

# Canova Hansen seasonal unit root test. An EViews add-in

Nicolás Ronderos

July 27, 2018

This document explains how to perform the test proposed by [2] using an EViews add-in. The Canova Hansen test can be seen as the KPSS test for seasonal time series. This document is divided in two sections, the first section explains theoretical details of the test and the second section shows examples using the add-in.

## 1 Canova Hansen Test

The test considers the null hypothesis of a stationary time series in the seasonal frequencies. Two equivalent deterministic seasonal representations are used in the test. One with seasonal dummy variables, and another with sine and cosine waves. For the seasonal representation the time series  $y_i$  is observed during  $n$  points in time, therefore if there are  $T$  years of data then  $n = Ts$ .

$$y_i = \mu + x_i' \beta + S_i + e_i \quad (1)$$

Where  $e_i \sim iid(0, \sigma^2)$ ,  $x_i$  is a vector of independent variables, [2] recommends that the first lag of  $y_i$  should be included as an independent variable. And also a deterministic seasonal term  $S_i$  which can be defined as  $d_i' \alpha$  i.e. a vector of  $s$  seasonal dummy variables (if the constant  $\mu$  is excluded) or as  $\sum_{k=1}^{s/2} \delta_k \sin(\frac{2\pi kt}{s}) + \gamma_k \cos(\frac{2\pi kt}{s})$  which include  $s - 1$  terms given that  $\sin(\pi t) = 0$ . Under the null hypothesis the coefficients of the seasonal term must be fixed through time, if they are not a seasonal unit root should be present.

### 1.1 Joint test for seasonal unit roots at all frequencies

The add-in handles the possibility of seasonal unit roots in quarterly or monthly data. To test the null of seasonal stationarity, i.e. no seasonal unit root exists in  $y_i$ , an LM test is proposed.

$$L_f = \frac{1}{n^2} \sum_{i=1}^n \hat{F}_i' (\hat{\Omega}^f)^{-1} \hat{F}_i \quad (2)$$

Where  $\hat{F}_i = \sum_{t=1}^i f_t \hat{e}_t$ , the term  $f_t$  is a matrix of size  $(s-1) \times 1$  and it contains the sinusoidal waves. The term  $\hat{e}_t$  is estimated by the OLS residuals of equation (1). The  $(s-1) \times (s-1)$  matrix  $\hat{\Omega}^f$  is the estimated variance-covariance matrix as follows.

$$\hat{\Omega}^f = \sum_{k=-m}^m w\left(\frac{k}{m}\right) \sum_{i=1}^n f_{i+k} \hat{e}_{i+k} f_i' \hat{e}_i \quad (3)$$

The matrix is estimated in this way to handle autocorrelation and heteroscedasticity in equation (1) as demonstrated by [1]. The function  $w(\cdot)$  is any Kernel function. The add-in can estimate  $\hat{\Omega}^f$  using the Bartlett, Parzen or Tukey-Hanning functions as defined below.

Kernel Functions	
Bartlett	$1 - \frac{ X }{m}$ for $\frac{ X }{m} \leq 1$
Parzen	$1 - \frac{6X^2}{m} + \frac{6 X ^3}{m}$ for $0 \leq  X  \leq 0.5$ and $2(1 - \frac{ X }{m})^3$ for $0.5 \leq  X  \leq 1$
Tukey-Hanning	$\frac{25}{12\pi^2} \frac{x^2}{m} \left( \frac{\sin(6\pi \frac{x}{5})}{\frac{6\pi x}{5}} - \cos(\frac{6\pi x}{5}) \right)$

The bandwidth of the Kernel in the add-in can be estimated automatically or can be fixed by the user in any integer.

## 1.2 Test for individual unit root at a specific seasonal frequency

In this case, the LM statistical can be calculated with the following expression.

$$L_{(\pi j/q)} = \frac{1}{n^2} \sum_{i=1}^n \hat{F}_{ji} (\hat{\Omega}_{jj}^f)^{-1} \hat{F}_{ji} \quad (4)$$

Where  $\hat{F}_{ji}$  is the corresponding cosine and sine waves of frequency  $w$  and  $\hat{\Omega}_{jj}^f$  is the corresponding block matrix for  $w < \pi$  and a scalar in the case that  $w = \pi$ .

## 1.3 Testing for nonconstant seasonal patterns

The representation using seasonal dummy variables can be used to test if the seasonal intercepts are stable across time. An LM statistical to test the stability of the  $a - th$  seasonal intercept  $a = 1, \dots, s$  is calculated as:

$$L_a = \frac{1}{\hat{\Omega}_{aa} n^2} \sum_{t=1}^n \hat{D}_{at} \quad (5)$$

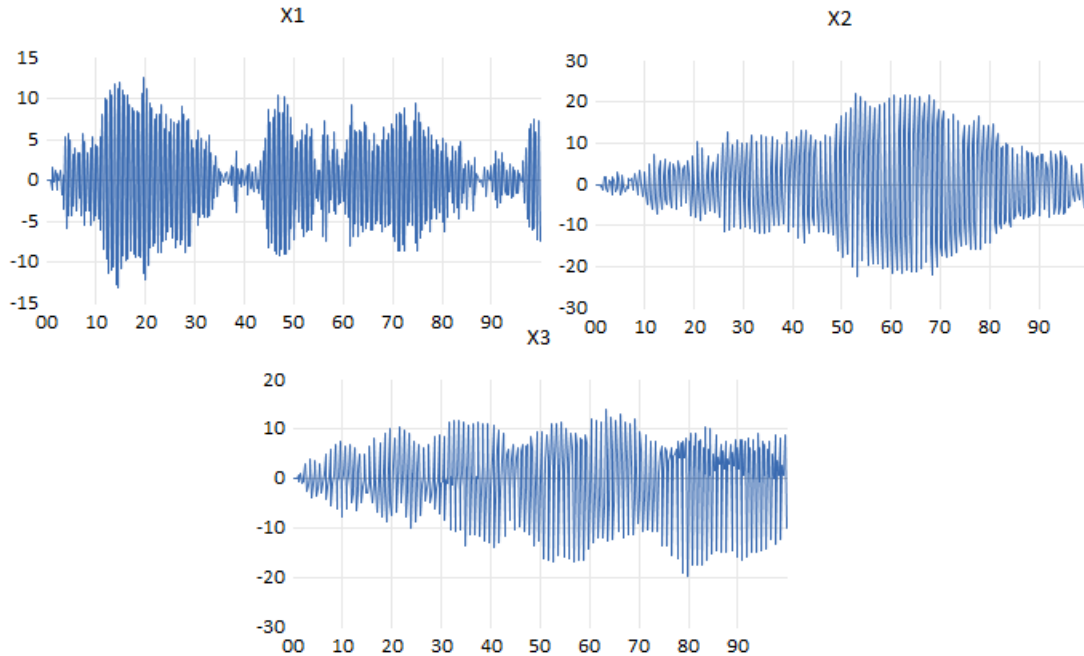
The term  $\hat{D}_{at}$  is the sum of the product between  $\hat{e}_t$  and the  $a - th$  seasonal intercept, i.e. the sum of the residuals of the  $a - th$  season. The scalar  $\hat{\Omega}_{aa}$  corresponds to the element of matrix  $\hat{\Omega}$ .

$$\hat{\Omega} = \sum_{k=-m}^m w\left(\frac{k}{m}\right) \sum_{i=1}^n d_{i+k} e_{i+k} d_i' e_i \quad (6)$$

## 2 Using the add-in

The add-in can be used via point and clicking, or via command line. Some examples are showed using simulated quarterly data, the time series are plotted in figure 1. A workfile with the data will appear after installing the add-in. The time series were simulated as follows with initial values equal to 0. Then random number generator is set with the command `rndseed 1`.

Figure 1: Simulated time series



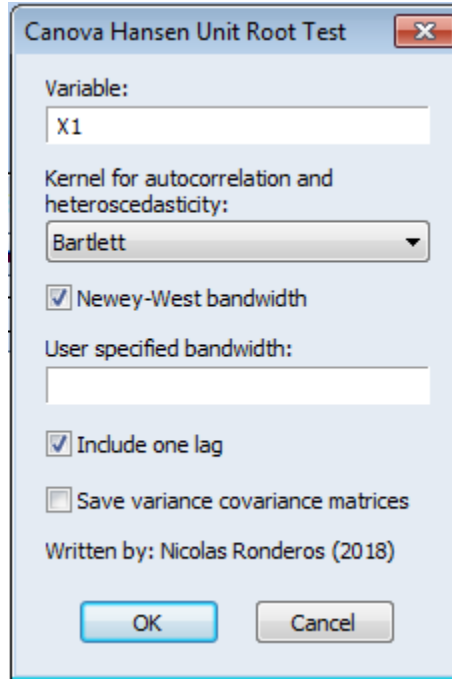
$$(1 + L)x_{1t} = e_t$$

$$(1 + L^2)x_{2t} = e_t$$

$$(1 + L)(1 + L^2)x_{3t} = e_t$$

The time series  $x_{1t}$  poses a seasonal unit root in frequency  $w = \pi$  or with a period of 2 quarters per cycle. The time series  $x_{2t}$  presents a seasonal unit root in frequency  $w = \frac{\pi}{2}$  or with a period of 4 quarters per cycle. And the time series  $x_{3t}$  exhibits the two previous seasonal unit roots. None of these time series present a unit root in the zero frequency, if a zero frequency unit root is present, then the time series must be differenced. After installing the add-in, the user has to click in the time series object proc->Add-ins->Canova Hansen to perform the test, then the window from figure 2 will appear.

Figure 2: Add-in window



To access the add-in via command line, the user has to follow the instructions given in table 1. The command defaults are the same as the ones shown in figure 2, with the exception if including the first lag of the dependent variable.

<i>series_name.canovahansen(options)</i>	
Options	
<i>Bartlett Kernel</i>	<i>Bartlett(Default)</i>
<i>Parzen Kernel</i>	<i>Parzen</i>
<i>Tukey Hanning Kernel</i>	<i>Tukey-Hanning</i>
<i>Include the first lag</i>	<i>il</i>
<i>User specified bandwidth</i>	<i>bandwidth=8 (Default Newey- West)</i>
<i>Save omega estimates</i>	<i>SO</i>

Table 1: Command line

For example, to perform the test on  $x_{1t}$  via command line the user has to use the following command `x1.canovahansen(bartlett)`. Figure 3 presents the results of the test, showing evidence of a seasonal a unit root at frequency  $\pi$ . The option to

include the first lag is ignored because if it is included the residuals of the test will not contain the negative unit root in the series. To apply the test on  $x_{2t}$  the user has to use the following command `x1.canovahansen(bartlett,il)`, also the user can change the kernel function. The results of the test are plotted in figure 4. There is evidence of a seasonal unit root in frequency  $\pi/2$ , and also it can be seen that the seasonal intercepts are not stable. Figure 5 outlines the results of the Canova Hansen test applied to  $x_{3t}$ , showing evidence of the two seasonal unit roots.. Note the joint test will reject the null hypothesis if only one seasonal unit root is present.

Figure 3: Canova Hansen test on  $x_{1t}$

Canova and Hansen (1995)								
H0: X1 is stationary at frequency W								
Frequencies	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
4 quarters per cycle	0.040296	2.0	1.070	0.898	0.749	0.670	0.610	0.469
2 quarters per cycle	1.751350	1.0	0.748	0.593	0.470	0.398	0.353	0.243
Joint test	1.772363	3.0	1.350	1.160	1.010	0.913	0.846	0.679
H0: The i-th seasonal intercept is stable								
Seasonal intercept	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
1	1.588842	1.0	0.748	0.593	0.470	0.398	0.353	0.243
2	1.553790	1.0	0.748	0.593	0.470	0.398	0.353	0.243
3	1.446423	1.0	0.748	0.593	0.470	0.398	0.353	0.243
4	1.545721	1.0	0.748	0.593	0.470	0.398	0.353	0.243

Figure 4: Canova Hansen test on  $x_{2t}$

Canova and Hansen (1995)								
H0: X2 is stationary at frequency W								
Frequencies	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
4 quarters per cycle	5.495808	2.0	1.070	0.898	0.749	0.670	0.610	0.469
2 quarters per cycle	0.009768	1.0	0.748	0.593	0.470	0.398	0.353	0.243
Joint test	5.502558	3.0	1.350	1.160	1.010	0.913	0.846	0.679
H0: The i-th seasonal intercept is stable								
Seasonal intercept	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
1	3.706788	1.0	0.748	0.593	0.470	0.398	0.353	0.243
2	2.214923	1.0	0.748	0.593	0.470	0.398	0.353	0.243
3	3.632463	1.0	0.748	0.593	0.470	0.398	0.353	0.243
4	2.101186	1.0	0.748	0.593	0.470	0.398	0.353	0.243

Figure 5: Canova Hansen test on  $x_{3t}$ 

Canova and Hansen (1995)

H0: X3 is stationary at frequency W

Frequencies	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
4 quarters per cycle	5.327569	2.0	1.070	0.898	0.749	0.670	0.610	0.469
2 quarters per cycle	2.248113	1.0	0.748	0.593	0.470	0.398	0.353	0.243
Joint test	5.945496	3.0	1.350	1.160	1.010	0.913	0.846	0.679

H0: The i-th seasonal intercept is stable

Seasonal intercept	LM Statistical	Critical values (D.F.)	1%	2.5%	5%	7.5%	10%	20%
1	2.626151	1.0	0.748	0.593	0.470	0.398	0.353	0.243
2	1.665502	1.0	0.748	0.593	0.470	0.398	0.353	0.243
3	0.430608	1.0	0.748	0.593	0.470	0.398	0.353	0.243
4	4.208409	1.0	0.748	0.593	0.470	0.398	0.353	0.243

## References

- [1] Donald WK Andrews. Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica: Journal of the Econometric Society*, pages 817–858, 1991.
- [2] Fabio Canova and Bruce E Hansen. Are seasonal patterns constant over time? a test for seasonal stability. *Journal of Business & Economic Statistics*, 13(3):237–252, 1995.