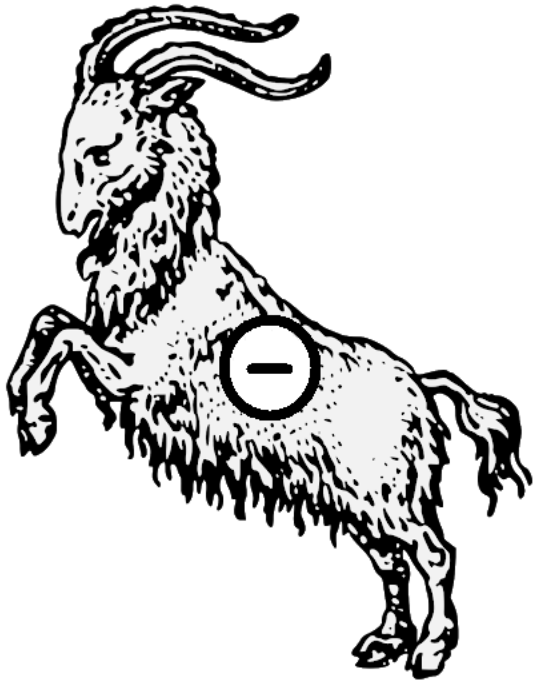

"CHARGE" ON AN ELECTRON

(MILLIKAN OIL DROP EXPERIMENT)

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Introduction:

The property we call "charge" is a basic property of physical bodies that cause attraction or repulsion between themselves and other charged bodies. Since charge is a result of the body having extra or missing electrons it follows that there must be some basic, fundamental unit of charge. This amount of charge, if negative, would be the charge of one electron. Its positive counterpart would be that of a proton.

Purpose:

The purpose of this experiment was to determine the charge of an electron.

Equipment List:

Oil drop apparatus

Atomizer

Microscope

Stopwatch

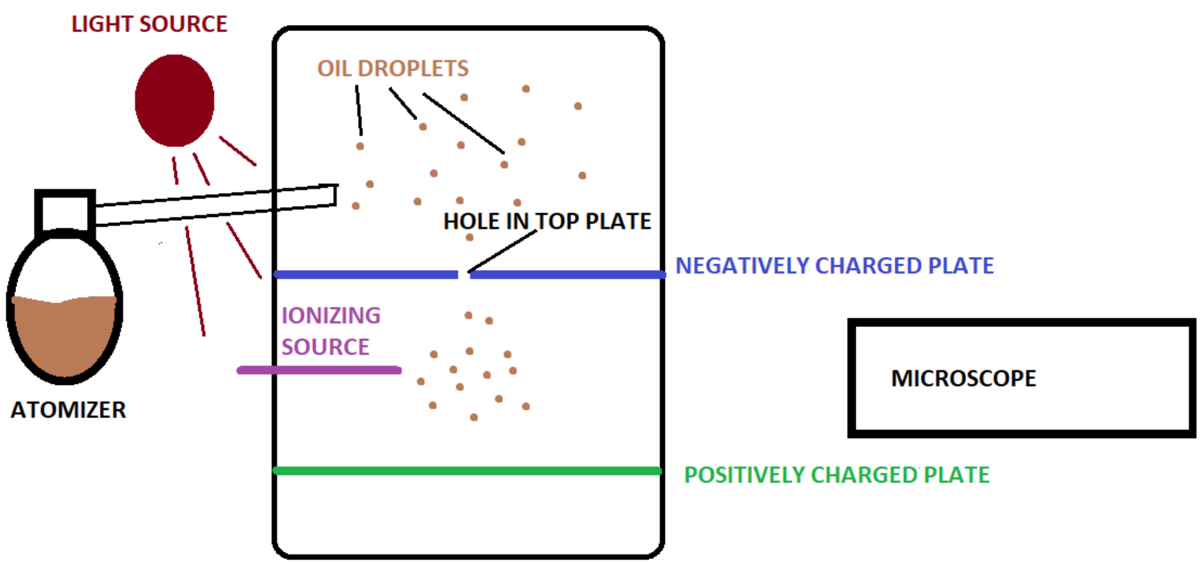
Voltmeter

Procedure:

From the atomizer, oil was aerosolized into the top portion of a cylindrical apparatus containing two oppositely-charged metal plates. An ionizing source was turned on and used to charge the oil particles. The particles drifted through the hole in the top plate and entered the space between the two charged plates. The oil droplets were illuminated with a light source and were viewed through a microscope and manipulated via the charged plates: the current was increased and decreased, thereby affecting the electrical field around charged droplets and causing them to move up or down in the viewing field. The polarities of the plates could be reversed, causing the charged droplets to move in the opposite direction. The droplets which were not charged at all did not react to these stimuli and fell to the bottom of the chamber.

A charged droplet which was found to be responsive to the charged plates by changing directions in response to reversing the polarity of the plates was isolated. Once that drop was captured, it was measured in "free-fall[1]", again in upward motion (when the polarity was one way), and in a downward motion (when polarity was reversed). By timing the drop in all three movements and measuring the distance travelled, it was possible to measure the force of the charge that was acting on the drops. By measuring the Voltage on the plates, it was possible to determine the charge of the drops.

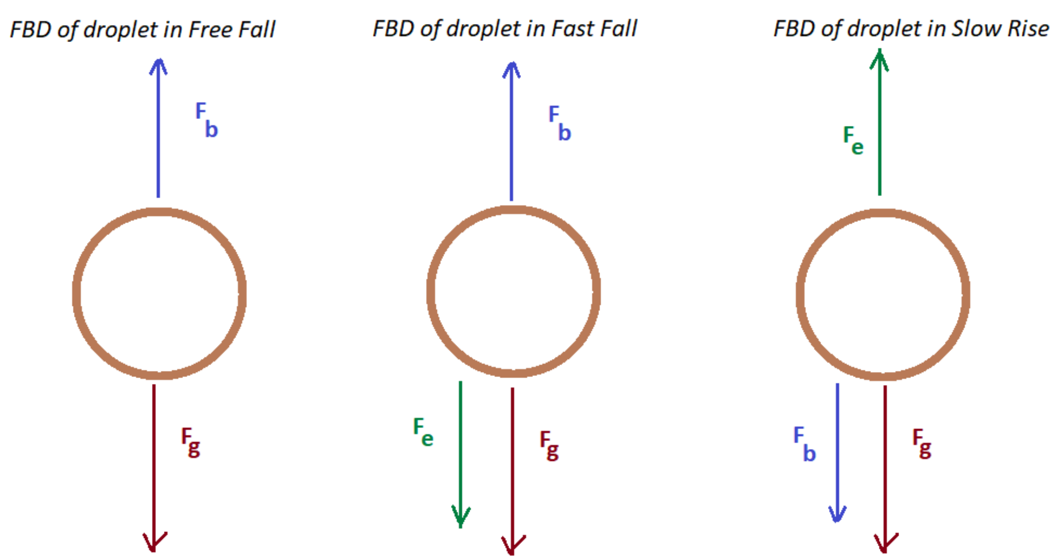
Diagram 1: Procedure Diagram



The atomizer was used to aerosol oil particles, as shown, which were viewed through the microscope.

[1] The term *"free-fall"* as used in this experiment does not refer to the term as defined in physics (an object under the constant acceleration of the gravitational force). The term "free fall" is used in this experiment to refer to an oil drop falling without any downward force other than the gravitational force. The actual physical term for this is *terminal velocity* which describes a state, such as that of the oil drop, in which the sum of the gravitational force, air resistance and buoyant force is zero. For this reason, the phrase "free-fall" is in quotation marks.

Diagram 2: Free-Body Diagram (FBD) of forces acting on oil droplets



In this diagram, $\mathbf{F_g}$ ≡ ***force of gravity,*** $\mathbf{F_b}$ ≡ ***buoyant force,*** **and** $\mathbf{F_e}$ ≡ ***applied force of electrical field.***

Data:

Data were collected by observing droplets moving downward and back up under the influence of a current; "Fast Fall" and "Slow Rise", respectively. "Free fall" data were collected by observing oil droplets fall without any added interference. Five droplets were observed; each one was used to collect seven data points for each "Fast Fall", "Slow Rise" and "Free Fall". The time it took for each droplet to travel 2.9 mm was recorded in seconds. The measured times for each of these movements for all five drops are in the following table:

Table 1 – Measured Times (in seconds)

Fast Fall	*Drop 1*	*Drop 2*	*Drop 3*	*Drop 4*	*Drop 5*
1	12.46	8.58	2.35	6.68	4.43
2	12.47	8.50	2.34	6.76	4.37
3	12.42	8.54	2.34	6.74	4.35
4	12.44	8.53	2.35	6.80	4.34
5	12.45	8.50	2.34	6.76	4.38
6	12.40	8.53	2.33	6.72	4.38
7	12.44	8.59	2.34	6.79	4.32
Slow Rise					
1	17.20	10.62	2.48	17.12	43.70
2	17.15	10.55	2.48	17.31	34.70
3	17.22	10.54	2.46	17.12	34.67
4	17.20	10.51	2.48	17.17	39.02
5	17.20	10.52	2.46	17.03	35.29
6	17.17	10.60	2.48	17.12	34.70
7	17.22	10.52	2.47	17.17	34.95
Free Fall					
1	89.59	89.08	89.68	22.34	9.95
2	89.65	89.64	90.05	22.41	10.00
3	89.62	89.15	89.54	22.48	10.03
4	89.48	89.29	89.45	22.37	9.97
5	89.61	89.32	90.03	22.07	10.03
6	89.62	89.32	89.62	22.33	10.02
7	89.72	89.58	89.64	22.47	10.03

Calculations:

I **The mean times and their standard deviations**

a. The mean time

For all five drops, each type of movement ("free-fall", fast-fall, and slow-rise) was measured seven times. The "average time" used in the chart is the statistical mean which is the following formula:

$t_{avg} = \frac{t_1+t_2+\cdots+t_n}{n}$ where *n* is defined as the number of trials.

For drop #3 the measured time for "free fall" was:

$$\frac{89.68\ s+90.05\ s+89.54\ s+89.45\ s+90.03\ s+89.62\ s+89.64\ s}{7} = 89.72\ s$$

b. The standard deviation (σ)

$\sigma = \sqrt{\frac{(t_1-t_{avg})^2+(t_2-t_{avg})^2\ldots\ldots+(t_n-t_{avg})^2}{n}}$ where n is the number of trials.

For drop #3 in the "free-fall":

$$\sqrt{\frac{(89.68\ s-89.72\ s)^2+(90.05\ s-89.72\ s)^2+(89.54\ s-89.72\ s)^2+(89.45\ s-89.72\ s)^2+(90.03\ s-89.72\ s)^2+(89.62\ s-89.72\ s)^2+(89.64\ s-89.72\ s)^2}{7}} = 0.217\ s$$

II **The charge on each drop**

The charge on each drop was measured using the following formula:

$Charge\ (in\ Coloumbs) = K\frac{(v_{ff}+v_{sr})\sqrt{v_g}}{2\ V}$ where K is a constant value (in this experiment K=$1.3655 \times 10^{-10}\frac{CV}{(\frac{m}{s})^{\frac{3}{2}}}$)

Because $velocity\ (in\ \frac{m}{s}) = \frac{distance\ (in\ meters)}{time\ (in\ seconds)}$ and the distance travelled in each of the trials of this experiment is the same quantity (*d*), the equation can be written in terms of *d* and *t.*

$$K\frac{(v_{ff}+v_{sr})\sqrt{v_g}}{2\ V} = K\frac{\left(\frac{d}{t_{ff}}+\frac{d}{t_{sr}}\right)\sqrt{\frac{d}{t_g}}}{2V}$$

$$q = K\frac{\left(\frac{d\,t_{sr} + dt_{ff}}{t_{ff}t_{sr}}\right)\sqrt{\frac{d}{t_g}}}{2V}$$

$$q = K\frac{d\left(\frac{t_{sr} + t_{ff}}{t_{ff}t_{sr}}\right)\frac{\sqrt{d}}{\sqrt{t_g}}}{2V}$$

$$q = K\frac{d^{\frac{3}{2}}(t_{sr} + t_{ff})}{2V\ t_{ff}t_{sr}t_g^{\frac{1}{2}}}$$

For drop #3, the measured "free-fall" time was 89.72 s, the measured slow-rise time was 2.47 s and the measured fast-fall time was 2.34 s. The Voltage used was 491.3 V. The distance travelled was 2.9 mm as in every trial.

Given $t_{ff} = 2.34\ s$; $t_{sr} = 2.47\ s$; $t_g = 89.72\ s$; $V = 491.3\ \mathrm{V}\ and$ and $\mathrm{K} = 1.3655\ \times\ 10^{-10}\frac{C\mathrm{V}}{(\frac{m}{s})^{\frac{3}{2}}}$

$$q = 1.3655\ \times 10^{-10}\frac{C\mathrm{V}}{(\frac{m}{s})^{\frac{3}{2}}}\cdot\frac{(2.9\ \times\ 10^{-3}\ m)^{\frac{3}{2}}\,(2.47\ s + 2.34\ s)}{2\cdot 491.3\ \mathrm{V}\ \cdot\ 2.34\ s\ \cdot 2.47\ s\ \cdot\ \sqrt{89.72\ s}}$$

$q = 1.907 \times 10^{-18}\ C$

III The margin of uncertainty

Each measured value has its own margin of uncertainty. For the measured times, the margin of uncertainty is the standard deviation of their means. The distance (2.9 mm) has the margin of ±.05 mm. The measured Voltage has a margin of ±10 V. To calculate how this potential margin of error affects the calculated charge, the total margin of error was calculated. The total margin of error is the square root of the sum of the squares of each of these amounts multiplied by the partial derivative of the formula differentiated for that particular value. In other words:

$$uq = \sqrt{\left(\frac{\partial}{\partial t_{ff}} K \frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ut_{ff}\right)^2 + \left(\frac{\partial}{\partial t_{sr}} K \frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ut_{sr}\right)^2 + \left(\frac{\partial}{\partial t_g} K \frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ut_g\right)^2 + \left(\frac{\partial}{\partial d} K \frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ut_{ff}\right)^2 + \left(\frac{\partial}{\partial V} K \frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ut_{ff}\right)^2}$$

$$uq = K\sqrt{\left(-\frac{d^{\frac{3}{2}}}{2\mathrm{V} t_{ff}^2 t_g^{\frac{1}{2}}} ut_{ff}\right)^2 + \left(-\frac{d^{\frac{3}{2}}}{2\mathrm{V} t_{sr}^2 t_g^{\frac{1}{2}}} ut_{sr}\right)^2 + \left(-\frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{4V\, t_{ff} t_{sr} t_g^{\frac{3}{2}}} ut_g\right)^2 + \left(\frac{3d^{\frac{1}{2}}(t_{sr}+t_{ff})}{4V\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} ud\right)^2 + \left(-\frac{d^{\frac{3}{2}}(t_{sr}+t_{ff})}{2\mathrm{V}^2\, t_{ff} t_{sr} t_g^{\frac{1}{2}}} u\mathrm{V}\right)^2}$$

For drop #3, the measured values are:

d = 2.9 mm (2.9×10^{-3} m) t_g = 89.72 s t_{ff} = 2.34 s t_{sr} = 2.47 s

The uncertainty values are:

ud = ±0.05 mm (5.0×10^{-5} m) ut_g = ±0.217 s ut_{sr} = ±0.009 s ut_{ff} = ±0.006 s uV = ±10 V

For these values, the equation above yields the value $uq = 6.294 \times 10^{-20}$ *C*. Given the calculated charge of $q = 1.907 \times 10^{-18}$ *C*, the calculated uncertainty means that the drop's charge can be given as $1.907 \pm 0.063 \times 10^{-18}$ *C.*

IV **Deriving the fundamental charge**

To determine the fundamental charge, it is necessary to find a number which, when multiplied by some set of integers, will yield the values obtained for the charge of the drops. To do this, the method used was to experiment with every possible value to determine if some integer set would multiply by it to yield the values calculated for the drops. For every possible value between 1.0×10^{-19} *C* and 2.5×10^{-19} *C* the proposed number of electrons was calculated (by dividing calculated q by said value). The nearest integer to that calculated value was determined (N_i). For each drop the difference was calculated between the nearest integer (N_i) and the calculated (q/Z). Finally, the sum of all five differences was calculated for all five drops for each proposed value of *Z.*

In other words:

N_i = int (q_i/Z)

$\varepsilon_i = \left|N_1 - \frac{q_1}{Z}\right|$

$\Sigma = \varepsilon_1 + \varepsilon_2 + \ldots.. + \varepsilon_5$

For the proposed charge value of 1.60×10^{-19} *C* using the calculated values for $q_1 - q_5$ (and factoring out the '$\times 10^{-19}$'):

$$\Sigma = \left|int\left(\frac{3.177}{1.60}\right) - \left(\frac{3.177}{1.60}\right)\right| + \left|int\left(\frac{15.90}{1.60}\right) - \left(\frac{15.90}{1.60}\right)\right| + \left|int\left(\frac{19.10}{1.60}\right) - \left(\frac{19.10}{1.60}\right)\right| + \left|int\left(\frac{22.30}{1.60}\right) - \left(\frac{22.30}{1.60}\right)\right| + \left|int\left(\frac{17.40}{1.60}\right) - \left(\frac{17.40}{1.60}\right)\right|$$

$\Sigma = 0.327$

Results:

Table 2 – Calculated Values

Avg Times					
free-fall	89.62	89.38	89.72	22.36	10.01
fast fall	12.44	8.532	2.340	6.762	4.357
slow rise	17.19	10.54	2.472	17.15	35.56
Std Devs					
σ free-fall	0.07815	0.1875	0.2556	0.1510	0.02422
σ fast-fall	0.02422	0.03312	0.006325	0.02994	0.02422
σ slow-rise	0.02805	0.03286	0.009832	0.09223	1.714
Voltage (in V)	491.1	150	491.3	208.4	499
q ($\times 10^{-19}$ C)	**3.177**	**15.90**	**19.10**	**22.30**	**17.40**
uq ($\times 10^{-19}$ C)	**±0.105**	**±1.14**	**±0.630**	**±1.22**	**±0.583**
Minimum value-q ($\times 10^{-19}$ C)	3.072	14.80	18.40	21.10	16.80
Maximum value-q ($\times 10^{-19}$ C)	3.282	17.10	19.70	23.50	18.00

The charges (q) and uncertainties (uq) were calculated as explained in “Calculations” parts II and III. To determine the charge of the electron the following table (Tables 3a and 3b) were constructed using the method explained in “Calculations” part IV.

Table 3a – Proposed Values for q_e $(Z \times 10^{-19}\,C)$

Z (C)	Drop 1 q/Z	N_1	Drop 2 q/Z	N_2	Drop 3 q/Z	N_3	Drop 4 q/Z	N_4	Drop 5 q/Z	N_5	ε_1	ε_2	ε_3	ε_4	ε_5	Σ
1.000	3.177	3	15.900	16	19.100	19	22.300	22	17.400	17	0.177	0.100	0.100	0.300	0.400	1.077
1.100	2.888	3.	14.455	14	17.364	17	20.273	20	15.818	16	0.112	0.455	0.364	0.273	0.182	1.385
1.200	2.648	3	13.250	13	15.917	16	18.583	19	14.500	15	0.353	0.250	0.083	0.417	0.500	1.603
1.300	2.444	2	12.231	12	14.692	15	17.154	17	13.385	13	0.444	0.231	0.308	0.154	0.385	1.521
1.400	2.269	2	11.357	11	13.643	14	15.929	16	12.429	12	0.269	0.357	0.357	0.071	0.429	1.484
1.500	2.118	2	10.600	11	12.733	13	14.867	15	11.600	12	0.118	0.400	0.267	0.133	0.400	1.318
1.600	1.986	2	9.938	10	11.938	12	13.938	14	10.875	11	0.014	0.063	0.063	0.063	0.125	0.327
1.700	1.869	2	9.353	9	11.235	11	13.118	13	10.235	10	0.131	0.353	0.235	0.118	0.235	1.072
1.800	1.765	2	8.833	9	10.611	11	12.389	12	9.667	10	0.235	0.167	0.389	0.389	0.333	1.513
1.900	1.672	2	8.368	8	10.053	10	11.737	12	9.158	9	0.328	0.368	0.053	0.263	0.158	1.170
2.000	1.589	2	7.950	8	9.550	10	11.150	11	8.700	9	0.412	0.050	0.450	0.150	0.300	1.362
2.100	1.513	2	7.571	8	9.095	9	10.619	11	8.286	8	0.487	0.429	0.095	0.381	0.286	1.678
2.200	1.444	1	7.227	7	8.682	9	10.136	10	7.909	8	0.444	0.227	0.318	0.136	0.091	1.217
2.300	1.381	1	6.913	7	8.304	8	9.696	10	7.565	8	0.381	0.087	0.304	0.304	0.435	1.512
2.400	1.324	1	6.625	7	7.958	8	9.292	9	7.250	7	0.324	0.375	0.042	0.292	0.250	1.282
2.500	1.271	1	6.360	6	7.640	8	8.920	9	6.960	7	0.271	0.360	0.360	0.080	0.040	1.111

Clearly, the number 0.327 ($\times\, 10^{-19}\,C$) is the minimum value for Σ meaning that 1.60 $\times\, 10^{-19}\,C$ yields values that are the closest fit to the calculated charges for all of the drops. To make the calculation somewhat more precise, the following is the same table with values for $Z = (1.60 \pm .08) \times 10^{-19}\,C$ at increments of $.001 \times 10^{-19}$.

Table 3b

$Z \times 10^{-19}\,C$	q_1/Z	N_1	Q_2/Z	N_2	q_3/Z	N_3	q_4/Z	N_4	q_5/Z	N_5	ε_1	ε_2	ε_3	ε_4	ε_5	Σ
1.520	2.090	2	10.461	10	12.566	13	14.671	15	11.447	11	0.090	0.461	0.434	0.329	0.447	1.761
1.530	2.076	2	10.392	10	12.484	12	14.575	15	11.373	11	0.076	0.392	0.484	0.425	0.373	1.750
1.540	2.063	2	10.325	10	12.403	12	14.481	14	11.299	11	0.063	0.325	0.403	0.481	0.299	1.569
1.550	2.050	2	10.258	10	12.323	12	14.387	14	11.226	11	0.050	0.258	0.323	0.387	0.226	1.243
1.560	2.037	2	10.192	10	12.244	12	14.295	14	11.154	11	0.037	0.192	0.244	0.295	0.154	0.921
1.570	2.024	2	10.127	10	12.166	12	14.204	14	11.083	11	0.024	0.127	0.166	0.204	0.083	0.603
1.580	2.011	2	10.063	10	12.089	12	14.114	14	11.013	11	0.011	0.063	0.089	0.114	0.013	0.289
1.590	1.998	2	10.000	10	12.013	12	14.025	14	10.943	11	0.002	0.000	0.013	0.025	0.057	0.096
1.600	1.986	2	9.938	10	11.938	12	13.938	14	10.875	11	0.014	0.063	0.063	0.063	0.125	0.327
1.610	1.973	2	9.876	10	11.863	12	13.851	14	10.807	11	0.027	0.124	0.137	0.149	0.193	0.629
1.620	1.961	2	9.815	10	11.790	12	13.765	14	10.741	11	0.039	0.185	0.210	0.235	0.259	0.928
1.630	1.949	2	9.755	10	11.718	12	13.681	14	10.675	11	0.051	0.245	0.282	0.319	0.325	1.223
1.640	1.937	2	9.695	10	11.646	12	13.598	14	10.610	11	0.063	0.305	0.354	0.402	0.390	1.514
1.650	1.925	2	9.636	10	11.576	12	13.515	14	10.545	11	0.075	0.364	0.424	0.485	0.455	1.802
1.660	1.914	2	9.578	10	11.506	12	13.434	13	10.482	10	0.086	0.422	0.494	0.434	0.482	1.917
1.670	1.902	2	9.521	10	11.437	11	13.353	13	10.419	10	0.098	0.479	0.437	0.353	0.419	1.786
1.680	1.891	2	9.464	9	11.369	11	13.274	13	10.357	10	0.109	0.464	0.369	0.274	0.357	1.573

From the table above, the minimum value for Σ is 0.096 (× 10^{-19} C) which is the value yielded by the proposed value of $Z = 1.590 \times 10^{-19}$ C.

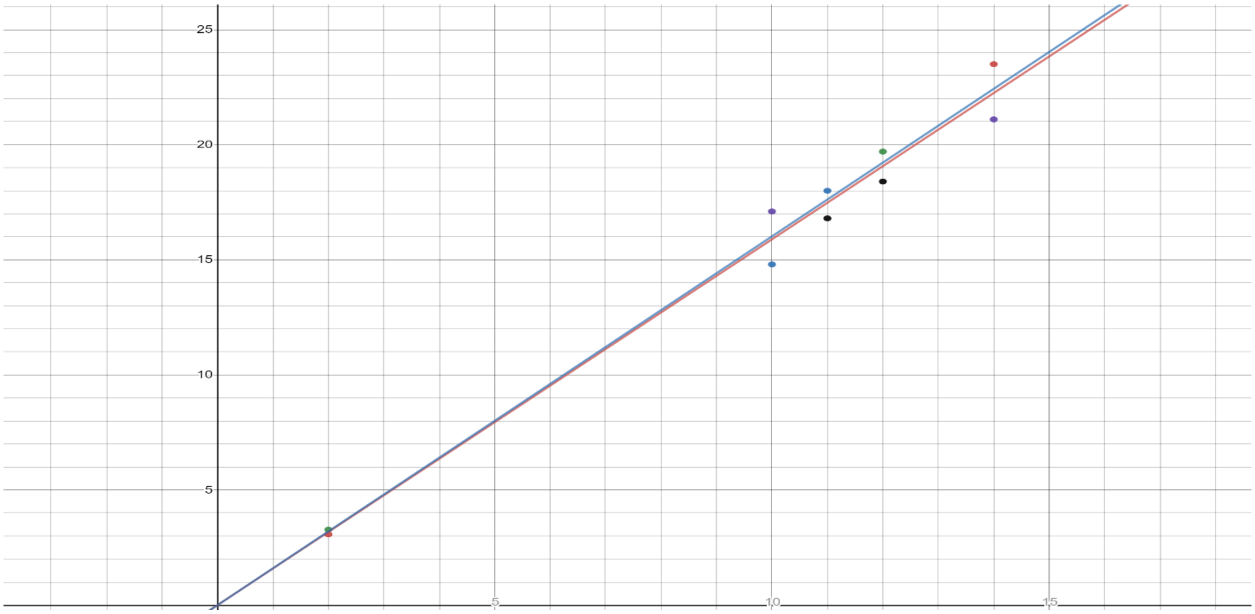
This value - 1.590 × 10^{-19} C – can be multiplied by a set of integers to yield the calculated charges on all five of the drops.

	Drop 1	Drop 2	Drop 3	Drop 4	Drop 5
Calculated charge (× 10^{-19} C)	3.177±.105	15.90±1.14	19.10±.630	22.3±1.22	17.40±.583
Number of electrons *e*	2	10	12	14	11
***e* × 1.59**	3.177	15.90	19.08	22.26	17.49

Each of the values are clearly within the minimum and maximum values which are listed on Table 2.

In the graph below, the red line represents the value calculated in this experiment (1.59 × 10^{-19} C) and the blue line represents Millikan's number. For each drop, the calculated maximum and minimum were put on the graph as points. As the graph shows, both lines go between the maximum and minimum values for each of the drops.

Graph 1



Charge vs. number of electrons at calculated value of q_e.

Conclusion:

The purpose of this experiment was to determine the charge of an electron. The charge was calculated to be 1.590 × 10^{-19} *C*. This value fits entirely within the margin of uncertainty. The range of values for *e* (the charge on an electron) based on the results of this experiment, can be stated to be in the range of [1.535, 1.636] × 10^{-19} *C.* Therefore, any value in the above range can be said to agree with the results of the experiment.

When Roger Millikan performed this experiment, the value he calculated was 1.602 × 10^{-19} *C.* This value is also within the range of values calculated in this experiment as shown in the graph above.

The sources of error in this experiment (as mentioned briefly in Calculations) were in the following: time, distance and Voltage. The operation of the stopwatch was based upon subjectively judging when the droplet passed the marks on the viewing field for 2.9mm. Even in the event of judging this perfectly, the accuracy of the subsequent interactions with the stopwatch were dependent on the user's hand-eye coordination, which is delayed to some degree. The distance was measured to the tenth of a millimeter, which leaves a margin of uncertainty of ±.05 mm. The Voltmeter must be assumed to be accurate to ±10 V.

Perhaps the very slight difference between our calculation and that accepted by science came about because of the delay on the part of the experimenters before "hitting the stopwatch". In other words, the slight "lag time" for the humans conducting the experiment to detect the drop reaching the end point and to stop the timing device might have slightly skewed the times longer, resulting in a slightly lesser value for q.

This experiment allowed the field of physics to "charge" into the future.

APPENDIX:

The team conducting this experiment made an effort to collect the data using real equipment as described in ***Procedure*** and ***Equipment.*** However, they were unable to isolate and keep track of the oil drops long enough to properly gather significant and useful data for the purpose of this experiment. For this reason, the actual experiment was conducted using a software model *Millisim* which was designed to model the real-life phenomena which this experiment is to observe and measure.

Millisim is an awesome software program ©Dave Stoddard which simulates the Millikan Oil Drop Experiment.