UNIVERSITY OF AUCKLAND
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ENGINEERING
DEPARTMENT OF ELECTRICAL, COMPUTER, AND SOFTWARE ENGINEERING

ELECTENG 332

Notes on Control Systems

Dear god help me, not another one...

by

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Contents

Chapter 1

Module 1: Basics of Signals and Systems

Learning Outcomes

- Uniqueness of the Exponential Signal
- Concept of Engineering Infinity
- Concept of Complex Frequency
- Classification of Signals: Energy & Power
- Classification of System
- What is a Control System
- Classification of a Control System: Open-loop & Closed-loop

1.1 Topic 1: Importance of Exponential Functions

The Exponential function, written as either e^{ax} or e^{at} depending on whether it is $f(t)$ or $f(x)$, has properties that make it mathematically unique.

1. The derivative (rate of change) of the exponential function is the exponential function itself. More generally, this is a function whose rate of change is proportional to the function itself.
$$\frac{\mathrm{d}e^{ax}}{\mathrm{d}x} = ae^{ax}$$
2. The integral of the exponential function is also the exponential function itself.
$$\int e^{ax}\,\mathrm{d}x = \frac{1}{a}e^{ax}$$

1.2 Topic 2: Concept of Engineering Infinity

Consider a signal e^{-at}. The time constant for this signal is $T = \frac{1}{a}$. Theoretically, the signal is meant to decay to zero as time approaches infinity, i.e.

$$\lim_{t \to \infty} e^{-at} = 0$$

But in practice, this is not the case, as it's value will be very very very small, after five time constants $5T$. *This is the Concept of Engineering Infinity.*

1.3 Topic 3: Concept of Complex Frequency

Complex frequency is found commonly in electrical engineering. It is often notated as $j\omega$ or $s = \sigma \pm j\omega$. These frequencies always come in pairs, so the use of $\pm$ is implicit to this, as complex numbers have complex conjugates, i.e. $s = \sigma + j\omega$ has the conjugate $s = \sigma - j\omega$.

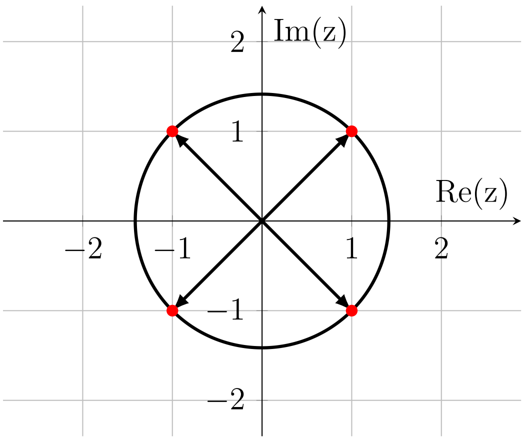


Figure 1.1: Plot of the circle $|z| = \sqrt{2}$ to show conjugates of complex numbers.

This also is backed up by De Moivre's Formula which is defined mathematically as:

$$\forall x \in \mathbb{R} \text{ and } \forall n \in \mathbb{Z} : e^{jnx} = \cos(nx) + j\sin(nx)$$

Or more generally for our applications:

$$e^{jx} = \cos(x) + j\sin(x) \text{ where } x \in \mathbb{R} \text{ (x is a real number) and } j \equiv i = \sqrt{-1}$$

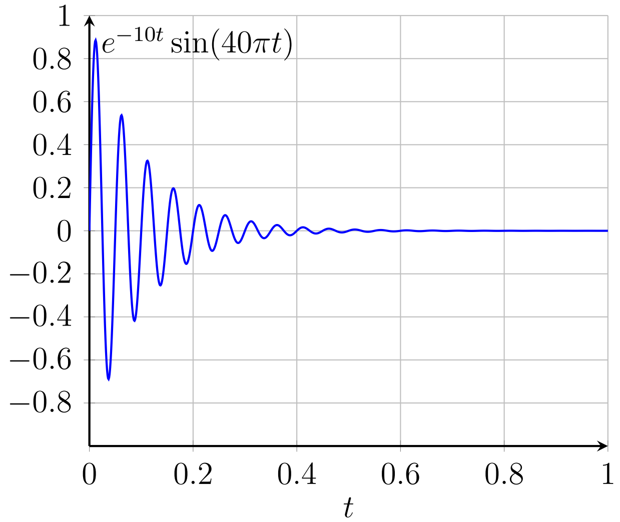
This means that:

$$\boxed{\textit{A complex frequency } j\omega \textit{ represents a pure sinusoidal signal of frequency } \omega \text{ rad/s}}$$

For example, if a signal has a complex frequency $j314$ rad/sec, then this corresponds to a pure sinusoid of frequency 314 rad/sec (i.e. 50 Hz). Furthermore:

A complex frequency $s = \sigma + j\omega$ represents an exponentially damped signal of frequency ω rad/s, and decays/amplifies at a rate decided by σ.

For example, the signal $e^{-10t}\sin(40\pi t)$ would look as:



1.4 Topic 4: What are Signals?

Chapter 2

Module 2: Mathematical Modelling of Dynamic Systems

2.1 Topic 1: [Topic Name]

2.2 Topic 2: [Another Topic Name]

Chapter 3

Module 3: Concept of Block Diagram Representation & Characteristics of Feedback Systems

Chapter 4

Module 4: Time Domain Analysis of Linear Systems

Chapter 5

Stability Analysis of Linear Systems