Homework #1 **MEAM 5460 - Spring 2023**
Assigned: 02/07/2023, Due: 02/14/2023

You are part of a team working on the preliminary design of an electric propulsion Group 3 (based on US military standards) Unmanned Aerial System (UAS) near the upper limit of the gross weight bucket (544 kgs) for that class. As part of that evaluation you have been tasked to provide an estimate of the hover performance metrics (Single rotor Disk loading, Power Loading, Ideal power and total power as well as Required power of the aircraft).

The rotor group have published (see data below) an estimated Figure of Merit (FM) variation curve for the rotors of the various concepts. Furthermore, the power system group has estimated a 95% percent efficiency of the complete power system (from battery to rotor). You are looking for an awesome performance review this year and therefore have decided to perform your evaluation at Standard Sea Level (ρ = 0.002378 slugs/ft^3) and at 2000 ft (ρ = 1.15490 N*s^2/m^4) above Mean Sea Level (MSL). A senior engineer has also recommended you provide relevant insight as to which vehicle(s) you would pick (if any) for further analysis.

The aircraft configurations your team is considering:

1.) Single main rotor/Tail rotor configuration
  a. Main rotor radius: 7.0 ft = 2.1336 m
  b. Number of blades: 6
  c. Chord: 0.33 ft = 0.100584 m
  d. Omega: 955 RPM = 100.007 rad/sec
  e. Assume tail rotors requires 15% additional power
2.) Quadcopter configuration
  a. Rotor radius: 0.61 m
  b. Number of blades: 3
  c. Chord: 2.0 inches =
  d. Omega: 350 rad/sec = 3342.25 RPM
3.) Tandem rotor configuration
  a. Rotor radius: 4.5 ft = 1.3716 m
  b. Number of blades: 5
  c. Chord: 7.62 cm = 0.0762 m
  d. Omega: 155 rad/sec = 1480 RPM
4.) Hexacopter configuration
  a. Rotor radius: 1.6 ft = 0.48768 m
  b. Number blades: 4
  c. Chord: 1.5 inches = 0.0381 m
  d. Omega: 3342 RPM = 350 rad/sec

Blade loading coeff.

C_T/σ	FM
0	0
0.01	0.2
0.02	0.4
0.03	0.5
0.04	0.55
0.05	0.57
0.06	0.58
0.07	0.59
0.08	0.6
0.09	0.6
0.1	0.6
0.11	0.6
0.12	0.6

Altitude = 2000 ft = 609.6 m
$\rho = 1.15490\ kg/m^3$
MSL: $\rho_0 = 1.225577\ kg/m^3 = 0.002378\ slug/ft^3$

MEAM5450 Project 2

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Abstract

https://github.com/nruhl25/HoveringVehicleDesign

1 Introduction

Figure 1 shows our group's experimental data for Figure of Merit vs Blade Loading Coefficient, as well as an interpolating cubic spline.

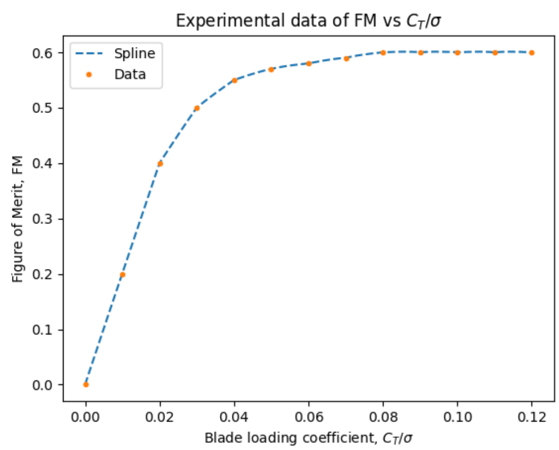


Figure 1: Figure of Merit vs Blade Loading Coefficient

In the following analysis, I compare the single rotor performance of the 4 different aircraft configurations in hover, each of which have the specifications given in the problem assignment on the first page of this report. In order to calculate the single rotor performance metrics, we define the thrust supplied by each rotor as

$$T_r = \frac{T_t}{N_r} \tag{1}$$

where N_r is the number of lifting rotors and $T_t = W$ is the weight of the aircraft (m=544 kg). The coefficient of thrust is then

$$C_T = \frac{T_r}{\rho A_r (\Omega R)^2} \tag{2}$$

where R is the rotor radius, A_r is the area of the rotor disk, Ω is the angular velocity of the blades, and ρ is air density. The Figure of Merit, FM, represents the ideal power divided by the actual power required to generate the thrust needed to hover. We can write the figure of merit as

$$\text{FM} = \frac{P_{ideal}}{P_{actual}} = \frac{C_T^{3/2}}{\sqrt{2}C_P} \tag{3}$$

where C_P is the coefficient of power, which could be decomposed into profile power and induced power as shown in Eq. 4, although this decomposition is not necessary in the analysis of overall rotor performance when data for FM is known (we will use Eqn. 6).

$$C_P = C_{P0} + C_{Pi} = \frac{\sigma C_{d0}}{8} + \frac{\kappa C_T^{3/2}}{\sqrt{2}}. \tag{4}$$

In this equation, C_{d0} is the coefficient of drag at zero and small angles of attack, κ is an experimentally derived quantity that captures that non-idealities of momentum theory, and σ is the rotor solidity, or the fraction of the rotor disc area "wetted" by N_b rotor blades of chord length c,

$$\sigma = \frac{N_b c \pi R}{A_r}. \tag{5}$$

2 Methodology

Since rotor solidity σ and thrust per lifting rotor T_r that each rotor must generate in hover are known, we can calculate the "single rotor" blade loading coefficient, C_T/σ, for each of the aircraft configurations. Using C_T/σ, we can then interpolate the data shown in Figure 1 to determine the FM.

Once FM is known, Eq. 3 can be re-arranged to determine the *actual* coefficient of power that must be supplied by each of the rotors.

$$C_p = \frac{C_{p,ideal}}{\text{FM}} \tag{6}$$

where we the ideal power coefficient is defined as

$$C_{p,ideal} = \frac{C_T^{3/2}}{\sqrt{2}} \tag{7}$$

The power loading, PL, is the thrust per power of the rotor and represents a dimensional measure of rotor efficiency. The power loading is equal to

$$\text{PL} = \frac{T}{P_{actual}} = \frac{C_T}{\Omega R C_P}. \tag{8}$$

Furthermore, we define the disk loading, or the thrust carried per unit area of the rotor disc, as

$$\text{DL} = \frac{T_r}{A_r} \tag{9}$$

Lastly, the total power consumed by rotor configuration can be determined by Eq. 10. Since our power group estimates 95% efficiency from power to rotor, the actual coefficient of power will be inflated by 5%:

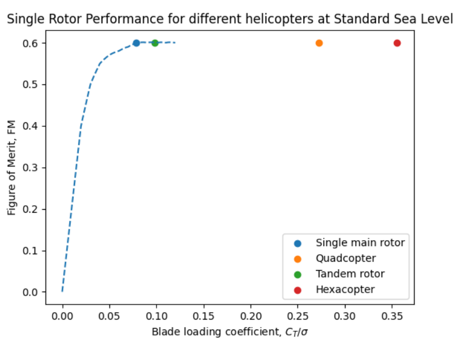
$$P_{total} = \left(\frac{1}{0.95}\right) N_r C_p \rho A_r (\Omega R)^3 \tag{10}$$

Additionally, the above formula must be adjusted for the single/main rotor configuration, for which the tail rotor requires an additional 15% power.

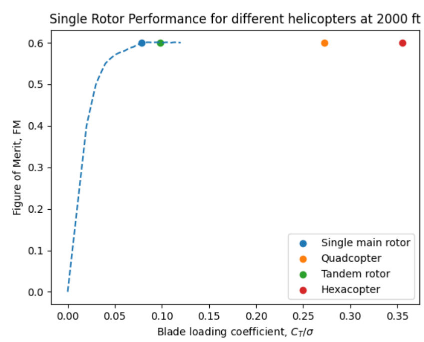
3 Results

Figure 2 shows the single rotor values of C_T/σ and FM for each of the aircraft configurations. It is interesting that since each of the aircraft have $C_T/\sigma > 0.6$, the Figure of Merit has already started to plateau at ~ 0.6.

I have performed the analysis at both Standard Sea Level (SSL) and at an altitude of 2000 ft (Mean Sea Level, MSL). The value of C_T/σ is consistently larger by $\sim 6.2\%$ at MSL, which means that the rotor must generate 6.2% more thrust in order to hover. Because of the large blade loading, the Figure of Merit, or the power efficiency, remains almost identical at both altitudes (Figure 4).



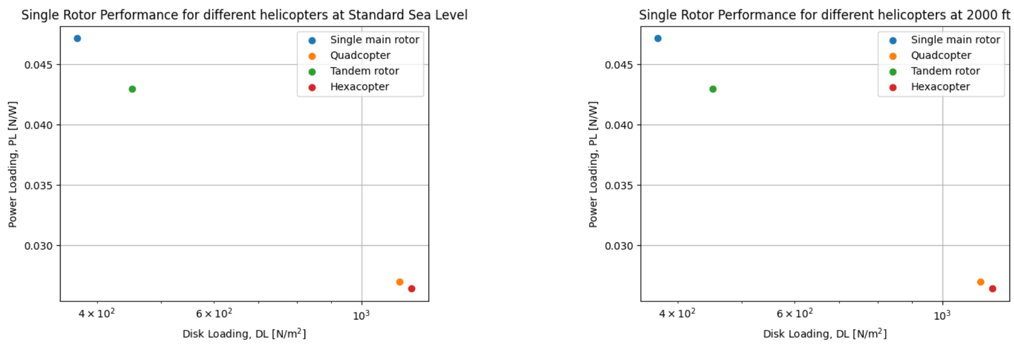
(a) FM vs C_T/σ at SSL



(b) FM vs C_T/σ at MSL

Figure 2: Figure of Merit vs Blade Loading Coefficient

Figure 3 shows a plot of PL versus DL for the 4 aircraft configurations.



(a) Power Loading (hover efficiency) vs Disk Loading at SSL

(b) Power Loading (hover efficiency) vs Disk Loading at MSL

Figure 3: Power Loading vs Disk Loading

The tables of Figure 4 shows the single rotor and total system power requirements for the 4 different aircraft configurations in hover, both at SSL and MSL.

Performance Analysis at Standard Sea Level								
Configuration	C_T/σ	FM	C_P	DL [N/m^2]	PL [N/W]	P_ideal (rotor) [kW]	P_actual (rotor) [kW]	P_total (system) [kW]
Single/main rotor	0.07428	0.59536	0.00065	373.15738	0.04825	65.84607	110.59900	133.88300
Quadcopter	0.25690	0.60000	0.00344	1141.29615	0.02781	28.78877	47.98128	202.02643
Tandem Rotor	0.09218	0.60085	0.00087	451.47433	0.04427	36.21349	60.27028	126.88481
Hexacopter	0.33516	0.60000	0.00717	1190.41095	0.02723	19.60113	32.66855	206.32767

Performance Analysis at Mean Sea Level								
Configuration	C_T/σ	FM	C_P	DL [N/m^2]	PL [N/W]	P_ideal (rotor) [kW]	P_actual (rotor) [kW]	P_total (system) [kW]
Single/main rotor	0.07882	0.59915	0.00071	373.15738	0.04714	67.83077	113.21161	137.04564
Quadcopter	0.27262	0.60000	0.00376	1141.29615	0.02699	29.65650	49.42750	208.11581
Tandem Rotor	0.09782	0.60058	0.00095	451.47433	0.04296	37.30502	62.11455	130.76748
Hexacopter	0.35567	0.60000	0.00784	1190.41095	0.02643	20.19194	33.65322	212.54669

Figure 4: Power requirements of different aircraft configurations

4 Analysis of Results

1. As shown in Figure 2, the blades of the Quadcopter and Hexacopter will stall because their blade loading is larger than C_T/σ¿0.10-0.12, so they should not be further considered as viable options. Moreover, they would require a much larger overall power than the other two configurations.

2. It is important to note that FM should only be compared at similar disk loadings, but since all the FM values are very similar, this metric is not very insightful for these configurations.

3. The Single/main rotor orientation has the largest PL and is therefore the most efficient in hover. As can be seen in Figure 4, the power loading of the Tandem rotor configuration is about $\sim$10% smaller than the Single/main rotor.

4. The Tandem rotor turns out to require about 5-6% less overall power than the Single/main rotor configuration at both MSL and SSL, especially since it does not have

the extra tail rotor (Figure 4).

5. However, the Tandem rotor configuration may be slightly more risky than the Single/main rotor, since the blades operate with higher blade loading. At MSL, the blade loading is 0.097, which could be very close to the stalling regime. Additionally, this mean that the tandem rotor orientation will have less climb potential than the single/main rotor.

5 Recommendations

I would look further into the Tandem rotor configuration and the Single/main rotor configuration. The Tandem rotor configuration would be more power efficient than the Single/main rotor, but it might not necessarily be the best option. Choosing between the two configurations will require further defining operational requirements, predicting the blade loading coefficient at which our blades will stall, and deciding whether 6% less required power is worth the extra risk and extra control complexity associated with the Tandem rotor configuration.