ELECTENG 332

Notes on Control Systems

Dear god help me, not another one...

by

Nicholas Russell

August 10th, 2024

Department of Electrical, Computer, and Software Engineering

Faculty of Engineering

University of Auckland

Contents

Chapter 1

Module 1: Basics of Signals and Systems

Learning Outcomes

- Uniqueness of the Exponential Signal
- Concept of Engineering Infinity
- Concept of Complex Frequency
- Classification of Signals: Energy & Power
- Classification of Systems
- What is a Control System
- Classification of a Control System: Open-loop & Closed-loop

1.1 Topic 1: Importance of Exponential Functions

The Exponential function, written as either e^{ax} or e^{at} depending on whether it is $f(t)$ or $f(x)$, has properties that make it mathematically unique.

1. The derivative (rate of change) of the exponential function is the exponential function itself. More generally, this is a function whose rate of change is proportional to the function itself.
$$\frac{\mathrm{d}e^{ax}}{\mathrm{d}x} = ae^{ax}$$
2. The integral of the exponential function is also the exponential function itself.
$$\int e^{ax}\,\mathrm{d}x = \frac{1}{a}e^{ax}$$

1.2 Topic 2: Concept of Engineering Infinity

Consider a signal e^{-at}. The time constant for this signal is $T = \frac{1}{a}$. Theoretically, the signal is meant to decay to zero as time approaches infinity, i.e.

$$\lim_{t \to \infty} e^{-at} = 0$$

But in practice, this is not the case, as its value will be very, very small after five time constants $5T$. This is the *Concept of Engineering Infinity.*

1.3 Topic 3: Concept of Complex Frequency

Complex frequency is found commonly in electrical engineering. It is often notated as $j\omega$ or $s = \sigma \pm j\omega$. These frequencies always come in pairs, so the use of $\pm$ is implicit to this, as complex numbers have complex conjugates, i.e. $s = \sigma + j\omega$ has the conjugate $s = \sigma - j\omega$.

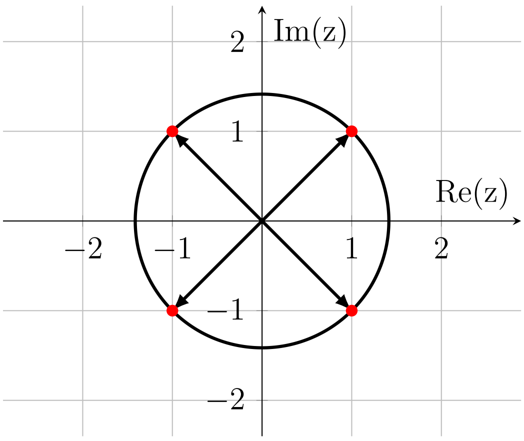

Figure 1.1: Plot of the circle $|z| = \sqrt{2}$

This is also backed up by De Moivre's Formula, which is defined mathematically as:

$$\forall x \in \mathbb{R}, \quad \forall n \in \mathbb{Z},$$
$$e^{jnx} = \cos(nx) + j\sin(nx)$$

Or more generally for our applications:

$$e^{jx} = \cos(x) + j\sin(x)$$
$$\text{Where}\, x \in \mathbb{R}\, (x \text{ is Real})$$
$$\text{and}\, j \equiv i = \sqrt{-1}.$$

This means that:

A complex frequency $j\omega$ represents a pure sinusoidal signal of frequency ω rad/s

For example, if a signal has a complex frequency $j314$ rad/sec, then this corresponds to a pure sinusoid of frequency 314 rad/sec (i.e. 50 Hz). Furthermore:

A complex frequency $s = \sigma + j\omega$ represents an exponentially damped signal of frequency ω rad/s, and decays/amplifies at a rate decided by σ.

For example, the signal $e^{-10t}\sin(40\pi t)$ would look like this:

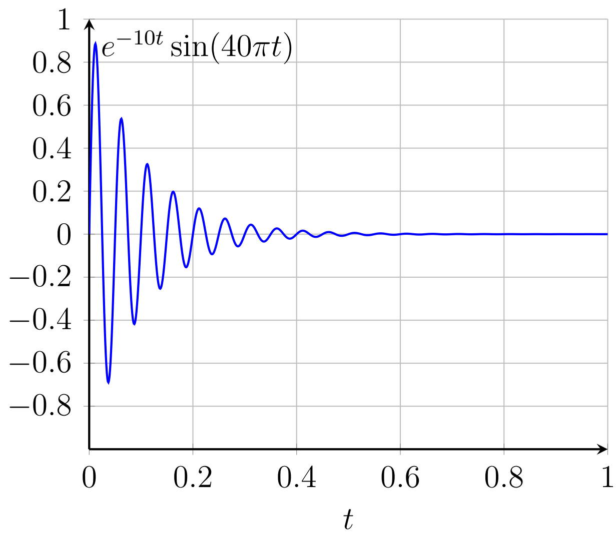

Figure 1.2: Plot of the signal $e^{-10t}\sin(40\pi t)$.

1.4 Topic 4: What are Signals?

Chapter 2

Module 2: Mathematical Modelling of Dynamic Systems

2.1 Topic 1: [Topic Name]

2.2 Topic 2: [Another Topic Name]

Chapter 3

Module 3: Concept of Block Diagram Representation & Characteristics of Feedback Systems

Chapter 4

Module 4: Time Domain Analysis of Linear Systems

Chapter 5

Stability Analysis of Linear Systems