You are working as part of a team responsible for executing a wind tunnel test of a new rotor design that your company has developed. As part of this effort, your technical lead engineer would like to have an analysis ready for expected results (expected $\theta_0, \theta_{1c}, \theta_{1s}, \alpha, Total\ Power$ to setup the rotor in tunnel 'trim' at hover, 25 and 50 knots). The test is to be performed for an articulated rotor (can be idealized as flap only, centrally hinged) with a radius of 5 ft operating at a tip speed of 650 ft/sec (chord .4 ft, 3 blades, -8 deg root to tip linear twist). The tunnel 'trim' is as follows:

1.) Use cyclic blade pitch inputs to zero out longitudinal and lateral flapping
2.) Use collective blade pitch input to obtain a certain $\frac{C_T}{\sigma}$ (target $0.005 \leq C_T \leq 0.01$)
3.) Use rotor shaft incident angle (α) to obtain specific C_X (Force along wind tunnel axis) (target $0.05 \leq C_X \leq 0.1$

As a simplification your team has decided to assume that blade flapping can be ignored in the inflow equation as well as to calculate C_X. You and your team are looking to win the engineer team of year away and to aid in that you are planning on presenting you results for a varied range of flap frequency.
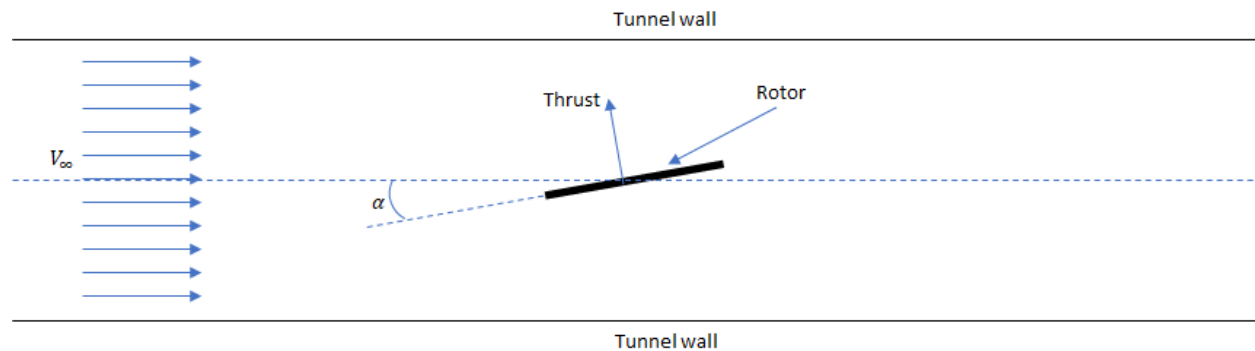
proj2_report

April 18, 2023

0.1 MEAM5460 Project 2: Trim Analysis

0.1.1 Nathaniel Ruhl

0.1.2 Spring 2023

0.1.3 Code: https://github.com/nruhl25/HoveringVehicleDesign/tree/main/proj2

0.1.4 Methodology

The goal of our "trim" is to determine the control inputs required to achieve a desired operational state. Our desired operational state consists of

1. Zeroing out longitudianal and lateral flapping, eg. $\beta_{1c} = \beta_{1s} = 0$
2. Acheiving a specified rotor thrust ($0.005 < C_T/\sigma < 0.01$)
3. Achieving a specified forward thrust ($0.05 < C_X < 0.1$, force along wind tunnel axis)

In order to acheive these outcomes, we can make use of the following controls. While the trim is highly couples, the numbers indicate which controls are mostly closely related to which outcomes:

1. Longitudinal and lateral cyclic pitch inputs, θ_{1c} and θ_{1s}
2. Input collective, θ_0
3. Rotor shaft incidence angle, α (I sometimes call this the "rotor-disc plane tilt angle")

The total power required to maintain the operating condition is derived from the coefficient of torque C_Q, for which the equation is given in the problem assignment. For this formula, we assume linear airfoils with a section coefficient of drag of $c_{d_0} = 0.01$ and a lift-curve slope of $c_{l_\alpha} = 2\pi$. Since $P = \Omega Q$, it can be shown that $C_P = C_Q$, and the total power is thus defined as

$$TotalPower = C_P \rho A v_{tip}^3$$

where the values of ρ, A, and v_{tip} are given in the problem assignment. The analysis below shows the required inputs and operating state of the rotor required zero out longitudinal and lateral flapping and to achieve a specificrange of values of C_T and C_X.

The system of equations in the problem assignment attached above are highly coupled, yet well-behaved non-linear equations that can be solved numerically. For the desired operational state, in which $\beta_{1c} = \beta_{1s} = 0$ and C_X and C_T are specified, we can defined a set of 5 equations and 5 uknowns. The uknown variables (which I refer to as "state variables" in my code) are

Relevant equations

$$F_x = T \sin \alpha$$

$$C_X = \frac{F_x}{0.5 \rho V_\infty^2 A}$$

$$\lambda = \mu \tan \alpha + \frac{C_T}{2\sqrt{\mu^2 + \lambda^2}}$$

$$C_T = \frac{\sigma c_{l_\alpha}}{2}\left[\frac{\theta_0}{3}\left(1 + \frac{3}{2}\mu^2\right) + \frac{\theta_{tw}}{4}(1 + \mu^2) + \frac{\mu}{2}\theta_{1s} - \frac{\lambda}{2}\right]$$

$$C_Q = \frac{\sigma c_{l_\alpha}}{2}\left[\frac{\lambda \theta_0}{3} + \frac{\lambda \theta_{tw}}{4} - \frac{\lambda^2}{2} - \frac{(\beta_{1c}^2 + \beta_{1s}^2)}{8} - \frac{\mu^2}{2}\left(\frac{\beta_0^2}{2} + \frac{3\beta_{1c}^2}{8} + \frac{\beta_{1s}^2}{8}\right) + \frac{c_{d_0}}{4 c_{l_\alpha}}(1 + \mu^2) - \frac{\mu \lambda \beta_{1c}}{2} - \frac{\mu \theta_0 \beta_{1s}}{3}\right]$$

$$\begin{bmatrix} \dfrac{8 v_\beta^2}{\gamma} & 0 & 0 \\[3mm] \dfrac{4}{3}\mu & \dfrac{8(v_\beta^2 - 1)}{\gamma} & \left(1 + \dfrac{\mu^2}{2}\right) \\[3mm] 0 & -\left(1 - \dfrac{\mu^2}{2}\right) & \dfrac{8(v_\beta^2 - 1)}{\gamma} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \end{bmatrix} = \begin{bmatrix} (1 + \mu^2)\theta_0 + \left(\dfrac{4}{5} + \dfrac{2}{3}\mu^2\right)\theta_{tw} + \dfrac{4}{3}\mu\theta_{1s} - \dfrac{4}{3}\lambda \\[3mm] \left(1 + \dfrac{\mu^2}{2}\right)\theta_{1c} \\[3mm] \dfrac{8}{3}\mu\theta_0 + 2\mu\theta_{tw} + \left(1 + \dfrac{3}{2}\mu^2\right)\theta_{1s} - 2\mu\lambda \end{bmatrix}$$

$\alpha \rightarrow Incidence\ angle$
$V_\infty \rightarrow Tunnel\ velocity$
$\mu \rightarrow Advance\ ratio$
$C_T \rightarrow Coefficient\ of\ thrust$
$C_Q \rightarrow Coefficient\ of\ torque$
$\rho \rightarrow 0.002378\ \dfrac{sl}{ft^3}$
$T \rightarrow Thrust$
$\lambda \rightarrow Total\ Inflow\ ratio$
$F_x \rightarrow Force\ along\ wind\ tunnel\ axis\ generated\ by\ rotor$
$C_X \rightarrow Coefficient\ of\ F_x$
$\sigma \rightarrow Solidity$
$v_\beta \rightarrow Non-dimensional\ flap\ frequency$
$\gamma \rightarrow Rotor\ lock\ number$
$\theta_{tw} \rightarrow Linear\ blade\ twist\ rate$
$\beta_0, \beta_{1c}, \beta_{1s} \rightarrow Blade\ coning, longitudinal\ and\ lateral\ flapping\ angle$
$\theta_0, \theta_{1c}, \theta_{1s} \rightarrow Rotor\ collective, lateral\ and\ longitudinal\ cyclic\ pitch\ angle$
$c_{l_\alpha} \rightarrow Airfoil\ lift\ curve\ slope\ (assume\ 2\pi)$
$c_{d_0} \rightarrow Airfoil\ steady\ coefficient\ of\ drag$
$A \rightarrow Rotor\ disk\ area$

$$\mathbf{x} = \begin{pmatrix} \beta_0 \\ \theta_0 \\ \theta_{1c} \\ \theta_{1s} \\ \lambda \end{pmatrix}.$$

The non-linear system of equations can be written as $f(\mathbf{x}) = 0$, where

$$f(\mathbf{x}) = \begin{pmatrix} A_{11}\beta_0 - c_1 \\ A_{21}\beta_0 - c_2 \\ A_{31}\beta_0 - c_3 \\ \lambda - \lambda_{RHS} \\ C_T - C_{T,RHS} \end{pmatrix}$$

The entries of the 3x3 matrix $\mathbf{A}$ and the 3x1 column vector $\mathbf{c}$ are given in the project assignment (I defined the matrix equation as $\mathbf{A}\beta = \mathbf{c}$). The root $\mathbf{x}^*$ can be solved by Newton's method, for which I have used `scipy.optimize.fsolve`. The algorithm is not very sensitive to initial guesses, and even $\mathbf{x_0} = [0; 0; 0; 0; 0]$ suffices.

The specifications of our articulated rotor are defined in the project assignment, and defined as global variables in `proj2.py`. Furthermore, in this analysis, I have used a typical Locke numer of $\gamma = 7$ since we do not know the moment of inertia of the aircraft.

I will discuss the results below and show snippets of my code, which can be adapted to perform further analysis.

```python
[1]: import numpy as np
     import importlib
     import matplotlib.pyplot as plt

     # local modules
     import proj2
     import plotter

     importlib.reload(proj2)
     importlib.reload(plotter)
     from proj2 import calc_trims_array, plot_rotor_trim, knots2mps, sigma,
      ↪calc_alpha
     from plotter import plot_trim_vary_flap_frequency, plot_trim_vary_velocity,
      ↪plot_rotor_disc_tilt
```

```python
[2]: # Goal values of CT and CX when performing the trim
     N_CT = 50
     N_CX = 4
     CT_norm_list = np.linspace(0.005, 0.01, N_CT)
     CT_list = sigma*CT_norm_list
     CX_list = np.linspace(0.05, 0.1, N_CX)
```

0.1.5 Trim Analysis in Hover ($v_\infty = 0$ knots, $\nu_\beta = 1.0$)

The first plot below shows the rotor disc tilt in hover, which is always $\alpha = 0°$. The subsequent set of 6 plots, which show each of the states solved in **x**, as well as the Total Power, versus C_T/σ show some interesting trends: - The rotor trim in hover does not depend on C_X at all, which is a good cross-check for the non-linear root-solving problem. Any non-zero value of C_X is incompatible with the hover condition. - Cyclic inputs are always zero. - In order to achieve a larger C_T/σ, blade collective must be increased in a nearly linear manner. The total inflow and total power increase in a similar way.

```
[3]: v_inf = knots2mps(0)   # m/s (since vtip is defined in m/s)
     nu_b = 1.0

     plot_rotor_disc_tilt(CX_list, CT_list, v_inf, nu_b)
```
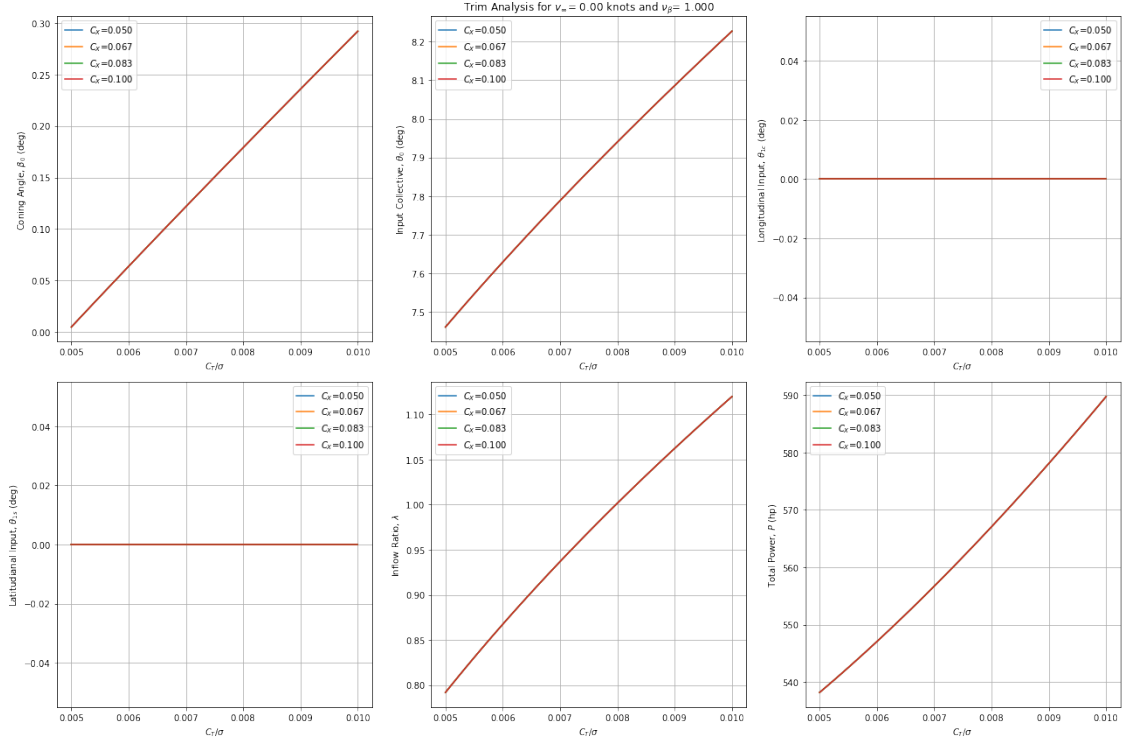

Required rotor rotor shaft incidence angle to achieve desired forward thrust (v_∞=0.00, ν_b = 1.000)
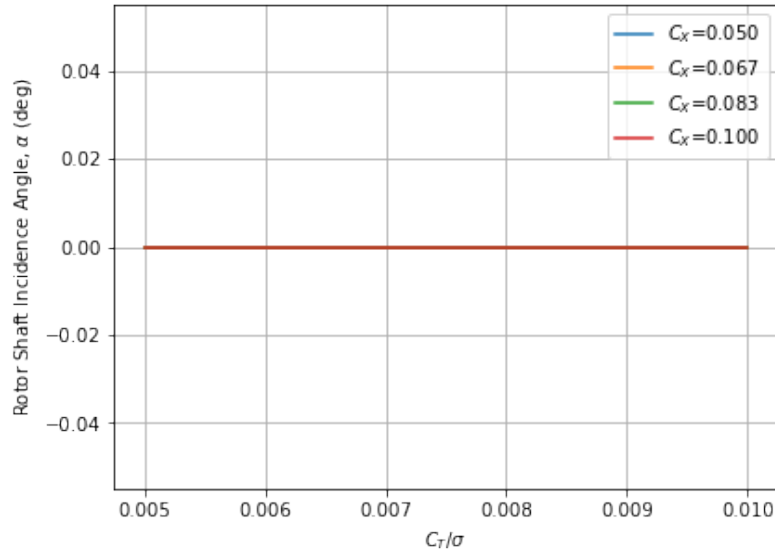
```
[4]: trims = calc_trims_array(CX_list, CT_list, v_inf, nu_b)
     trim1_tuple = (CX_list, CT_list, v_inf, nu_b, trims) # info relevant to trim1

     plot_rotor_trim(trim1_tuple)
```

3

0.1.6 Trim Analysis in Forward Flight ($v_\infty = 25$ knots, $\nu_\beta = 1.0$)

In order to achieve this speed of forward flight, the rotor disc must be oriented as shown in the plot below. It makes sense that as forward thrust (C_X) increases, the rotor incidence angle must increase. Moreover, if C_T/σ is increased, the rotor incidence angle does not have to be as large to achieve a constant C_X.

```
[5]: v_inf = knots2mps(25)  # m/s (since vtip is defined in m/s)
     nu_b = 1.0

     plot_rotor_disc_tilt(CX_list, CT_list, v_inf, nu_b)
```

Required rotor rotor shaft incidence angle to achieve desired forward thrust ($v_\infty = 25.00$, $\nu_b = 1.000$)
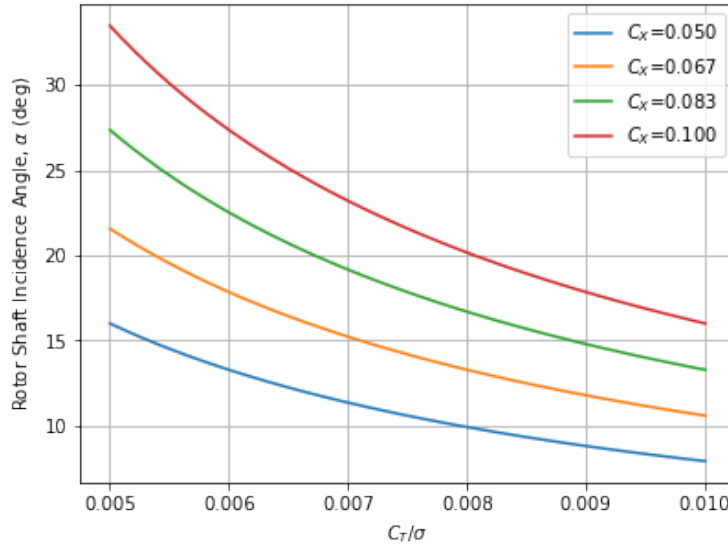
Unlike in hover, the curves in the 6 subplots change for different C_X values, and the plots are also more curved. We can see that as C_X increases, all of the controls must increase in magnitude. Notably, θ_{1s} becomes more negative, while the other curves become more positive. I am a little surprised that the magnitudes of θ_{1c} and θ_{1s} are so small. This may indicate an error in my code, as 2 classmates obtained larger cyclic input values, but the same overall behavior of these plots. Nevertheless, the directionality of θ_{1s} and θ_{1c} makes sense. In forward flight, the advancing side of the rotor sees a larger free-stream velocity and would therefore experience a larger lift force. In order to prevent flapping, a $-\theta_{1s}$ needs to be used to increase lift on the advancing side, but this also leads to a differential fore/aft lift generation. In order to cancel out this flapping, a θ_{1c} input must be used.

It is interesting that input collective decreases with C_T/σ. This is the opposite of what we saw in the trim for hover, where θ_0 and C_T/σ are directly proportional. I am not 100% sure about the mechanism behind this, but it is possible that the relative wind contributes so much to the inflow/thrust, that the blade sections themselves do not need to generate as much thrust (via increasing angle of attack). In the section "Effect of forward flight velocity", we will see at what forward velocity this change in behavior occurs.

Lastly, it is interesting to note that total power in forward flight is often less than in hover, and depends on the values of C_X and C_T to make a direct comparison.

```
[6]: trims = calc_trims_array(CX_list, CT_list, v_inf, nu_b)
     trim1_tuple = (CX_list, CT_list, v_inf, nu_b, trims) # info relevant to trim1

     plot_rotor_trim(trim1_tuple)
```

5

Trim Analysis for $v_\infty = 25.00$ knots and $\nu_\beta = 1.000$
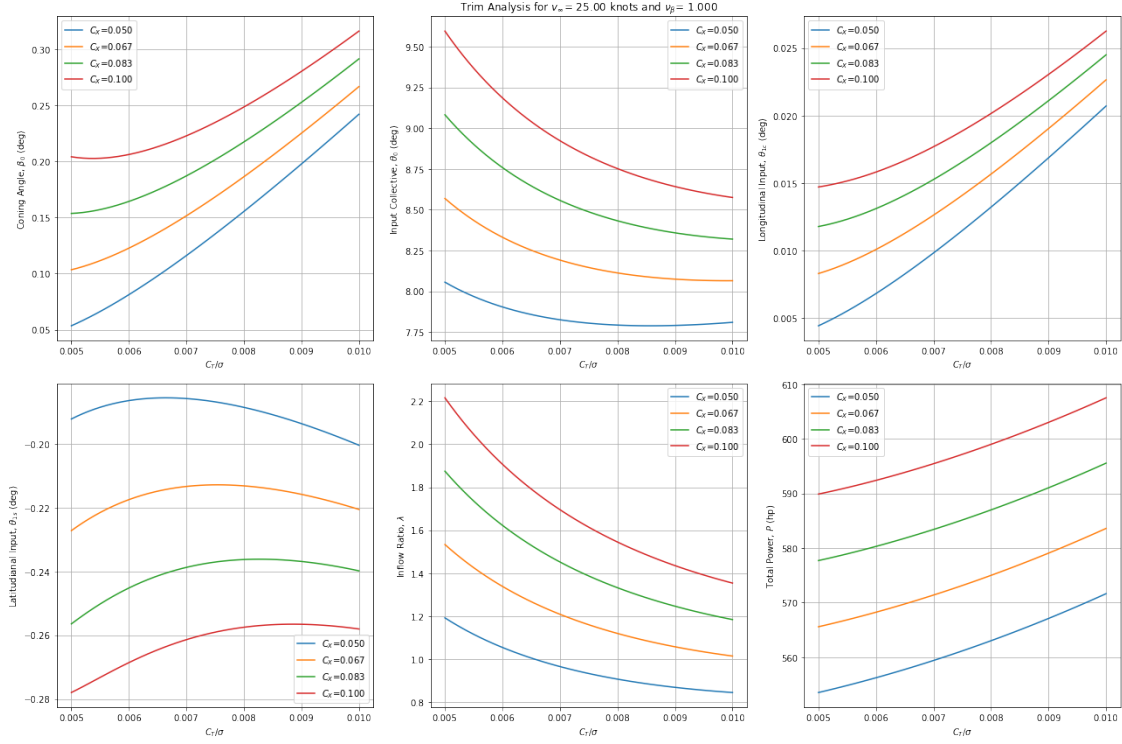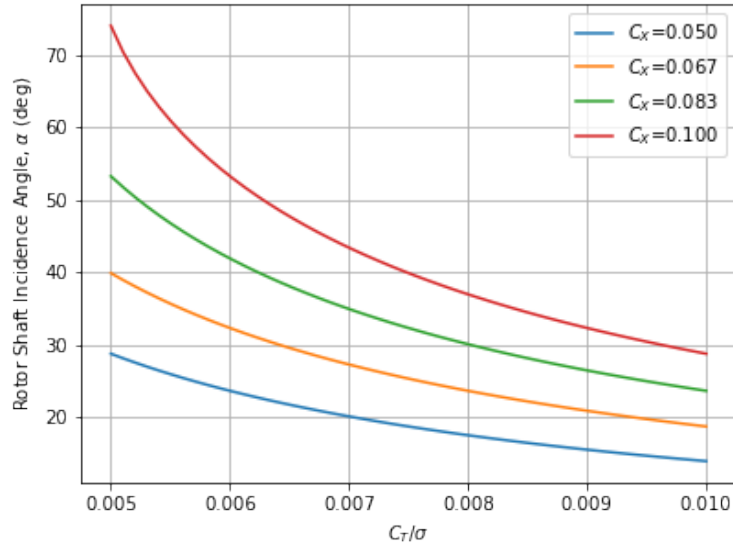
0.1.7 Trim Analysis in Forward Flight ($v_\infty = 33$ knots and $\nu_\beta = 1$)

I chose to include $v_\infty = 33$ knots because this is approximately the maximum v_∞ for which α exists (within the range of C_X). When we consider $v_\infty = 50$ knots below, we will see why α can become singular in forward flight, or equivalently, why helicopters have maximum forward flight velocities. The plots of θ_{1c} and θ_{1s} below start to show interesting behavior for large values of C_X and small values of C_T. This behavior leads up to the singularity in α.

```
[7]: v_inf = knots2mps(33)   # m/s (since vtip is defined in m/s)
     nu_b = 1.0

     plot_rotor_disc_tilt(CX_list, CT_list, v_inf, nu_b)
```
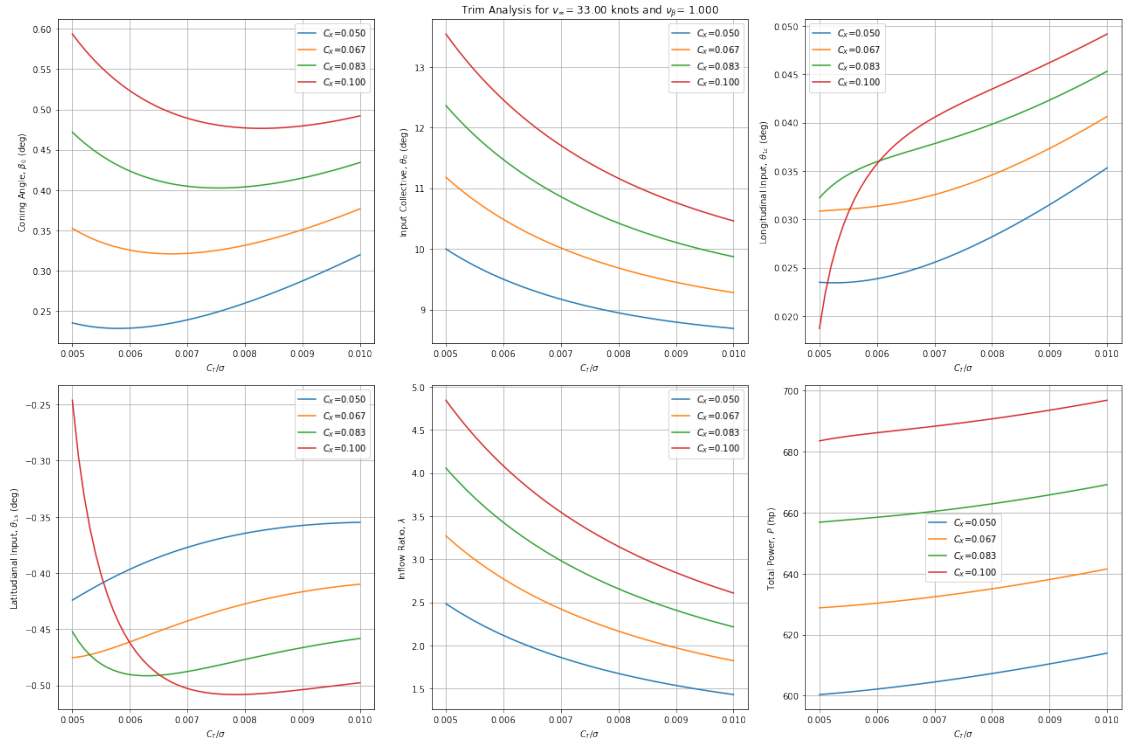
Required rotor rotor shaft incidence angle to achieve desired forward thrust ($v_\infty = 33.00$, $\nu_b = 1.000$)


```
[8]:  trims = calc_trims_array(CX_list, CT_list, v_inf, nu_b)
      trim1_tuple = (CX_list, CT_list, v_inf, nu_b, trims) # info relevant to trim1

      plot_rotor_trim(trim1_tuple)
```

0.1.8 Trim Analysis in Forward Flight ($v_\infty = 50$ knots and $\nu_\beta = 1$)

As can be seen in the error messages below, the arcsin() in the equation for α goes out of range because

$$\frac{0.5 C_X v_\infty^2}{C_T v_{tip}^2} > 1$$

. Moreover, the plot of α vs C_T/σ shows where the singularity occurs, near an angle of $\alpha = 1.4°$. For a given v_∞, we have a strict condition on the ratio of C_X to C_T which determines the maximum rotor shaft incidence angle/forward flight velocity:

$$\frac{C_X}{C_T} > \frac{v_{tip}^2}{0.5 v_\infty^2}$$

```
[9]: v_inf = knots2mps(50)   # m/s (since vtip is defined in m/s)
     nu_b = 1.0

     plot_rotor_disc_tilt(CX_list, CT_list, v_inf, nu_b)
```

```
/Users/nathanielruhl/Desktop/HOV_MEAM5460/HoveringVehicleDesign/proj2/proj2.py:3
1: RuntimeWarning: invalid value encountered in arcsin
  return np.arcsin((0.5*CX*v_inf**2)/(CT*vtip**2))
/Users/nathanielruhl/Desktop/HOV_MEAM5460/HoveringVehicleDesign/proj2/proj2.py:3
1: RuntimeWarning: invalid value encountered in arcsin
  return np.arcsin((0.5*CX*v_inf**2)/(CT*vtip**2))
/Users/nathanielruhl/Desktop/HOV_MEAM5460/HoveringVehicleDesign/proj2/proj2.py:3
1: RuntimeWarning: invalid value encountered in arcsin
  return np.arcsin((0.5*CX*v_inf**2)/(CT*vtip**2))
/Users/nathanielruhl/Desktop/HOV_MEAM5460/HoveringVehicleDesign/proj2/proj2.py:3
1: RuntimeWarning: invalid value encountered in arcsin
  return np.arcsin((0.5*CX*v_inf**2)/(CT*vtip**2))
```

Required rotor rotor shaft incidence angle to achieve desired forward thrust (v_∞=50.00, $\nu_b = 1.000$)
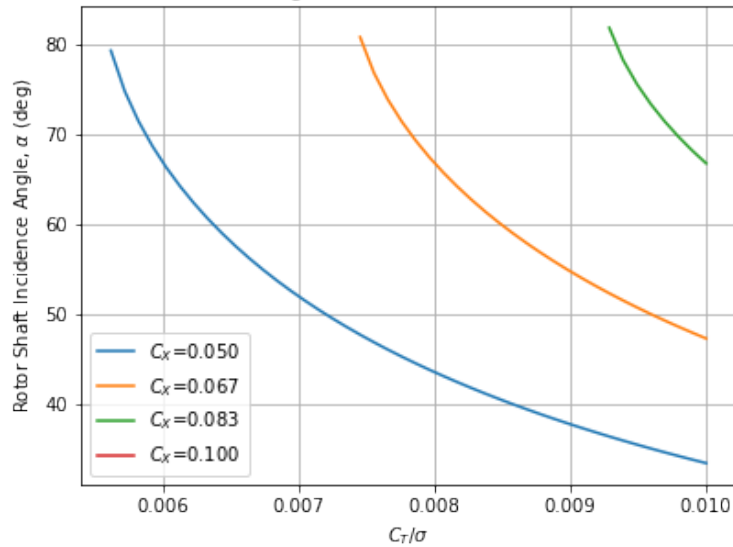

```
[10]:  trims = calc_trims_array(CX_list, CT_list, v_inf, nu_b)
       trim1_tuple = (CX_list, CT_list, v_inf, nu_b, trims) # info relevant to trim1

       plot_rotor_trim(trim1_tuple)
```

/Users/nathanielruhl/Desktop/HOV_MEAM5460/HoveringVehicleDesign/proj2/proj2.py:3
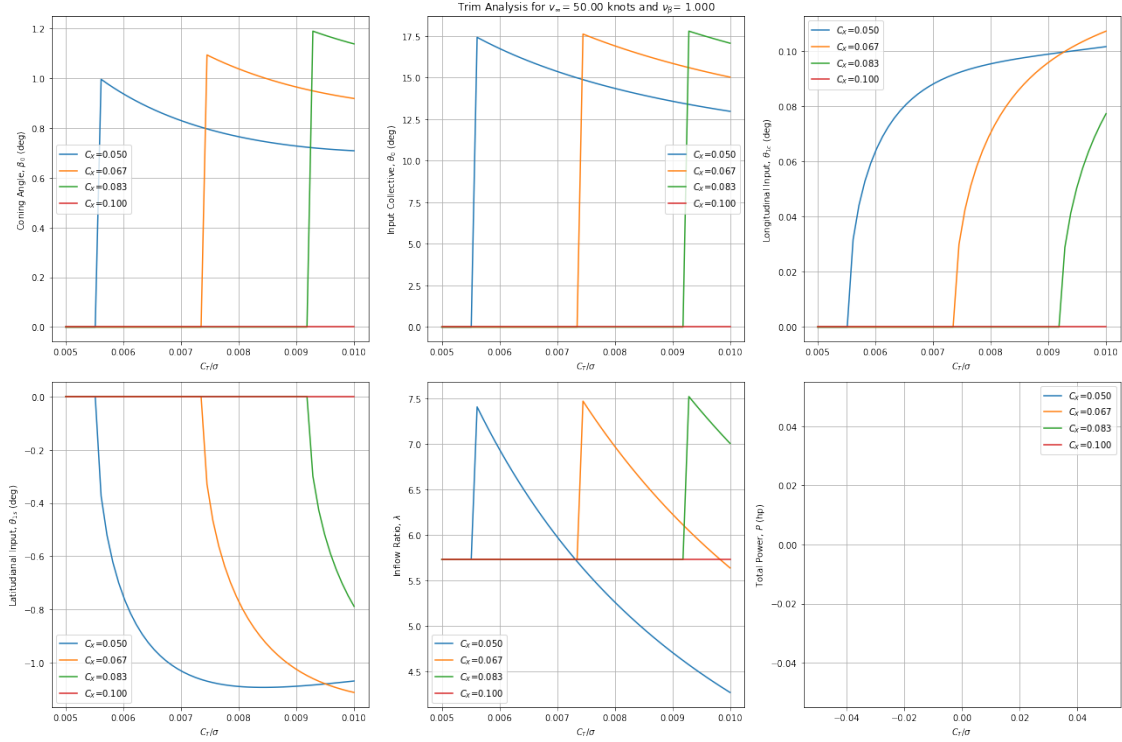1: RuntimeWarning: invalid value encountered in arcsin
  return np.arcsin((0.5*CX*v_inf**2)/(CT*vtip**2))
/opt/anaconda3/lib/python3.8/site-packages/scipy/optimize/minpack.py:175:
RuntimeWarning: The iteration is not making good progress, as measured by the
  improvement from the last ten iterations.
  warnings.warn(msg, RuntimeWarning)

Trim Analysis for v_∞ = 50.00 knots and ν_β = 1.000

0.1.9 Effect of varying non-dimensional flap frequency

The non-dimensional flap frequency $\nu_\beta = 1$ is an idealization for a centrally-hinged rotor and means that the natural frequency of flapping is equal to the rotational frequency of the rotor. In other words, the input-to-output response is once per revolution. In real life, the rotor hinge will not be at te exact center of the rotor, which adds stiffness to the system, and there is also extra stiffness caused by forward flight. For this reason ν_β may be slightly larger than one.

In order to consider the effects of non-dimension flap frequency, ν_β, we will consider $v_\infty = 25$ knots and $C_X = 0.05$. The effect of ν_β is strong, and the plots of β_0 and θ_{1c} do change significantly for different values of v_∞ and C_X. Therefore, it is worthwile to re-run these cells for different v_∞ and C_X, but the qualitative observations made for the other 4 plots are universal. θ_0, θ_{1s}, λ, and Total Power are independent of ν_β. This is extremely helpful to know when designing and operating the rotor system!

```
[11]: nu_b_list = np.linspace(1.0,1.12,4)
      plotter.plot_trim_vary_flap_frequency(nu_b_list, v_inf=knots2mps(25), CX=0.05)
```
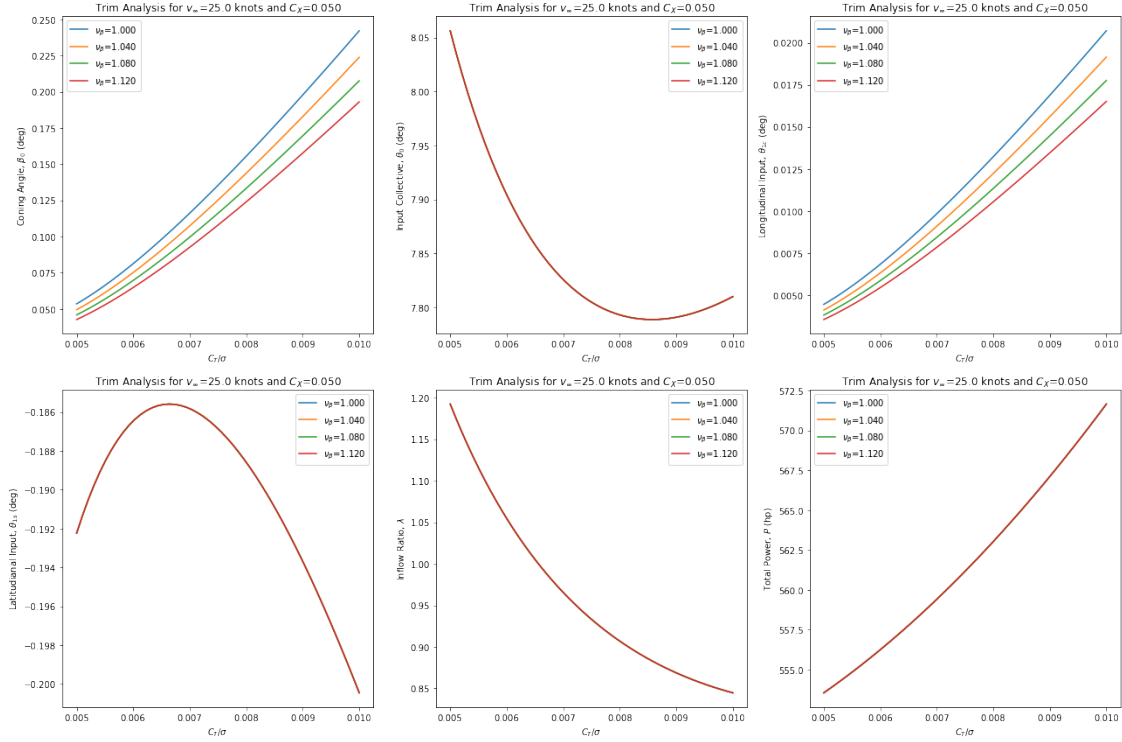
0.1.10 Effects of Forward Flight Velocity

The cell below shows the aforementioned phenomenon that when forward flight velocity passes ~ 23 knots, the general relationship between θ_0 and C_T flips directionality. As stated previously, my hypothesis is that the relative wind ends up contributing to thrust, and therefore the airfoils themselves do not need to generate as much thrust via increasing angle of attack.

```
[12]: v_inf_list = np.linspace(knots2mps(16),knots2mps(26), 5)
      plotter.plot_trim_vary_velocity(v_inf_list, CX=0.05, nu_b=1.0)
```