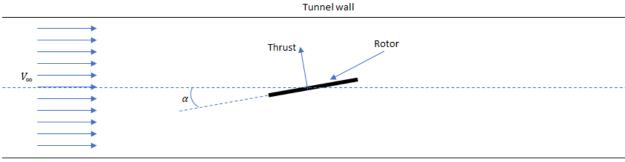
Assigned: 03/30/2023, Due: 04/17/2023

You are working as part of a team responsible for executing a wind tunnel test of a new rotor design that your company has developed. As part of this effort, your technical lead engineer would like to have an analysis ready for expected results (expected θ_0 , θ_{1c} , θ_{1s} , α , $Total\ Power$ to setup the rotor in tunnel 'trim' at hover, 25 and 50 knots). The test is to be performed for an articulated rotor (can be idealized as flap only, centrally hinged) with a radius of 5 ft operating at a tip speed of 650 ft/sec (chord .4 ft, 3 blades, -8 deg root to tip linear twist). The tunnel 'trim' is as follows:

- 1.) Use cyclic blade pitch inputs to zero out longitudinal and lateral flapping
- 2.) Use collective blade pitch input to obtain a certain $\frac{c_T}{\sigma}$ (target $0.005 \le C_T \le 0.01$)
- 3.) Use rotor shaft incident angle (α) to obtain specific C_X (Force along wind tunnel axis) (target $0.05 \le C_X \le 0.1$

As a simplification your team has decided to assume that blade flapping can be ignored in the inflow equation as well as to calculate C_X . You and your team are looking to win the engineer team of year away and to aid in that you are planning on presenting you results for a varied range of flap frequency.



Tunnel wall

Relevant equations

$$F_{\chi} = T \sin \alpha$$

$$C_{\chi} = \frac{F_{\chi}}{0.5\rho V_{\infty}^{2} A}$$

$$\lambda = \mu \tan \alpha + \frac{C_{T}}{2\sqrt{\mu^{2} + \lambda^{2}}}$$

$$C_{T} = \frac{\sigma c_{l_{\alpha}}}{2} \left[\frac{\theta_{0}}{3} \left(1 + \frac{3}{2} \mu^{2} \right) + \frac{\theta_{tw}}{4} (1 + \mu^{2}) + \frac{\mu}{2} \theta_{1s} - \frac{\lambda}{2} \right]$$

$$C_{Q} = \frac{\sigma c_{l_{\alpha}}}{2} \left[\frac{\lambda \theta_{0}}{3} + \frac{\lambda \theta_{tw}}{4} - \frac{\lambda^{2}}{2} - \frac{\left(\beta_{1c}^{2} + \beta_{1s}^{2}\right)}{8} - \frac{\mu^{2}}{2} \left(\frac{\beta_{0}^{2}}{2} + \frac{3\beta_{1c}^{2}}{8} + \frac{\beta_{1s}^{2}}{8} \right) + \frac{c_{d_{0}}}{4c_{l_{\alpha}}} (1 + \mu^{2}) - \frac{\mu \lambda \beta_{1c}}{2} - \frac{\mu \theta_{0} \beta_{1s}}{3} \right]$$

$$\left[\frac{8\nu_{\beta}^{2}}{3} - \frac{\theta_{0}^{2}}{3} + \frac{\beta_{0}^{2}}{3} + \frac{$$

$$\begin{bmatrix} \frac{8\nu_{\beta}^{2}}{\gamma} & 0 & 0 \\ \frac{4}{3}\mu & \frac{8(\nu_{\beta}^{2}-1)}{\gamma} & \left(1+\frac{\mu^{2}}{2}\right) \\ 0 & -\left(1-\frac{\mu^{2}}{2}\right) & \frac{8(\nu_{\beta}^{2}-1)}{\gamma} \end{bmatrix} \begin{bmatrix} \beta_{0} \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} (1+\mu^{2})\theta_{0} + \left(\frac{4}{5} + \frac{2}{3}\mu^{2}\right)\theta_{tw} + \frac{4}{3}\mu\theta_{1s} - \frac{4}{3}\lambda \\ \left(1+\frac{\mu^{2}}{2}\right)\theta_{1c} \\ \frac{8}{3}\mu\theta_{0} + 2\mu\theta_{tw} + \left(1+\frac{3}{2}\mu^{2}\right)\theta_{1s} - 2\mu\lambda \end{bmatrix}$$

 $\alpha \rightarrow Incidence \ angle$

 $V_{\infty} \rightarrow Tunnel\ velocity$

 $\mu \rightarrow Advance\ ratio$

 $C_T \rightarrow Coefficient \ of \ thrust$

 $C_Q \rightarrow Coefficient \ of \ torque$

$$\rho \to 0.002378 \frac{sl}{ft^3}$$

 $T \rightarrow Thrust$

 $\lambda \rightarrow Total\ Inflow\ ratio$

 $F_x \rightarrow$ Force along wind tunnel axis generated by rotor

 $C_X \to Coefficient of F_x$

 $\sigma \rightarrow Solidity$

 $v_{\beta} \rightarrow Non - dimensional flap frequency$

 $\gamma \rightarrow Rotor\ lock\ number$

 $\theta_{tw} \rightarrow Linear \ blade \ twist \ rate$

 $\beta_0, \beta_{1c}, \beta_{1s} \rightarrow Blade\ coning, longitudinal\ and\ lateral\ flapping\ angle$

 $\theta_0, \theta_{1c}, \theta_{1s} \rightarrow Rotor\ collective, lateral\ and\ longitudinal\ cyclic\ pitch\ angle$

 $c_{l_{\alpha}} \rightarrow Airfoil \ lift \ curve \ slope \ (assume \ 2\pi)$

 $c_{d_0} \rightarrow Airfoil steady coefficient of drag$

 $A \xrightarrow{\circ} Rotor disk area$