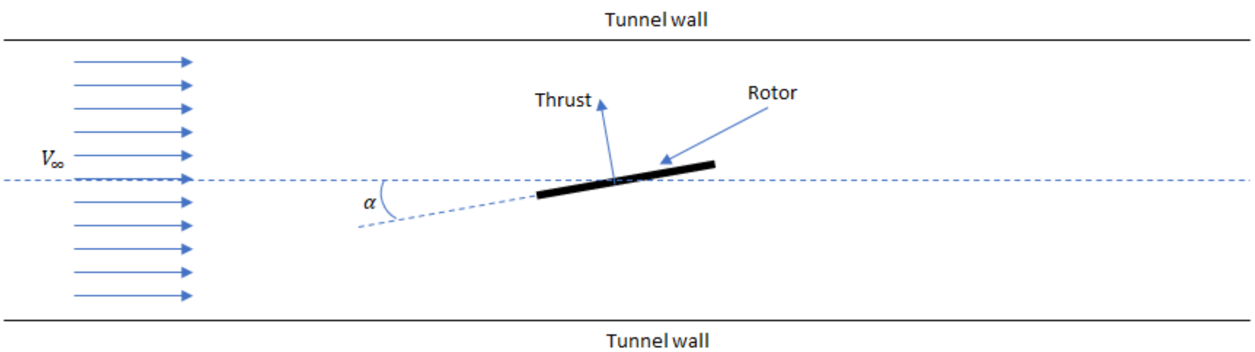
Homework #2 **MEAM 5460 - Spring 2023**

Assigned: 02/21/2023, Due: 02/28/2023

Your team is currently getting ready to setup a wind tunnel test to measure the induced velocity generated by the rotor of your company's awesome new VTOL vehicle. As part of the test you have been asked to provide an estimate of what the induced velocity and associated induced power to expect (both Imperial dimensional units). The wind tunnel rotor model diameter is 5 ft operating at a range of tip speeds between 400 to 525 ft/sec. The tests are to be performed for a constant thrust of 445 N and tunnel speed is to be varied from hover to 50 knots with the rotor setup for a -10.0, 0.0 and 10.0 deg incidence angle with respect to the tunnel axis (angle shown below is positive direction). To go the extra mile, you are planning on providing your comments on the trends observed in your report.



Relevant equations

$$\lambda = \mu \tan\alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

$$\frac{P}{P_h} = \frac{\mu}{\lambda_h}\tan\alpha + \frac{\lambda_h}{\sqrt{\mu^2 + \lambda^2}}$$

$$\lambda = \mu\, tan\, \alpha + \lambda_i$$

$$P_h = T v_h$$

$$\lambda_h = \sqrt{\frac{C_T}{2}}$$

$T \rightarrow Thrust$
$P \rightarrow Induced\ Power$
$P_h \rightarrow Induced\ Power\ in\ hover$
$\lambda \rightarrow Total\ Inflow\ ratio$
$\lambda_i \rightarrow Induced\ inflow\ ratio$
$\alpha \rightarrow Incidence\ angle$
$\mu \rightarrow Advance\ ratio$
$\lambda_h \rightarrow Induced\ inflow\ ratio\ in\ hover$
$v_h \rightarrow\ Induced\ inflow\ velocity\ in\ hover$
$C_T \rightarrow Coefficient\ of\ thrust$
$\rho \rightarrow 0.002378\ sl/ft\hat{}3$

MEAM5460 HW2

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Abstract

https://github.com/nruhl25/HoveringVehicleDesign

1 Introduction

In this report, I provide an estimate of the induced velocity, v_i, and total induced power, P, that we expect to measure in our upcoming tests of our awesome new VTOL vehicle. As stated in the problem formulation attached above, the rotor model diameter is 5 ft and it produces a thrust of 100.04 lbf. Since I am using momentum theory in this report, and we assume that the rotor generates a constant 100.04 lbf of thrust, tip speed does not come into play in this analysis. More specifically, we will see below that the dimensional induced velocity v_i is independent of tip speed, but the non-dimensional quantity λ_i is not. The results presented below are valid for the full range of tip speeds that we plan to test: from 400 ft/sec to 525 ft/sec.

In the analysis below, I calculate the induced velocity and induced power for 3 different incidence angles, $\alpha = -10.0°, 0.0°, \text{and } 10.0°$, and vary the tunnel speed, or freestream velocity v_∞, from zero to 50 knots (84.4 ft/sec).

2 Equations

The total inflow ratio is the algebraic sum of freestream velocity perpendicular to the rotor disc and induced inflow:

$$\lambda = \mu \tan \alpha + \lambda_i \tag{1}$$

where λ_i is the induced inflow ratio,

$$\lambda_i = \frac{v_i}{\Omega R}. \tag{2}$$

Momentum theory predicts the inflow ratio to be

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}} \tag{3}$$

where μ is the advance ratio and C_T is the coefficient of thrust:

$$\mu = \frac{v_\infty \cos(\alpha)}{\Omega R} \tag{4}$$

$$C_T = \frac{T}{\rho A(\Omega R)^2}. \tag{5}$$

Equation 3 must be solved iteratively for λ. In my Fortran code linked in the abstract, I use the Newton-Raphson method to solve for λ and defined the convergence criterion as

$$\epsilon = \left| \frac{\lambda_{n+1} - \lambda_n}{\lambda_{n+1}} \right| = 0.0005. \tag{6}$$

Once λ is solved, the *total* induced power can easily be determined by

$$P = P_h \left(\frac{\mu}{\lambda_h} \tan\alpha + \frac{\lambda_h}{\sqrt{\mu^2 + \lambda^2}} \right) \tag{7}$$

where the induced quantities in hover are defined as

$$\lambda_h = \sqrt{\frac{C_T}{2}} \tag{8}$$

$$P_h = T v_h \tag{9}$$

$$v_h = \lambda_h(\Omega R). \tag{10}$$

3 Results and Discussion

The trends for induced velocity v_i and induced power P that we expect to see on test day are shown in the Figures 1 and 2 below. I also included a plot of P/P_h in Figure 3 for ease of comparison to hover.

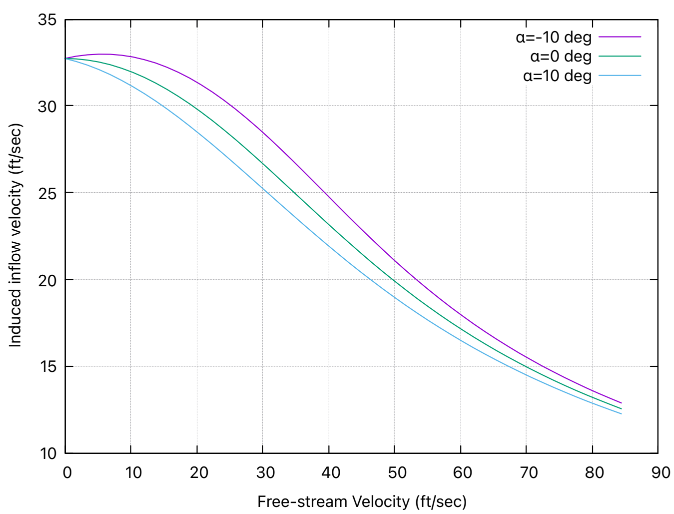

Figure 1: Induced velocity v_i as a function freestream velocity v_∞

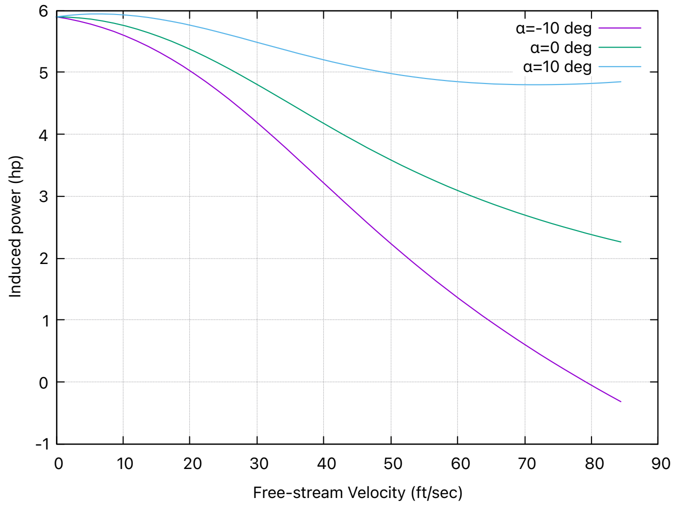

Figure 2: Induced power P as a function of freestream velocity v_∞

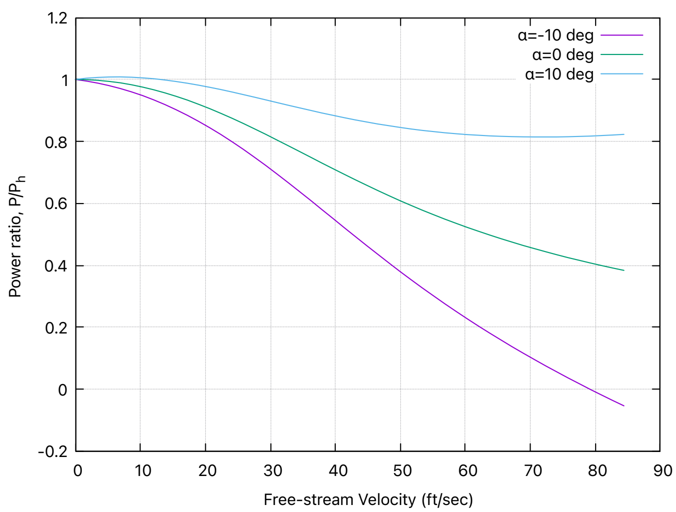

Figure 3: P/P_h as a function of freestream velocity v_∞

I present results for the dimensional quantities v_i and P not only because we are preparing for testing, but more specifically because these quantities are independent of tip speed (in terms of momentum theory). Therefore, these results apply for all ΩR from 400 ft/sec to 525 ft/sec. This independence can be seen explicitly by factoring ΩR out of Equations 3 and 7 (after substituting the expression for λ).

It is interesting to note that the non-dimensional induced inflow λ_i is not independent of tip speed; it is a function of both free-stream velocity and tip speed: $\lambda_i = f\left(\Omega R, v_\infty\right)$. For a given free-stream velocity (hence v_i is specified by Figure 1), the induced inflow ratio is inversely proportional to tip speed, as shown in the equation below. This formula can be used in conjunction with Figure 1 to calculate the induced inflow ratio for a given tip speed:

$$\lambda_i = \frac{v_i}{\Omega R}. \tag{11}$$

It is important to note that the induced velocity is largest for the the more negative incidence angles α (the assignment attached above clarifies the sign convention). Furthermore, Figure 2 shows that P drops below zero when v_∞ is greater than ~79.1 ft/sec. At this negative incidence angle and large freestream velocity, the net velocity at the rotor disk switches from downwards to upwards. The net inflow velocity at the rotor disk (positive defined as downwards) is equal to

$$v_{net} = \lambda \Omega R = v_i - v_\infty \cos\left(\frac{\pi}{2} + \alpha\right). \tag{12}$$

Figure 4 below shows v_{net} as a function of v_∞ for the three different incidence angles. It can be seen that v_{net} becomes negative at $v_\infty \approx 79.1$ ft/sec. The maximum upward velocity for the $-10°$ incidence angle is 1.67 ft/sec at $v_\infty = 84.4$ ft/sec. As shown in Figure 2, P becomes negative at $v_\infty \approx 79.1$ ft/sec, which indicates that the rotor is likely in a "windmill brake state" in which the rotor extracts power from the freestream. It is also possible that the rotor is in a "vortex ring state" (VRS), although this is less likely with the fore/aft-ward skew of the wake in non axi-symmetric flight. On test day, we will be able to confirm the operating state of the rotor, both by flow visualizations and by comparing experimental data to momentum theory predictions.

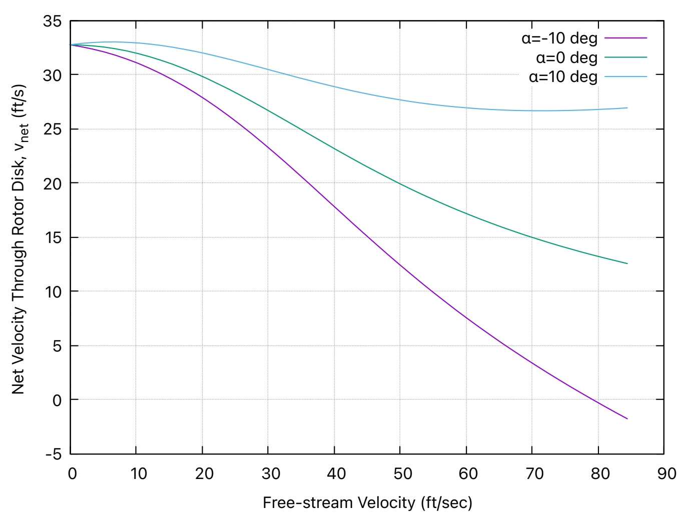

Figure 4: Net velocity through rotor disk v_{net} as a function of free-stream velocity v_∞. The shape of this plot resembles Figure 2, indicating that the total induced power is $P = Tv_{net}$.

One last observation from Figure 3 is that P/P_h monotonically decreases with freestream velocity for a wide range of incidence angles less than $\sim 10°$. However, we can see that for $\alpha = 10°$, P/P_h increases to above 1 for a small range of freestream velocities. I do not have a great explanation for this right now, and while it is something to keep in mind, it is a small effect and this increase in required power will not be an issue for the the vast majority of our operating range of $\alpha < 10°$.