The spherical bessel functions

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When solving the Helmholtz equation in spherical coordinates by separation of variables, the radial equation has the form

$$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \left(x^2 - n(n+1)\right) y = 0. \quad (1)$$

The two linearly independent solutions to this equation are called the spherical Bessel functionsj_n and $y_n n$, and are related to the ordinary Bessel functions J_n and Y_n by

$$j_n(x) = \sqrt{\frac{\pi}{2x}} J_{n+\frac{1}{2}}(x). \quad (2)$$

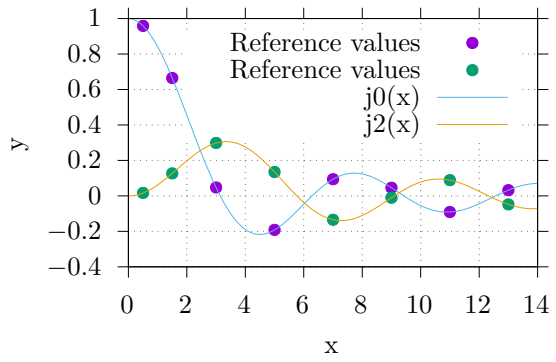

Figure 1: Illustration of the first and third spherical bessel functions.