UNIVERSITY OF AUCKLAND
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ENGINEERING
DEPARTMENT OF ELECTRICAL, COMPUTER, AND SOFTWARE ENGINEERING

ELECTENG 332

Notes on Control Systems

Dear god help me, not another one...

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Basics of Signals and Systems

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1.1 Topic 1: The Importance of the Exponential Function

The Exponential function, written as either e^{ax} or e^{at} depending on whether it is $f(t)$ or $f(x)$, has properties that make it mathematically unique.

1. The derivative (rate of change) of the exponential function is the exponential function itself. More generally, this is a function whose rate of change is proportional to the function itself.

$$\frac{\mathrm{d}e^{ax}}{\mathrm{d}x} = ae^{ax} \tag{1}$$

2. The integral of the exponential function is also the exponential function itself.

$$\int e^{ax}\,\mathrm{d}x = \frac{1}{a}e^{ax} \tag{2}$$

1.2 Topic 2: The Concept of Engineering Infinity

Consider a signal e^{-at}. The time constant for this signal is $T = \frac{1}{a}$. Theoretically, the signal is meant to decay to zero as time approaches infinity, i.e.

$$\lim_{t\to\infty} e^{-at} = 0 \tag{3}$$

But in practice, this is not the case, as its value will be very, very small after five time constants $5T$ (or 5τ). This is the ***Concept of Engineering Infinity***. The signal will never reach zero, but it will be so small that it can be considered zero for all practical purposes. This is a very important concept in control systems, as it allows us to simplify our calculations and analysis.

1.3 Topic 3: The Concept of Complex Frequency

Complex frequency is found commonly in electrical engineering. It is often notated as $j\omega$ or $s = \sigma \pm j\omega$. These frequencies always come in pairs, so the use of $\pm$ is implicit to this, as complex numbers have complex conjugates (normally notated by z^* or $\bar{z}$). i.e. $s = \sigma + j\omega$ has the conjugate $s = \sigma - j\omega$.

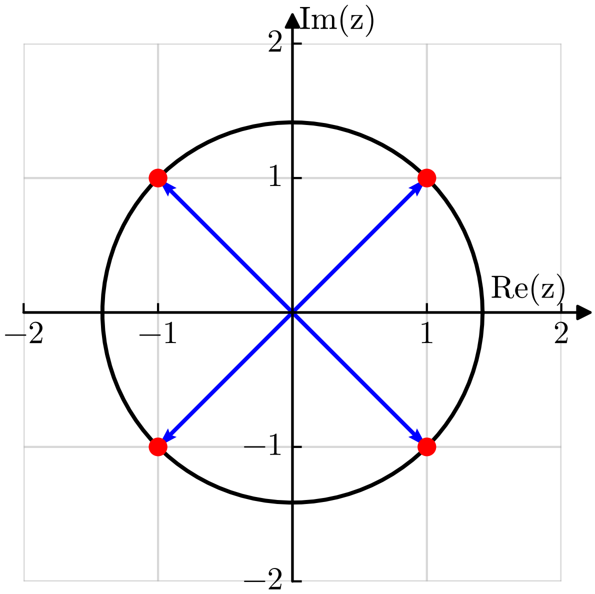

Figure 1.1: Argand diagram of $|z| = \sqrt{2}$

This is also backed up by De Moivre's formula which is defined mathematically as:

$$\forall x \in \mathbb{R}, \quad \forall n \in \mathbb{Z}, \tag{4}$$

$$e^{jnx} = \cos(nx) + j\sin(nx) \tag{5}$$

Or more generally for our applications (this is also known as Euler's formula):

$$e^{jx} = \cos(x) + j\sin(x) \tag{6}$$

$$\text{Where } x \in \mathbb{R} \, (x \text{ is real}) \tag{7}$$

$$\text{and } j \equiv i = \sqrt{-1} \tag{8}$$

This means that:

$\boxed{\textit{A complex frequency } j\omega \textit{ represents a pure sinusoidal signal of frequency } \omega \text{ rad/s}}$