Full Surplus Extraction and Consideration Sets

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Motivation

- Traditional mechanism design assumes agents fully understand the mechanisms they are interacting with, evaluating all available alternatives presented to them
- However, complexity embedded both in the environment and different mechanisms could make difficult or impossible for the agent to evaluate all alternatives
- Hence, natural to study environments with partial consideration in which deviations could only be "local"

What I do

- Revisit the classic mechanism design problem of (full) surplus extraction in a setting with bounded-rational agents which can only imitate a subset of types
- Characterize the conditions that guarantee full surplus extraction to be feasible in this setting

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Further details

$\Rightarrow$ My model mixes the reduced form model in McAfee and Reny (1992) and the partially verifiable types in Green and Laffont (1986)

$\Rightarrow$ It is general in the sense that it could be applied to any incentive problem (with transfers and quasilinear preferences)

$\Rightarrow$ I introduce *consideration sets* using partial verifiable types

Preview of results

- Some degree of linear independence required to guarantee full surplus extraction (but weaker than in Crémer and McLean (1985, 1988))

- *Inverse consideration sets* play a key role to determine whether it is feasible or not

Related literature

- **Mechanism design with correlation**: Crémer and McLean (1985, 1988), McAfee and Reny (1992), Farinha Luz (2013), Kramer (2020), Fu et al. (2021), Lopomo et al. (2022)
- **Partial verifiability**: Green and Laffont (1986), Strausz (2017), Reuter (2022)
- **Consideration sets**: Eliaz and Spiegler (2011), Manzini and Mariotti (2014), Caplin et al. (2018), Fershtman and Pavan (2022)
- **Behavioral mechanism design**: Eliaz (2002), de Clippel (2014), de Clippel et al. (2018)

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  3. A *consideration set* $C_t \subseteq T$

- C_t represent the types the agent with type t could convincingly imitate or evaluate in the mechanism

Model

- Here valuations, beliefs, and consideration sets are exogenous and fixed
- As in McAfee and Reny (1992), valuations and beliefs could come from a previous unmodeled interaction between the designer and the agent
- Considerations sets here also come from this previous interaction

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- A contract $x : \Omega \to \mathbb{R}$ determines transfers at each state
- A (direct) mechanism is defined by a collection of contracts $(x_t)_{t \in T}$, where x_t is the contract for type t

Agent's payoffs and incentive compatibility

- Given his type t and contract x, an agent obtains

$$v_t - \langle p_t, x \rangle$$

where $\langle p_t, x \rangle = \sum_{\omega \in \Omega} p_t(\omega) x(\omega)$

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  or equivalently,

$$\langle p_t, x_t \rangle \leq \langle p_t, x_{t'} \rangle, \quad \forall t \in T, t' \in C_t$$

The mechanism design problem

- We will be looking for an incentive compatible mechanisms that collects the full surplus from each type (in expectation), i.e.,

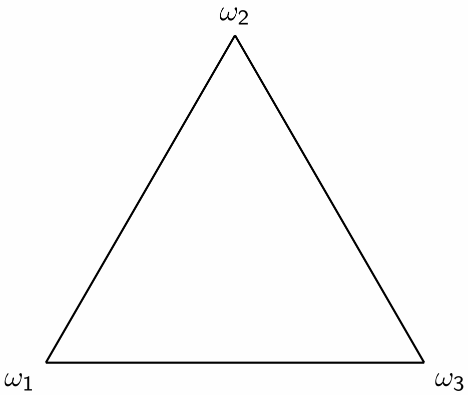
$$\langle p_t, x_t \rangle = v_t$$

  for each $t \in T$
- The goal is to characterize under which conditions such mechanism is guaranteed to exist (for any structure of valuations)

Main idea

- Starting from a mechanism $(x_t)_{t \in T}$, focus on a particular type t
- Consider an alternative contract $\hat{x}_t$ such that $\langle p_t, \hat{x}_t \rangle = 0$ and $\langle p_{t'}, \hat{x}_t \rangle > 0$ for $t' \neq t$
- Note the contract $(x_t + \hat{x}_t)$ makes no difference for t (in terms of incentives and revenue) but punishes every other type t' which tries to imitate t
- Can we find such $\hat{x}_t$?

Graphical representation of the main idea



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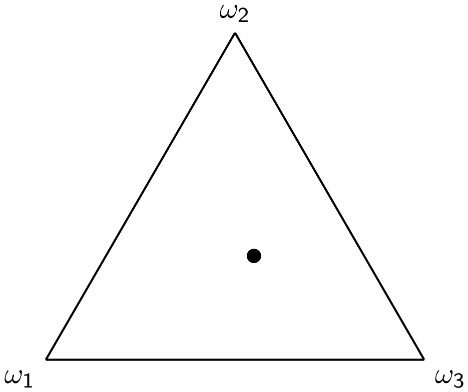

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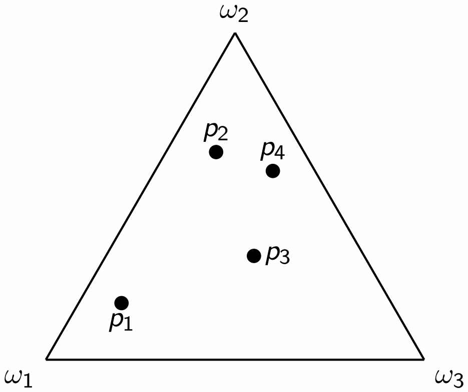

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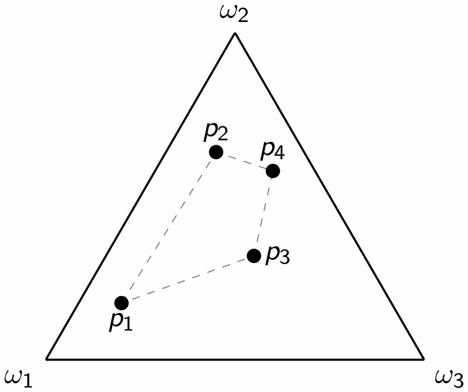

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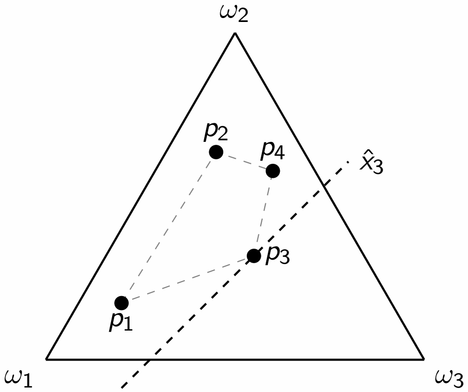

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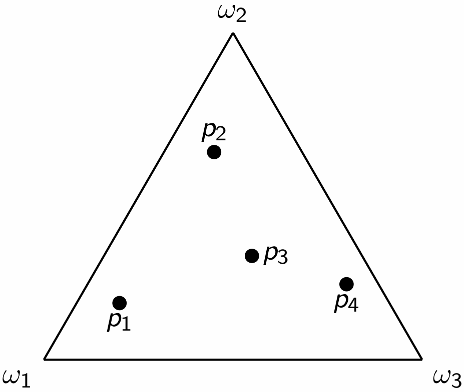

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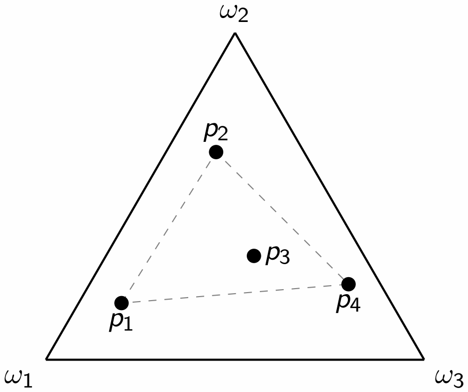

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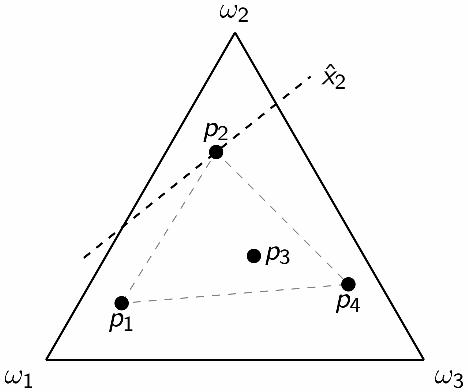

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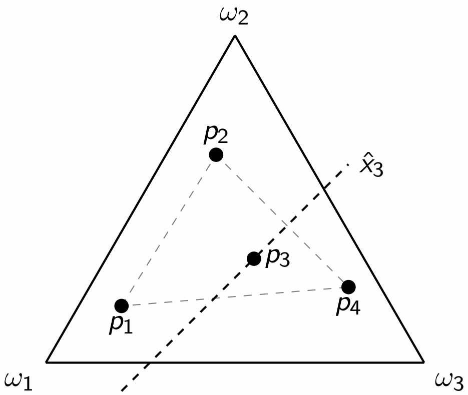


Set of beliefs and the CM condition

- Let P^X be the set of beliefs associated with a subset of types $X \subseteq T$, i.e., $P^X = \{p_t \in \Delta(\Omega) : t \in X\}$
- A set of beliefs P satisfies the CM condition if for any $p \in P$, $p \notin co\,(P \backslash \{p\})$

- Under full-consideration, Crémer and McLean (1985, 1988) showed that if P^T satisfies the CM then full surplus extraction is feasible

Inverse Consideration Sets

- The main result shows that the key elements to characterize are what I call *inverse consideration sets*, which represents the types that consider type t as a potential deviation
- Formally, the inverse consideration set for type t is defined as

$$D_t = \{t' \in T : t \in C_{t'} \text{ and } t' \neq t\}$$

- Using the inverse consideration sets, the incentive compatibility constraints could be rearranged as

$$\langle p_t, x_t \rangle \leq \langle p_{t'}, x_t \rangle\,, \quad \forall t \in T, t' \in D_t$$

Main result

Theorem

Suppose $p_t \notin co\left(P^{D_t}\right)$ for all $t \in T$. Then full surplus extraction is feasible.

- Note P^{D_t} is the set of beliefs associated with types t' which are not t but can pretend to be t
- While the structure of the inverse considerations sets D_t in key, there is no direct condition over C_t above
- P^T satisfying the CM condition is sufficient but not required

Sketch of proof

- Consider a particular type t
- We want z_t that allows us to separate t from the types that could pretend to be t
- That is,

$$\langle p_t, z_t \rangle = 0$$

$$\langle p_{t'}, z_t \rangle > 0, \quad \forall t' \in D_t$$

- If $p_t \notin co\left(P^{D_t}\right)$ then existence of such a z_t is guaranteed by Farkas' lemma
- Then, we build the contract for t as follows

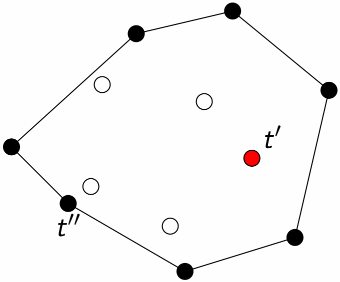
$$x_t = v_t + \alpha_t z_t$$

  where $\alpha_t \in \mathbb{R}_+$
- Note x_t satisfies $\langle p_t, x_t \rangle = v_t$ and, for α_t big enough, $\langle p_{t'}, x_t \rangle > v_{t'}$ for all $t' \in D_t$
- Repeating the process for all other types, we obtain a collection of contracts with the required characteristics

Sketch of proof

- Note that $\langle p_{t'}, x_{t'} \rangle = v_{t'}$ and $\langle p_{t'}, x_t \rangle > v_{t'}$ implies that incentive compatibility for type t' with respect to t is satisfied
- Hence, the collection of contracts identified here satisfies the incentive compatibility constraints with respect to the relevant consideration sets, and achieves full surplus extraction

Why inverse CS and not CS?



Why inverse CS and not CS?

- Full extraction fails here even if t' considers his own contract only: $C_{t'} = \{t'\}$
- Moreover, IC it fails as long as there is another type which prefers $x_{t'}$ to his own contract
- "Reducing" $C_{t'}$ does not alleviate this concern
- However, dropping t' from these types' considerations set does

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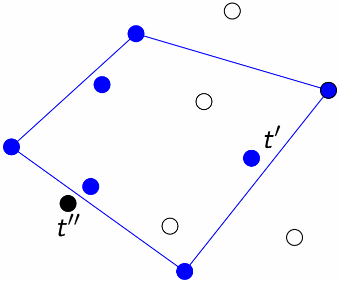

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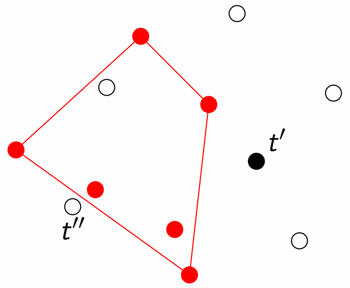

Some examples

- Local deviations: e.g. $C_t = \{t, t+1, t+2\}$
- Honest vs sophisticated types: $C_t = \{t\}$ for honest t and $C_t = T$ for sophisticated t
- Separable environments: $C_t \subseteq T_i$ for $t \in T_i$, and $\{T_i\}_i$ a partition of T

Local deviations in an insurance market

- Consider the problem of a insurance company facing a continuum of agents
- Let $\Omega = \{\omega_0, \omega_1\}$, with ω_0 and ω_1 the "safe" and "unsafe" states respectively
- $p_t = \Pr(\omega_1 | t)$ represents the probability of the unsafe state for each type, i.e., their risk
- Without loss, we will order types according their risk, so higher types have higher risk:

$$p_1 < p_2 < \ldots < p_N$$

- We denote by v_t the maximum or targeted surplus that the company could get in expectation from an agent of type t

Local deviations in an insurance market

- Suppose that each type could only partially falsify their true risk level
- In particular, type t could only pretend to have a higher risk "close" to their true risk level, but not lower
- So, type t could pretend to be types $t' = t + 1$ and $t' = t + 2$, but not $t' > t + 2$ nor $t' < t$

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- The question is whether the company could collect the targeted surplus in this setting
- We assume everybody is risk neutral, and that being insured by the company indeed has an implicit value which exceeds v_t for an agent with type t

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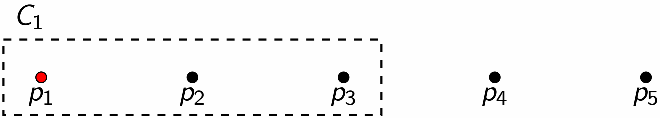

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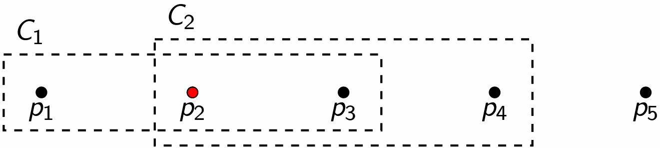

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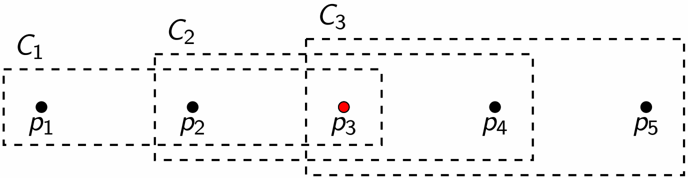

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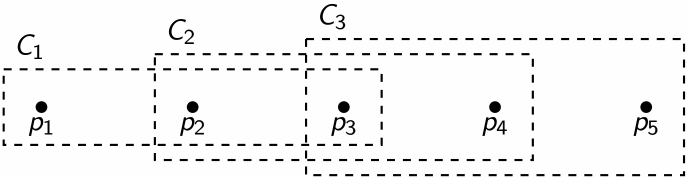

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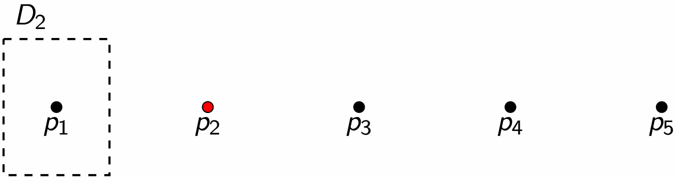

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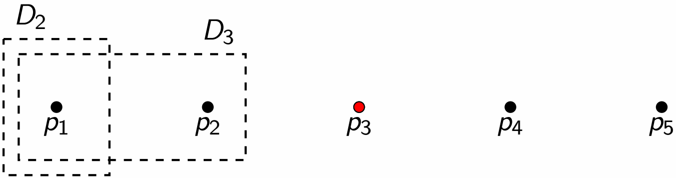

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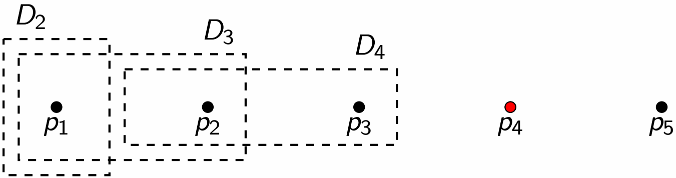

Local deviations in an insurance market

- Then, the associated consideration sets are
  - $C_t = \{t, t+1, t+2\}$ for $t < N-1$,
  - $C_{N-1} = \{N-1, N\}$, and
  - $C_N = \{N\}$
- while the inverse consideration sets are
  - $D_t = \{t-2, t-1\}$ for $t > 2$
  - $D_1 = \emptyset$
  - $D_2 = \{1\}$
- Clearly, for each D_t, $p_t \notin co\left(P^{D_t}\right)$, since $p_t > p_{t-1}$ for all t
- So, "full surplus extraction" is feasible here

Local deviations in an insurance market

- What about two-sided deviations?
- With a binary state, allowing deviations in an interval makes full surplus extraction impossible:

$$p_{t-1} < p_t < p_{t+1} \Rightarrow p_t = \alpha p_{t+1} + (1-\alpha) p_{t-1} \text{ for some } \alpha \in (0,1)$$

- However, if there are more than two states then full surplus extraction could be feasible for alternative orderings

Honest and sophisticated types

- Suppose that types could be classified into two groups: honest and sophisticated
- Honest types always report truthfully: $C_t = \{t\}$
- Sophisticated types are fully rational, and could deviate to any contract available: $C_t = T$

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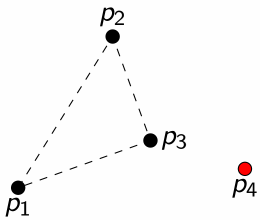
Corollary

Consider a environment with a set of honest types $H \subseteq T$. If $P^{T \setminus H}$ satisfies the CM condition, and $p_t \notin co\left(P^{T \setminus H}\right)$ for every $t \in H$, then full surplus extraction is feasible.

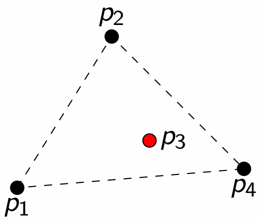
Honest and sophisticated types

- In this case, the condition identified in Theorem 1 could be separated in two conditions
- For sophisticated types, the standard convex independence condition is required since if full surplus extraction cannot be guaranteed ignoring types in B, it cannot be guaranteed after considering all types in T
- However with respect to honest types, the condition is relaxed, and the comparisons are only made with respect to the sophisticated types and not other behavioral types

Honest and sophisticated types

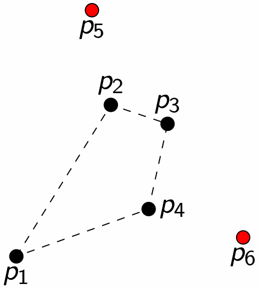


(a) Only $t = 4$ being an honest type



(b) Only $t = 3$ being an honest type

Honest and sophisticated types



An auction with behavioral types and correlation

- N bidders competing for a single item
- Valuations $(\theta_i)_{i\in\{1,2,\ldots,N\}}$ jointly distributed according to F
- Correlated valuations: $F(\cdot|\theta_i) \neq F(\cdot|\theta_i')$
- Payoff from allocation q and transfer x, given valuation θ_i: $\theta_i q - x$
- Efficient allocation: $q_i = 1$ iff $\theta_i > \theta_j, \forall j \neq i$
- Behavioral bidders can only report true valuation θ_i, no restrictions for other bidders
- Behavioral status determined by their valuation, i.e., there are some valuations associated only with behavioral bidders

An auction with behavioral types and correlation

- Let's focus on the problem of a single bidder i
- Mapping the elements of this auction into the notation of the main model:

$$\begin{array}{ll} \theta_i \to t & F(\theta_{-i}|\theta_i) \to p_t(\omega) \\ \theta_{-i} \to \omega & E_{\theta_{-i}}(\theta_i q_i(\theta)|\theta_i) \to v_t \end{array}$$

Proposition

Consider the auction environment. Let B_i the set of behavioral types for bidder i. If for all bidders $i \in N$, and valuations $\theta_i \in \Theta_i$,

$$F(\cdot|\theta_i) \notin co(\{F(\cdot|\theta_i') : \theta_i' \notin B_i \text{ and } \theta_i' \neq \theta_i\})$$

then the optimal mechanism achieves full surplus extraction.

Separable environments

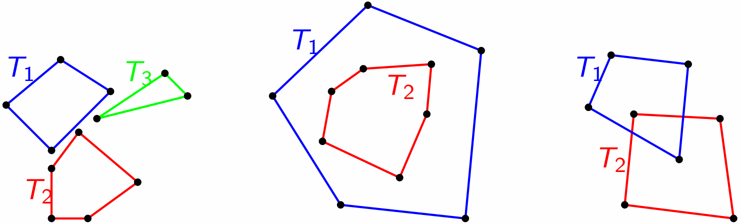
- We say that an environment is separable if there exist a partition of T, $\{T_1, T_2, ...\}$ such that $C_t \subseteq T_i$ for all $t \in T_i$
- Hence, in a separable environment types could be separated into groups in which types could only deviate among types in the same group

Corollary

Consider a separable environment indexed by $\mathcal{I}$. If P^{T_i} satisfies the CM condition for each $i \in \mathcal{I}$, then full surplus extraction is feasible.

- This result only identifies a sufficient condition for full surplus extraction in this case

Separable environments



(a) Fully separated groups

(b) Convex hull of a group containing another group

(c) Overlapping convex hull for different groups

Also in the paper...

- Why revelation principle fails in this context
- An example in which consideration sets are explicitly determined by a previous interaction and are based in the allocation associated with different types

Concluding remarks

- I studied a mechanism design setting with correlation and a bounded-rational agent
- I characterized the conditions required to guarantee full surplus extraction in this setting
- While framed as revenue maximization, results apply to the implementation of any payoff structure or allocation rule more broadly

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Main contribution

· Extending McAfee and Reny (1992) to a flexible behavioral environment, relaxing full-consideration

· Proposing a model which decouples *consideration sets* and *inverse consideration sets*, showing the importance of the later

Summary

McAfee and Reny (1992) + Green and Laffont (1986)

Types	$\rightarrow$	valuations	$+$	beliefs	$+$	consideration sets
t		v_t		p_t		C_t

Inverse consideration set: $D_t = \{t' \neq t : t \in C_{t'}\}$

Theorem: Suppose $p_t \notin co\left(P^{D_t}\right), \forall t \in T$. Then full surplus extraction is feasible.

Thanks!

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