum value of the total profit by changing the values in the decision variable table, but constrained to the raw material x, raw material y, and liquid waste disposal constraints. The production levels that the model returned were 4500 pounds of the primary product, 6000 pounds of product k, and 1500 pounds of specially treated liquid waste. In using these production levels, all raw materials are used and all liquid waste is properly disposed of. These production level produces a loss of \$7,062.50.

Sensitivity Analysis

Having used Excel and the Solver Tool, we were able to get a sensitivity report for the model which evaluates how changes in certain parameters would affect the optimal solution. The sensitivity report contains tables pertaining to the objective function (total profit) and constraints (raw materials)

Sensitivity Analysis of Objective Coefficients

The table containing the sensitivity analysis for the objective coefficients is shown below.

Product/Treatment	Final Value	Objective Coefficient	Allowable Increase	Allowable Decrease
Primary Product Production Level	4500	2.491666667	1E+30	0.88333333327755
Secondary Product K Production Level	6000	-0.066666667	0.441666666663878	0.0583333333343035
Secondary Product M Production Level	0	0.185416667	0.220833333331939	1E+30
Special Treatment of Liquid Waste Production Level	1500	-0.25	0.1166666667	0.220833333331939

Looking at the sensitivity analysis table for the objective coefficients, the final value column represents the optimal production level values for each product or treatment given the parameters. These optimal production level values represent the levels at which the company should produce at to get the optimal level of profit. The object coefficients represent the unit profit for each pound produced for each product or treatment. As mentioned earlier the unit profit is the difference between the revenue per pound of product and the costs per pound of product. The important columns within this table are the allowable increase and allowable decrease columns. These columns represent the range of values that the objective coefficients for which the current production level remains optimal assuming that the only change that is made is the single objective coefficient that you are changing. So the values for which the current optimal production level remains optimal is: primary product production level: [1.60833333, inf.); product k: [-0.125, 0.375], product m: (-inf., 0.40625], special liquid waste treatment [-0.47083333, -0.13333333]

Sensitivity Analysis of Raw Materials

The table containing the sensitivity analysis for the raw materials is shown below.

Raw Material	Final Value	Shadow Price	Constraint Right Hand Side	Allowable Increase	Allowable Decrease
Raw Material X	7500	0.116666667	7500	1500	3000
Raw Material Y	9000	1.0625	9000	6000	1500

Looking at the sensitivity analysis table for the raw materials available, the important column is the shadow price. For this problem, the shadow price represents the amount the objective (in this case profit) will increase by increasing the raw materials that are available. For each additional pound of raw material x that is made available, the profit for the upcoming month will increase just over \$0.11. For each additional pound of raw material y that is made available, the profit for the upcoming month is increase by about \$1.06. Interpreting this result, it would be more beneficial to try and increase raw material y than raw material x due to the shadow price having a larger and positive magnitude increase in the profit.

Recommendations

Our model indicates that a production level of 4500 pounds of the primary product, 6000 pounds of product k, and 1500 pounds of specially treated liquid waste be undertaken. As a result of undertaking this production level the company will report a loss of \$7,062.50 for the upcoming month. The Smith-Brown Industrial Chemicals Inc. (SBIC) accountant recommended that the company eliminate the alternative product k since it costs more than they will be able to earn back. The accountant is correct in that the negative profitability of product k since product k has a unit profit of -\$0.07. However, the use of product k allows for the maximum profit given the liquid waste disposal requirement.

With the primary product has the highest unit profit, all available resources are first devoted to the primary product. Doing this produces 4500 pounds of primary product and liquid waste, while using all 9000 pounds of the raw material y and 4500 pounds of raw material x. The next avenue to go down is the disposal of the liquid waste. We cannot use product m since all the raw material y has been used, this leaves the only possibilities as treating the liquid waste or producing product k. Since product k losses less money for the company, the most amount of product

k should be produced, which is 6000 pounds of product k. The 6000 pounds of product k uses the rest of the raw material x supply and leaves 1500 pounds of liquid waste to be specially treated.

The recommendations that we would make for the management at SBIC would be to possibly lower the administrative or refinery fixed costs. The production level returns \$30,450.00 in revenue while only costing \$20,012.50. The reason for the loss of \$7,062.50 is that the monthly total fixed costs of \$17,500.00 causes the company to end up losing money. So, if the company could find a way to lower the fixed costs that they incur monthly, then the company might start seeing some profits.

Reflection

The tools needed for this project included Excel and the Solver tool. Previous classes, namely BUSA 245 and MATH 430, helped us to learn and get comfortable using Excel to build models and to use the solver tool. However, in using these tools you really need to understand the problem and relationships between the variables to build a successful model. With the goal of maximizing profits based on previous monthly data and estimations of parameters, we use what we knew to calculate unit profit values for the different products and relate that to the production level values to retrieve the objective. This probably was the most challenging part, but once we understood how the different variables interacted, we were able to create the production level overview table that displays where the materials are being devoted to and in a way serves as a tool to check out work. In the end, we did have prior experience with these types of problems from previous classes, but these types of problems can cause one to get stumped if one doesn't see the connections properly.