Worksheet
APPM 5630 Spring 2025
Advanced Convex Optimization

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Instructions Solve the following exercises using CVX (MATLAB) or cvxpy (Python). Use the following matrices:

$$A = \begin{bmatrix} 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 & 8 & 2 \\ 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 & 9 & 3 \\ 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 & 10 & 4 \\ 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 & 11 & 5 \\ 5 & 10 & 4 & 9 & 3 & 8 & 2 & 7 & 1 & 6 \end{bmatrix}$$

$$y = [1, 2, 3, 4, 5]^T$$

In Python, you can create A using `A = np.tile( np.arange(1,12), 5 )[:-5].reshape((5,-1),order='F')` and in Matlab you can create it using `A = reshape( mod( (1:m*n)-1, 11 )+1, 5, 10 )`.

Exercises

1. Solve $\min_x \|x\|_2$ subject to $\|Ax - y\|_2 \le 0.1$. The minimum should be 0.294216.
2. Solve $\min_x \|x\|_2^2$ subject to $\|Ax - y\|_2 \le 0.1$. The minimum should be 0.0865633.
3. Solve $\min_x \|x\|_1$ subject to $\|Ax - y\|_2 \le 0.1$. The minimum should be 0.787669.
4. Re-solve the equation in Exer. 1 and request the dual variable λ, and use this value so that your solution of $\min_x \|x\|_2 + \lambda\|Ax - y\|_2$ coincides with that of Exer. 1.
5. Matrices in objective: Solve $\min_{x\in\mathbb{R}^5} g(A - x\mathbb{1}^T)$ where $g(A) = \sum_{j=1}^n \sqrt{\sum_{i=1}^m A_{ij}^2}$ is the sum of column ℓ_2 norms. The minimum should be 63.9551.
6. Matrices in objective: Solve $\min_{x\in\mathbb{R}^5} h(A - x\mathbb{1}^T)$ where $h(A) = \|A\|$ is the spectral norm of A (largest singular value). The minimum should be 14.3922.
7. Matrix variables: Solve $\min_{X\in\mathbb{R}^{5\times 10}} \|X - A\|_F$ subject to $\mathbb{1}_5^T X \mathbb{1}_{10} = 1$. The minimum should be 40.1637.
8. Matrix variables: Let B be the first 5 columns of A. Solve $min_{X\in\mathbb{R}^{5\times 5}} \|X - B\|_F$ subject to $X \succeq 0$ (i.e., X is positive semi-definite). The minimum should be 14.436.
9. Bonus: in cvxpy, solve problem 1 but for multiple right-hand-side vectors y (draw each one randomly). Set y to be a "parameter" in cvxpy, and see how much that speeds up repeated solves.

Remarks With CVX and Matlab, note that if X is a matrix, then `norm(X,1)` and `norm(X(:),1)` are different (same for the 2 norm), as the former gives you the operator norm. The CVX command `norms(X,1)` is also different — try all three versions with the A matrix given above.

In Python, be careful with vectors of size `(n,)` vs vectors of size `(n,1)` since they do not play together well!