Monopolistic Screening with Buyers who Sample (or Price Discrimination when Buyers Sample)

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Motivation

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  - E.g., PCs (Sovinsky Goeree (2008)), bank accounts (Honka et al (2017)), health markets (Abaluck & Adams-Prassl (2021)), experiments (Aguiar et al (2022))
- Not many models of markets with both adverse selection and information frictions

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Main question: what’s the optimal menu in this case?

Spoilers!

Model: Seller designs a menu of offers as in Mussa & Rosen (1978) but...

- Menu is unknown for buyers
- Buyers can only sample offers from the menu
- Finally, buyers decide whether to purchase one of the sampled product if any

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Main results

- Single sample $\Rightarrow$ optimal menu always contains a single element
- Two samples $\Rightarrow$ optimal menu never contains only two offers; it is always asymmetric

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  - Valuations are private information

Preferences and menus

- If a buyer with valuation θ purchases a good of quality q and price p, his utility is

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- Seller designs a menu of quality-price pairs (offers)

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- Existence requires upper limit on the size of the menu $\overline{m}$
- We focus on the case of large $\overline{m}$

Benchmarks

Benchmark: Efficient allocation

- The efficient allocation involves maximizing the surplus for each type of buyer,i.e.,

$$q_i^* = \arg\max_q \theta_i q - \phi(q) \quad \Rightarrow \quad \phi'(q_i) = \theta_i$$

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- For $\phi(q) = q^2/2$, this takes the simple form

$$q_i^* = \theta_i$$

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q_h^{mr} &= \theta_h & \qquad p_h^{mr} &= \theta_h q_h^{mr} - (\theta_h - \theta_l) q_l^{mr} \\
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- Let $\mu_h^{mr} = \frac{\theta_l}{\theta_h}$ the critical value that determines which form is better

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- If μ_h is small, then too many sales are lost due to the mismatch

Optimal menu with a single sample

Theorem 1

Consider the problem with a single sample. Generically, the optimal menu contains a single offer. Moreover, this offers takes the form $(q_i^*, \theta_i q_i^*)$ for some type θ_i. ▸ *Proof*

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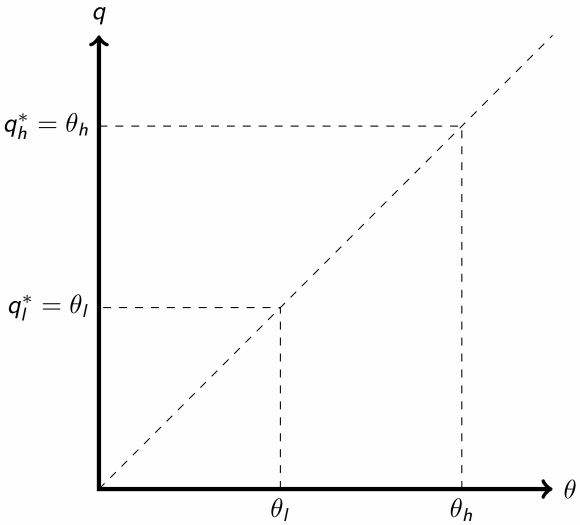
▸ Proof

Intuition:

- No comparison between offers since there is only one sample
- No incentive compatibility
- Participation constraints determines structure of each offer
- Resulting program is linear in offers
- Finally, one offer is better than the others

Optimal menu in pictures



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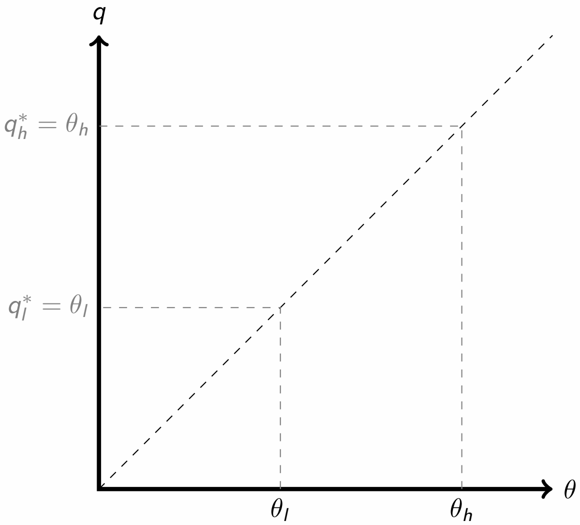

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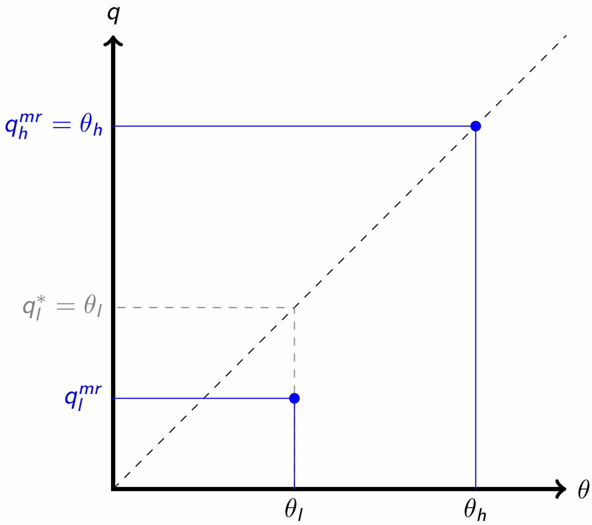

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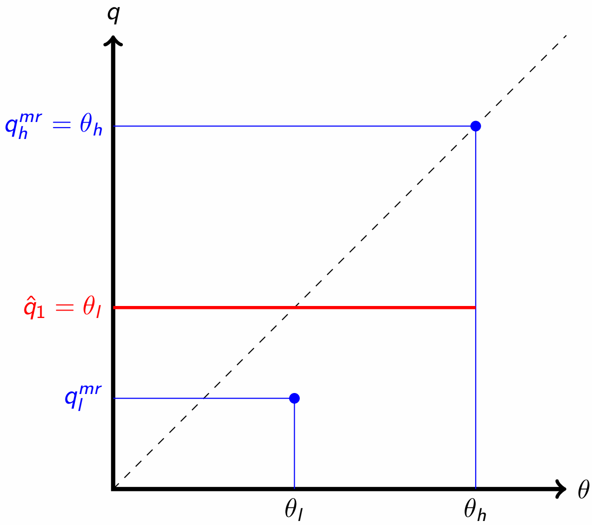

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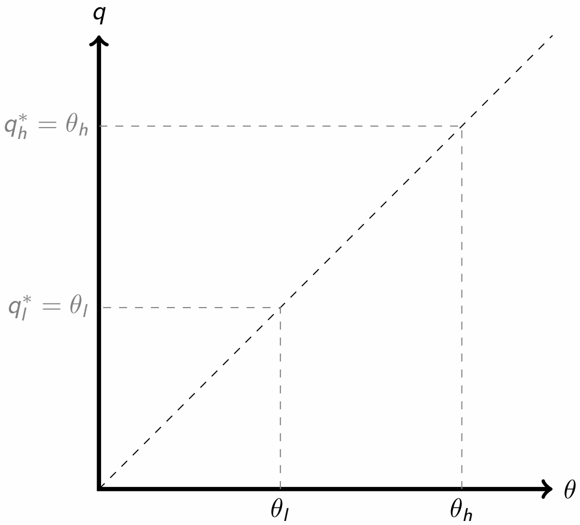

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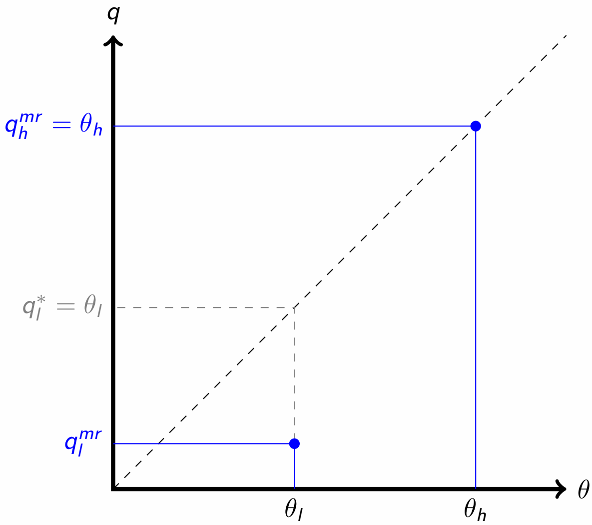

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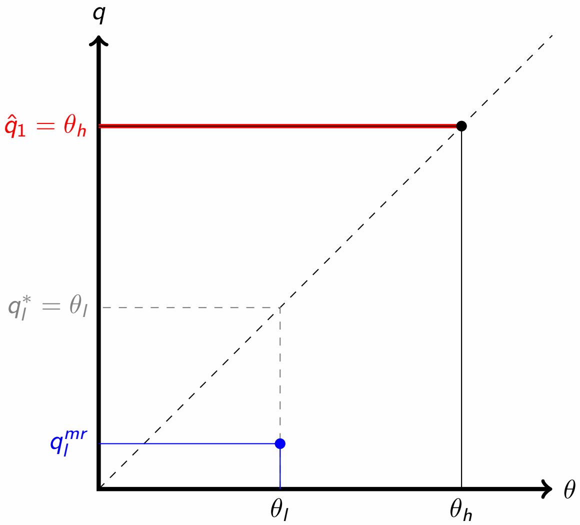


Comparative statics with a single sample

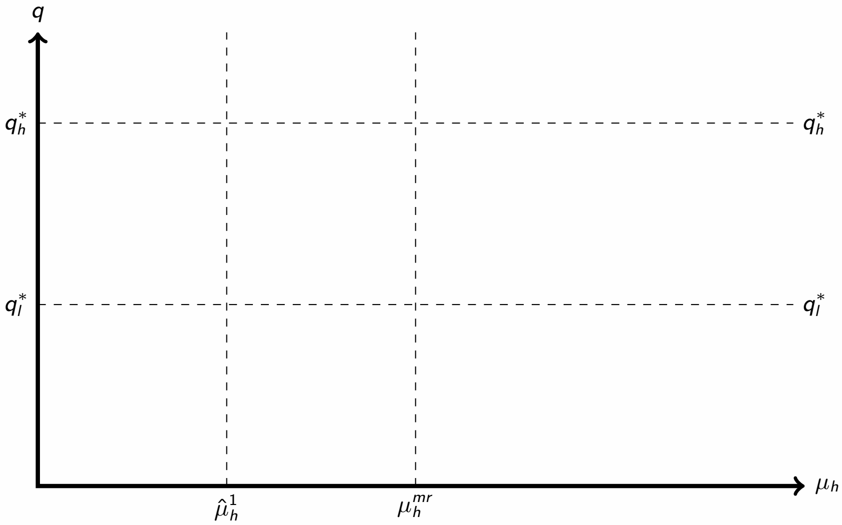
Proposition 1

Fix θ_l and θ_h. There is a unique threshold $\hat{\mu}_h^1 \in (0,1)$ such that for $\mu_h > \hat{\mu}_h^1$ the optimal menu contains only offer $(q_h^*, \theta_h q_h^*)$, while for $\mu_h < \hat{\mu}_h^1$ the optimal menu contains only $(q_l^*, \theta_l q_l^*)$.

Proposition 2

$\hat{\mu}_h^1 < \mu_h^{mr}$.

Quality provision for different levels of μ_h



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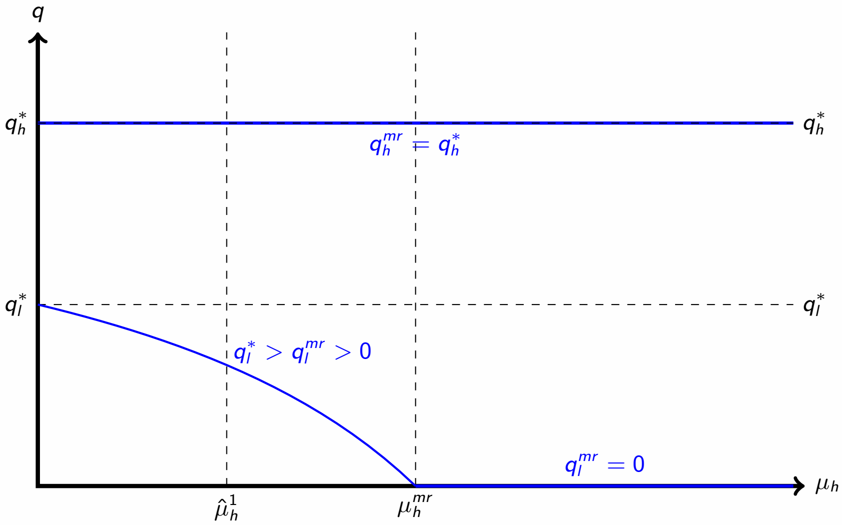

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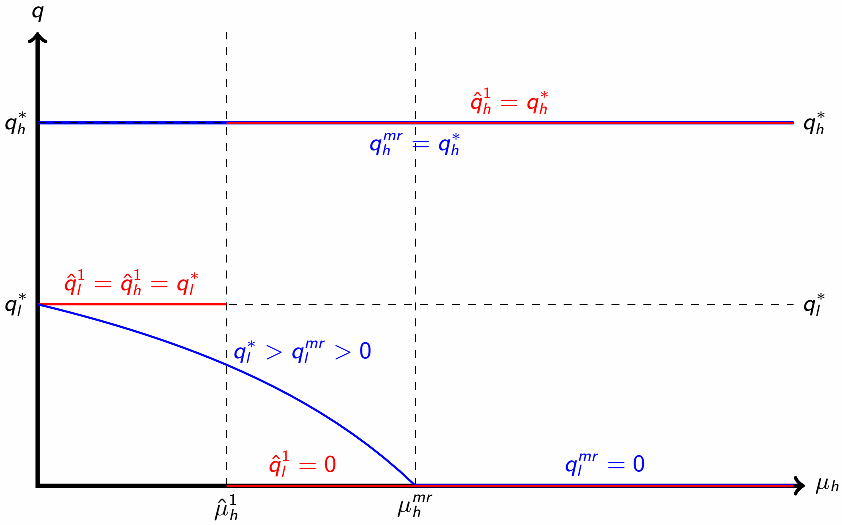
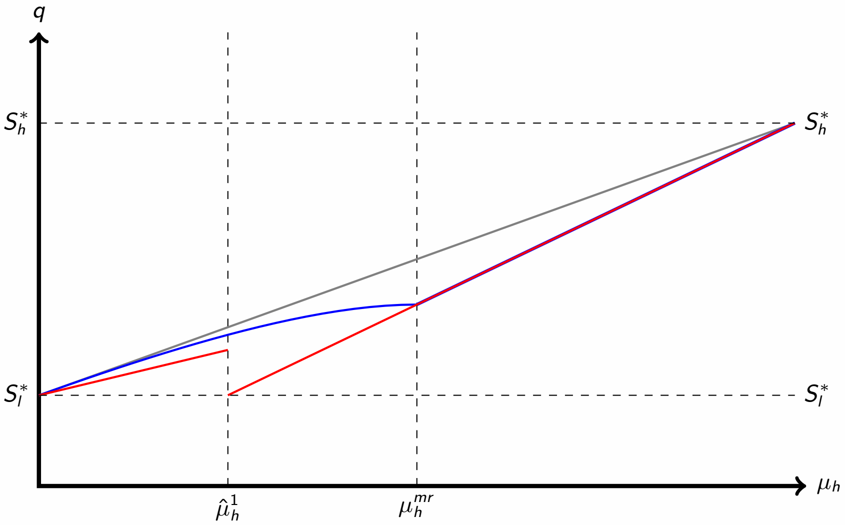
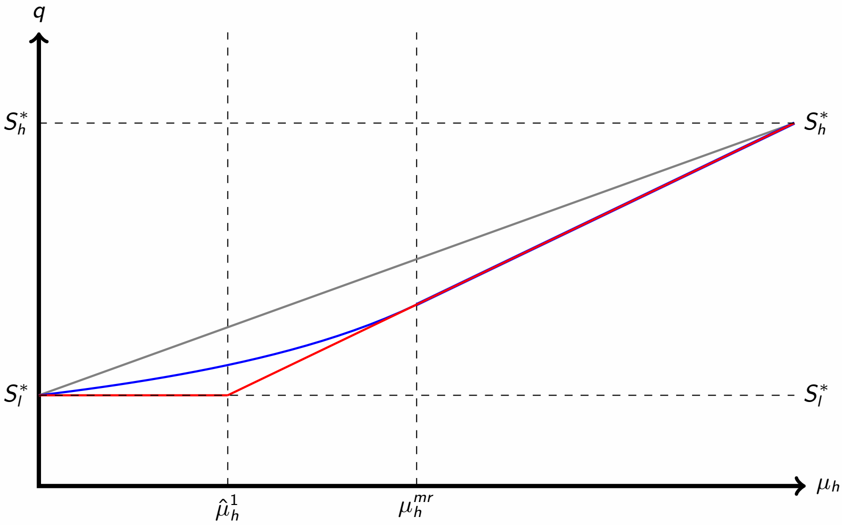


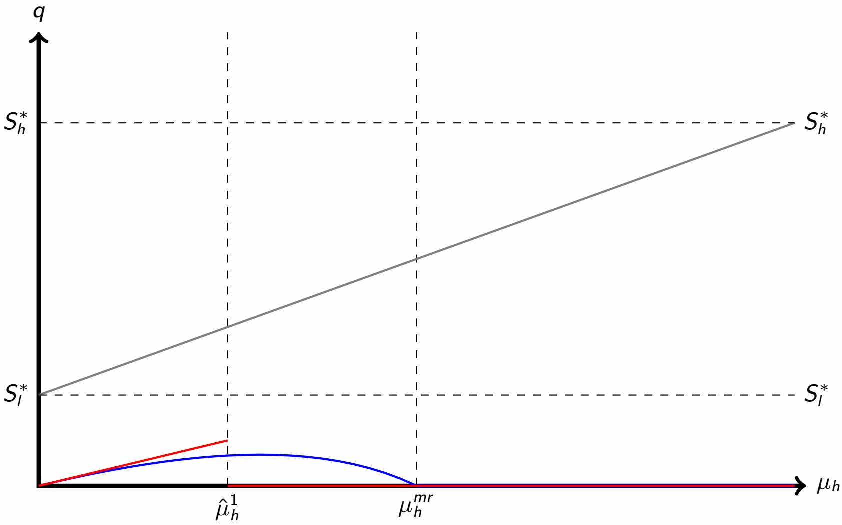
Welfare for different levels of μ_h



Profits for different levels of μ_h



Consumer surplus for different levels of μ_h



Two samples

The problem with two samples

- We know turn to the case of two samples

Lemma 1

Consider the problem with two samples. Suppose the optimal menu contains only two offers (q_a, p_a) and (q_b, p_b), and the maximum menu size $\overline{m}$ is large. Then, the profits of the menus $\{(q_a, p_a)\}$ and $\{(q_b, p_b)\}$ must be the same.

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- Since two different offers could be observed with positive probability, incentive compatibility constraints would play a role
- The first result shows that in an optimal menu, the offers must satisfy a particular characteristic

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- Then, overall profits are
$$\frac{1}{4}r_a + \frac{1}{2}r_{ab} + \frac{1}{4}r_b$$
- For this to be optimal, $r_{ab} > \max\{r_a, r_b\}$ must hold o.w. offering only one would be optimal

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- If the fraction of offers b is small enough, then buyers are more likely to observe $\{a, b\}$ instead of only **b** $\Rightarrow$ overall gain from including **b**
- Moreover, if $r_a = r_b$, there is no cost on including b hence optimal to maximize the probability of $\{a, b\}$ being observed, i.e., balance both offers in the menu

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$$\text{"Loss"} \quad r_a \to r_b$$

- If the fraction of offers b is small enough, then buyers are more likely to observe $\{a, b\}$ instead of only $\mathbf{b}$ $\Rightarrow$ overall gain from including $\mathbf{b}$
- Moreover, if $r_a = r_b$, there is no cost on including b hence optimal to maximize the probability of $\{a, b\}$ being observed, i.e., balance both offers in the menu
- If $r_a > r_b$ then the cost of including b makes better to bias towards the better offer a.

Main result with two samples

Theorem 2

Consider the problem with two samples. Suppose the cost function is $\phi(q) = \frac{q^\eta}{\eta}$ and the menu size $\overline{m}$ is large enough. Then, the optimal menu never contains only two offers. ▸ *Proof*

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- Fixing the size of the menu $= 2$, Lemma 1 and an optimality condition define a system of 2 equations with and 1 unknown
- The solutions for each equation are generically incompatible if $\phi(q) = \frac{q^\eta}{\eta}$

Optimal menu with two samples: characterization

Optimal menu is characterized by the following two equations (for $\overline{m}$ large enough)

$$\phi'(q_l) = \theta_l - \frac{(1 - x_l^2)\mu_h}{x_l^2 + 2x_l(1 - x_l)\mu_l}(\theta_h - \theta_l)$$

$$\frac{x_h}{x_l} = \frac{\mu_h}{\mu_l}\left(\frac{S_h^* - (\theta_h - \theta_l)q_l}{S_l(q_l)} - 1\right)$$

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Proposition?

Suppose the optimal menu exhibits screening. Then,

(i) There is a positive relationship between the quality provided to low valuation buyers and the fraction of offers tailored to low valuation buyers

(ii) If the proportion of high valuation buyers increase, then quality provided to low valuation buyers decreases as well as the fraction of offers tailored to this type of buyers

Extensions

Extensions

- More than two valuations (single sample)
- Collection of menus (single sample)
- Heterogeneity in sample sizes

◂ Jump to Related lit.

More than two valuations (and a single sample)

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Proposition 3

Consider the problem with more than two valuations and a single sample. Suppose Assumption 1 holds. Then, the optimal menu contains a single offer.

Collection of menus

- We consider a setting in which each "offer" is a menu of quality-price pairs

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Proposition 4

Consider the problem with collection of menus and a single sample. Suppose Assumption 2 holds. Then, the optimal mechanism contains a single menu.

Heterogeneity in sample sizes

- Suppose a fraction β of buyers observe two samples, while a fraction $1 - \beta$ observe only one sample.

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Lemma 2

Consider the problem with heterogeneous sample sizes. Suppose the optimal menu contains only two offers (q_a, p_a) and (q_b, p_b), and the maximum menu size $\overline{m}$ is large. Then, the profits of the menus $\{(q_a, p_a)\}$ and $\{(q_b, p_b)\}$ must be the same.

Proposition 5

Consider the problem with heterogeneous sample sizes. Suppose the cost function is $\phi(q) = \frac{q^\eta}{\eta}$ and the menu size $\overline{m}$ is large enough. Then, the optimal menu never contains only two offers.

Other extensions with more than one sample

- Lemma 1 also extends to settings with more samples and/or more valuations

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Other extensions with more than one sample

- Lemma 1 also extends to settings with more samples and/or more valuations
- Theorem 2: ???
- Collection of menus with multiple samples is challenging as relevant constraints become messy

Related literature

- **Product line design and price discrimination:** Mussa & Rosen (1978)
    - *Single offer optimal:* Sandmann (2022), Bergemann et al (2022), Doval & Skreta (2023)
- **Sampling equilibrium:**
  Osborne & Rubinstein (1998, 2003), Spiegler (2006), García-Echeverri (2021)
- **Information frictions + Asymmetric Information:**
  Villas Boas (2004), Garrett et al (2018), Lester et al (2019)
- **Search:**
  Burdett & Judd (1983), Ursu et al (2021), Safonov (2022), Fershtman & Pavan (2022)

EconCS:

- **Revenue maximization with samples:**
  Dhangwatnotai et al (2015), Babaioff et al (2018), Daskalakis & Zampetakis (2020), Fu et al. (2021)
- **Menu-size complexity:**
  Hart & Nisan (2017, 2019) Bergemann et al (2021)

Concluding remarks

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Contribution

- Novel application of non-measurable uncertainty and samples (for buyers instead of sellers)
- Not many models with both adverse selection and information frictions

Concluding remarks

- I proposed a model of asymmetric information and information frictions due to bounded-rationality
- Optimal menu has a single offer if buyers could sample once, and never has only two offers if they sample twice.

Contribution

- Novel application of non-measurable uncertainty and samples (for buyers instead of sellers)
- Not many models with both adverse selection and information frictions

Takeaways: interaction of asymmetric information and bounded-rationality/information frictions lead to non-trivial changes in the product line

- Single sample: noise could destroy incentives to provide variety
- Two samples: optimal menu is typically asymmetric, and number of samples $\neq$ number of offers

Summary

Seller designs menu $\rightarrow$ Buyers sample offer(s) and decide whether to purchase a sampled offer or not

Thanks!

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Theorem 1

Consider the problem with a single sample. Generically, the optimal menu contains a single offer. Moreover, this offers takes the form $(q_i^*, \theta_i q_i^*)$ for some type θ_i.

Theorem 2

Consider the problem with two samples. Suppose the cost function is $\phi(q) = \frac{q^2}{2}$ and the menu size $\overline{m}$ is large enough. Then, the optimal menu never contains only two offers.

Suggestions?

- Two equations, one unknown

$$\phi'(q_l) = \theta_l - \frac{3\mu_h}{1+2\mu_l}(\theta_h - \theta_l)$$

$$\theta_\ell q_l - \phi(q_l) = \mu_h \left(\theta_h q_h^* - \phi(q_h^*) - (\theta_h - \theta_l) q_l\right)$$

- Two equations, two unknowns

$$\phi'(q_l) = \theta_l - \frac{(1-x_l^2)\mu_h}{x_l^2 + 2x_l(1-x_l)\mu_l}(\theta_h - \theta_l)$$

$$x_l = \frac{1}{1 + \frac{\mu_h}{\mu_l}\left(\frac{\theta_h q_h^* - \phi(q_h^*) - (\theta_h - \theta_l) q_l}{\theta_l q_l - \phi(q_l)} - 1\right)}$$

Sketch of Proof of Theorem 1 (and Proposition 3)

return to Theorem 1

- Step 1: only "efficient" offers are included in the menu: (θ, θ^2) for some θ
  - No incentive compatibility constraints since only single offer is observed each time
  - If offer with quality q is drawn, for which last type accepting is θ, optimal to price it at $p = \theta q$
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  - Then, if offer is accepted by $\theta' \geq \theta$, optimal to match efficient quality provision for θ
- Step 2: given that only offers of this form are offered optimal menu is determined by a linear problem
  - Solution involves assigning all mass to "best" offer only

Proof of Theorem 2

return to Theorem 2

- If an optimal menu contains only two offers, then it must maximize

$$\frac{1}{4}(\theta_l q_l - \phi(q_l)) + \frac{1}{2}\left(\mu_l(\theta_l q_l - \phi(q_l)) + \mu_h(\theta_h q_h - \phi(q_h) - (\theta_h - \theta_l)q_l)\right)$$
$$+\frac{1}{4}\mu_h(\theta_h q_h - \phi(q_h) - (\theta_h - \theta_l)q_l)$$

- FOC involves

$$\begin{aligned} q_h &= q_h^* \text{ (``no distortion at the top'')} \\ \phi'(q_l) &= \theta_l - \frac{3\mu_h}{1+2\mu_l}(\theta_h - \theta_l) \end{aligned} \tag{1}$$

- Then from Lemma 1, we also have

$$\theta_l q_l - \phi(q_l) = \mu_h(\theta_h q_h^* - \phi(q_h^*) - (\theta_h - \theta_l)q_l) \tag{2}$$

- Equations (1) and (2) are generically incompatible for $\phi(q) = \frac{q^\eta}{\eta}$