Full Surplus Extraction and Consideration Sets

Nicolas Pastrian

March 21, 2022

University of Pittsburgh

Introduction

Motivation

- In a standard mechanism design setting, we will require that each agent type has no incentives to imitate any other type
- However, considering all potential deviations could be costly or even impossible for an agent
- Hence, natural to look at environments in which deviations could only be "local"
- First ingredient: restricting each type to deviate only locally

Motivation

- Private information lead agents to retain informational rents
- However, it is well known that in settings with correlation all rents could be extracted even if agents hold private information
- This result is usually referred as *full surplus extraction*
- The key condition to guarantee full surplus extraction was identified by Crémer and McLean (1985,1988)
- Second ingredient: correlated information

Motivation

- Here, I revisit this setting in a “bounded-rationality” environment
- In particular, I will look into a simple model with *considerations sets* in the classic mechanism design problem of full surplus extraction
- Consideration sets will be fixed and independent of the (current) *“mechanism”*
- Using this model, I identify the key condition that guarantees full surplus extraction to be feasible

Consideration sets

- Consideration set: "alternatives which consumers actively consider before making their final purchase decision" (Monash University, Marketing Dictionary)
- Related ideas: awareness sets, evoked sets, choice sets

Consideration sets

- Here, I use the consideration set as the set of types a particular type could imitate or pretend to be
- Restrictive but natural in a direct mechanisms environment
- This consideration sets could be justified as a product of informational frictions, bounded-rationality, partially verifiable messages, or costly information acquisition.

Overview

- My model is a modified version of the reduced form approach in McAfee and Reny (1992): single agent, exogenous (correlated) information, no allocation
- The innovation is the inclusion of consideration sets limiting what different types could report
- Main result: characterization of the conditions that guarantee full surplus extraction in an environment where types can only imitate a subset of types

Model

Model

- There is a single agent (or a continuum of agents) with finitely many types $t \in T$
- Each type t is associated with 3 elements: v_t, p_t, and C_t

Model

1. $v_t \in \mathbb{R}_+$ is the valuation/surplus/informational rents which comes from an unmodeled stage or mechanism
2. $p_t \in \Delta(\Omega)$ is a probability distribution/belief over a finite set of exogenous states Ω
    - We assume different types hold different beliefs

$$p_t \neq p_{t'} \text{ if } t \neq t'$$

3. $C_t \subseteq T$ is a set of types a particular type could imitate
    - We will assume $t \in C_t$ and refer to C_t as the consideration set of type t
    - This set is assumed to be exogenous but in this context could also thought as defined by the interaction in the previous unmodeled stage/mechanism

Model

- For example, for an auction
  - v_t could be the expected utility of a bidder with valuation t in a second price auction,
  - p_t his beliefs over the valuation of other bidders ω, and
  - C_t the valuations he could pretend to have if his true valuation is t

Model

- A contract $x : \Omega \to \mathbb{R}$ is a mapping from states into transfers, with $x(\omega)$ the transfer required in state ω
- The payoff for type t and contract x is

$$v_t - \langle p_t, x \rangle$$

where $\langle p_t, x \rangle = \sum_{\omega \in \Omega} p_t(\omega) x(\omega)$

- A (direct) mechanism $\mathbf{x} = \{x_t : t \in T\}$ is a collection of contracts, one for each type $t \in T$

Incentive compatibility and consideration sets

- Given a mechanism $\mathbf{x}$, agent chooses best contract given his type t and his set of potential deviations C_t
- Hence, incentive compatibility requires to be computed only among types in his consideration set, i.e.,

$$x_t \in \arg\min_{t' \in C_t} \langle p_t, x_{t'} \rangle$$

The mechanism design problem

- We are interested on whether the designer is able to extract all the surplus from the agent using a mechanism $\mathbf{x}$.
- Formally, we say a mechanism achieves full surplus extraction if for all $t \in T$

$$\langle p_t, x_t \rangle = v_t$$

- We say full surplus extraction is feasible if there exists an incentive compatible mechanism $\mathbf{x}$ which achieves full surplus extraction

Full surplus extraction if $C_t = T$ for all $t \in T$

- In a fully rational setting, Crémer and McLean have identified the key condition for full surplus extraction
- Being that the set of beliefs for all types must be linearly independent (convex independence)
- What we will do next is to provide the characterization if types can only deviate among subset of types

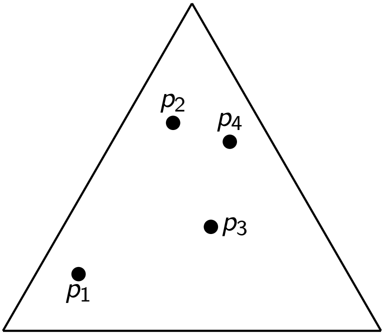
Beliefs and the CM condition

Definition

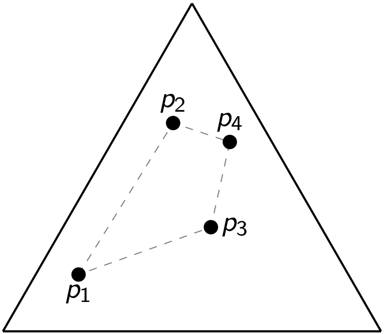
A set of beliefs P satisfies the CM condition if for any $p \in P$, $p \notin co(P \backslash \{p\})$

- $co(P)$ denotes the convex hull of the set P
- For any subset of types $S \subseteq T$ we denote by P^S the set of beliefs for types in S
- Then Crémer and McLean's result could be expressed as requiring that the set P^T satisfies the CM condition
- With consideration sets, P^T satisfying the CM condition is a sufficient condition but very far from necessary

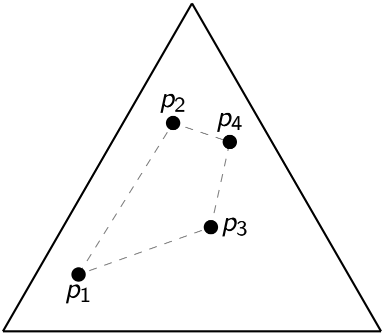
In a picture



In a picture



In a picture

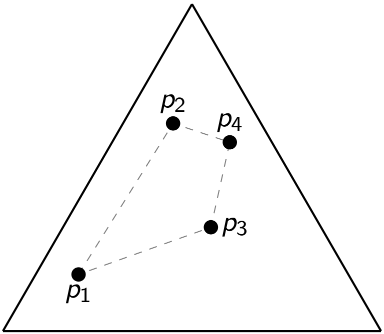


- P^T is convex independent, hence full surplus extraction could be guaranteed

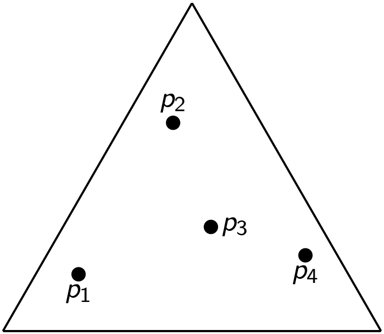
Potential deviators

- In order to determine whether full surplus extraction could be guaranteed or not we need to analyze a particular collection of sets...
- Not the set of potential deviations but its "inverse": the set of potential **deviators**
- Let's denote by $D_t = \{t' \in T : t \in C_{t'} \text{ and } t' \neq t\}$ the set of potential deviators for t

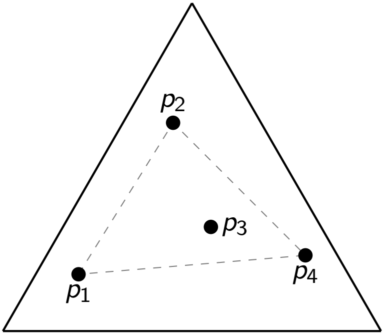
In pictures



In pictures

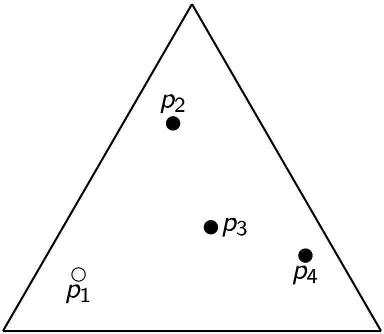


In pictures

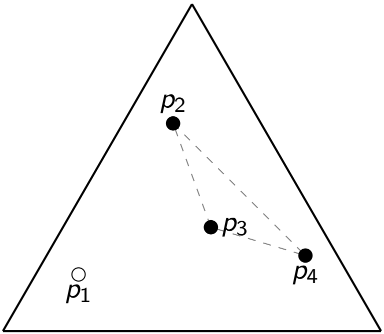


Not convex independent

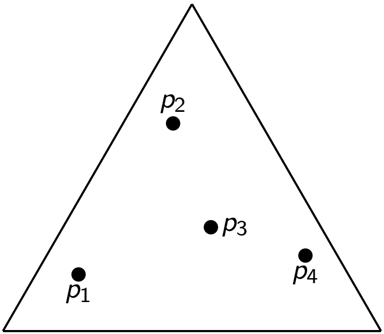
In pictures



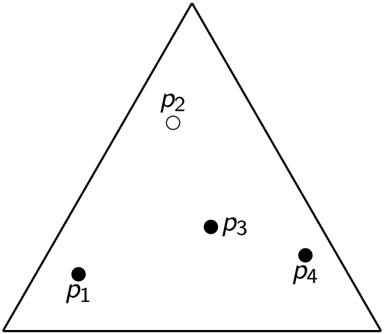
In pictures



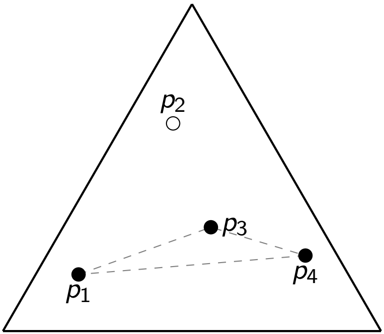
In pictures



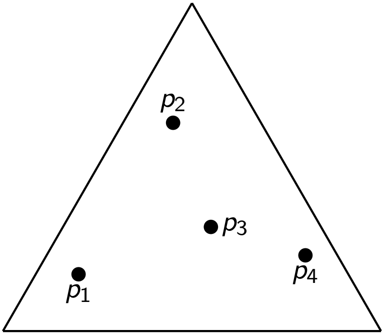
In pictures



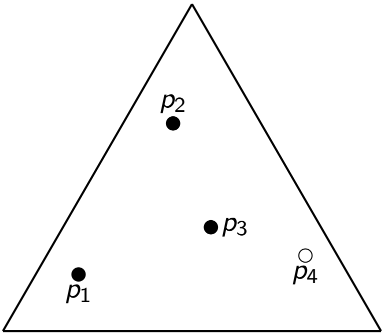
In pictures



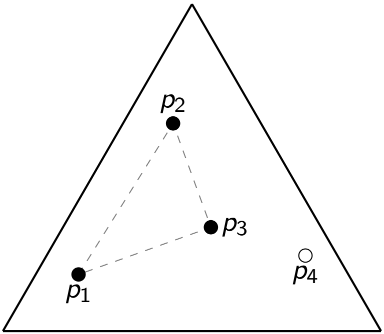
In pictures



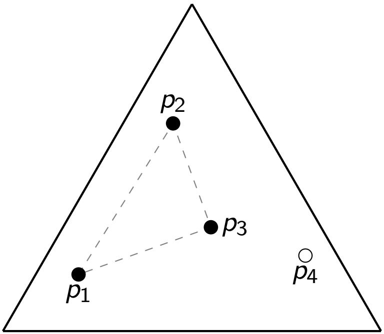
In pictures



In pictures

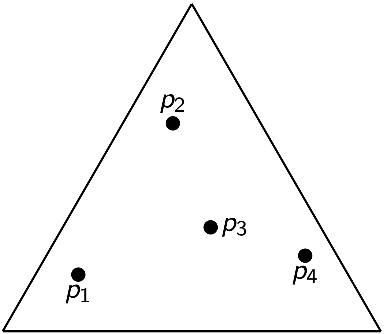


In pictures

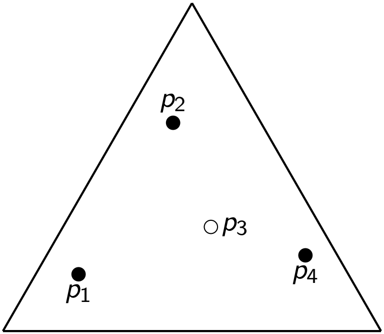


Types 1, 2 and 4 could be separated from the other types

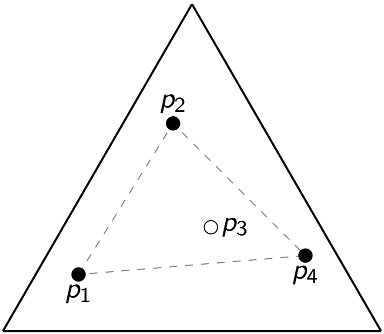
In pictures



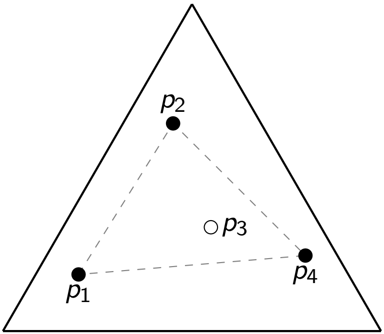
In pictures



In pictures

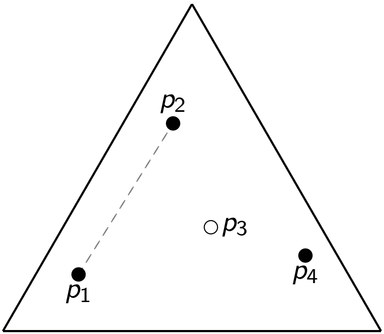


In pictures



Type 3 cannot be separated if all types could imitate him

In pictures



If type 4 cannot imitate 3, then type 3 could be separated from his deviators

From the picture

- The same result holds if any other type can't imitate type 3 here
- Note that what types could be imitated by type 3 doesn't matter
- The consideration set itself is not relevant
- Only its inverse is key

Main result

Theorem

Suppose $p_t \notin co\left(P^{D_t}\right)$ *for all* $t \in T$. *Then full surplus extraction is feasible*

Sketch of proof

- Consider a particular type t
- We want z_t that allows us to separate t from the types that could pretend to be t
- That is,

$$\langle p_t, z_t \rangle = 0$$

$$\langle p_{t'}, z_t \rangle > 0, \quad \forall t' \in D_t$$

- If $p_t \notin co\left(P^{D_t}\right)$ then existence of such a z_t is guaranteed by Farkas' lemma

Sketch of proof

- Then, we build the contract for t as follows

$$x_t = v_t + \alpha_t z_t$$

where $\alpha_t \in \mathbb{R}_+$

- Note x_t satisfies $\langle p_t, x_t \rangle = v_t$ for all and $\langle p_{t'}, x_t \rangle > v_{t'}$ for $t' \in D_t$ (for α_t big enough)
- Repeating the process for all other types, we obtain a collection of contracts with the characteristics described above

Sketch of proof

- Note that $\langle p_{t'}, x_{t'} \rangle = v_{t'}$ and $\langle p_{t'}, x_t \rangle > v_{t'}$ implies that incentive compatibility for type t' with respect to t is satisfied
- Hence, the collection of contracts identified here satisfies the incentive compatibility constraints with respect to the relevant consideration sets, and achieves full surplus extraction

An example

An example with local deviations

- Let $\Omega = \{\omega_0, \omega_1\}$, with ω_0 and ω_1 the "safe" and "unsafe" states respectively
- $p_t = \Pr(\omega_1 | t)$ represents the probability of the unsafe state for each type, i.e., their risk
- Without loss, we will order types according their risk, so higher types have higher risk:

$$p_1 < p_2 < ... < p_N$$

An example with local deviations

- We will assume that each type could only partially falsify their true risk level
- In particular, type t could only pretend to have a higher risk "close" to their true risk level, but not lower
- So, type t could pretend to be types $t' = t + 1$ and $t' = t + 2$, but not $t' > t + 2$ nor $t' < t$

An example with local deviations

- Then, the associated consideration sets are
  - $C_t = \{t, t+1, t+2\}$ for $t < N-1$,
  - $C_{N-1} = \{N-1, N\}$, and
  - $C_N = \{N\}$
- The corresponding deviators sets are
  - $D_t = \{t-2, t-1\}$ for $t > 2$
  - $D_1 = \emptyset$
  - $D_2 = \{1\}$
- Clearly, for each D_t, $p_t \notin co\left(P^{D_t}\right)$, since $p_t > p_{t-1}$ for all t
- So, "full surplus extraction" is feasible here

Sub-environments

Sub-environments

- The example above shows that some structure could be given to the conditions that guarantee full surplus extraction in this context...
- However, interpreting the conditions could be difficult in general settings
- I will present two sub-environments in which the conditions could be easier to interpret:
  - An environment with honest types, and
  - Separable environments

An environment with honest types

An environment with honest types

- Suppose that types could be classified into two groups: behavioral and sophisticated
- Behavioral types always report truthfully: $C_t = \{t\}$
- Sophisticated types are rational, and could deviate to any contract available: $C_t = T$

Corollary

Consider a environment with a set of honest/behavioral types $B \subseteq T$. If $P^{T \setminus B}$ satisfies the CM condition, and $p_t \notin co\left(P^{T \setminus B}\right)$ for every $t \in B$, then full surplus extraction is feasible.

An environment with honest types

- In this case, the condition identified in Theorem 1 could be separated in two conditions
- For sophisticated types, the standard CM condition is required since if full surplus extraction cannot be guaranteed ignoring other types, it cannot be guaranteed after considering all types
- However with respect to behavioral types, the condition is relaxed, and the comparisons are only made with respect to the sophisticated types and not other behavioral types

More results for honest types

- We have can provide two complementary results in this environment

More results for honest types

Corollary

Consider a particular behavioral type $b \in B$. Let x_{-b} be an incentive compatible mechanism for types $t \neq b$. If $p_b \notin co(P^{T \setminus B})$ then there exists a contract x_b such that the contract menu (x_b, x_{-b}) is incentive compatible and $\langle p_b, x_b \rangle = v_b$.

- In short, the condition over the beliefs of a behavioral type allows for full extraction for that particular type even if not possible for sophisticated types

More results for honest types

Proposition

Suppose $P^{T\backslash B}$ satisfies the CM condition. If for all $t \in B$ either $p_t \notin co\left(P^{T\backslash B}\right)$ or

$$v_t \geq \sum_{t' \notin B} \lambda_t(t')v_t,$$

where $\lambda_t \in \Delta(T\backslash B) : p_t = \sum_{t' \notin B} \lambda_t(t')p_{t'}$, then full surplus extraction is feasible.

- If the condition over behavioral types fails, full extraction still feasible if their valuations are "big enough"

Separable environments

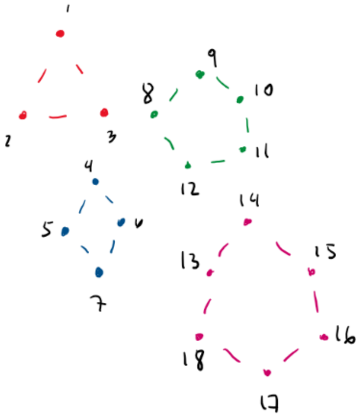
Separable environments

- We say that an environment is separable if there exist a partition of T, $\{T_1, T_2, ...\}$ such that $C_t \subseteq T_i$ for all $t \in T_i$
- Hence, in a separable environment types could be separated into clusters in which types could only deviate among types in the same cluster

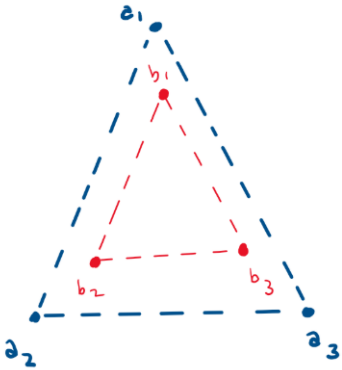
Corollary

Consider a separable environment indexed by $\mathcal{I}$. If P^{T_i} satisfies the CM condition for each $i \in \mathcal{I}$, then full surplus extraction is feasible.

Pictures of separable environments



Pictures of separable environments



Some comments

- Note neither the example nor the environment with honest types are separable
- Hence, there are interesting environments captured by the general model which are not separable

Related literature and Conclusion

Related literature

- Classics: Myerson (1981), Cremer and McLean (1985, 1988), McAfee and Reny (1992)
- Genericity of full surplus extraction: Heifetz and Neeman (2006), Barelli (2009), Chen and Xiong (2011, 2013)
- Recent full surplus extraction: Farinha Luz (2013), Lopomo, Rigotti, and Shannon (2020, 2021), Krahmer (2020), Fu et al (2021), Albert et al. (2022)
- "Behavioral" mechanism design: Eliaz (2002), Severinov and Deneckere (2006), Saran (2011), De Clippel, Saran and, Serrano (2018)
- Consideration sets: Eliaz and Spiegler (2011), Manzini and Mariotti (2014), Fershman and Pavan (2022)

Concluding Remarks

- We characterize the conditions required to guarantee full surplus extraction with "local" deviations
  - I found that the key element to characterize is the set of potential deviators
  - I provide two simpler sub-environments in which the characterization could be applied (*an environment with honest types* and *separable environments*)

Concluding Remarks

- Next steps
  - Beyond the reduced form model
  - General mechanism design problem
  - Endogenous consideration sets

Thanks!