Designing Mechanisms with Bounded-Rational Agents

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Motivation

- Relaxing the assumption that agents always perfectly optimize in mechanism design settings
- Motived by evidence that people deviate from equilibrium even for dominant strategy mechanisms
- While repetition improves equilibrium prediction power, low frequency interactions, as well changing conditions (and mechanisms) could limit such improvements in practice
- Goal is to study how mechanism design could incorporate this reasoning in the implementation of reforms to incentive systems

3 papers

- Screening with Behavioral Types
- Surplus Extraction with Behavioral Types
- Screening Agents Who Sample

Some common features across models

- Bounded-rationality behavior present in agents choices
- Mechanisms are interpreted as menus
- Such menus are not fully observable by all agents
- Agents only evaluate alternatives they observed
- What is observable is “exogenous” and fixed

Screening with Behavioral Types

Idea

- Start with the simplest model: monopolistic screening problem
- Some agents report truthfully regardless of the mechanism being used
- "IPV" setting
- Focus on direct mechanisms (for most of the paper)

Model

- Seller provides a product of quality $q \geq 0$ at cost $c(q) = q^2/2$
- Continuum of buyers
  - Fraction α has high valuation θ_H, fraction $1 - \alpha$ has low valuation $\theta_L > 0$
  - Fraction γ is completely strategic (S), fraction $1 - \gamma$ is behavioral (B)
- Valuation and behavioral status are independent
- We assume behavioral types always report their information truthfully, regardless of the mechanism implemented
- All buyers have quasilinear preferences

$$\theta \cdot q - t$$

with $t \in \mathbb{R}$ transfer/price paid to the seller

Model

- We restrict to direct mechanism

$$\Gamma = (q_i^j, t_i^j)_{i\in\{L,H\}, j\in\{S,B\}}$$

- q_i^j is quality received by reported type (θ_i, j)
- t_i^j is the price paid by reported type (θ_i, j)
- We will refer to a quality-price pair as a contract
- Seller's goal is to maximize her expected profits designing Γ among IC and IR mechanisms
- A mechanism is IC if strategic buyers have no incentives to deviate to the contract of other type of buyers (given their valuation)
- A mechanism is IR if buyers get non-negative utility given their valuation, behavioral status, and personalized contract

Incentive compatibility and individual rationality

More formally:

- A mechanism is incentive compatible (IC) if it satisfies

$$\theta_i q_i^S - t_i^S \geq \theta_i q_{i'}^j - t_{i'}^j,$$

for $i, i' \in \{L, H\}$ and $j \in \{S, B\}$.

  - Only strategic types could evaluate all contracts

- A mechanism is individually rational (IR) if

$$\theta_i q_i^j - t_i^j \geq 0$$

for $i \in \{L, H\}$ and $j \in \{S, B\}$

  - Here individual rationality implies all buyers could always opt out from the mechanism after they have looked at their realized allocation

Benchmarks

- Benchmark # 1: full information mechanism

$$q_i^j = \theta_i$$

$$t_i^j = \theta_i q_i^j = \theta_i^2$$

- Benchmark # 2: optimal mechanism without behavioral buyers

$$q_L = \max\left\{\theta_L - \frac{\alpha}{1-\alpha}\left(\theta_H - \theta_L\right), 0\right\}$$

$$q_H = \theta_H$$

$$t_L = \theta_L q_L$$

$$t_H = \theta_H q_H - (\theta_H - \theta_L) q_L$$

Optimal mechanism

Proposition

The optimal mechanism is given by

$$q_L^j = \max\left\{\theta_L - \frac{\gamma\alpha}{1-\alpha}\left(\theta_H - \theta_L\right), 0\right\}$$

$$q_H^j = \theta_H$$

$$t_i^j = \theta_i q_i^j$$

$$t_H^S = \theta_H q_H^S - (\theta_H - \theta_L) q_L^B$$

- All buyers with low valuation receive quality below the efficient level and pay the same price
- All buyers with high valuation receive the efficient quality level but they pay differentiated prices
- Everyone but strategic buyers with high valuation get zero rents

Intuition of the result

- Seller would like to extract the rents of everyone but he can't
- Extracting from behavioral buyers is easier
- However, providing the efficient quality level to behavioral buyers with low valuation face same problem faced with strategic ones
- That is, strategic buyers with high valuation would prefer such contract due to complementarity
- So, she needs to pool low valuation buyers

Pricing interpretation

The proposed mechanism could be interpreted in a pricing setting as follows (assuming no exclusion)

- Two versions (qualities) of the same product are offered
- Premium version is provided at the efficient high quality level, with two differentiated prices
  - Full normal price paid "simple" (behavioral) buyers
  - Discounted price paid by "sophisticated" (strategic) buyers
- Cheap version provided at an inefficient low quality, single price offered

Exclusion and comparative statics

Proposition

Fix θ_L, θ_H and α. There exists $\bar{\gamma}$ such that for $\gamma < \bar{\gamma}$ there is no exclusion

- With behavioral players, exclusion is observed less often
- Moreover, if there are enough of them we always get no exclusion

Proposition

Both welfare and seller's profits are increasing in the fraction of behavioral buyers (decreasing in γ).

- Rents from behavioral buyers could be extracted easily which increase profits
- It also increase welfare by increasing the quality provided to low valuation buyers

Extensions

- Ex-ante participation constraints
- Information acquisition
- Constrained message space

Related literature

- Severinov and Deneckere (2006) studies screening with honest buyers in a continuous setting, characterize the optimal *password* mechanism with exhibits no exclusion
- Saran (2011) studies bilateral trade with honest players and shows honest bidders allows for cross subsidies that could improve efficiency
- Ostrizek and Shishkin (2019) studies screening where seller designs both mechanisms and frames
- "Behavioral" mechanism design: Eliaz (2002), De Clippel et al (2018)
- Robust mechanism design: Bergemann and Morris (2005), Carroll (2019)

Concluding remarks

- We present a model of screening where some buyers always report truthfully
- We have shown that in the optimal mechanism only two quality levels are offered but three prices are used in order to increase revenue from behavioral buyers
- First step on a more general setup

Future ideas

- Extend behavioral assumptions on uninformed / behavioral agents
- Extend environment beyond screening and profit maximization
- Dynamic model
    - Incentives to create initial mechanism in a certain way
    - Incentives to update mechanism at a certain pace
    - Conditions for learning
- Beyond binary valuations

Surplus Extraction with Behavioral Types

Motivation

- In a mechanism design setting where information is independent: private information leads agents to retain informational rents
- Myerson (1981) and others have shown that if instead information is correlated then extracting all the rents is possible
- This is usually called *full extraction*
- Cremer and McLean (1988) identify the key "independence" condition under which full extraction is possible

Informal description of behavioral types

- In this paper, we include behavioral types in this standard setting
- We focus on a particular class of behavioral types
- That is, types who doesn't react optimally to incentives and always reveal their private information
- This is a simple but very strong assumption: allows for perfect identification of each behavioral type
    - Best setting for the designer
    - Good starting point

Overview

- We consider a reduced form approach (McAfee and Reny (1992), Lopomo, Rigotti and Shannon (2020))
  - Single agent
  - Informational rents generated from unmodeled stage
  - Exogenous states
  - Correlation represented through beliefs over states
  - No allocation in the current stage, only transfers ("contract")
- Finite environment (types, states)
- We characterize the key conditions to guarantee full extraction with behavioral types

Model

- Finite set of agent's types T
- Finite set of states Ω
- Each type t associated with
  - Informational rents $v_t \in \mathbb{R}_+$
    - These rents comes from an unmodeled stage (e.g., second price auction)
  - Beliefs $p_t \in \Delta(\Omega)$
    - Correlation between types and states if $p_t \neq p_{t'}$
- A contract $c : \Omega \to \mathbb{R}$ is a mapping from states into transfers, with $c(\omega)$ the transfer required in state ω
- A contract menu $\mathbf{c} = \{c_t : t \in T\}$ is a collection of contracts, one for each type
- The agent has quasilinear preferences

$$v_t - \langle p_t, c_{t'} \rangle$$

where $\langle p_t, c_{t'} \rangle = \sum_{\omega \in \Omega} p_t(\omega) c_{t'}(\omega)$

Introducing behavioral types

- We allow some types in T to be *behavioral*
- As before, we assume behavioral types always reveal their type regardless of the contracts offered.
- That is, they not need to satisfy any incentive compatibility constraint.
- Let $B \subseteq T$ be the set of behavioral types.
- Similarly, let $S = T \backslash B$ be the set of *strategic* types.

The designer's problem

- We are interested on whether the principal/designer is able to extract all the informational rents from the agent using a contract menu **c**.
- Formally, a contract menu achieves full extraction if for all $t \in T$

$$\langle p_t, c_t \rangle = v_t$$

- A contract menu is incentive compatible if each strategic type chooses his cost minimizing contract, i.e., for all $s \in S$

$$c_s \in \arg\min_{t \in T} \langle p_s, c_t \rangle$$

- We say full extraction with behavioral types is feasible if there exists an incentive compatible contract menu **c** which achieves full extraction

CM condition

Definition
A set of beliefs P satisfies the CM condition if for any $p \in P$, $p \notin co(P \backslash \{p\})$

- Without behavioral types, Crémer and McLean (1988) have identified the key condition for full extraction
- Being that the sets of beliefs for all types, P^T, must satisfy the CM
- We provided the characterization if behavioral types are also present

Main result

Theorem

Full extraction with behavioral types is feasible if

1. P^S *satisfies the CM condition, and*
2. *For all types* $b \in B$, $p_b \notin co(P^S)$

- This imposes no restrictions on the structure of v_t
- CM over strategic types still necessary
- Condition over behavioral types slightly relaxed

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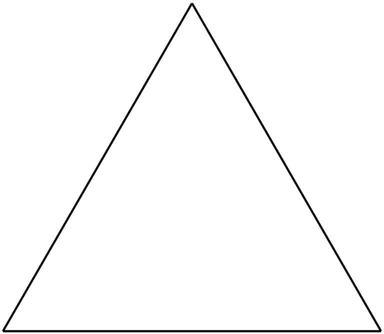

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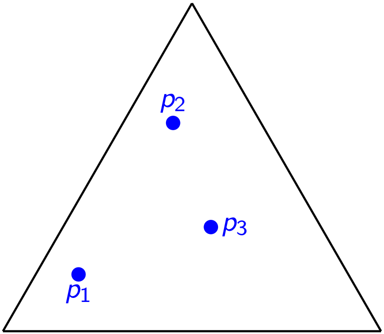

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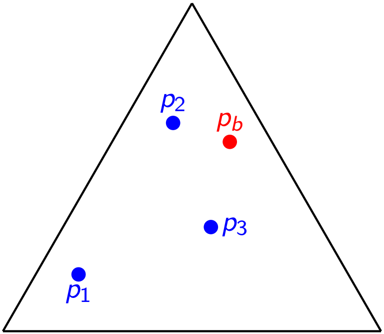

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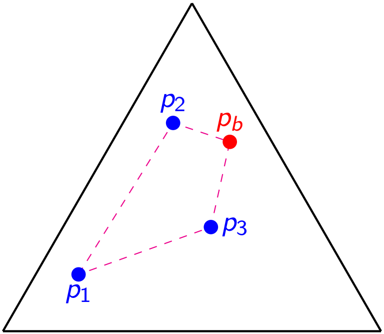

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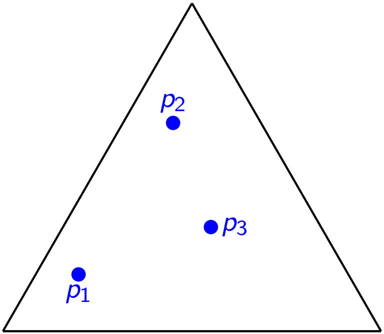

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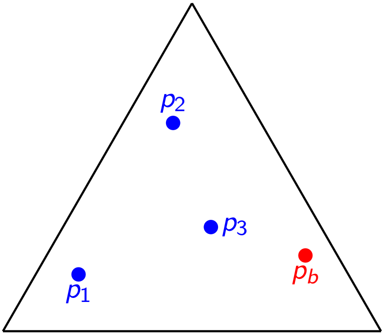

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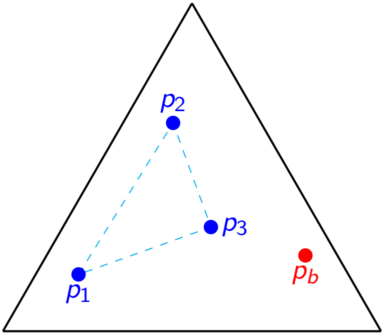

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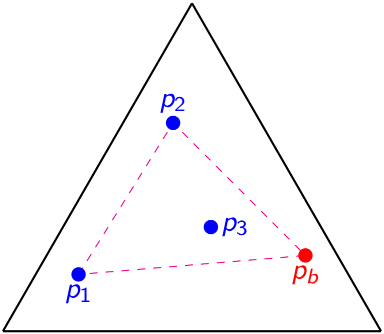

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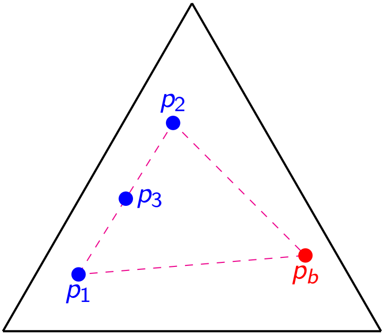

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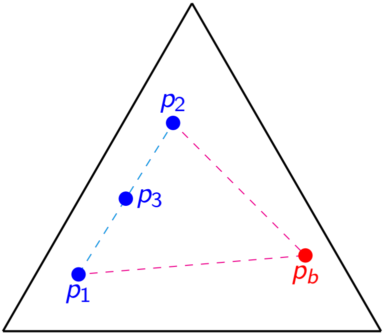

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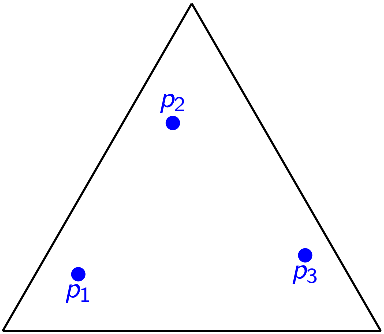

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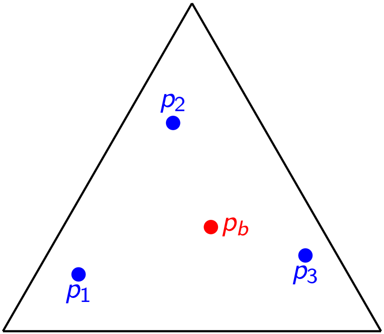

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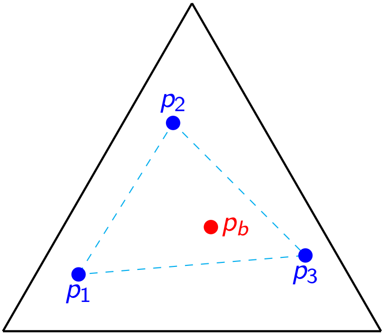

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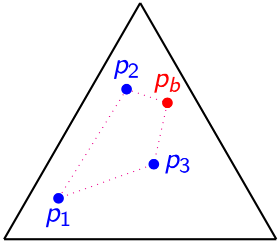
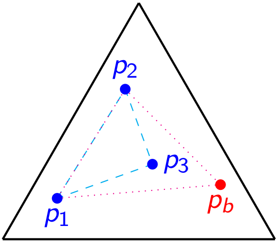
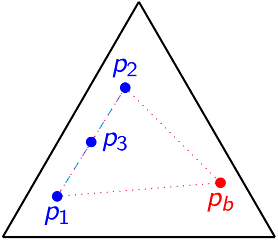


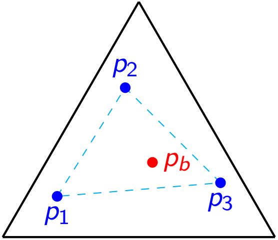
(a) CM/MR theorem applies



(b) Main theorem applies



(c) Condition (i) fails



(d) Condition (ii) fails

From the main theorem

- Without loss to consider $|B| = 1$ since contract for b is independent from contract for $b' \neq b$
- Moreover, contract offered to b is independent of the contracts offered to any other type $t \in T : t \neq b$

What if conditions in the Theorem fail?

Corollary

Consider a particular behavioral type $b \in B$. Let c_{-b} be an incentive compatible contract menu for types $t \neq b$. If $p_b \notin co(P^S)$ then there exists a contract c_b such that the contract menu (c_b, c_{-b}) is incentive compatible and $\langle p_b, c_b \rangle = v_b$.

- In short, Condition 2 allows for full extraction from behavioral types even if not possible for strategic types (i.e., Condition 1 fails)

What if conditions in the Theorem fail?

Proposition

Suppose P^S satisfies the CM condition. Let $\hat{B} = \{b \in B : p_b \in co(P^S)\}$. Then, full extraction with behavioral types is feasible if for each $b \in \hat{B}$

$$v_b \geq \sum_{s \in S} \lambda^b(s) v_s,$$

where $\lambda^b \in \Delta(S) : p_b = \sum_{s \in S} \lambda^b(s) p_s.$

- If Condition 2 fails, full extraction still feasible if informational rents of behavioral types are "big enough"

Related literature

- Myerson (1981): shows an example where correlation lead to full extraction in a simple finite environment
- Cremer and McLean (1988): auction environment
- McAfee and Reny (1992): reduced form setting, full extraction with finite types; virtual full extraction with continuous types
- Lopomo, Rigotti, and Shannon (2020): revisit full and virtual full extraction with finite and continuous types
- Krahmer (2020): mech design + info design
- Fu et al (2021): unknown correlated distribution but samples still allow for full extraction

Concluding Remarks

- We characterize full surplus extraction in the presence of behavioral types
  - We identify a relaxation of the standard convex independence condition that guarantees full extraction
  - Full extraction is easier in this environment but still doesn't comes for free as some conditions are required
- Future steps
  - Alternative behavioral assumptions on behavioral types
  - Beyond reduced form approach
  - Necessary conditions for full extraction

Screening Agents Who Sample

Warning

- Very preliminary, incomplete and unfinished
- All comments and suggestions will be extremely appreciated

Motivation

- As consumers, we received many offers for buying a lot of different products
- However, limited attention or even choice overload, could lead buyers to not evaluate all alternatives presented to them
- Moreover, conjecturing what is offered as opposed to evaluating what is offered could be quite unrealistic
- Here I study a model in which buyers don't evaluate all the available alternatives presented by a seller
- Instead buyers sample some of the alternatives and evaluate their characteristics
- This captures a model of search and discovery that could not be controlled neither by the seller nor the buyer completely

Related literature

- Hart and Nisan (2019): study complexity as the number of entries in the menu representation of a mechanism; simple mechanisms work fine if types are independent, not so well if there is correlation
- Dhangwatnotai et al (2015): building upon Bulow and Klemperer (1996) result, they show that using a random bid as reserve guarantees $1/2$ of the revenue for "all" distributions
- Fu et al (2021): unknown correlated distribution but access to samples $\rightarrow$ full extraction is feasible (with bounded sample size).
- More than one sample: not necessarily better and proofs are challenging (Babaioff et al (2018), Daskalakis and Zampetakis (2020))

Related literature

- Search: Diamond (1971), Stahl (1989), Ellison and Ellison (2009), Ellison and Wolitzky (2012)
- No competition and no obfuscation: monopolist face buyers with a noisy search technology and have no way to affect it
- No price dispersion: in the solution a single quality-price pair is offered

Model

- Single seller, continuum of buyers
- Everyone is risk neutral
- Seller produce good of quality q at cost $q^2/2$
- Buyer utility is quasilinear

$$\theta \cdot q - t$$

  with θ his valuation and t the price he pays to the seller
- Valuations takes two values: $\theta_H > \theta_L > 0$

Model

- We will refer to a quality-price pair as a *contract*
- A mechanism or menu in this setting is then a collection of contracts
- Buyers will not observe the complete mechanism, instead they will sample contracts from the menu of available contracts (uniformly) at random
- Sampling in this context will refer to the procedure used by buyers to observe contracts

Model

- We will focus on the single sample case: buyers draw a single contract from the menu
- After drawing a contract, buyers decide whether they accept or reject the offer
- Rejection lead to zero payoff for both the buyer and the seller
- Note that agents here are bounded-rational and cannot conjecture what will be on the menu to decide how much to search

Useful reference points

Given the assumptions,

- Full information, no sampling, "efficient" contract for type θ offers quality θ and charges θ^2
- The optimal mechanism if buyers observe all contracts has at most two entries either,
  - it provides

$$(\theta_H, \theta_H^2 - (\theta_H - \theta_L)q_L) \text{ and } (q_L, \theta_L q_L)$$

    with $q_L = \theta_L - \alpha(\theta_H - \theta_L)$,
  - or offers (θ_H, θ_H^2) only

Main result (so far)

Proposition

Consider the single sample problem with binary valuations. In the optimal mechanism, the seller always prefer to offer a single contract (q^*, t^*)*. Moreover, such contract is either accepted only by the high valuation buyers which obtain zero expected utility, or it is accepted by all buyers and only the high valuation buyers obtain a positive payoff. In the first case, efficient quality is provided but only to high valuation buyers, while in the second case only the low valuation buyers receive the efficient quality.*

Main result (so far)

- Even though she could offer two differentiated contracts, as in the setting without sampling, the seller will never choose to do so
- That is, the seller prefers to reduce variety as a response to the noise in buyer's choices

Sketch of proof

- The argument behind the proof is straightforward
- First, we show that only full information contracts will be used by the seller
- Consider a contract (q, t) such that $t \neq \theta q$ for neither $\theta = \theta_L$ nor $\theta = \theta_H$
- We will show that it is optimal for the seller to replace such contract for other contract of the form (θ, θ^2), i.e., a full information contract

Sketch of proof

- Let $\hat{\theta}$ be the lowest type accepting contract (q, t), that is, the lowest θ such that $\theta \cdot q - t \geq 0$
- Note that replacing (q, t) by $(\hat{\theta}, \hat{\theta}^2)$ increases profits since it maximizes the value collected from type $\hat{\theta}$
- Hence, only contracts of the full information form are offered in an optimal menu

Sketch of proof

- It remains to show that the seller will always prefer to offer only one of the two contracts
- To show this, we consider an auxiliary problem in which the seller chooses the probability each contract is sampled

$$p \cdot \alpha \left(\theta_H^2 - \frac{\theta_H^2}{2} \right) + (1 - p) \left(\theta_L^2 - \frac{\theta_L^2}{2} \right)$$

where p is the probability contract (θ_H, θ_H^2) is sampled

Sketch of proof

- Note that this problem is linear, hence the single contract solution follows directly by comparing the profits generated by each contract
- This also implies that the uniform sampling assumption is without loss

Quick comparative statics

- Note that (θ_H, θ_H^2) is offered for $\alpha > \left(\frac{\theta_L}{\theta_H}\right)^2$, and (θ_L, θ_L^2) is offered otherwise
- Hence, higher proportion of high valuation buyers favors the high contract
- Similarly, a higher θ_H translates to higher chances of having the high contract as the solution
- Note that $\theta_H < 2\theta_L$ guarantees offering two contracts to be optimal if there is no sampling
- However, even under this assumption the optimal menu with a single sample could take any of the two forms described above

What about welfare? Part 1: maximizing welfare

- Note that in terms of welfare, the price dimension is irrelevant since there is no incentive compatibility issues
- Hence, we only care about the quality provided in each contract
- We note first that the welfare maximizing mechanism also uses a single contract
- It either only serves the high type efficiently ($q = \theta_H$) or provides the average quality to everyone ($q = \alpha\theta_H + (1-\alpha)\theta_L$)
- If we assume $\theta_H < 2\theta_L$ then the welfare maximizing contract always provide the average quality

What about welfare? Part 2: welfare under the profit maximizing mechanism

- First observation is that the welfare under the profit maximizing solution is always suboptimal (as long as $\theta_H < 2\theta_L$, weakly if not)
- Obviously, it also exhibits inefficiencies when compared with the full information-no sampling solution

Some results with two samples

- Determining the optimal mechanism with more than one sample is more difficult
- If we restrict the seller to use at most two different contracts then the solution could involve
  - using a single high contract,
  - using a single low contract, or
  - using two different contracts
- Moreover, these 3 types of menus could arise (as a function of the proportion of high valuations buyers α) even assuming $\theta_H < 2\theta_L$ (which ruled out exclusion without sampling)

Some results with two samples

- Next steps:
  - What if we allow the seller to add copies of the mechanisms into the pool (i.e., deciding the probability each contract is drawn)?*
  - Are two contracts enough?

Targeted menus

- In the previous model, each type has the same chances of observing each contract
- What if we allow the seller to target each type with different menus?
- In the extreme case, the problem becomes trivial
- If the seller could offer personalized menus, she would offer a menu with a differentiated single contract to each type and extract all their rents
- Next steps: what if targeting is imperfect (noisy)?

More than two valuations in the single sample case

- Extending the main result beyond binary valuations is straightforward
- Indeed, with N different valuations $\theta_1 < \theta_2 < ... < \theta_N$
  - Only full information form contracts are offered
  - Moreover, the optimal menu involves using a single contract
- It also extends to continuous valuations directly as long as we consider finite size menus

Concluding remarks

- I have characterized the optimal mechanism offered by seller when buyers sample once from the alternatives offered by her
- The main result being that this noisy technology involve reducing variety in favor of offering a single option
- Moreover, the optimal menu induces inefficiency by either excluding too much

Future directions

- Full characterization for more than one sample
- Targeted menus with noise
- Insightful suggestions from the audience?

Final final comments

- We have reviewed 3 models of mechanism design with bounded-rational agents
- They offer 3 different approaches to address the issue on designing mechanisms when perfect rationality fails
- Still only first steps into more complex environments

Thanks!