5 Alice and Bob Play Nim

Alice and Bob are playing a game on a sequence of n nonnegative integers $\langle a_1, \ldots, a_n \rangle$ with the following rules:

- Players alternate taking turns, with Alice playing first.
- In each turn a player selects a positive integer a_i from the sequence, and replaces it with $a_i - z$, where z is any divisor of a_i (including both 1 and a_i).
- If the current player has no valid move (that is, if the sequence consists of all zeros), this player loses the game.

For example, if the initial sequence is $(1, 5, 4)$, Alice can subtract 2 from the third number in the sequence (since 2 is a divisor of 4) resulting in the sequence $(1, 5, 2)$. Bob could then subtract 5 from the middle number, resulting in $(1, 0, 2)$, and so on.

For any given input sequence, one of the two players has a winning strategy. Let $p(n, m)$ denote the probability that Alice has a winning strategy for a random game of the form $(a_1, \ldots, a_n)$, under the assumption that each a_i is chosen uniformly at random from the interval $[0, m]$. Write a program that, given any number m computes

$$P(m) \; = \; \lim_{n \to \infty} p(n, m).$$

You may assume that the limit always exists and the value of $P(m)$ is a rational number, that is, it can be expressed as x/y, where x and y are relatively prime integers (share no common factors). Output your answer as the pair of integers "`x y`".

You may assume that $1 \leq n \leq 10^5$ and $1 \leq m \leq 10^7$.

Input and output files can be found at: `http://challengebox.cs.umd.edu/2019/Nim`

Input:

The input consists of a single line that contains just the number m.

Output:

The output is the space-separated pair "`x y`" such that $P(m) = x/y$.

Example:

If $m = 1$, then each starting sequence is a bit string. It is easy to see that Alice wins if the number of 1's in the bit string is odd, and Bob wins if if the number of 1's is even. By symmetry, both Alice and Bob have equal probabilities of winning. Thus, $P(m) = 1/2$ and the output is "`1 2`".

Input:	Output:
`1`	`1 2`