Ijsselmeer Lake Problem

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Executive Summary

- If the Ijsselmeer is kept at its current volume of 15 km^3, then it will be fresh water in approximately 5.9 years.
- If the Ijsselmeer decreases in volume at a constant rate over the next 28 years to 9.5 km^3, then it will be fresh water in approximately 6.3 years.

Introduction

As the Netherlands moves to dam off the *Zuiderzee* from the *Noordzee*, there are concerns from local fisherman, who make their living off *Zuiderzee* [2], as to the ambitious projects effect on the fish population. The main concern is that once the *Zuiderzee* is cut off from the open ocean and is only fed by freshwater rivers, such as the Ijseel, it will eventually become a freshwater body. It is our job to explore just how long this process will take in two scenarios. The first assuming no change in the lake volume and the second assuming some land reclamation.

Assumptions

We can start by exploring the base assumptions that were made in order to obtain a relatively simple, analytic solution to our problem. Our first assumption that we make is that since the River Ijssel is freshwater we can ignore what ever salt and mineral content it might have. Next we know that, along with the River Ijssel, there are a number of smaller tributaries that feed into the Ijsselmeer not to mention the chance of rainwater adding to it, but since the River Ijssel is by far the largest, we assume all other sources of water are negligible. Then we can assume that the *Afsluitdijk* will control the outflow to approximately match the inflow from the River Ijssel because otherwise the Ijsselmeer would either flood or start to drain. Lastly we need the volume of the Ijsselmeer in order to tackle this problem but the measurement given to us is the surface area. To get our volume we will assume that the depth is approximately equal to the average depth of the Ijsselmeer which is about 5 meters. [1]

In summary:

- The River Ijssel has a negligible salt content.
- The water inflow from all other sources are negligible.
- The *Afsluitdijk* makes inflow and outflow equal.
- The depth is approximately equal to the average depth of 5 meters.

Variables and Values

Here we lay out all of our variables and values used for both the constant and differential volume problems.

- $x(t)$ - the amount of salt in km^3 at time t
- t - the time in years
- x_0 - the amount of salt in km^3 at time 0
- I - the outflow of salt water in $\frac{km^3}{year}$
- V - the total volume of the lake in km^3
- V_0 - the initial volume of the lake in m^3
- m - the rate of change of volume of the lake

Salinity With Constant Volume

For this part of the problem we use all the base assumptions outlined earlier along with the additional assumption that the volume of the Ijsselmeer will remain constant for this entire period.

In order to get our differential equation, we start with the idea that the change in the amount of salt Δx over time Δt is equal to the difference between the amount going in and the amount going out.

$$\Delta x = in - out \tag{1}$$

Since the River Ijssel is the only intake we need to consider and we know that it's fresh water, we can say the the inflow of salt is zero. For the outflow of salt we will take the concentration of salt, multiply it by the outflow of the saltwater, and then multiply it by the change in time. We'll see the that dimensional analysis of this term breaks down to volume of salt.

$$\Delta x = 0 - I(\frac{x}{V})\Delta t \tag{2}$$

$$\Delta x = -\frac{Ix}{V}\Delta t \tag{3}$$

We can now move Δt to the other side of the equation and evaluate it as $\Delta t \to 0$ in order to evaluate the changing salt values over a series of infinitesimal time scales which gives us a differential equation.

$$\frac{dx}{dt} = -\frac{Ix}{V} \tag{4}$$

Our equation is first order, linear, and autonomous so it is separable.

$$\int \frac{dx}{x} = \int -\frac{I}{V} dt \tag{5}$$

$$\ln(x) = -\frac{I}{V}t + C \tag{6}$$

$$x(t) = Ce^{-\frac{I}{V}t} \tag{7}$$

We now have an Initial Value Problem with the amount of salt at the initial time $x(0)$ is equal to x_0. We will substitute in the condition, $x(0) = x_0$ to find the arbitrary constant C.

$$x_0 = Ce^{-\frac{I}{V}(0)} \tag{8}$$

$$x_0 = C \tag{9}$$

Giving us our function:

$$x(t) = x_0 e^{-\frac{I}{V}t} \tag{10}$$

Finally we can solve for t.

$$t = -\frac{V \ln\left(\frac{x}{x_0}\right)}{I} \tag{11}$$

Salinity With Differential Volume

Now for this part of our problem we no longer assume a constant volume and instead assume that over the next 28 years there is a constant decrease in volume as the land is reclaimed with polders. For differential volume we can start again at the change in salt Δx with the added consideration of changing volume when we calculate the concentration.

$$\Delta x = 0 - I(\frac{x}{V_0 + mt})\Delta t \tag{12}$$

$$\Delta x = -\frac{Ix}{V_0 + mt}\Delta t \tag{13}$$

Again we'll move Δt to the other side of the equation and evaluate it as $\Delta t \to 0$ in order to get our differential equation.

$$\frac{dx}{dt} = -\frac{Ix}{V_0 + mt} \tag{14}$$

Though this time it is not autonomous, it is still separable.

$$\int \frac{dx}{x} = \int -\frac{I}{V_0 + mt} dt \tag{15}$$

$$\ln(x) = -\frac{I}{m} \ln{(V_0 + mt)} + C \tag{16}$$

$$x(t) = Ce^{-\frac{I}{m} \ln{(V_0 + mt)}} \tag{17}$$

And solve for C using the initial condition $x(0) = x_0$.

$$x_0 = Ce^{-\frac{I}{m} \ln{(V_0 + m(0))}} \tag{18}$$

$$\frac{x_0}{e^{-\frac{I}{m} \ln{(V_0)}}} = C \tag{19}$$

Giving our function:

$$x(t) = \frac{x_0}{e^{-\frac{I}{m} \ln{(V_0)}}} e^{-\frac{I}{m} \ln{(V_0 + mt)}} \tag{20}$$

We can then solve for t.

$$t = \frac{e^{-\frac{m \ln{(\frac{x}{x_0})} - I \ln V_0}{I}} - V_0}{m} \tag{21}$$

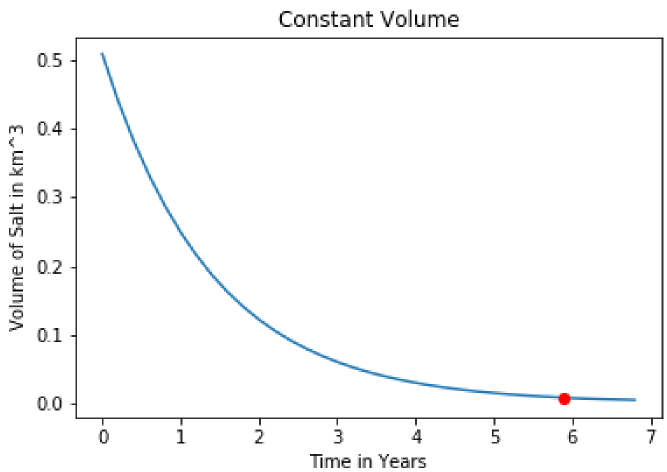
Results

Constant Volume

Finally to find out how long it will take for the Ijsselmeer to become freshwater (salinity $< 0.05\%$) with constant volume we'll put the following values into equation 11:

- $V = (3000km^2)(0.005km) = 15km^3$
- $x = (15km^3)(0.0005) = 0.0075km^3$
- $x_0 = (15km^3)(0.034) = 0.51km^3$
- $I = 10.72\frac{km^3}{Year}$

Giving us approximately 5.9 years before the Ijsselmeer is freshwater.

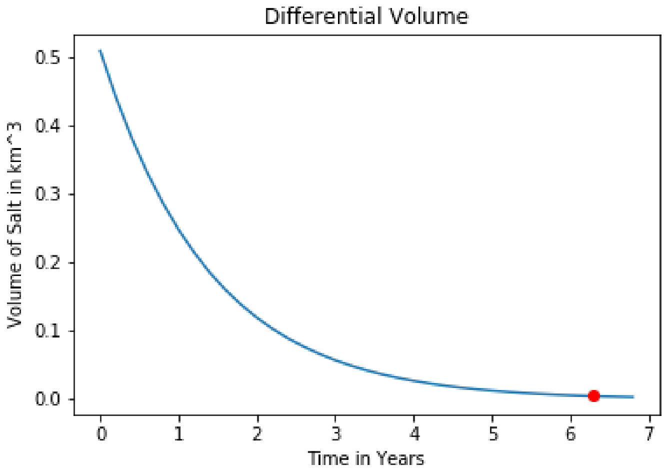


Differential Volume

We can do the same thing for differential volume by putting in the following values to Equation 21:

- $V_0 = (3000km^2)(0.005km) = 15km^3$
- $x = (9.5km^3)(0.0005) = 0.00475km^3$
- $x_0 = (15km^3)(0.034) = 0.51km^3$
- $I = 10.72\frac{km^3}{Year}$
- $m = \frac{15km^3 - 9.5km^3}{0Y - 28Y} = -0.196\frac{km^3}{Y}$

Giving us approximately 6.3 years before the Ijsselmeer is freshwater.



Conclusion

In a scenario with no land reclamation the desalination of the Ijseelmeer will take 5.9 years while reclaiming land during the process increases that time to 6.3 years. At first glance a faster desalination process seems ideal but it is likely that the extended time is a safer option. Desalinating the Ijseelmeer over a slightly longer time could make for a smoother transition for fish population which in turn may help diminish or at least delay the economic repercussions for the local fishing economy. The extra land reclaimed during this collapse can then be used to support and possibly even grow fishing communities despite the eventual collapse of the saltwater fish populations.

Sources

[1] *Ijssel*. URL: `https://en.wikipedia.org/wiki/IJssel`.

[2] *IJsselmeer: The Largest Lake in Western Europe*. URL: `https://www.lakepedia.com/lake/ijsselmeer.html`.