Surplus Extraction with Behavioral Types* PRELIMINARY AND INCOMPLETE

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This version: September 4, 2021
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Abstract

We reexamine the surplus extraction problem in a mechanism design setting with behavioral types. We focus on an extreme class of behavioral types which always perfectly reveal their private information. However, incentive compatibility constraints remain necessary for strategic types with respect to the contract of all types. We characterize the sufficient conditions that guarantee full extraction in a finite version of the reduced form environment of McAfee and Reny (1992). The standard convex independence condition identified in Crémer and McLean (1988) remains necessary only among the beliefs of strategic types, while a weaker condition is required for the beliefs of behavioral types.

1 Introduction

In mechanism design settings, private information leads agents to retain informational rents if information is independently distributed. However, we know since Myerson (1981) that under correlation it is possible to extract all the informational rents from agents. Hence, in presence of correlation, private information not necessarily lead to agents obtaining information rents. This

*I thank Luca Rigotti and Richard Van Weelden for providing guidance in this project. I also thank Svetlana Kosterina, Benjamin Matta, participants of the Microeconomic Theory Brownbag at University of Pittsburgh, the 2021 Pennsylvania Economic Theory Conference and the 32nd Stony Brook International Conference on Game Theory for useful comments.

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result is what is usually called *full (surplus) extraction*. Crémer and McLean (1988) have identified the key independence condition that guarantees full extraction (convex independence) in mechanism design settings.

In this project we examine the full surplus extraction problem considering the presence of *behavioral types*. That is, types that don't respond optimally to incentives. We focus on a particular extreme case of such behavioral types: types which don't react to the mechanisms implemented and always reveal their private information perfectly. While extreme, this assumption allows for a simple characterization and also works as a benchmark for the designer problem since it gives him the most advantaged setting to operate. The key feature of this assumption is that it allows for a perfect identification of each behavioral type and allows us to characterize their behavior regardless of the mechanism implemented. While extracting rents from this types should be easier, incentive compatibility still requires their contract to be non-attractive to strategic types, imposing constraints to the designer.

We consider a reduced form environment similar to the one used by McAfee and Reny (1992) and Lopomo et al. (2020), where in an unmodeled stage an agent is left with informational rents which depends on his private information. There is also an exogenous source of uncertainty in which the current stage contracts could condition on. We will refer to the current stage contracts simply as contracts and ignore any reference to the mechanism that generates the informational rents. The main differences with respect the model in McAfee and Reny (1992) are that we introduce a class of behavioral types which always report their private information truthfully and focus on a finite type space.

The remaining of this note is organized as follows. We describe the model and the main result in Section 2. We present some extra results in Section 3. In Section 4 we apply our main result in an auction environment with correlated valuations. Finally, we provide a short review of related literature in Section 5.

2 Model

There is a finite set of types T and a finite set of states Ω. Each type t is associated with a valuation (i.e., informational rents) $v_t \in \mathbb{R}_+$ and beliefs $p_t \in \Delta(\Omega)$. We define a contract c as a mapping from states into transfers, such that $c(\omega) \in \mathbb{R}$ is the transfer required in state $\omega \in \Omega$. A contract menu $\mathbf{c}$ is a collection $\{c_t : t \in T\}$ such that $c_t : \Omega \to \mathbb{R}$, i.e., a collection of contracts, one for each type.

There is a single agent with quasilinear preferences. Hence, from a contract

c, type t's payoff is given by

$$v_t - \langle p_t, c \rangle$$

where $\langle p_t, c \rangle$ denotes the expected value of c under p_t, that is

$$\langle p_t, c \rangle = \sum_{\omega \in \Omega} p_t(\omega)\, c(\omega).$$

We are interested on whether the principal or designer is able to extract all the remaining rents from the agent using a contract menu $\mathbf{c}$[1].

We introduce the definition of full extraction formally below.

Definition 1. *A contract menu* $\mathbf{c}$ *achieves full extraction if for all* $t \in T$

$$\langle p_t, c_t \rangle = v_t$$

In the traditional setting it is required that all types prefer their own contract to the contract offered to others. We depart from the standard model by introducing a particular class of *behavioral* types. In particular, a behavioral type here is a type for which no incentive compatibility constraint is considered. That is, a behavioral type always report his type truthfully. While this assumption could seem extreme, we claim that this is without loss due to a revelation principle argument as long as the strategy of the behavioral types is independent of the mechanism implemented.

Let $B \subseteq T$ be the set of behavioral or unsophisticated types. Similarly, let $S = T \backslash B$ be the set of *standard, sophisticated or strategic* types.

Since in our model behavioral types don't respond to incentives, we will require incentive compatible constraints only for strategic types. Moreover, we will use a restrictive incentive compatibility notion, requiring that potential deviations have no impact in the informational rents of the agent. That is, we will require that each (strategic) type chooses his cost minimizing contract.

Definition 2. *A contract menu* $\mathbf{c}$ *is incentive compatible if for each strategic type* $s \in S$*,*

$$c_s \in \arg\min_{t \in T} \langle p_s, c_t \rangle$$

We will be looking for a incentive compatible contract menu that fully extracts the informational rents from the agent.

Definition 3. *Full extraction with behavioral types is feasible if there exists a incentive compatible contract menu* $\mathbf{c}$ *which achieves full extraction*

[1] However, as Börgers (2015) notes, the focus on surplus extraction is arbitrary and the same results could be applied to implement any particular profile of payoffs or allocations.

Crémer and McLean (1988) have shown that in a setting without behavioral types full extraction is feasible if the set of beliefs satisfies the independence condition below.

Definition 4. *A set of beliefs P satisfies the CM condition if for any $p \in P$, $p \notin co(P \backslash \{p\})$*

This condition is known as the convex independence condition, and it is a linear independence condition over the set of beliefs. It also coincides with the more general condition of probabilistic independence used by McAfee and Reny (1992) and Lopomo et al. (2020) if applied to a setting with finite types as the model we use here.

We assume that different types hold different beliefs, that is, $p_t \neq p_{t'}$ if $t \neq t'$, and denote by P^X the set of beliefs associated to types in $X \subseteq T$.

We state our main result below.

Theorem 1. *Full extraction with behavioral types is feasible if*

(i) P^S satisfies the CM condition, and

(ii) For all types $b \in B$, $p_b \notin co(P^S)$

Proof. Step 1: from (i), we now from Crémer and McLean (1988) that we can find a contract that fully extract if we restrict types to S. Notice that such contract remains incentive compatible and reaches full extraction among types in S as long as the contracts offered to types in B doesn't generate incentives to any type in S to deviate.

Step 2: now, we construct the contract for types in B. Consider a single behavioral type b. For this type, it suffices to find z_b such that

$$\langle p_b, z_b \rangle = 0$$

$$\langle p_s, z_b \rangle > 0, \quad \forall s \in S$$

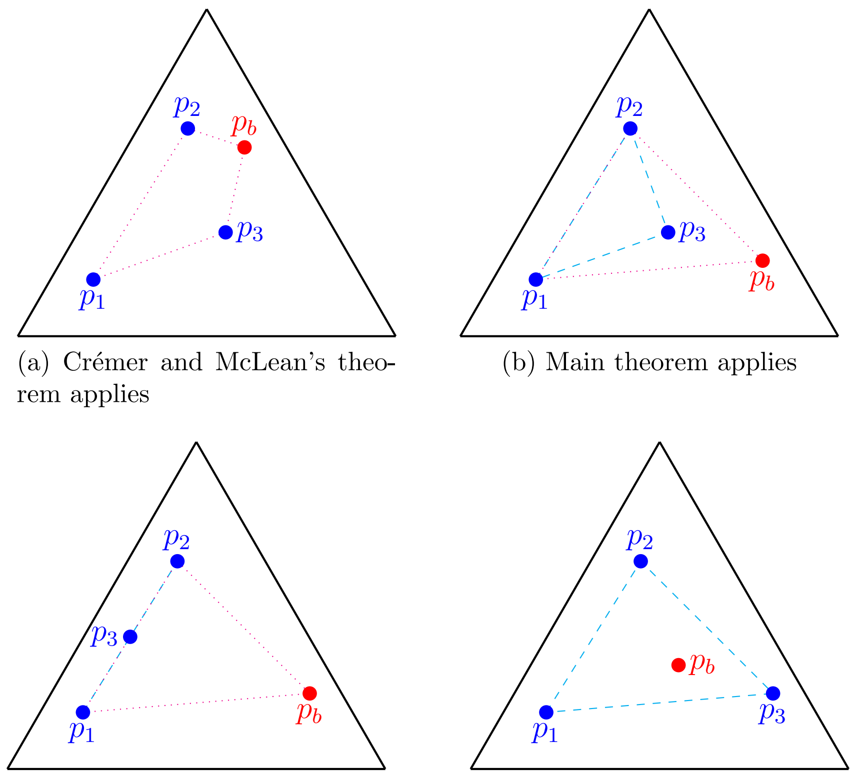
Due to condition (ii), the solution to this system of inequalities is guaranteed by a separation argument (see for example Farkas' lemma). Hence, we can use z_b to construct the extracting contract for the behavioral type in a similar way we construct the contract for types in S. In particular,

$$c_b = v_b + \alpha_b z_b$$

with $\alpha_b = \max_{s \in S} \frac{v_s - v_b}{\langle p_s, z_b \rangle}$ satisfies all the required conditions.

Note that the contract designed for type b has no impact on the contract of any other type in B or S. Hence, we can repeat the process for all others types in B to construct the contract for each remaining type.

□



(a) Crémer and McLean's theorem applies

(b) Main theorem applies

(c) Condition (i) fails

(d) Condition (ii) fails

Figure 1: Representation of an environment with three states, three strategic types (1,2, and 3), and a single behavioral type (b). In (a), P^T satisfies the CM condition so full extraction is feasible. In (b), only P^S satisfies the CM condition; since $p_b \notin co(P^S)$, full extraction remains feasible. In (c), P^S no longer satisfies the CM condition (condition (i) in the main theorem fails), so full extraction cannot be guaranteed. Finally, in (d) P^S satisfies the CM condition but $p_b \in co(P^S)$ (condition (ii) fails), hence full extraction cannot be guaranteed.

Notice that from the proof above, it is without loss to look at environments where $|B| = 1$ since the problem for each behavioral type could be looked at in complete isolation from other behavioral types. This is possible since there is no "cross" incentive compatibility conditions.

So, in the presence of the behavioral types studied here, a slightly relaxed convex independence conditions is required to guarantee full extraction. This highlight in particular that incentive compatibility and full extraction are not entirely isolated features of a mechanism. Note that the second condition on the theorem above allows us to separate the behavioral type from strategic types. This translates into a particular direction of incentive compatibility: that no strategic type wants to deviate to the contract of the behavioral type considered. This is in contrast to the first condition which requires, as in the standard extraction problem, to consider incentive compatibility in both directions. This is why the augmented condition required for behavioral types is weaker than the one required over strategic types (i.e., the standard CM condition).

As was argued before, having P^S satisfying the CM condition is a necessary condition for full extraction even with behavioral types. Hence, considering environments where this condition is violated will automatically rule out the possibility of fully extracting.

Clearly, if the CM condition is satisfied not only over P^S but over the whole set of beliefs P^T, then fully extraction holds trivially since the same proof from the original work from Crémer and McLean (1988) works in this context. If such condition only holds over P^S and in addition $p_b \notin co(P^S)$, then full extraction is still feasible. Hence, the conditions identified in Theorem 1 relaxes the condition required for full extraction without behavioral types. However, if $p_b \in co(P^S)$ full extraction fails again even if the only types violating the independence condition are behavioral and do not need to satisfy any incentive compatibility constraints themselves. The reason why full extraction fails here is that even if these types don't need to satisfy incentive compatibility constraint, other types could be attracted by their contracts, limiting the power of the designer to extract rents from them.

3 Further results

In this section we present two more results for the general setting.

The first result in this section is a direct consequence of Theorem 1, and shows that the same contract works for behavioral types even if full extraction could not be achieved for strategic types.

Corollary 1. *Consider a particular behavioral type $b \in B$. Let c_{-b} be an incentive compatible contract menu for types $t \neq b$. If $p_b \notin co(P^S)$ then there exists a contract c_b such that the contract menu (c_b, c_{-b}) is incentive compatible and $\langle p_b, c_b \rangle = v_b$*

Proof. Follows directly from the proof of Theorem 1. □

This result is analogous to the result in Börgers (2015) which shows that if the convex independence condition holds then any allocation rule could be made incentive compatible. Here we show that implementing full extraction for a particular behavioral type b requires us to check only if his beliefs are in the convex hull of the beliefs of the strategic types, regardless of what could be achieved from other types.

The second result in this section characterizes full extraction when the second condition in Theorem 1 fails, imposing some conditions over the valuations of the agent.

Proposition 1. *Suppose P^S satisfies the CM condition. Let $\hat{B} = \{b \in B : p_b \in co(P^S)\}$. Then, full extraction with behavioral types is feasible if for each $b \in \hat{B}$*

$$v_b \geq \sum_{s \in S} \lambda^b(s) v_s,$$

where $\lambda^b \in \Delta(S) : p_b = \sum_{s \in S} \lambda^b(s) p_s$.

Proof. The contracts for types $s \in S$ and $b \in B \backslash \hat{B}$ remain the same as in Theorem 1.

For types $b \in \hat{B}$, there are two cases to consider:

1. If $v_b \geq \max_{s \in S} v_s$ then a flat constant contract $c_b = v_b$ doesn't violate any incentive compatibility condition and extract all rents from type b.

2. For $v_b < \max_{s \in S} v_s$ we construct the contract for b as follows.

   First, $p_b \in co(P^S)$ implies there exist weights $(\lambda_s)_{s \in S}$ such that $\sum_{s \in S} \lambda_s = 1$, $\lambda_s \geq 0$ for all $s \in S$ and $p_b = \sum_{s \in S} \lambda_s p_s$.

   Let $v_{max} = \max_{s \in S} v_s$ and $\overline{v}_b = \sum_{s \in S} \lambda_s v_s$. Now, consider the contract

   $$c_b = \alpha_b \sum_{s \in S} \lambda_s c_s + (1 - \alpha_b) v_{max}$$

   with $\alpha_b = \frac{v_b - \overline{v}_b}{v_{max} - \overline{v}_b}$. Note $\langle p_b, c_b \rangle = v_b$ and $\langle p_s, c_b \rangle \geq v_s$ for all $s \in S$. Hence c_b fully extract the rents from b and doesn't violate any incentive compatibility constraint.

Repeating this process for all other $b' \in \hat{B}$ we obtain a feasible contract menu which achieves full extraction. □

This result shows how full extraction is maintained if we replace the condition over the beliefs of the behavioral types for a condition over their payoffs. While the condition is restrictive, it is less extreme than the condition required for fully strategic types which replaces the inequality in the proposition above with an equality.

4 Auction application

In this section we introduce an auction environment to illustrate the main result. We start by formally describe the auction model, then we use a single bidder reduction and apply our main theorem and characterize the fully extracting mechanism.

We consider a standard private values auction environment with correlation: there is a single item which could be allocated to one of $n \geq 2$ bidders. The set of buyers is denoted by N. Each bidder has a valuation θ_i for the item. This valuation is each buyer private information, and hence only known to himself. There are a finite set of potential valuations for each bidder i, which we denote by Θ_i. We also define $\boldsymbol{\Theta} = \times_{i \in N} \Theta_i$ and $\Theta_{-i} = \times_{j \neq i} \Theta_j$, with general elements $\boldsymbol{\theta}$ and θ_{-i} respectively. There is a common prior F over the vector of valuations $\boldsymbol{\theta}$, i.e., $F \in \Delta(\boldsymbol{\Theta})$.

We are interested in case of correlated valuations, so we do not impose any independent distributions assumption over F. This implies that a bidder i with valuation θ_i holds beliefs $F(\cdot|\theta_i) \in \Delta(\Theta_{-i})$ over the valuations of the other bidders.

As in the general model of Section 2, we will introduce some *behavioral types* among the bidders. Here a behavioral type will be determined by his valuation-belief pair. We denote by $B_i \subseteq \Theta_i$ the set of behavioral types for player i. Here behavioral types will always report truthfully while non-behavioral types will report what is best for them. Invoking the revelation principle, we will focus on direct revelation mechanisms without loss.

We will restrict attention to the symmetric case in which all bidders share the same space of valuations, that is for any i and j, $\Theta_i = \Theta_j = \Theta$, and their beliefs are also symmetric, that is for any $t \in \Theta$ and $\omega \in \Theta^{n-1}$, $F(\theta_{-i} = \omega|\theta_i = t) = F(\theta_{-j} = \omega|\theta_j = t)$ for all i and j. Moreover, we will assume $B_i = B_j = B$ for all bidders i and j as well.

We will further assume that each valuation generate a different distribution over the valuations of the other bidders so $F(\cdot|\theta_i = t) \neq F(\cdot|\theta_i = t')$ for all $t, t' \in \Theta$.

Note that by our symmetry assumption, each valuation will not only determine the beliefs but also the degree of sophistication of a particular bidder of type θ. Hence, if he has valuation $\theta \in B$, then the bidder will hold beliefs $F(\cdot|\theta)$ and always report truthfully, while if his valuation is $\theta' \notin B$ then his beliefs are $F(\cdot|\theta')$ and he is fully strategic.

We proceed to introduce a single bidder reduction of the auction resented above. This reduction will allows us to use the main theorem above to solve for the optimal auction.

Since we are interested in cases where full surplus extraction is feasible, we will fix the allocation rule to be the one maximizing the total surplus, i.e., the efficient allocation in which the bidder with the highest valuation gets the item. Moreover, we will assume that any tie is resolved in favor of a particular bidder i and focus on the analysis of this bidder[2].

We also be using the same notation from Section 2 for the elements of the single bidder reduction, so we can use Theorem 1 directly.

In particular, we denote valuations of bidder i by t and the vector of valuations of the other bidders different from i by ω. We let the beliefs of type t be $p_t(\omega) = F(\theta_{-i} = \omega|\theta_i)$.

We will denote the gross expected utility of bidder i with valuation t by v_t. Hence, under the efficient allocation rule

$$v_t = t \left(\sum_{\theta_{-i}:\max_{j\neq i} \theta_j \leq t} F(\theta_{-i}|\theta_i = t) \right).$$

Finally, $c_t(\omega)$ will represent the transfer made by the bidder if his reported valuation is t and the valuations reported by other bidders is ω.

In order to apply Theorem 1, we will need to impose some conditions over the beliefs of the bidder. In particular, we impose that for all $t \in \Theta$

$$p_t \notin co(p_{t'} : t' \notin B \text{ and } t' \neq t).$$

Note that this condition is equivalent to the two conditions in Theorem 1. Hence, we can apply directly Theorem 1 to guarantee full surplus extraction in this setting. Moreover, we can use the construction in the proof of Theorem 1 to compute the transfers required to achieve it in this setting. Since this transfer rule is incentive compatible and extracts all the surplus under the surplus maximizing allocation, the optimal mechanism will indeed extract all the informational rents in expectation as long as the condition above holds. We state this result formally in the corollary below, using the original notation for the auction environment.

Proposition 2. *Consider the auction environment. Let B_i the set of behavioral types for bidder i. If for all bidders i, and valuations $\theta_i \in \Theta_i$,*

$$F(\cdot|\theta_i) \notin co(\{F(\cdot|\theta_i') : \theta_i' \notin B_i \textit{ and } \theta_i' \neq \theta_i\})$$

then the optimal mechanism achieves full surplus extraction.

[2] We do this only for simplicity, everything extends directly to any alternative tie-breaking rule as usual.

Proof. Follows from Theorem 1 and the single bidder reduction characterized above. □

5 Related literature

Myerson (1981) characterize the optimal auction assuming bidder's valuation are continuous and independently distributed, but also construct an example with discrete valuations and correlation where full extraction is feasible.

Crémer and McLean (1988) characterize the full extraction problem in an auction environment with correlation and discrete types. They originally identify the convex independence condition, and shows that such condition is key to obtain full extraction.

The studies above restrict the space of types to the set of payoff types or small type spaces. More general type spaces are considered by Farinha Luz (2013) which studies the surplus extraction problem in rich type spaces. The author characterizes an upper bound to the revenues studying a relaxed problem and shows that under a linear independence condition this upper bound is achieved by the optimal mechanism in the complete problem.

A different approach is taken by McAfee and Reny (1992) which studies the surplus extraction problem with both discrete and continuous types using a reduced form approach. Recently, Lopomo et al. (2020) revisits both the continuous and discrete problems studied in McAfee and Reny (1992) providing an alternative proof of the original results and considering infinite menus instead of finite menus as in the original work. In this paper we also use a reduced form approach but restrict to finite type spaces.

Fu et al. (2021) shows that the full extraction results holds even if the correlated distribution is unknown but the designer has access to samples from the true distribution. They show that the solution involve using the samples as a correlating device instead of a learning process.

Krähmer (2020) studies the joint problem of designing the information structure and the mechanism, and shows that extracting the full surplus is possible (under partial information control) if beliefs satisfy a convex independence condition.

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