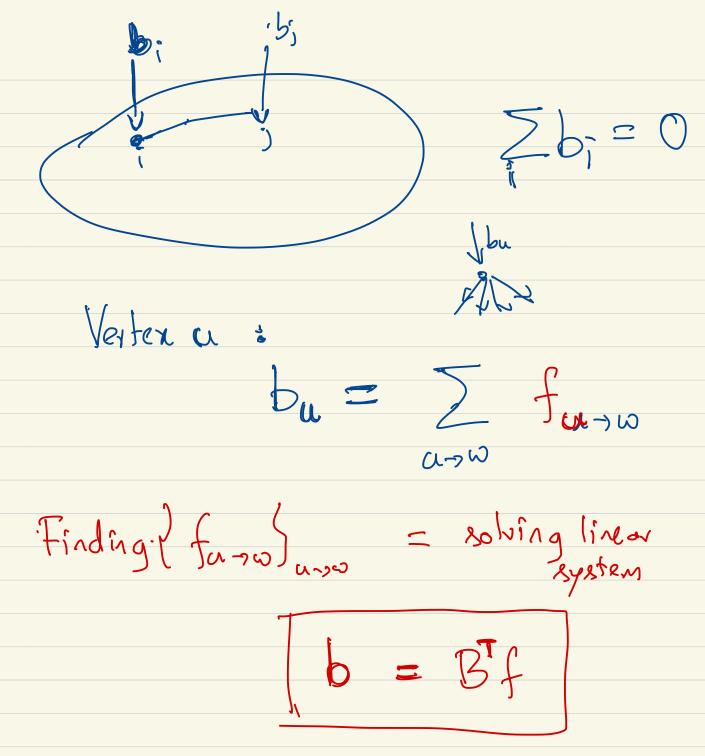
- KVL/KPL Sum of potential differences on acycle -> II2. R12 + I23. R23 + I34. R39 + I45 R95 +17181=0 - Ohms law: VA MULL OC TVA-Vo= IR Way The Carte of t General Flow

VA-Vc = I=IAC'RAC VA-Vc= IAB'RAB+IB'RSC = 2

Potentials > l'Corrents on all edges B: ¿Voltages] Rn - Rm celges verter \_A >B 001000=1000 N = flows Set of electrical Flows = column spor(B) 



colspon(B) Subspace of electrical 16000 Circulations No correct leaving or entering the network. + (irculations Space = Electrical A) 15600

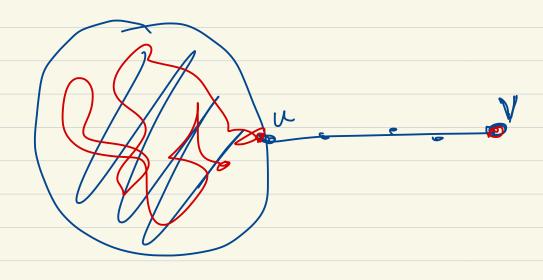
Suppose f lectrical floors (=) Licol Spon (B) (=) B<sup>7</sup>f = 0 (=) boundary currents Energy of a flow = \[ \frac{2}{\lambda\_{u=w}} = \lambda \int \lambda \rangle Thmo Suppose we fix bER Among all flows (not just electrical) h nt Bh=b electrical flow minimises energy

Proof: Given any flow h sit  $B^T h = b$ h= he + hc e lectrical circulation "B"h = BThe + BThc Bhe = Bh=b Energy of h, 11 h 11 = ( | hell + 1/hell > || he||2

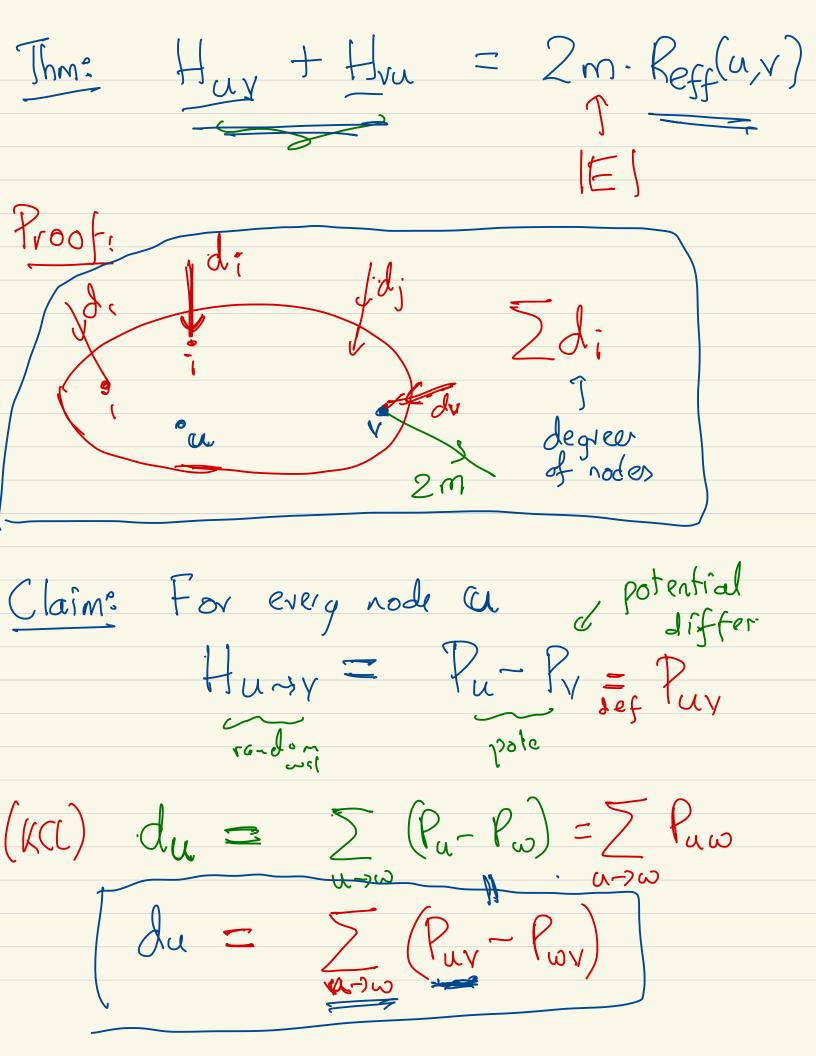
Effective Resistances 2 Ref(A,B) energy of onit Reff (A,B) of covert into A leaving B potential differences Adding edges con only Corallary, reduce effective venirs don €

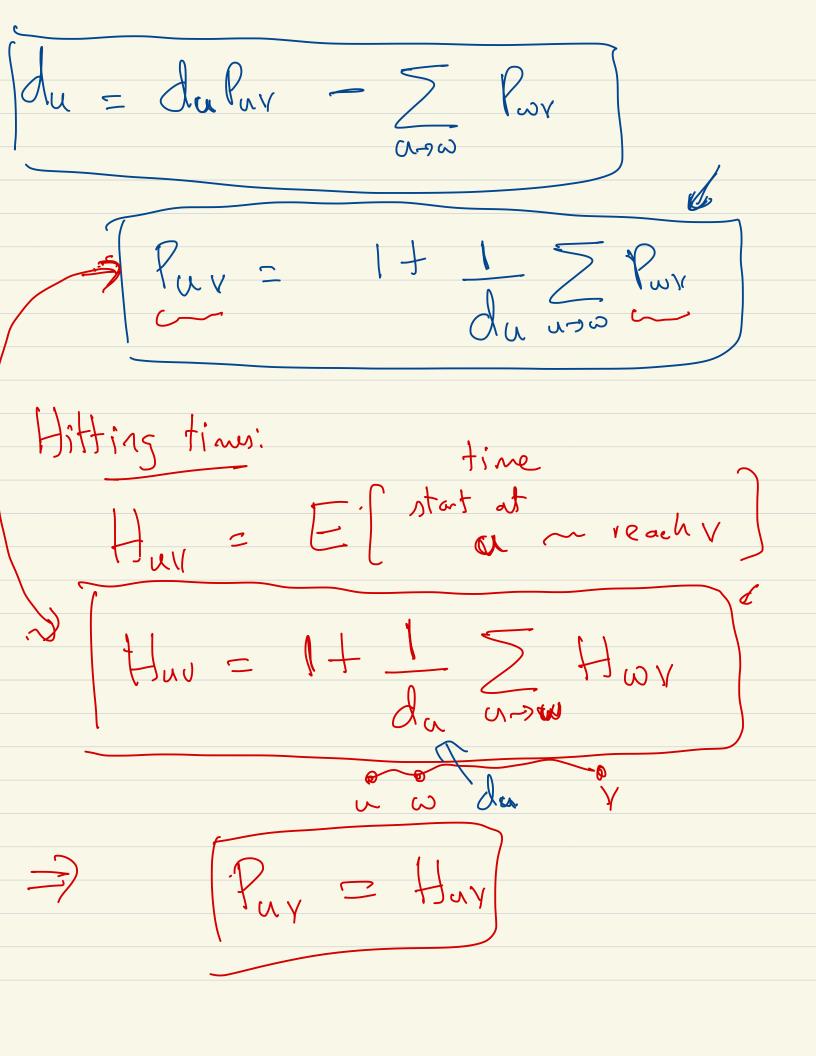
## Hitting Time: (u->v)

= Expected time taken by ar.walk storting at a to reach V.



Huv = lorge Hun = gradh





di (A) Ø REVERSE ALC CURRENTS (c)

A+C)

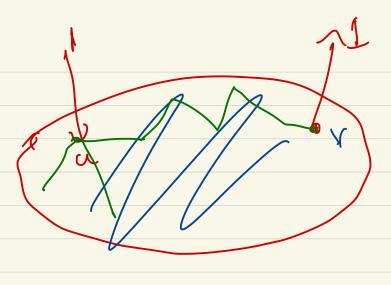
2m

 $P_{uv} = (2m) \cdot R_{tff}(u,v)$ 

Hant Hyou = 2m REFF (a, V)

Corollan: REFF (4,4) satisfy
Corollan: REFF (4,4) Ratisfy finiongle inequality (metric on groph)
(metric on groph)
Proof: 2m: Reff(u,v) = expected time
$R_{EFF}(uv) + R_{EFF}(vw) > R_{EFF}(u,w)$
Yedse (u,v)
REFELUIV) < 1
THE PP C 1 / C I
=> Huy + Hua = 2.m REFF(u,y)
$\leq 2m$

In any graph G Given a gropha, let Ma = uniform distribution over spoming trees of growth



Pick a uniform sponning tree T

- Route 1 unit from a >v

In tree T

For

Eff = electrical flow takes 1 unit

Potential difference between u~v Reff (a, x) =

If 
$$(u,v) \in tree T$$

$$f_{T}(u,v) = 1$$

$$u \rightarrow v \notin tvee T$$

$$f_T(u,v) = 0$$

$$\sum_{G,v} R_{eff}(\alpha,v) = n-1$$

