Homework 1 and 2
APPM 5630 Advanced Convex Optimization, Spring 2025

Due date: Friday, Jan. 31 2025, before midnight, via Gradescope

Instructor: Prof. Becker
Revision date: 1/16/2025

Theme: Convex sets, convex functions

Instructions Collaboration with your fellow students is allowed and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks. Please write down the names of the students that you worked with. An arbitrary subset of these questions will be graded.

Homework submission instructions at github.com/stephenbeckr/convex-optimization-class/tree/master/Homeworks. You'll turn in a PDF (either scanned handwritten work, or typed, or a combination of both) to **gradescope**, using the link from the Canvas assignment.

Note This homework is a bit heavier on theory (HW1) than computation (HW2). As we progress through the semester, the computational part will increase slightly and the theory part will decrease slightly.

Reading Read chapters 1–2 in [BV2004] (and chapter 3 if you have time). The book [BV2004] ("Convex Optimization" by Boyd and Vandenberghe) is freely available at Stephen Boyd's website

Homework 1

Do problems 1–3, and then do problem 4 *or* problem 5.

Problem 1: [BV2004] Problem 2.2: Show that a set is convex if and only if its intersection with any line is convex. Show that a set is affine if and only if its intersection with any line is affine.

Problem 2: [BV2004] Problem 2.8: which of the following sets S (a–d, see book) are polyhedra? If possible, express S in the form $S = \{x | Ax \preceq b, Fx = g\}$.) *Note: it may be useful to know something about the concept of dual norms (Appendix A.1.6 in [BV2004]) and the related Hölder inequality (3.1.9 in [BV2004]); you can review other linear algebra facts via my handwritten "appendix" notes.* For this problem, you don't need to give rigorous proofs (unless you want to), but give some kind of reasoning.

Problem 3: [BV2004] Problem 2.12: which of the following sets (a–g, see book) are convex? Like the previous problem, you don't need to give rigorous proofs (unless you want to), but give some kind of reasoning.

Problem 4: [BV2004] Problem 2.13: consider the set of rank-k *outer products*, defined as $\{XX^T \mid X \in \mathbb{R}^{n\times k}, \operatorname{rank} X = k\}$. Describe its conic hull in simple terms.

Note: the *conic hull* of a set C is the set of all *conic combinations* of points in C, and is also the smallest *convex cone* containing C. It is *not* just the smallest (possibly non-convex) *cone* containing C.

Problem 5: [BV2004] Problem 2.35: a matrix $X \in \mathrm{SS}^n$ is called *copositive* if $z^T X z \geq 0$ for all $z \geq 0$.

a) Verify that the set of copositive matrices is a proper cone

b) Then find its dual cone. [Note: you can either find the dual cone yourself, and give a brief justification for why it is the dual cone; or if you are stuck on finding the dual cone, you can ask the instructor for what the dual cone is, but then your answer should give a more complete proof justifying why this is indeed the dual cone.]

Homework 2

Problem 1: Using the white wine data from the Spanish wine quality database `https://archive.ics.uci.edu/ml/datasets/Wine+Quality` hosted at the UCI machine learning repository, compare ordinary least-squares regression

$$\beta_2 = \operatorname{argmin} \|y - X\beta\|_2$$

with more robust ℓ_1 regression (cf. p. 294 §6.1.1 in [BV2004])

$$\beta_1 = \operatorname{argmin} \|y - X\beta\|_1$$

Report on the differences and make observations (e.g., are the estimators significantly different? are there outliers, and if so, does removing the outliers change the estimators?), and probably include a brief plot. You don't need to make a lengthy report, but give at least a few sentences of discussion. You may use any software you wish to solve the regression problems, e.g., `cvx` in Matlab or `cvxpy` in Python. [We'll do a demonstration of these software packages in class]

What to turn in? Please turn in a short printed document, with plots and text answering the questions above, and print relevant snippets of code (e.g., the part of the code that solves the problems — I don't need to see all of the plotting code, for example). If you have bugs in your code, I won't be able to help you when I grade, but you can always ask at office hours.