ELECTENG 332

Notes on Control Systems

Dear god help me, not another one...

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Introduction

These notes are compiled from the lectures of ELECTENG 332: Signals and Systems at The University of Auckland. They are intended as a personal reference to assist with assignments, exam preparation, and understanding key concepts in control systems and signal analysis.

The notes cover various topics, including signal modeling, feedback systems, and time-domain analysis, which are crucial for the analysis and design of control systems. Feel free to add personal insights or additional references as you study these materials.

Organisation of the Notes

The notes are divided into modules, with each section corresponding to specific topics covered in the course. Here's how they are organized:

- **Module 1: Basics of Signals and Systems**
  Key topics include time-domain representation, Laplace transforms, and system responses.
- **Module 2: Mathematical Modeling of Dynamic Systems**
  Covers the mathematical modeling of electrical and mechanical systems using block diagrams and differential equations.
- **Module 3: Block Diagrams & Feedback Systems Overview**
  Discusses the fundamentals of feedback control, including system stability, noise reduction, and transient response characteristics.
- **Module 4: Time Domain Analysis of Linear Systems**
  Provides a detailed analysis of transient and steady-state responses, including rise time, settling time, and peak overshoot.
- **Module 5: Stability Analysis of Linear Systems**
  Focuses on analyzing the stability of linear systems using different criteria such as the Routh-Hurwitz criterion.

How I Use These Notes

These notes are a living document that I update as I gain a deeper understanding of the material. Here's how I use them:

- **Quick Reference**: For quick lookups, the Table of Contents helps navigate directly to the relevant section.
- **In-Depth Study**: For exam preparation, I revisit each module, ensuring I understand each concept before moving on.
- **Personal Insights**: I add thoughts, extra readings, and questions for further study.

Exercises and examples are included to enhance the understanding of more complex topics, such as feedback control and stability analysis.

Part I

Module 1

Module 1:

Basics of Signals and Systems

Learning Outcomes

- Uniqueness of the Exponential Signal
- Concept of Engineering Infinity
- Concept of Complex Frequency
- Classification of Signals: Energy & Power
- Classification of Systems
- What is a Control System
- Classification of a Control System: Open-loop & Closed-loop

Module 2:

Topic 1: The Importance of the Exponential Function

The Exponential function, written as either e^{ax} or e^{at} depending on whether it is $f(t)$ or $f(x)$, has properties that make it mathematically unique.

1. The derivative (rate of change) of the exponential function is the exponential function itself. More generally, this is a function whose rate of change is proportional to the function itself.

$$\frac{\mathrm{d}e^{ax}}{\mathrm{d}x} = ae^{ax} \tag{1}$$

2. The integral of the exponential function is also the exponential function itself.

$$\int e^{ax}\,\mathrm{d}x = \frac{1}{a}e^{ax} \tag{2}$$

Module 3:

Topic 2: The Concept of Engineering Infinity

Consider a signal e^{-at}. The time constant for this signal is $T = \frac{1}{a}$. Theoretically, the signal is meant to decay to zero as time approaches infinity, i.e.

$$\lim_{t \to \infty} e^{-at} = 0 \tag{1}$$

But in practice, this is not the case, as its value will be very, very small after five time constants $5T$ (or 5τ). This is the ***Concept of Engineering Infinity***. The signal will never reach zero, but it will be so small that it can be considered zero for all practical purposes. This is a very important concept in control systems, as it allows us to simplify our calculations and analysis.

Module 4:

Topic 3: The Concept of Complex Frequency

Complex frequency is found commonly in electrical engineering. It is often notated as $j\omega$ or $s = \sigma \pm j\omega$. These frequencies always come in pairs, so the use of $\pm$ is implicit to this, as complex numbers have complex conjugates (normally notated by z^* or $\bar{z}$). i.e. $s = \sigma + j\omega$ has the conjugate $s = \sigma - j\omega$.

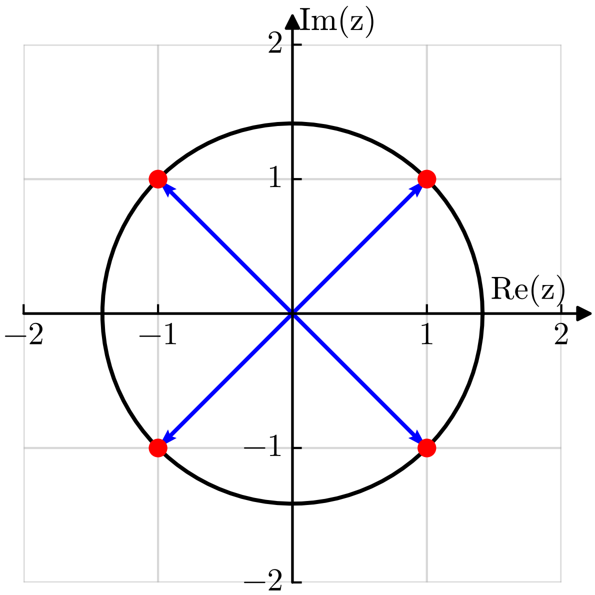

Figure 4.1: Argand diagram of $|z| = \sqrt{2}$

This is also backed up by De Moivre's formula which is defined mathematically as:

$$\forall x \in \mathbb{R}, \quad \forall n \in \mathbb{Z}, \tag{1}$$

$$e^{jnx} = \cos(nx) + j\sin(nx) \tag{2}$$

Or more generally for our applications (this is also known as Euler's formula):

$$e^{jx} = \cos(x) + j\sin(x) \tag{3}$$

$$\text{Where } x \in \mathbb{R} \, (x \text{ is real}) \tag{4}$$

$$\text{and } j \equiv i = \sqrt{-1} \tag{5}$$

This means that:

A complex frequency $j\omega$ represents a pure sinusoidal signal of frequency ω rad/s

For example, if a signal has a complex frequency $j314$ rad/s, then this responds to a pure sinusoid of frequency 314 rad/s (i.e. 50 Hz).

Furthermore:

A complex frequency $s = \sigma + j\omega$ represents an exponentially damped signal of frequency $j\omega$ rad/s, and decays/amplifies at a rate decided by σ.

For example, the signal $f(t) = e^{-10t}\sin(40\pi t)$ would look like this: