Monopolistic Screening with Buyers Who Sample

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Motivation

- Consumers are exposed to a huge variety of products
- Computational constraints make it impossible for them to have access and evaluate all available alternatives
- Sellers determine product line and pricing taking these limitations into consideration

Goal

- A simple model that
  - captures the tradeoff of increasing variety
  - considers limited computational capacity of consumers
- Analyze the model from the designer/seller perspective

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What I do

$\Rightarrow$ I propose a model in which buyers cannot evaluate all available alternatives presented by the seller

$\Rightarrow$ Instead, they only sample some of the alternatives and then evaluate them

$\Rightarrow$ The main question is how the optimal menu/mechanism looks like in this context

Spoiler alert!

Model: Mussa and Rosen (1978) $+$ boundedly rational buyers

$\downarrow$

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- Single sample result shows how "noise" could drive seller to remove (effective) variety

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Two samples $\Rightarrow$ optimal menu has size $\neq 2$

- Two samples shows that number of samples $\neq$ number of offers in general

Related literature

- Product line design and pricing: Mussa and Rosen (1978), Doval and Skreta (2022)
- Revenue maximization with samples: Dhangwatnotai et al (2015), Babaioff et al (2018), Daskalakis and Zampetakis (2020), Fu et al (2021)
- Sampling/$S(1)$ equilibrium: Osborne and Rubinstein (1998, 2003), Spiegler (2006), García-Echeverri (2021)
- Search: Weitzman (1979), Doval (2018), Ursu et al (2021), Safonov (2022), Fershtman and Pavan (2022)

Main differences with previous literature

- In revenue maximization with samples papers is the seller who faces uncertainty about buyers valuations and uses sampling to improve the mechanism
  - Here uncertainty comes from the menu offered by the seller and it is experienced by buyers
- Search papers focus on optimal choice procedure for a given environment
  - Here behavior of buyers as given and focus on the design problem instead

Model

- Single seller producing a good of quality $q \geq 0$ at cost $q^2/2$
- Continuum of buyers with private valuations $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L > 0$
- Proportion of high valuation buyers $\alpha \in (0, 1)$

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- Seller designs a (finite) menu of quality-price pairs in order to maximize her expected profits
- I will refer to a quality-price pair (q, p) as an offer
- If a buyer with valuation θ accepts an offer (q, p) then he gets payoff $\theta q - p$ while the seller gets $p - q^2/2$
- If the buyer rejects the offer then both parties get zero

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- The number of samples will be exogenous and fixed
- Outside option (i.e., $(0, 0)$) always available and never counted as part of the menu (Hart and Nisan (2019))
- Since duplicating all offers makes no difference, I focus on menus with minimum size

Timing

Seller designs menu

↓

Buyers sample from the menu

↓

Buyers decide whether to accept
one of the sampled offers

- Buyers learn valuation before accepting/rejecting offers
- No uncertainty about value of sampled alternative

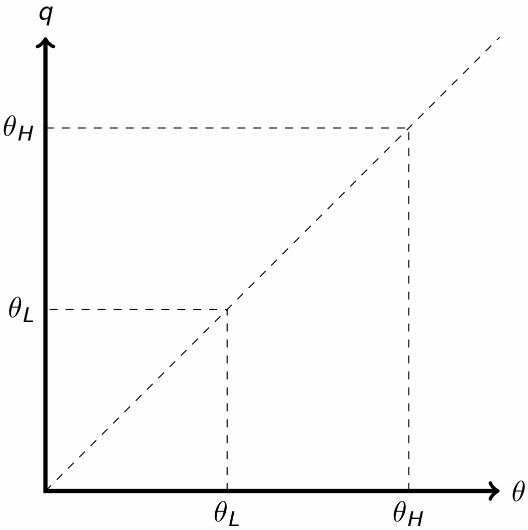
Revisiting the standard model

- Menu is fully observed by the buyers
- The efficient allocation matches quality with the valuation of each type of buyer, i.e., $q = \theta$ for a buyer of type θ
- Profit maximizing menu takes one of two forms either
  - contains only (θ_H, θ_H^2) (and $(0, 0)$), or
  - contains two offers:

$$(\hat{q}_L, \theta_L \hat{q}_L) \text{ and } (\theta_H, \theta_H^2 - (\theta_H - \theta_L)\hat{q}_L)$$

where $\hat{q}_L = \theta_L - \frac{\alpha}{1-\alpha}(\theta_H - \theta_L) < \theta_L$

Standard model in pictures



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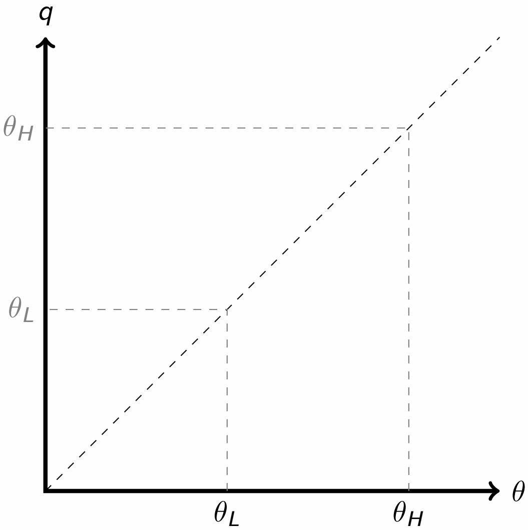

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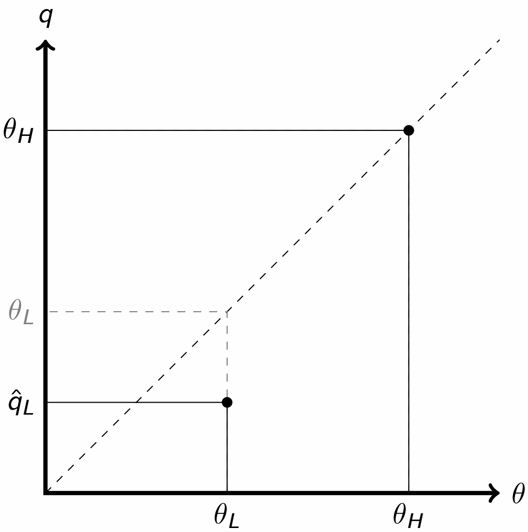

The problem with a single sample

- Given a menu of size m, each offer will be observed with probability $1/m$
- Since each buyer will only be able to draw a single sample, they will only able to compare such sampled offer with their outside option of rejecting the offer
- Hence, there will be no relevant incentive compatibility constraint to satisfy

Main result with a single sample

Proposition

Consider the single sample problem with two valuations. In the optimal menu, the seller always prefer to include a single offer (q^*, p^*).

- Hence, in an environment with a single sample, the *effective variety* offered is reduced
- However, the menu could still contain several offers but all of them must be identical

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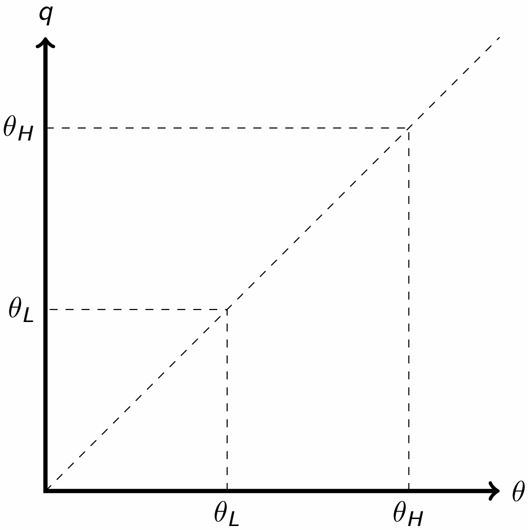
- However, the menu could still contain several offers but all of them must be identical
- Result extends directly beyond binary valuations

Sketch of proof

- Step 1: only "efficient" offers are included in the menu: (θ, θ^2) for some θ
  - No incentive compatibility constraints since only single offer is observed each time
  - If offer with quality q is drawn, for which last type accepting is θ, optimal to price it at θq
  - Then, if offer accepted by $\theta' \geq \theta$, optimal to match efficient quality provision for θ

- Step 2: given that only offers of this form are offered optimal menu is determined by a linear problem
  - Solution involves assigning all mass to "best" offer only

Optimal menu with a single sample (in pictures)



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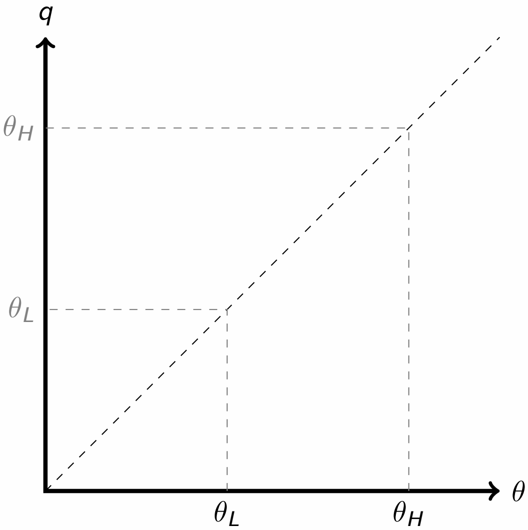

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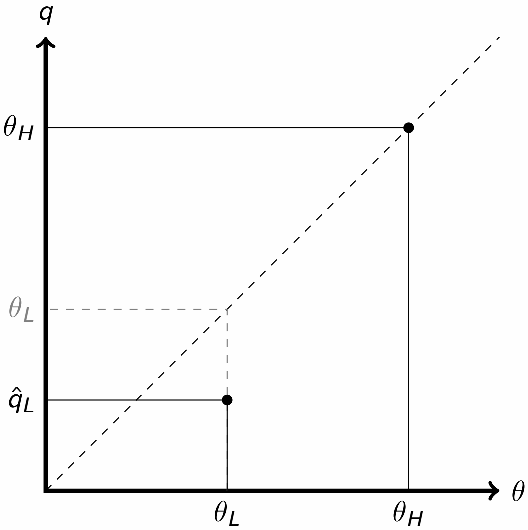

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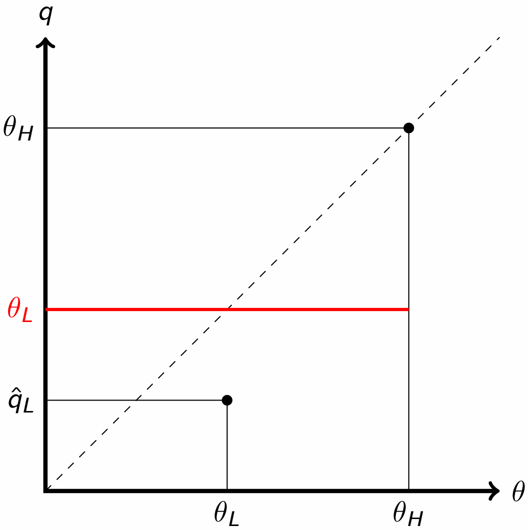

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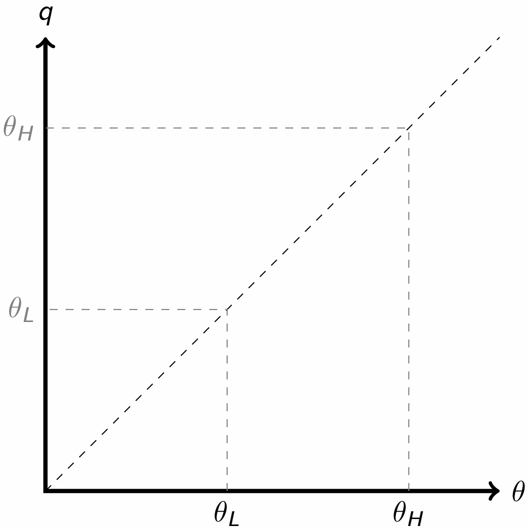

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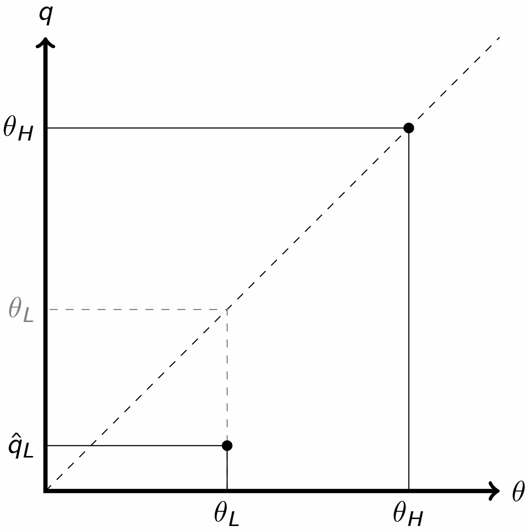

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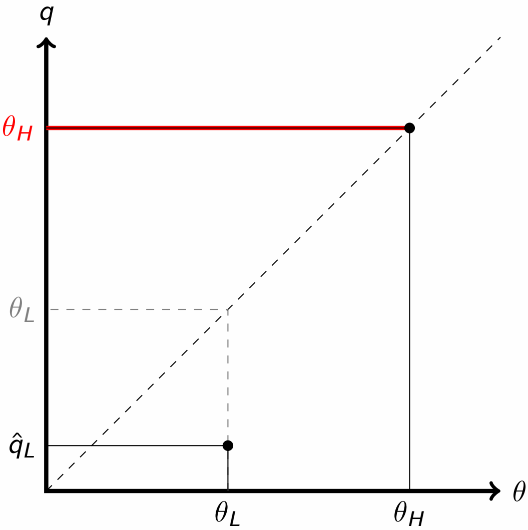

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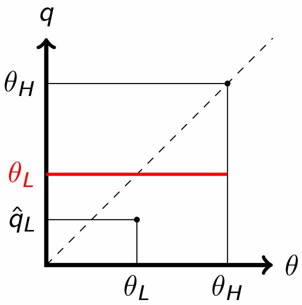
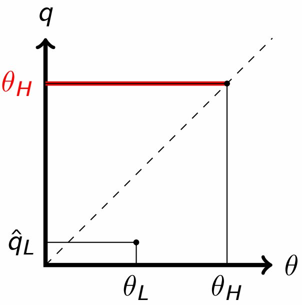


(a) $\alpha < \hat{\alpha}$: optimal menu offers only $q = \theta_L$ (red). All buyers accept the offer.



(b) $\alpha > \hat{\alpha}$: optimal menu offers only $q = \theta_H$ (red). Only buyers with valuation θ_H accept the offer.

Two samples

- For a menu of size m, buyers will observe a single offer i with probability $1/m^2$, and two offers j and k with probability $2/m^2$
- Since more than one alternative would be evaluated with positive probability (unless all offers are identical), there would be relevant incentive compatibility constraints to satisfy now
- This makes the characterization of the optimal menu challenging...

Results with two samples

Lemma

Consider the problem with two samples. Suppose that the optimal menu contains only two offers (q_a, p_a) and (q_b, p_b). Then, the expected profits by menus $\{(q_a, p_a)\}$ and $\{(q_b, p_b)\}$ must be the same.

Proposition

Consider the problem with two samples and two valuations. Then, the optimal menu does not contain only two different offers.

Intuition behind Lemma

- Fix (q_a, p_a) and (q_b, p_b)
- Let R_i the value generated for the seller if buyers observe $i = a, b, ab$
- Let x the probability **a** is drawn
- Consider the following problem for the seller

$$\max_x x^2 R_a + (1-x)^2 R_b + 2x(1-x) R_{ab}$$

- If exists, the interior solution is

$$x^* = \frac{1}{1 + \frac{R_{ab} - R_b}{R_{ab} - R_a}}$$

- Note, $x^* = 1/2 \iff R_a = R_b$

Intuition behind Lemma

- A necessary condition is $R_{ab} > \max\{R_a, R_b\}$ (i.e., there must be gains from using a menu)
- Assume $R_a \geq R_b$
- Starting from a menu only containing **a**, including **b** induces...

$$\begin{array}{ll} \text{“Gain”} & R_a \to R_{ab} \\ \text{“Loss”} & R_a \to R_b \end{array}$$

- x^* balances this tradeoff
  - If **b** drawn with small probability ϵ, more likely to observe $\{a, b\}$ instead of **b** only $\Rightarrow$ overall gain from including **b**
  - If $R_a = R_b$, no cost of including **b**, so optimal to maximize prob. of $\{a, b\}$
  - If $R_a > R_b$, then costly to include **b** and having both with same probability is too costly $\Rightarrow$ optimal to “bias” toward **a**

From Lemma to Proposition and beyond

- For two valuations and two samples, I could show that never optimal to set **a** and **b** such that $R_a = R_b$ (up to a very specific set of parameters)

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- For two valuations and two samples, I could show that never optimal to set **a** and **b** such that $R_a = R_b$ (up to a very specific set of parameters)

- Note lemma doesn't really depend on binary valuations
- Lemma could also be extended beyond 2 samples directly
- Extending the proposition to a more general structure is still work in progress

Concluding remarks

- I presented a model in which a seller interact with boundedly rational buyers which cannot observe the menu designed by her and instead get samples from it
- I showed that the optimal menu when buyers have access to a single sample involves including a single offer, matching the best contract for one type of buyers
- In the case of two samples, I showed that the optimal menu cannot contain only two alternatives, each sampled with probability $1/2$

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What is next?

$\Rightarrow$ Full characterization for two or more samples

$\Rightarrow$ Study the effect of competition on the seller problem

$\Rightarrow$ Allow the seller to use targeted menus/ads

$\Rightarrow$ Alternatives settings: taxes and social insurance systems

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Thanks!

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