European Journal of Operational Research 230 (2013) 26–41

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

A stochastic aggregate production planning model in a green supply chain: Considering flexible lead times, nonlinear purchase and shortage cost functions

S.M.J. Mirzapour Al-e-hashem [a,*], A. Baboli [b], Z. Sazvar [b,c]

[a] EMLYON Business School, 23 Ave. Guy de Collongue, Ecully, Lyon, France
[b] INSA-Lyon, DISP Laboratory, Villeurbanne F-69621, France
[c] Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 13 July 2011
Accepted 22 March 2013
Available online 11 April 2013

Keywords:
Supply chain management
Aggregate production planning
Green principles
Quantity discount
Nonlinear shortage cost
Demand uncertainty

ABSTRACT

In this paper we develop a stochastic programming approach to solve a multi-period multi-product multi-site aggregate production planning problem in a green supply chain for a medium-term planning horizon under the assumption of demand uncertainty. The proposed model has the following features: (i) the majority of supply chain cost parameters are considered; (ii) quantity discounts to encourage the producer to order more from the suppliers in one period, instead of splitting the order into periodical small quantities, are considered; (iii) the interrelationship between lead time and transportation cost is considered, as well as that between lead time and greenhouse gas emission level; (iv) demand uncertainty is assumed to follow a pre-specified distribution function; (v) shortages are penalized by a general multiple breakpoint function, to persuade producers to reduce backorders as much as possible; (vi) some indicators of a green supply chain, such as greenhouse gas emissions and waste management are also incorporated into the model. The proposed model is first a nonlinear mixed integer programming which is converted into a linear one by applying some theoretical and numerical techniques. Due to the convexity of the model, the local solution obtained from linear programming solvers is also the global solution. Finally, a numerical example is presented to demonstrate the validity of the proposed model.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

1.1. Green supply chain

Awareness of the environment has been increasing in the last few decades. More people are conscious of the world's environmental problems such as global warming, toxic substance usage, and a decrease of non-renewable resources. In response to this threat, a number of organizations are applying 'green' principles, such as using environmentally friendly raw materials and recycled paper for packaging, and reducing the use of fossil fuels (Mirzapour Al-e-hashem et al., 2011c). These green principles have been expanded to many areas of work, including supply chains. Green Supply Chain Management (GSCM) has been emerging in the last few years (Markovitz-Somogyi et al., 2009). Adding the 'green' concept to the 'supply chain' concept creates a new paradigm where the supply chain will have a direct relation to the environment. This is interesting because historically, these two paradigms have been in head-on collision with each other (Srivastava, 2007). This idea covers every stage in manufacturing and in the life cycle of product, i.e. from product design to recycling (see for example Diabat and Govindan, 2011; Wang et al., 2011; Zhu and Sarkis, 2011; Eltayeb et al., 2011; Yeh and Chuang, 2011; Bai and Sarkis, 2010; Sheu, 2008; Kumar et al., 2012). Also, Min and Kim (2012) have presented a comprehensive review on green supply chain researches by studying more than five hundred papers.

Green logistics as a sub set of green supply chain has been expanding in the last few years (Srivastava, 2007). One comprehensive literature review of studies on green logistic is Dekker et al. (2012). They state that respect to the environment, transportation is the most visible aspect of supply chains. They examine four choices with respect to transportation which are supported by Operations Research models, namely, mode choice (or modal split), use of intermodal transport, equipment choice and fuel choice. The former choice in transport is the mode of transportation (mode choice), viz. transport by plane, ship, truck, rail, barge or pipelines. Each mode has different characteristics in terms of costs, transit time, accessibility, and also different environmental performance. In present study we use this method to embed the environmental issue in our model.

* Corresponding author. Address: EMLYON Business School, 23 Ave. Guy de Collongue, Ecully cedex 69134, Lyon, France. Tel.: +33 627 41 44 39.

E-mail addresses: mirzapour@em-lyon.com (S.M.J. Mirzapour Al-e-hashem), armand.baboli@insa-lyon.fr (A. Baboli), z.sazvar@insa-lyon.fr (Z. Sazvar).

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.03.033

One of the most important questions in green logistics is how to identify preferred solutions balancing environmental and business concerns (Quariguasi Frota Neto et al., 2009). Quariguasi Frota Neto et al. (2009) have emphasized that Improving environmental quality comes at a cost, so the question is which trade-offs occur between the environmental impacts of an economic activity and its costs, and what are best solutions balancing ecological and economic concerns? The aim is to determine solutions in which environmental damage can only be decreased if costs are increased. These solutions are called eco-efficient. The idea of exploring best alternatives is based on Pareto-optimality (Huppes and Ishikawa, 2005).

1.2. Aggregate production planning (APP)

One of the well known subjects that could be addressed in GSCM is aggregate production planning (APP), an operational activity that draws up an aggregate plan for the production process, in advance of 3–18 months, to give an idea to management as to what quantity of materials and other resources are to be procured and when, so that the total cost of operations of the organization is kept to the minimum over that period. Many researchers have focused on APP from several years ago (Shi and Haase, 1996) and numerous APP models with differing degrees of difficulty have been developed in the last decades. Since Holt et al. (1955) presented the approach for the first time; scholars have proposed numerous models to help solve APP problems, each with their own supporters and detractors. In a comprehensive study, Nam and Logendran (1992) reviewed APP models from 140 journal articles and 14 books, and classified them into optimal and near-optimal models. Hanssman and Hess (1960) proposed a model based on the linear programming approach using a linear cost function of the decision variables. Haehling (1970) extended the Hanssman and Hess (1960) model for multi-product, multi-stage production systems in which optimal disaggregation decisions can be made under capacity constraints. Masoud and Hwang (1980) presented three Multi Criteria Decision Making Methods (MCDMs): Goal Programming (GP), sequential multi-objective problem and the step method. These methods are applied to solve APP problems by maximizing profits, and minimizing changes in workforce levels, inventory investments and backorders. Goodman (1974) proposed a GP model which approximates the original nonlinear cost structure of Holt's model by linear terms and solves it applying a variant of the simplex method. Baykasoglu (2001) has extended Masud and Hwang's model with more constraints such as sub-contractor selection and setup times. The integration of APP problems with other planning problems has been considered, for instance scheduling problems (Buxey, 1993; Foote et al., 1988), manpower planning problems (Mazzola et al., 1998), long set up time problems (Porkka et al., 2003; Xue et al., 2011) and supply chain planning (Ramezanian et al., 2012; Jamalnia and Soukhakian, 2009; Mirzapour Al-e-hashem et al., 2012, Mirzapour Al-e-hashem et al., 2011a,b).

1.3. Motivation

However, the wide range of studies devoted to the APP and its integration with the other supply chain challenges, it is becoming increasingly difficult to ignore the gap of environmental aspect in this respect. According to the literature, to the best of our knowledge, while Mirzapour Al-e-hashem et al. (2011c) is the first attempt to adopt green concepts to aggregate production planning in deterministic condition, the present study is among the first to embed the concept of green logistic in aggregate production planning under uncertainty.

We incorporate the concept of green supply chain in two ways: 1 – mode choice; by considering transport mode decision variables (mode choice) to reduce GHGs. We assume an interrelationship between lead time and transportation mode: the shorter the lead times, the more expensive transportation will be, while also increasing GHGs and 2 – waste management; obviously a supply chain is also characterized by the products it supplies. Mirzapour Al-e-hashem et al. (2011b) consider only the inventory aspects of products, but we take a more comprehensive view. The point is that some products are friendlier to the environment than others. In other words, we assume each unit of product is associated with a percentage of waste, and we limit the total amount of waste produced by each factory.

Previous models dealing with supply chains mostly employ a simplified transportation cost function such as linear term in the objective function (see e.g. Ng et al., 2001 and Leblanc et al., 2004). This approach can be justified by assuming that these costs are part of the unit price (and therefore are managed by the supplier) or are fixed and incorporated into the ordering costs (Leenders et al., 2002). Despite the fact that linear (constant unit) transportation costs are desirable from a simplicity and tractability standpoint, they are often far from accurate (Carter et al., 1995). Thomas and Griffin (1996), Ertogral et al. (2007) and Hill and Galbreth (2008) identify the impact of complex shipping costs as an issue that has not been adequately addressed in the literature. In this paper we assume that the vehicle type can be selected among various available vehicle types and the transportation cost depends directly on the associated fixed and variables cost of selected vehicles.

Vendors usually offer quantity discounts to encourage the customer to order more, and the producer intends to discount the unit production cost if the amount of production is large. This study solves a nonlinear APP model capable of treating various quantity discount functions simultaneously, including linear, single breakpoint, step, and multiple breakpoint functions. In addition, we consider the shortage cost function as a multi-breakpoint nonlinear function to persuade the decision maker (DM) to reduce the backorders as much as possible.

Finally by introducing a new theoretical linearization technique and utilizing the previously available ones, the proposed nonlinear model is approximated to a solvable linear mixed integer program to obtain a global optimum. In the end, using the two-stage stochastic programming approach, a numerical example is analyzed to validate the model.

The rest of the manuscript is organized as follows: in Section 2, the problem description and model assumptions are presented. In Section 3 the model formulation is described and in Section 4 solution procedure is presented. The validity and applicability of the proposed model are demonstrated by an example in Section 5. Finally, conclusions and future research are presented in Section 6.

2. Problem description

Aggregate production planning is a traditional problem that firms have to deal with. APP is concerned with the determination of production, inventory, and workforce levels to meet varying demand requirements over a medium-term planning horizon that ranges from 3 to 18 months. The planning horizon is often divided into several periods. Typically, the physical resources of the firm are assumed to be fixed during the planning horizon of interest. Given the external demand requirements, the planning effort is oriented toward the best utilization of those resources (see Gallego, 2001).

The proposed multi-site APP problem in a supply chain can be described as follows:

There are K suppliers, J factories (plant), and I customers. In order to fulfill the uncertain demand of customers, each factory produces several product items grouped into N product families. Here we call this the product. Each product is made from only one raw material and should be supplied by the suppliers. Production costs (fixed and variable) of a certain item at different sites can be different. The procurement cost of the raw materials depends on the order size and can vary, that is, the larger the order, the lower the procurement cost per unit, while the basic price could be different among different suppliers. Each factory is characterized by its own inventory and production capacities. The latter is limited to the available workers and the allowed amount of regular working time and overtime. As all factories, suppliers and customers are spread out geographically, the transportation cost from suppliers to factories and from factories to customers can vary. The unit transportation cost is a function of the vehicle type that should be selected from the available vehicle types.

Given that storing products in customers' zones is possible, and that backorders will be penalized by a nonlinear structure cost function, the problem is to determine: (1) the quantity of product n manufactured at factory j to fulfill the stochastic demand of customer zone i in each period of time; (2) the quantity of raw material n provided by supplier k in all periods to fulfill the net requirements of factory j with regard to lead time; (3) the number of workers hired and fired in each period; (4) the quantity of product n stored at factory j and customer zone i in each period; (5) the type of vehicle used for transportation of the products; (6) the amount of demand in each customer zone not met in each period, so that the total loss along the entire supply chain will be minimized regarding some green principles such as greenhouse gas emission and waste management.

2.1. Notation

Variables	
XR_{njt}	the amount of product type n produced at plant j in regular time of period t
XO_{njt}	the amount of product type n produced at plant j in overtime of period t
L_{jt}	available workers at plant j in period t
F_{jt}	the number of workers fired at plant j in period t
H_{jt}	the number of workers hired at plant j in period t
$I1_{njt}$	inventory level of product type n at plant j at the end of period t
$I2_{nit}$	inventory level of product type n at demand point i at the end of period t
XQ_{nkjgt}	number of units of product type n shipped from supplier $k(1, 2, \ldots, K)$ to plant j in period t by vehicle type $g(1, 2, \ldots, G)$
YQ_{njigt}	number of units of product type n shipped from plant j to demand point i in period t by vehicle type g
b_{nit}	shortage of product n in demand point i in period t
Y_{njt}	a binary variable that determines whether plant j produces product type n in period t
XV_{kjgt}	the number of vehicle type g used for transportation between supplier k and factory j
YV_{jigt}	the number of vehicle type g used for transportation between factory j and customer zone i
Parameters	
D_{nit}/D^s_{nit}	demand for product n $(1, 2, \ldots, N)$ at demand point $i(1, 2, \ldots, I)$ in period $t(1, 2, \ldots, T)$/under scenario $s(1, \ldots, S)$
CR_{jn}	the variable production cost for product type n in regular time at plant j
CO_{jn}	the variable production cost for product type n in overtime at plant j
PC_{nj}	the fixed production cost of product type n at plant j
P_{ni}	sale price of product n at demand point i
SC_j	labor cost at factory j
a_{nj}	manpower needs to produce one unit of product type n at plant j
FC_j	firing cost at factory j
HC_j	hiring cost at factory j
$CI1_{nj}$	inventory holding cost for product n at plant j
$CI2_{ni}$	inventory holding cost for product n at demand point i
α	allowed fraction of overtime
TV_g	the variable transportation cost of vehicle type g
TF_g	the fixed transportation cost when vehicle type g is selected for transportation
$d1_{kj}$	distance between supplier k and factory j
$d2_{ji}$	distance between factory j and customer zone i
CM_{nk}	cost of product type n provided by supplier k
$CP1_j$	product storage capacity at plant j
$CP2_j$	product storage capacity at customer zone i
CS_{nk}	number of product type n that could be provided by supplier k in each period
v_n	volume of a unit product type n (m^3)
V_g	the capacity of vehicle type g (m^3)
GHG_g	CO_2 emissions for vehicle type g per distance unit
GHl_{jt}	allowed greenhouse gas emission level in period t by factory j
WT	allowed amount of waste produced by each factory
wp	percentage of waste produced by product n in factory j
LT_{nkjg}	lead time required for shipping product type n from supplier k to plant j using vehicle type g
LT_{njig}	lead time required for shipping product type n from plant j to demand point i using vehicle type g
π_{ni}	shortage cost of product n at demand point i
ρ^{ξ}	occurrence probability of scenario s $(1, \ldots, S)$

2.2. Deterministic model

Objective function terms

Labor cost

$$\sum_{j,t} SC_j \cdot L_{jt} + \sum_{j,t} FC_{jt} \cdot F_{jt} + \sum_{j,t} HC_{kj} \cdot H_{jt} \tag{1}$$

Inventory cost

$$\sum_{n,j,t} CI1_{nj} \cdot I1_{njt} + \sum_{n,i,t} CI2_{ni} \cdot I2_{nit} \tag{2}$$

Transportation cost

$$\sum_{k,j,i,g,t} TF_g \cdot (XV_{kjgt} + YV_{jigt}) + \sum_{k,j,g,t} TV_g d1_{kj} XV_{kjgt} + \sum_{j,i,g,t} TV_g d2_{ji} \cdot YV_{jigt} \tag{3}$$

Procurement, production cost

$$\sum_{n,j,t} PC_{jt} \cdot Y_{njt} + \sum_{i,j,g,t} CR_{jg} \cdot XR_{njt} + \sum_{i,j,g,t} CO_{jg} \cdot XO_{njt} + \sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt}) \cdot XQ_{nkjgt} \tag{4}$$

Sales income

$$\sum_{n,j,i,g,t} P_{ni} \cdot YQ_{njigt} \tag{5}$$

Shortage cost

$$\sum_{n,i,t} \pi_{ni}(B_{nit}) \cdot B_{nit} \tag{6}$$

$$\begin{aligned} \text{Min}Z = & \sum_{j,t} SC_j \cdot L_{jt} + \sum_{j,t} FC_j \cdot F_{jt} + \sum_{j,t} HC_j \cdot H_{jt} + \sum_{n,j,t} CI1_{nj} \cdot I1_{njt} \\ & + \sum_{n,i,t} CI2_{ni} \cdot I2_{nit} + \sum_{k,j,i,g,t} TF_g \cdot (XV_{kjgt} + YV_{jigt}) + \sum_{k,j,g,t} TV_g d1_{kj} XV_{kjgt} \\ & + \sum_{j,i,g,t} TV_g d2_{ji} \cdot YV_{jigt} + \sum_{n,j,t} PC_{jt} \cdot Y_{njt} + \sum_{n,j,g,t} CR_{jn} \cdot XR_{njt} \\ & + \sum_{n,j,g,t} CO_{jn} \cdot XO_{njt} + \sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt}) \cdot XQ_{nkjgt} \\ & + \sum_{n,i,t} (\pi_{ni}(B_{nit}) \cdot B_{nit}) - \sum_{n,j,i,g,t} P_{ni} \cdot YQ_{njigt} \end{aligned} \tag{7}$$

s.t.

$$I2_{ni(t-1)} + \sum_{j,g} YQ_{njig(t-LTjig)} - D_{nit} - B_{ni(t-1)} = I2_{nit} - B_{nit} \quad \forall n,i,t \tag{8}$$

$$I1_{nj(t-1)} + \sum_{k,g} XQ_{nkjg(t-LTkjg)} - \sum_{i,g} YQ_{njigt} = I1_{njt} \quad \forall n,j,t \tag{9}$$

$$\sum_{n} a_{nj} \cdot XO_{njt} \leqslant \alpha \cdot L_{jt} \quad \forall j,t \tag{10}$$

$$\sum_{n} a_{nj} \cdot XR_{njt} \leqslant L_{jt} \quad \forall j,t \tag{11}$$

$$L_{jt} = L_{j(t-1)} + H_{jt} - F_{jt} \quad \forall j,t \tag{12}$$

$$\sum_{k,g} XV_{kjgt} \cdot GHG_g \cdot d1_{kj} + \sum_{i,g} YV_{jigt} \cdot GHG_g \cdot d2_{ji} \leqslant GHl_{jt} \quad \forall j,t \tag{13}$$

$$\sum_{n,t} (XR_{njt} + XO_{njt}) \cdot wp_{nj} \cdot \leqslant WT_j \quad \forall j \tag{14}$$

$$\sum_{j,g} XQ_{nkjgt} \leqslant CS_{nk} \quad \forall n,k,t \tag{15}$$

$$\sum_{n} I1_{njt} \leqslant CP1_j \quad \forall j,t \tag{16}$$

$$\sum_{n} I2_{nit} \leqslant CP2_i \quad \forall i,t \tag{17}$$

$$(XV_{kjgt} - 1)V_g \leqslant \sum_{n} \upsilon_n \cdot XQ_{nkjgt} \leqslant XV_{kjgt} V_g \quad \forall k,j,g,t \tag{18}$$

$$(YV_{jigt} - 1)V_g \leqslant \sum_{n} \upsilon_n \cdot YQ_{njigt} \leqslant YV_{jigt} V_g \quad \forall j,i,g,t \tag{19}$$

$$\frac{Y_{jt}}{M} \leqslant \sum_{n} (XO_{njt} + XR_{njt}) \leqslant Y_{jt} \cdot M \quad \forall j,t \tag{20}$$

$$XO_{njt} + XR_{njt} \leqslant \sum_{k,g} XQ_{nkjg(t-LTkjg)} \quad \forall n,j,t \tag{21}$$

$$\sum_{i,g} YQ_{njigt} \leqslant XO_{njt} + XR_{njt} \quad \forall n,j,t \tag{22}$$

$$\begin{aligned} & L_{jt}, F_{jt}, H_{jt}, XV_{kjgt}, YV_{jigt} \textit{integer} \\ & \cdot I1_{njt}, I2_{nit}, XQ_{nkjgt}, YQ_{njigt}, XR_{njt}, XO_{njt} \geqslant 0 \\ & Y_{jt} \in \{0,1\} \end{aligned} \tag{23}$$

where Eq. (7) is the objective function of the proposed model and tries to minimize total losses of APP in the supply chain including human resources costs (salary, hiring and firing costs), inventory holding costs in both factories and customers' zones, transportation costs, production costs (regular and overtime), procurement costs, and shortage costs from which total sales income is conducted. Constraint (8) is an inventory and backorder balance equation in customer zones, constraint (9) is an inventory balance equation for factories, constraints (10) and (11) specify the upper limits of regular and overtime in factories, constraint (12) is a balance equation for workforce level and ensures that the number of available workers is equal to the number of workers in previous periods in addition to the change of workforce level in the current period. Constraints (13) and (14) limit the greenhouse gas emission of transportation issues and waste produced during the production process to a predetermined level. The GHG limit introduced could be interpreted as an ethical boundary fixed by the corporate strategy or could be a threshold over it the firm might pay extra taxes because of its emission ratio. Constraint (15) ensures that the order size released for each supplier is limited by its capacity. Constraints (16) and (17) determine the upper limits of inventory capacity in factories and customer zones. Constraints (18) and (19) calculate the number of each vehicle type used for transportation of the products between suppliers and factories and also between factories and customer zones. Constraint (20) is an auxiliary constraint to guarantee that the binary variable Y_{jt} takes value 1, once at least one product is produced in an associated factory, otherwise it equals zero. This variable is used in the objective function to consider the fixed setup cost for each factory. Constraint (21) restricts the number of products produced in factories in regular or overtime to the received number of raw materials in the current period. Constraint (22) limits the number of transferred products to the number of products produced in regular or overtime in the current period. Constraint (23) defines the variable types.

Remark 1. The GHG and waste limits introduced can be interpreted as an ethical boundary set by the corporate strategy or as a threshold over which the firm might pay extra taxes because of its emission ratio. In fact, the waste produced during the production process (wp) in our model is not tangible, contrary to defective products that force a plant to produce more than usual or to consider associated costs as an objective function. In our model the waste is of the same nature as GHGs. For example, some products in industry produce contaminated water during their production process. In some cases, the governmental rules limit the amount of industrial waste and emissions authorized, in order to protect the environment against their harmful effects. In fact wp is the rate at which each product produces these environmentally destructive impacts.

2.3. Quantity discount function

As noted above, vendors usually offer quantity discounts to encourage the customer to order more, and the producer discounts the unit production cost if the amount of production is large. In this research we propose an APP model capable of treating various quantity discount functions simultaneously, including linear, single breakpoint, step, and multiple breakpoint functions. A multiple breakpoint quantity discount function depicted in Fig. 1 can usually be represented as follows:

$$CM_{nk}(XQ_{nkjgt}) = \begin{cases} CM_1 + r_1(XQ_{nkjgt} - XQ_1) & \text{if } XQ_1 \leqslant XQ_{nkjgt} \leqslant XQ_2 \\ CM_2 + r_2(XQ_{nkjgt} - XQ_2) & \text{if } XQ_2 \leqslant XQ_{nkjgt} \leqslant XQ_3 \\ \vdots & \\ CM_{m-1} + r_{m-1}(XQ_{nkjgt} - XQ_{m-1}) & \text{if } XQ_{m-1} \leqslant XQ_{nkjgt} \leqslant XQ_m \end{cases} \tag{24}$$

where r_i is the slope when the quantity ordered is betweenXQ_i and XQ_{i+1}, and m means there are $m-1$ line segments in $CM(XQ_i)$. CM_{nk} is the unit purchase cost, but it is not constant to allow us to simply measure total purchase cost such as $CM_{nk} \times XQ_{nkjgt}$. In case of quantity discount, the purchase cost is a function of quantity

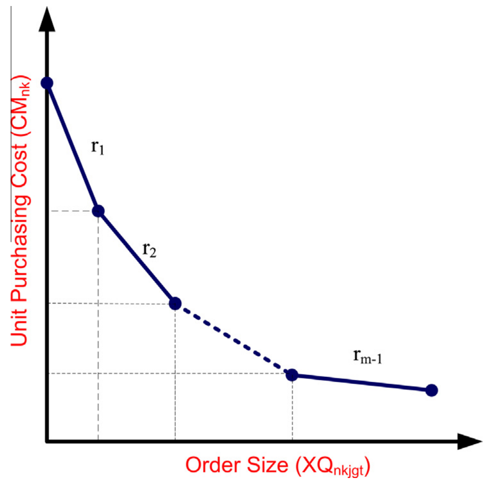


Fig. 1. Multiple breakpoint quantity discount function.

ordered (see Fig. 1), it therefore must be written as: $CM_{nk}(XQ_{nkjgt})\text{-}\times XQ_{nkjgt}$, in which $CM_{nk}(XQ_{nkjgt})$ means that CM is a function of XQ. Since the price and quantity discount functions are variable, the total cost function of the APP model is then formulated as a nonlinear form which is difficult to solve for finding a global solution (see Eq. (4)). In order to solve the APP model for obtaining a global solution, this study applies the linearization techniques presented in the next section.

2.4. Nonlinear shortage cost function

Due to the difficulty of dealing with nonlinear functions, many APP models assume that the shortage cost is penalized by a linear function. We consider the shortage cost function as a multiple breakpoint nonlinear function to persuade the decision maker (DM) to reduce the backorders as much as possible. In this way, more backorders lead to more and more shortage costs; in other words, the slope of the penalty function gradually increases by the amount of backorders.

The relationship between backorders and shortage costs depicted in Fig. 2 could be formulated as follows:

$$\pi_{ni}(B_{nit}) = \begin{cases} \pi_{ni1} + r_{ni1}(B_{nit} - B_1) & \text{if } B_1 \leqslant B_{nit} \leqslant B_2 \\ \pi_{ni2} + r_{ni2}(B_{nit} - B_2) & \text{if } B_2 \leqslant B_{nit} \leqslant B_3 \\ \vdots & \\ \pi_{ni(q-1)} + r_{niq}(B_{nit} - B_{q-1}) & \text{if } B_{q-1} \leqslant B_{nit} \leqslant B_q \end{cases} \tag{25}$$

3. Linearization of nonlinear multi-breakpoint functions

3.1. Quantity discount function

To linearize the multiple breakpoint discount function $CM_{nk}(XQ_{nkjgt})$, we introduce a binary variable (t), and also convert variable XQ_{nkjgt} to $m-1$ independent variables as $\overset{m}{XQ}_{nkjgt}$, where; $XQ_{nkjgt} = \sum_{m'=1}^{m-1} \overset{m'}{XQ}_{nkjgt}$, so Eq. (24) can be rewritten as follows:

$$CM_{nk}(XQ_{nkjgt}) = \begin{cases} (CM_1 - r_1XQ_1) + r_1\overset{(1)}{XQ}_{nkjgt} & \text{if } XQ_1 \leqslant \overset{(1)}{XQ}_{nkjgt} \leqslant XQ_2 \\ (CM_2 - r_2XQ_2) + r_2\overset{(2)}{XQ}_{nkjgt} & \text{if } XQ_2 \leqslant \overset{(2)}{XQ}_{nkjgt} \leqslant XQ_3 \\ \vdots & \\ (CM_{m-1} - r_{m-1}XQ_{m-1}) + r_{m-1}\overset{m-1}{XQ}_{nkjgt} & \text{if } XQ_{m-1} \leqslant \overset{(m-1)}{XQ}_{nkjgt} \leqslant XQ_m \end{cases} \tag{26}$$

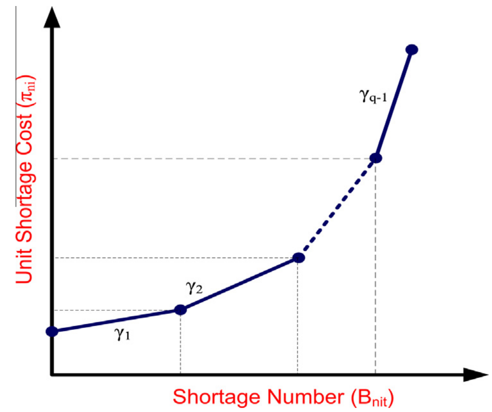


Fig. 2. Multiple breakpoint shortage cost function.

Therefore, the last term of the Eq. (4) in the objective function ($CM_{nk}(XQ_{nkjgt})\,.\,XQ_{nkjgt}$) could be expanded as

$$CM(XQ)\cdot XQ = \begin{cases} (CM_1 - r_1XQ_1)\overset{(1)}{XQ} + r_1\overset{(1)}{XQ}{}^2 & \text{if } XQ_1 \leqslant \overset{(1)}{XQ} \leqslant XQ_2 \\ (CM_2 - r_2XQ_2)\overset{(2)}{XQ} + r_2\overset{(2)}{XQ}{}^2 & \text{if } XQ_2 \leqslant \overset{(2)}{XQ} \leqslant XQ_3 \\ \vdots & \\ (CM_{m-1} - r_{m-1}XQ_{m-1})\overset{(m-1)}{XQ} + r_{m-1}\overset{(m-1)}{XQ}{}^2 & \text{if } XQ_{m-1} \leqslant \overset{(m-1)}{XQ} \leqslant XQ_m \end{cases} \tag{27}$$

As proved in Appendix A, with the help of $m-1$ constraints, the equivalent mathematical structure of $\sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt})\cdot XQ_{nkjgt}$ can be written as

$$\sum_{n,k,j,g,t}\sum_{m'=1}^{m-1}\left[(CM_{nkm'} - r_{nkm'}XQ_{nkm'})\overset{(m')}{XQ}_{nkjgt} + r_{nkm'}\overset{(m')}{XQ}{}^2_{nkjgt}\right] \tag{28}$$

s.t.

$$XQ_{nk(m-1)}t_{nkjgt(m-1)} \leqslant \overset{(m-1)}{XQ}_{nkjgt} \leqslant XQ_{nkm}t_{nkjgt(m-1)} \quad \forall n,k,j,g,t,m \tag{29}$$

$$XQ_{nkjgt} = \sum_{m'=1}^{m-1}\overset{m'}{XQ}_{nkjgt} \quad \forall n,k,j,g,t \tag{30}$$

$$\sum_{m'=1}^{m-1} t_{nkijgm'} = 1 \quad \forall n,k \tag{31}$$

There is also another linearization technique proposed by Tsai (2007) for multiple breakpoint functions. We have applied Tsai's method for comparison with our proposed method. As shown in Appendix B, the equivalent linear form of the Eq. (24) based on Tsai's techniques is simplified as follows:

$$\sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt})XQ_{nkjgt} = \sum_{n,k,j,g,t}\sum_{m'=1}^{m-1}\overset{(m')}{Z}_{nkjgt} \tag{32}$$

s.t.

$$\sum_{m'=1}^{m-1} XQ_{nkm'}t_{nkjgtm'} \leqslant XQ_{nkjgt} \leqslant \sum_{m'=1}^{m-1} XQ_{nk(m'+1)}t_{nkjgtm'} \tag{33}$$

$$\sum_{m'=1}^{m-1} t_{nkjgtm'} = 1 \tag{34}$$

$$(CM_{nkm'} - r_{nkm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt} - (1 - t_{nkjgtm'})M \leqslant z_{nkjgtm'} \tag{35}$$

$$z_{nkjgtm'} \leqslant (CM_{nkm'} - r_{nkm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt} + (1 - t_{nkjgtm'})M \tag{36}$$

$$-t_{nkjgtm'}M \leqslant z_{nkjgm'} \leqslant t_{nkjgtm'}M \tag{37}$$

Comparing the two methods, we have: In our proposed method (28)–(31), the nonlinear term $\overset{(m')}{XQ}_{nkjgt}{}^2$ is located in the objective function, while in the second method (32)–(37), this nonlinear term is located in the constraints. Despite of the fact that in both methods $n \times k \times j \times g \times t \times \ (m-1)$ extra binary (t) and nonzero ($\overset{(m)}{XQ}$ or Z) variables are used for linearization, in the proposed method merely four different classes of constraints will be imposed on the primitive model while in Tsai's (2007) method, seven new different classes of constraints will be added to the model. This comparison proves that the proposed method is preferred due to the smaller scale of the equivalent problem.

3.2. Shortage cost function

Using the proposed linearization method we convert the multiple breakpoint shortage cost function to an equivalent mathematical programming structure as follows:

$$\sum_{n,i,t}\pi_{ni}(B_{nit})\cdot B_{nit} = \sum_{n,i,t}\sum_{q'=1}^{q-1}\left[(\pi_{niq'} - \gamma_{niq'}B_{niq'})\overset{(q')}{B}_{nit} + \gamma_{niq'}\overset{(q')}{B}{}^2_{nit}\right] \tag{38}$$

s.t.

$$B_{ni(q-1)}\tau_{nit(q-1)} \leqslant \overset{(q-1)}{B}_{nit} \leqslant B_{niq}\tau_{nit(q-1)} \quad \forall n,i,t,q \tag{39}$$

$$\sum_{q'=1}^{q-1}\tau_{nitq'} = 1 \quad \forall n,i \tag{40}$$

$$B_{nit} = \sum_{q'=1}^{q-1}\overset{(q')}{B}_{nit} \quad \forall n,i \tag{41}$$

where q is the number of segments, τ is a binary variable and γ is the slope of the segments.

These equivalent mathematical programs for quantity discount function and shortage cost are still not linear because of the existence of the nonlinear terms $\left(\overset{(m')}{XQ}_{nkjgt}{}^2, \overset{(q')}{B}_{nit}{}^2\right)$ in the model.

In order to linearize the model we apply a famous technique, namely, piecewise linear programming. According to this technique (Chang, 2002; Li and Yu, 1999), to linearize a non-convex function $f(x)$ we can use Eq. (42) as follows:

$$L(f(x)) = f(a_1) + s_1(x-a_1) + \sum_{l'=1}^{l-1}\frac{s_{l'} - s_{l'-1}}{2}(|x-a_{l'}| + x - a_{l'}), a_m \geqslant x \geqslant \alpha > 0 \tag{42}$$

where $L(f(x))$ is the piecewise linear form of the$f(x)$ and, l is the number of divisions. The more divisions there are, the more precise the estimation will be, and the more computational complexities there will be. As shown in Fig. 3, S_l is the slope of the lines, computed using the following equation:

$$s_l = \frac{f(a_{l+1}) - f(a_l)}{a_{l+1} - a_l} \tag{43}$$

According to Yu and Li (2000) and Li and Yu (1999), for a minimization problem, for $s_l > s_{l-1}$ (shortage penalty function), the linear equivalent for the absolute term could be written as follows:

$$\text{Min } L(f(x)) = f(a_1) + s_1(x-a_1) + \sum_{l'=2}^{l-1}(s_{l'} - s_{l'-1})(\xi_{1l'} + x - a_{l'}) \tag{44}$$

$$\text{s.t. } x - a_l + \xi_{1l} \geqslant 0, \quad \xi_{1l} \geqslant 0 \tag{45}$$

Also, for $s_l < s_{l-1}$ (quantity discount function), the linear equivalent formulation for the absolute term could be written as follows:

$$\text{Min } L(f(x)) = f(a_1) + s_1(x-a_1) + \sum_{l'=2}^{l-1}(s_{l'} - s_{l'} - 1)(-\xi_{2l'} + u_{l'}a_{l'} + x - a_{l'}) \tag{46}$$

$$\text{s.t. } \xi_{2l} \geqslant x + M(u_l - 1), \quad \xi_{2l} \geqslant 0, \quad u_l \in \{0,1\} \tag{47}$$

Therefore, by using Eqs. (44) and (46), the nonlinear terms $\overset{(m')}{XQ}_{nkjgt}{}^2$ and $\overset{(q')}{B}_{nit}{}^2$ are replaced by the equivalent piecewise linear form of $L\left(\overset{(m')}{XQ}_{nkjgt}{}^2\right), L\left(\overset{(q')}{B}_{nit}{}^2\right)$. Finally, by considering the constraints (45) and (47), the linear equivalent deterministic APP model for the supply chain could be written as follows:

$$\begin{aligned}\text{Min } Z = &\sum_{j,t}SC_j\cdot L_{jt} + \sum_{j,t}FC_j\cdot F_{jt} + \sum_{j,t}HC_j\cdot H_{jt} + \sum_{n,j,t}CI1_{nj}\cdot I1_{njt}\\ &+\sum_{n,i,t}CI2_{ni}\cdot I2_{nit} + \sum_{k,j,i,g,t}TF_g\cdot(XV_{kjgt} + YV_{jigt})\\ &+\sum_{k,j,g,t}TV_g d1_{kj}XV_{kjgt} + \sum_{j,i,g,t}TV_g d2_{ji}\cdot YV_{jigt} + \sum_{n,j,t}PC_{jt}\cdot Y_{njt}\\ &+\sum_{n,j,t}CR_{jn}\cdot XR_{njt} + \sum_{n,j,t}CO_{jn}\cdot XO_{njt}\\ &+\sum_{n,k,j,g,t}\sum_{m'=1}^{m-1}\left[(CM_{nkm'} - r_{nkm'}XQ_{nkm'})\overset{(m')}{XQ}_{nkjgt} + r_{nkm'}L\left(\overset{(m')}{XQ}_{nkjgt}{}^2\right)\right]\\ &+\sum_{n,i,t}\sum_{q'=1}^{q-1}\left[(\pi_{niq'} - \gamma_{niq'}B_{niq'})\overset{(q')}{B}_{nit} + \gamma_{niq'}L\left(\overset{(q')}{B}_{nit}{}^2\right)\right] - \sum_{n,j,i,g,t}P_{ni}\cdot YQ_{njigt}\end{aligned} \tag{48}$$

s.t.

$$L\left(\overset{(q)}{B}_{nit}{}^2\right) = f(a_1) + s_1\left(\overset{(q)}{B}_{nit} - a_1\right) + \sum_{l'=2}^{l-1}(s_{l'} - s_{l'-1})(\xi_{1nitql'} + \overset{(q)}{B}_{nit} - a_{l'}) \tag{49}$$

$$\overset{(q)}{B}_{nit} - a_l + \xi_{1nitql} \geqslant 0, \quad \forall n,i,t,q,l \tag{50}$$

$$\begin{aligned}L\left(\overset{(m')}{XQ}_{nkjgt}{}^2\right) = &f(a_1) + s_1(\overset{(m)}{XQ}_{nkjgt} - a_1)\\ &+\sum_{l'=2}^{l-1}(s_{l'} - s_{l'-1})(-\xi_{2nkjgtml'} + u_{nkjgtml'}a_{l'} + \overset{(m)}{XQ}_{nkjgt} - a_{l'})\end{aligned} \tag{51}$$

$$\xi_{2nkjgtml} \geqslant \overset{(m)}{XQ}_{nkjgt} + M(u_{nkjgtml} - 1), \quad \forall n,k,j,g,t,m,l \tag{52}$$

$$\overset{(m)}{XQ}_{nkjgt}, \overset{(q)}{B}_{nit}, \xi_{1nitql}, \xi_{2nkjgtml} \geqslant 0, u_{nkjgtml}, \tau_{nitq}, t_{nkjgtm} \in \{0,1\} \tag{53}$$

And Constraints 8, (9)–(23), (29)–(31), (39)–(41), (43)

3.3. Flexible lead time

In most of the previous researches, the lead time is usually assumed to be fixed. Consequently, the rational relationship between lead time and transportation mode is not taken into account, to

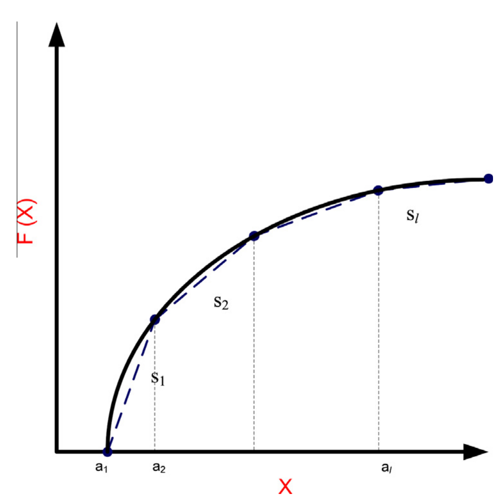


Fig. 3. Piecewise linear estimation.

avoid complexity in the modeling. In this research the inverse relationship between transportation cost and lead time is assumed as depicted in Fig. 4.

On the other hand, our proposed model is capable of determining the vehicle type. The more expensive the selected vehicle, the shorter the lead time will be. There are also some interrelationships between the lead time and greenhouse gas emission levels; both are usually determined by some standard tables. For example if we compare aircraft and water freight as two different vehicle types, aircraft have a shorter lead time in comparison with water freight and, conversely, water freight has a lower level of greenhouse gas emissions compared to aircraft. As the model shows, the lead time is embedded as an index of period, which itself has indices of factory and supplier (or customer) in addition to the index of vehicle type.

Fig. 5 determines the greenhouse gas emission level for different vehicle types against their capacities. It is taken from the website of "Network for Transport and Environment" (NTM), which is a nonprofit organization initiated in 1993 and aimed at establishing a common base of values on how to calculate the environmental performance of various modes of transport (road, rail, sea and air).

4. Solving procedure

In the first step we solve the proposed model with deterministic demand and report the solution. After that we consider a two stage stochastic programming approach to deal with demand uncertainty. In the two-stage framework, the supply, production and inventory (in factories) decisions for the current time period are made 'here-and-now' prior to the resolution of uncertainty, whereas the decisions for the rest of the time periods, such as distribution, shortage and inventory (in customer zones) are postponed in a 'wait-and-see' mode until after the uncertainties are revealed.

First-stage (FS) decision variables:

$$FS = \sum_{j,t} SC_j \cdot L_{jt} + \sum_{j,t} FC_j \cdot F_{jt} + \sum_{j,t} HC_j \cdot H_{jt} + \sum_{n,j,t} CI1_{nj} \cdot I1_{njt} + \sum_{k,j,g,t} TF_g \cdot XV_{kjgt} + \sum_{k,j,g,t} TV_g d1_{kj} XV_{kjgt} + \sum_{n,j,t} PC_{jt} \cdot Y_{njt} + \sum_{n,j,t} CR_{jn} \cdot XR_{njt} + \sum_{n,j,t} CO_{jn} \cdot XO_{njt} + \sum_{n,k,j,g,t} \sum_{m'=1}^{m-1} \left[(CM_{nkm'} - r_{nkm'} XQ_{nkm'}) \overset{(m')}{XQ}_{nkjgt} + L\left(r_{nkm'} \overset{(m')}{XQ}{}^2_{nkjgt} \right) \right] \tag{54}$$

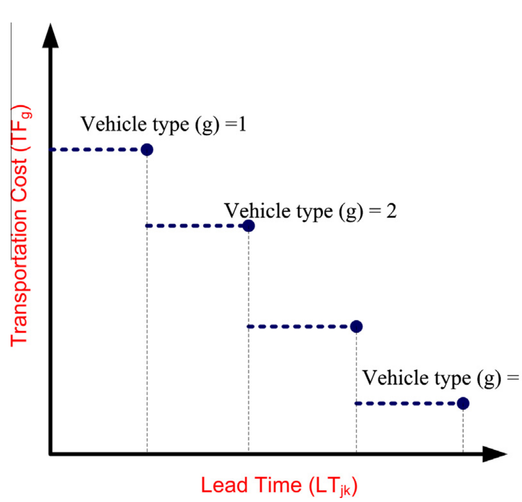


Fig. 4. Relationship between lead time and transportation cost.

Second-stage (SS) decision variables:

$$SS_s = \sum_{n,i,t} CI2_{ni} \cdot I2^s_{nit} + \sum_{j,i,g,t} TF_g YV^s_{jigt} + \sum_{j,i,g,t} TV_g d2_{ji} \cdot YV^s_{jigt} + \sum_{n,i,t} \sum_{q'=1}^{q-1} \left[(\pi_{niq'} - \gamma_{niq'} B_{niq'}) \overset{(q')}{B}{}^s_{nit} + L\left(\gamma_{niq'} \overset{(q')}{B}{}^{s\,2}_{nit} \right) \right] - \sum_{n,j,i,g,t} P_{ni} \cdot YQ^s_{njigt} \tag{55}$$

where s is the index of scenarios.

So the objective function for scenario s can be simplified as:

$$Z_s = FS + SS_s \tag{56}$$

Therefore, the two-stage stochastic optimization model is as follows:

$$\text{Min } Z = FS + E(SS_s) \tag{57}$$

s.t.

$$I2^s_{ni(t-1)} + \sum_{j,g} YQ^s_{njig(t-LTjig)} - D^s_{nit} - B^s_{ni(t-1)} = I2^s_{nit} - B^s_{nit} \quad \forall n,i,t,s \tag{58}$$

$$\sum_{k,g} XV_{kjgt} \cdot GHG_g \cdot d1_{kj} + \sum_{i,g} YV^s_{jigt} \cdot GHG_g \cdot d2_{ji} \leqslant GHl_{jt} \quad \forall j,t,s \tag{59}$$

$$\sum_{k,g,t} XV_{kjgt} \cdot EU_g \cdot d1_{kj} + \sum_{i,g,t} YV^s_{jigt} \cdot EU_g \cdot d2_{ji} \leqslant EUl_j \quad \forall j,s \tag{60}$$

$$\sum_{n} I2^s_{nit} \leqslant CP2_i \quad \forall i,t,s \tag{61}$$

$$(YV^s_{jigt} - 1)V_g \leqslant \sum_{n} v_n \cdot YQ^s_{njigt} \leqslant YV^s_{jigt} V_g \quad \forall j,i,g,t,s \tag{62}$$

$$\sum_{i,g} YQ^s_{njigt} \leqslant XO_{njt} + XR_{njt} \quad \forall n,j,t,s \tag{63}$$

$$B_{ni(q-1)} \tau^s_{nit(q-1)} \leqslant \overset{(q-1)}{B}{}^s_{nit} \leqslant B_{niq} \tau^s_{nit(q-1)} \quad \forall n,i,t,q,s \tag{64}$$

$$\sum_{q'=1}^{q-1} \tau^s_{nitq'} = 1 \quad \forall n,i,s \tag{65}$$

$$B^s_{nit} = \sum_{q'=1}^{q-1} \overset{(q')}{B}{}^s_{nit} \quad \forall n,i,t,s \tag{66}$$

$$L\left(\overset{(q)}{B}{}^{s\,2}_{nit} \right) = f(a_1) + s_1 \left(\overset{(q)}{B}{}^s_{nit} - a_1 \right) + \sum_{l'=2}^{l-1} (s_{l'} - s_{l'-1}) \left(\xi_{1nitql'} + \overset{(q)}{B}{}^s_{nit} - a_{l'} \right), \quad \forall n,i,t,q,s \tag{67}$$

$$\overset{(q)}{B}{}^s_{nit} - a_l + \xi^s_{1nitql} \geqslant 0, \quad \forall n,i,t,q,l,s \tag{68}$$

and constraints 10, (11), (12), (15), (16), (18), (21), (23), (29)–(31), (43), (51), (50)–(53)

To apply the two-stage stochastic programming, the scenario-based approach is used to represent the uncertainties. To reduce the number of scenarios, we use a Monte Carlo sampling approach to generate the scenarios. Each scenario is then assigned the same probability with the sum of the probabilities for all the scenarios equal to 1. For example, if we use Monte Carlo sampling to generate 10 scenarios, the probability of each scenario is given as 0.1. The number of scenarios is determined using a statistical method to obtain solutions within specific confidence intervals for a desired level of accuracy. This method is very useful for scenario reduction, specifically for large-sized problems (see You et al., 2009).

5. Experimental results

A typical multi-national company is willing to plan its aggregate production planning. The planning time horizon is assumed to be six periods. The number of product families is assumed to be 4. The production and distribution network under consideration consists of three production sites F_1, F_2 and F_3 which are spread out geographically, and three customer centers located in three different cities C_1, C_2 and C_3. Raw materials are supplied by three

Vehicle type	Shipment weight [ton]	Distance [km]	Transport work [tkm]	CO_2 [kg]	NO_x [g]	HC [g]	CO [g]	PM [g]
Van petrol	5.0	120,00	600,00	319,20	408,00	114,00	3294,00	<0.01
Van diesel	5.0	120,00	600,00	621,00	1638,00	102,00	570,00	120,00
Small truck	5.0	120,00	600,00	151,20	1728,00	114,00	498,00	42,00
Medium truck	5.0	120,00	600,00	106,20	834,00	36,00	162,00	18,00
Heavy truck	5.0	120,00	600,00	74,40	582,00	24,00	108,00	12,00
Tractor + 'city trailer'	5.0	120,00	600,00	73,80	606,00	24,00	120,00	12,00
Truck + trailer	5.0	120,00	600,00	44,40	366,00	12,00	72,00	6,00
Tractor + semitrailer	5.0	120,00	600,00	37,80	306,00	12,00	60,00	6,00
Tractor + megatrailer	5.0	120,00	600,00	34,80	282,00	12,00	78,00	<0.01
Truck + semitrailer	5.0	120,00	600,00	34,20	276,00	12,00	78,00	6,00

Vehicle type	Shipment weight [ton]	Distance [km]	Transport work [tkm]	CO_2 [kg]	NO_x [g]	HC [g]	CO [g]	PM [g]
Intercontinental passenger aircraft (belly)	5.0	120,00	600,00	321,60	1506,00	6,00	96,00	<0.01
Continental passenger aircraft (belly)	5.0	120,00	600,00	517,20	1656,00	96,00	768,00	<0.01
Regional passenger aircraft (belly)	5.0	120,00	600,00	766,20	2166,00	138,00	4860,00	<0.01
Intercontinental freight aircraft	5.0	120,00	600,00	233,40	8916,00	6,00	24,00	<0.01
Regional freight aircraft	5.0	120,00	600,00	1074,60	402,00	2490,00	17976,00	<0.01
Continental freight aircraft	5.0	120,00	600,00	748,80	3780,00	84,00	834,00	<0.01

Vehicle type	Shipment weight [ton]	Distance [km]	Transport work [tkm]	CO_2 [kg]	NO_x [g]	HC [g]	CO [g]	PM [g]
Container 7000 TEU	5.0	120,00	600,00	6,36	126,00	6,00	12,00	6,00
Container 11 000 TEU	5.0	120,00	600,00	6,08	114,00	6,00	12,00	6,00
Container 1400 TEU	5.0	120,00	600,00	9,24	174,00	6,00	18,00	12,00
RoRo 2000 Lanemeter	5.0	120,00	600,00	22,62	846,00	12,00	78,00	24,00
RoRo 3000 Lanemeter	5.0	120,00	600,00	35,82	654,00	12,00	60,00	18,00
General cargo 10000 dwt	5.0	120,00	600,00	8,84	216,00	6,00	18,00	12,00

Vehicle type	Shipment weight [ton]	Distance [km]	Transport work [tkm]	CO_2 [kg]	NO_x [g]	HC [g]	CO [g]	PM [g]
Electric train, average size, EU	5.0	120,00	600,00	<0.01	18,00	<0.01	<0.01	6,00
Electric train, shuttle, EU	5.0	120,00	600,00	<0.01	18,00	<0.01	<0.01	6,00
Electric train, volume, EU	5.0	120,00	600,00	<0.01	24,00	<0.01	<0.01	12,00
Electric train, bulk, EU	5.0	120,00	600,00	<0.01	18,00	<0.01	<0.01	6,00

Fig. 5. GHG emission level for different vehicle types (road, rail, sea, air).

suppliers located at S_1, S_2 and S_3. We first assume that the demand is deterministic, as shown in Table 1. And then solve the deterministic model accordingly. In the next step the demand is assumed to follow a normal distribution with the expected value and standard deviation equal to 400 and 100, respectively. The suppliers' capacity is assumed to be 350. The cost items as well as the other data are presented in Tables 2–7. This numerical example is roughly inspired from a real case which is completely described in Mirzapour Al-e-hashem et al. (2011b).

All computations were run using IBM ILOG CPLEX 12.2 on a PC Pentium IV-2.5 GHz Core 2 Duo processor and 2 GB RAM DDR under win VISTA. Presented hereunder are the resulting solutions for which we have relied on a set of the above-mentioned data.

The regular time and the overtime devoted by each factory at each period to produce different product types are reported in Table 8. The first column indicates the production state in regular time and overtime, the second column shows the active sites for each product, and the third one determines the product type. As shown in Table 8, type 1 products are produced only at factory 3, type 2 products are planned for factories 1 and 2, type 3 products are assigned to all factories, and type 4 products are produced at both factories 2 and 3. However, all factories have used their regular times as much as possible to produce the products, and the largest proportion of the overtime at factories 1 and 2 is assigned to type 2 products.

The interactions of supply chain components are reported in Tables 9 and 10. The blank cells similar to other unreported relevant data are equal to 0. In Table 9, the interaction between suppliers and factories and the transportation mode are presented: the first column indicates the supplier, and the second and third columns show which factories are served by that supplier and for which product types. For example, the second row determines that 90, 133, 117, 50 and 298 units of type 3 products are transported from supplier 1 to factory 1, at consecutive periods 1 to 6, except period 4 (i.e. at period 4, there is no interaction between supplier 1 and factory 1). Moreover, the transportation mode is also determined: for instance, the units transported from supplier 1 to factory 1 at period 2 are shipped partially by vehicle type 1 (33 units) and

Table 1
Demand forecast.

		Customer 1 Period						Customer 2 Period						Customer 3 Period					
		1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
Product	1	360	360	290	390	320	460	140	380	440	280	410	320	470	430	360	360	370	380
	2	380	370	440	480	420	80	560	520	450	530	580	330	370	460	260	540	390	500
	3	340	230	220	460	340	380	490	170	210	640	400	520	310	380	530	370	400	340
	4	550	550	400	320	360	350	540	320	430	210	330	390	490	370	380	570	370	400

Table 2
Regular and overtime production cost, holding cost in factories and manpower-hour needed for each units of products.

		CR Factory			CO Factory			CI1 Factory			a Factory		
		1	2	3	1	2	3	1	2	3	1	2	3
Product	1	19.2	19.2	9.6	38.4	38.4	19.2	6	6	3	0.0375	0.025	0.01875
	2	6.4	12.8	19.2	12.8	25.6	38.4	2	4	6	0.0313	0.0313	0.025
	3	19.2	16	22.4	38.4	32	44.8	6	5	7	0.0375	0.025	0.0375
	4	19.2	16	12.8	38.4	32	25.6	6	5	4	0.0375	0.0125	0.0125

Table 3
Price and holding cost in customer's zones.

		CI2 Customer			P Customer		
		1	2	3	1	2	3
Product	1	6	6	4	37	36.5	37.5
	2	4	4	6	29	30	30.5
	3	6	5	5	43	44	45
	4	6	5	4	25	26	26

Table 4
Distances between supply chain entities.

		d2 (km) Customer			d1(km) Supplier		
		1	2	3	1	2	3
Factory	1	25	100	150	25	200	100
	2	200	55	120	100	55	400
	3	100	400	300	150	120	300

partially by vehicle type 2 (100 units). In Table 10 the interactions between factories and customers, as well as the preferred vehicle type, are presented and can be interpreted as in Table 9. As expected, most customer demands (factories' requirements) are served by their closest factories (suppliers). In exceptional cases, however, shortages occur and the savings in other costs do not offset charges for disconnecting the supply chain. Table 11 provides an insight into each objective function component and its proportion of the total cost of the supply chain.

As seen in Table 11, the shortage cost is 1.5% of the total cost, while the transportation cost is near to 16%. This means that when the decision makers are not concerned about GHGs, the shortage cost is not particularly high in comparison with the transportation cost and the inventory cost. However, as discussed in the following section, it plays a significant role in offsetting the extra charges entailed by GHGs.

5.1. Sensitivity analysis

In this section we provide insights through a sensitivity analysis done for the greenhouse gas emission level against the profit of the supply chain, for the deterministic problem. We first solve the problem by relaxing the greenhouse gas emission constraint. This way, as expected, the maximum profit incurred. We then tighten the right hand side of this constraint step by step and analyze the impact of the greenhouse gas emission level on the profit as well as transportation cost. The results are depicted in Fig. 6.

As seen in Fig. 6, when the GHG constraint is relaxed, the maximum profit will be gained and the worst value for GHGs will be obtained (16,648). This could be explained by the fact that the model tries to take advantage of the more spacious vehicles to combine the customers' demands (factories' requirements) in successive periods, to save transportation costs and avoid shortage costs. It is definitely not explained relation to the GHGs. In fact, the optimum solution is a tradeoff between the transportation and inventory cost (+production cost) on the one hand, and the lost sale and shortage cost, on the other. When we impose an upper

Table 5
Purchasing cost parameters.

		CM Discount level				R Discount level			
		0–100	100–200	200–500	>500	0–100	100–200	200–500	>500
Product	1	6	5	4	3	−0.01	−0.01	−0.003	−0.003
	2	7.5	6.5	5.5	4.5	−0.01	−0.01	−0.003	−0.003
	3	8	7	6	5	−0.01	−0.01	−0.003	−0.003
	4	5.5	5	4.5	4	−0.005	−0.005	−0.002	−0.003

Table 6
Shortage cost parameters.

		π Shortage level				γ Shortage level			
		0–100	100–500	500–1000	>1000	0–100	100–500	500–1000	>1000
Product	1	8	11	26	46	0.03	0.0375	0.04	0.041
	2	10	13	28	48	0.03	0.0375	0.04	0.043
	3	12	15	30	50	0.03	0.0375	0.04	0.045
	4	15	17	32	52	0.02	0.0375	0.04	0.047

Table 7
Lead Times between factories and customers and between factories and suppliers.

		LT_2									LT_1								
		Customer 1 Vehicle type			Customer 2 Vehicle type			Customer 3 Vehicle type			Supplier 1 Vehicle type			Supplier 2 Vehicle type			Supplier 3 Vehicle type		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Factory	1	0	2	1	0	1	1	0	2	1	0	1	1	0	1	1	0	1	1
	2	0	1	2	0	2	1	0	1	1	0	1	1	0	1	1	0	1	2
	3	0	1	1	0	1	1	0	1	2	0	1	1	0	1	1	0	1	1

Table 8
Regular time and overtime assigned in each factory to produce different products (method = 1: regular time, method = 2: overtime).

Method	Factory *i*	Product *j*	Period *t*					
			1	2	3	4	5	6
1	1	2	550.0	619.87	621.35	626.15	638.17	348.0
1	1	3	288.33	122.66	177.33	100.0	56.60	298.39
1	2	2	147.32	195.48	171.4	213.11	175.32	438.04
1	2	3	760.0	717.33	732.66	850.0	800.0	551.6
1	2	4	850.0	814.93	844.47	505.52	700.0	540.0
1	3	1	1000.0	1400.0	1200.0	1250.0	1560.0	2021.53
1	3	3	219.0	–	262.66	200.0	265.0	–
1	3	4	500.0	595.06	400.0	810.0	150.0	310.0
2	1	2	240.0	230.12	228.65	223.85	211.82	211.82
2	2	2	328.6	328.6	328.6	286.9	324.7	61.90
2	3	1	–	–	–	–	–	668.5

Table 9
Interactions of supply chain components (suppliers and factories).

Supplier *i*	Factory *j*	Product *j*	Period *t*					
			1	2	3	4	5	6
1	1	2	350(1)	350(1)	350(1)	350(1)	350(1)	350(1)
1	1	3	90(2)	33(1) 100(2)	17(1) 100(2)	–	50(1)	298(1)
1	2	3	260(1)	217(1)	233(1)	350(1)	300(1)	52(1)
1	2	4	350(1)	315(1)	350(1)	–	200(1)	40(1)
1	3	1	350(2)	350(1)	350(2)	350(2)[a]	350(2)	350(1)
1	3	4	–	35(1)	–	350(1)	150(1)	310(1)
2	2	2	476(1) 24(2)	500(1)	500(1)	500(1)	500(1)	500(1)
2	2	3	500(1)	500(1)	500(1) 5(2)	500(1)	500(1)	500(1)
2	2	4	500(1)	500(1)	495(1)	500(1)	500(1)	500(1)
2	3	1	500(1)	500(2)	500(2)	500(2)	500(2)	500(1)
3	1	2	440(1)	500(1)	500(1)	500(1)	500(1)	210(1)
3	1	3	208(1)	–	–	200(1)	7(1)	–
3	3	1	500(1) 200(2)	500(1) 200(2)	500(1) 200(2)	200(1) 500(2)	210(1) 490(2)	500(1)
3	3	3	219(1)	–	263(1)	–	265(1)	–
3	3	4	500(1) 60(2)	500(1)	400(1)	460(1)	–	–

[a] It means that 350 units of product type 1 are transported by vehicle type 2, from supplier 1 to factory 3 during period 4.

limit for greenhouse gas emission levels (8340), as expected, the profit falls rapidly from 203,540 to 190,500 (∼6.4% reduction). It then reaches a state in which, even if the limitation is tightened, the profit and transportation cost decreases gradually with an approximately fixed slope. For instance, when the GHG upper limit is reduced from 8340 to 7900 (∼5.3%), the associated profit

Table 10
Interactions of supply chain components (factories and customers).

Factory *j*	Customer *c*	Product *j*	Period *t*					
			1	2	3	4	5	6
1	1	2	360(1) 16(2)	370(1) 24(2)	424(1) 40(2)	456(1)	380(1)	80(1) 150(2)
1	1	3	208(1)	123(1)	117(1)	100(1)	57(1)	298(1)
1	2	2	414(1)	456(1)	386(1)	394(1)	470(1)	330(1)
2	2	2	126(1)	64(1)	64(1)	136(1)	110(1)	–
2	2	3	470(1)	170(1) 160(2)	210(1)	480(1)	400(1)	414(1)
2	2	4	445(1)	395(1)	430(1)	210(1)	330(1)	290(1)
2	3	2	350(1)	460(1)	260(1)[a] 176(2)	364(1)	390(1)	500(1)
2	3	3	290(1)	380(1)	523(1) 7(2)	370(1)	400(1)	140(1)
2	3	4	405(1)	420(1)	380(1) 35(2)	296(1)	370(1)	250(1)
3	1	1	340(1) 90(2)	270(1) 290(2)	390(2)	320(2)	410(2)	–
3	1	3	112(1) 107(2)	–	103(1) 160(2)	200(1)	265(1)	–
3	1	4	485(1)	595(1)	400(1)	320(1) 250(2)	110(1) 40(2)	310(1)
3	2	1	120(1)	380(1) 30(2)	410(1) 40(2)	240(1) 330(2)	80(1) 320(2)	–
3	3	1	450(1)	430(1)	360(1)	360(1)	370(1) 380(2)	2690(2)
	3	4	15(1)	–	–	240(1)	–	–

[a] It means that 260 units of product type 2 are transported by vehicle type 1, from factory 2 to customer 3 during period 3.

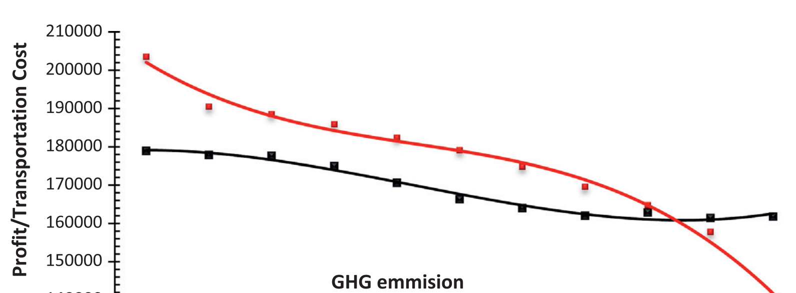
Table 11
Objective function components for deterministic problem

	Total cost	Profit	Labor cost	Inventory and production cost	Transportation cost	Shortage cost	Purchase cost
Value	968,338	101,270	250,770	419,740	153,690	14,948	129,190
Percentage	100	10.46	25.9	43.35	15.87	1.54	13.34

decreases from 190,500 to 188,540 (~1.0% reduction). This is approximately the same as the case in which the GHG upper limit decreases more and reaches the level of 7500, where the profit decreases from 188,540 to 185,890 (~1.4% drop). This part of the diagram may be more interesting because it prompts the decision makers to reduce GHGs considerably (~6%) without costing so much (lower than 1.5% of the profit). This state remains unchanged until the GHG drops to 6700. Thereafter, by tightening the GHG limitation more, the transportation cost still decreases in a linear manner, but the profit decreases with an increasing slope (collapse). In fact this GHG level (6700) is a turning point for the profits, where the decreasing slope of profit reduction (before this point) changes to an increasing slope (after this point). The fact behind this is that high amounts of greenhouse gas emissions are firstly avoided by the model through proper choices such as appropriate suppliers and vehicles, where sequential orders are combined to use the advantage of quantity discount and reduce transportation frequencies to offset the extra costs. But in order to avoid more GHG emissions, these aforementioned strategies are no longer useful. This reduction will therefore be possible only if some orders are partially cancelled, which entails a high risk of unfulfilled demand. Hence, because of the nonlinear form of the shortage penalty function, there will inevitably be some essential cost for the supply chain, and consequently the profit will decrease considerably. This is why the transportation cost is still decreasing gradually even after the turning point, while the profit is rapidly collapsing.

As previously mentioned the GHG and waste limits introduced can be interpreted as an ethical boundary set by the corporate strategy or as a threshold over which the firm might pay extra taxes because of its emission ratio. This Figure can also be interpreted as a Pareto set in which the decision maker can select the most preferred solution among them, according to his/her preferences.

We also perform another sensitivity analysis for the waste: as expected, tightening the total waste restriction leads to a lower profit margin. The configuration of the production rate is changed accordingly. In other words, the production rate of the products with a higher density of waste is reduced and, conversely, the production rate of the products with a lower density of waste in-



	16648	8340	7900	7500	7100	6700	6255	5840	5420	5000	4150
Transportation cost	178920	177860	177650	174940	170620	166310	164000	162020	162870	161440	161800
Profit	203540	190500	188540	185890	182360	179120	174840	169600	164770	157810	140450

■ Transportation cost ■ Profit

Fig. 6. Transportation cost and profit against tightening of the greenhouse gas emission limitation.

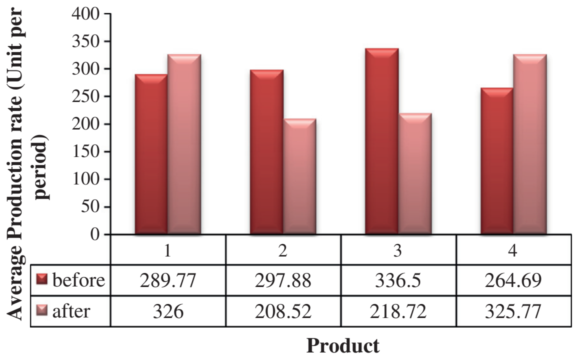


Fig. 7. Production rates before and after the waste limitation.

creases or at least remains unchanged. Fig. 7 compares the average production rate of the product families before and after considering the waste limitation.

The percentage of waste (wp) produced by type 1, 2, 3 and 4 products is supposed to be equal to 0.02, 0.05, 0.03 and 0.01, respectively, and the prices associated with them are assumed to be 37, 30, 44 and 26, respectively. So, in this typical example the type 2 product has the largest potential to create waste, in comparison with the other ones. On the other hand, the company is more interested in producing a type 3 product, because of the relatively higher margin it has and because its demand never decreases. Therefore, according to Fig. 7, before applying the waste upper bound limitation, roughly speaking, the model tries to meet the products' demands regarding their margins. For example, the average production rate for a type 3 product is 336.5. When the upper limit for the total waste is applied, the proposed model tries to reduce the waste and keep the profit level unchanged as much as possible. This is why the production rate for a type 3 product is decreased to 218.72 after we limit the total waste.

5.2. Sample size determination

In this research a scenario-based approach is selected to deal with uncertainty. One of the significant concerns in this method is how to identify the number of scenarios that meet the pre-specified confidence level. We therefore propose a statistical method to estimate the number of scenarios in the following. The number of scenarios is specified by the preferred level of accuracy of the solution, which can be calculated by the confidence interval of the expected total cost. The confidence interval can be calculated as follows. The Monte Carlo sampling variance estimator of the result for a stochastic programming problem, which is independent of the probability distribution of the uncertain parameters, is given by

$$S(n) = \sqrt{\frac{\sum_{s=1}^{n}(E(Z) - Z_s)^2}{n-1}} \tag{69}$$

where n is the number of scenarios and Z_s is the total cost of scenario s. Then the confidence interval of $1-\alpha$ is given as

$$\left[E(Z) - \Phi_{\alpha/2}\frac{S(n)}{\sqrt{n}}, E(Z) + \Phi_{\alpha/2}\frac{S(n)}{\sqrt{n}}\right] \tag{70}$$

where $\Phi_{\alpha/2}$ is a quantity for which the following equation for a standard normal random variable $\varphi \approx N\ (\mu = 0, \sigma = 1)$ will be satisfied.

$$\Pr(\varphi \leqslant \Phi_{\alpha/2}) = 1 - \alpha/2 \tag{71}$$

For example, for $\alpha = 0.05$, $\Phi_{\alpha/2}$ will be 1.96 according to the standard normal distribution tables.

On the other hand, if we are given the sampling estimator $S(n)$ and the maximum possible error (er) for confidence level $1-\alpha/2$, the minimum required number of scenarios can be determined by,

$$n' \geqslant \left[\frac{\Phi_{\alpha/2}S(n)}{er}\right]^2 \tag{72}$$

Therefore, to determine the number of scenarios n', we first solve the stochastic programming model with a small number of scenarios n (i.e. $n = 10$). To estimate the value of sampling estimator $S(n)$ by using Eqs. (69) and (72) we can determine the required number of scenarios (n') for a desired confidence interval.

So, we generate 10 ($n = 10$) sample scenarios to determine the required number of scenarios for confidence level 95% using Eqs. (69)–(72).

The values of objective function components for these ten scenarios are shown in Table 12.

Table 12 shows that for each scenario different levels of profit are earned, and that the total cost components such as workforce cost, transportation cost, shortage cost, purchase cost and inventory holding cost vary among different scenarios. In fact the variability that exists in profits and costs makes the decision more complicated. For instance, in Scenario 2 the purchase cost is not very high, whereas in Scenario 1 the maximum purchase cost is obtained. The highest profit level in the scenarios generated will be in Scenario 4. The variability of the shortage, purchase and transportation costs, as well as that of the profits, is depicted in Fig. 8 so that they can be compared more precisely. Profit is depicted as representative of the outcomes of all interactions between supply chain components. Transportation cost is almost synonymous with GHG emissions. The shortage cost is a customer-oriented measure and shows the service level. Finally, the purchase cost is highlighted because of the discount option incorporated in the model.

Due to uncertainty and the resulting variability, the number of scenarios should be wisely designed to absorb these fluctuations and to avoid time-consuming computations.

Table 12
Objective function components under different scenarios.

Scenario	Total cost	Profit ($-Z$)	Workforce cost	Transportation cost	Shortage cost	Purchase cost	Inventory and production cost
1	992,805	83,638	263,960	155,490	18,135	13,1040	424,180
2	945,580	81,796	242,290	154,800	15,765	12,4500	408,225
3	963,829.4	10,4210	246,500	157,030	7229.4	12,9110	423,960
4	944,579	12,0510	253,110	152,060	2974	12,8630	407,805
5	968,527	77,199	252,900	154,830	21,877	127,190	411,730
6	967,735	89,273	244,840	156,350	15,765	127,580	423,200
7	958,540	93,833	251,140	155,030	13,685	127,050	411,635
8	945,948	99,038	242,830	154,180	4048	126,420	418,470
9	934,048	118,190	23,9610	150,340	8881	126,260	408,957
10	981,056	62,610	251,920	157,850	23,816	128,400	419,070
Expected value	960264.74	93029.7	248910	154796	13217.54	127618	415723.2
Standard deviation	18211.311	18118.2342	7178.2805	2235.8503	7213.9265	1797.5403	6760.7712

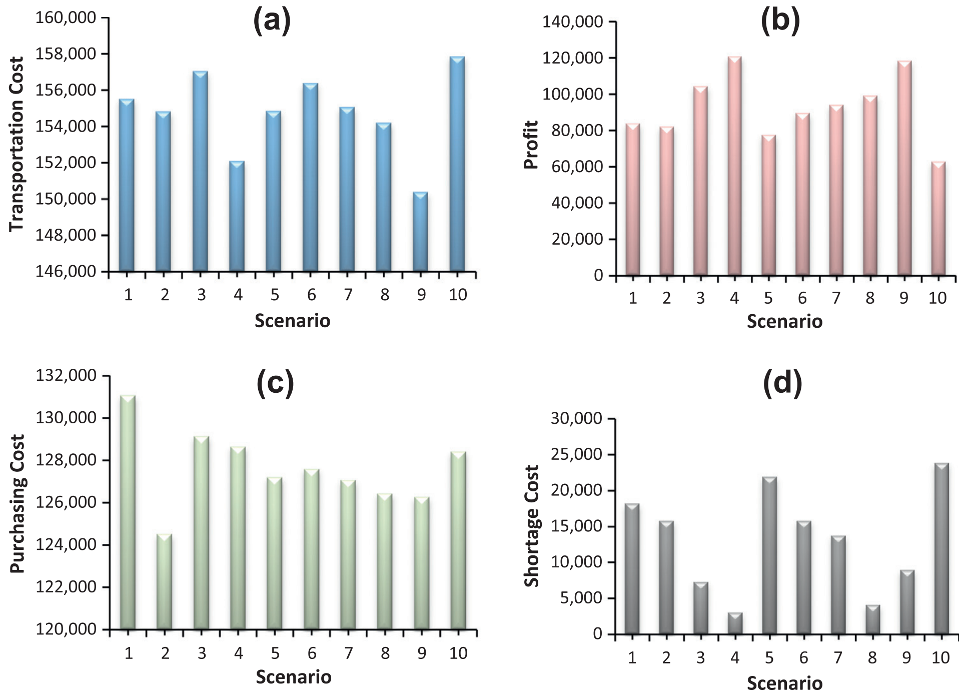


Fig. 8. Objective function components under different scenarios.

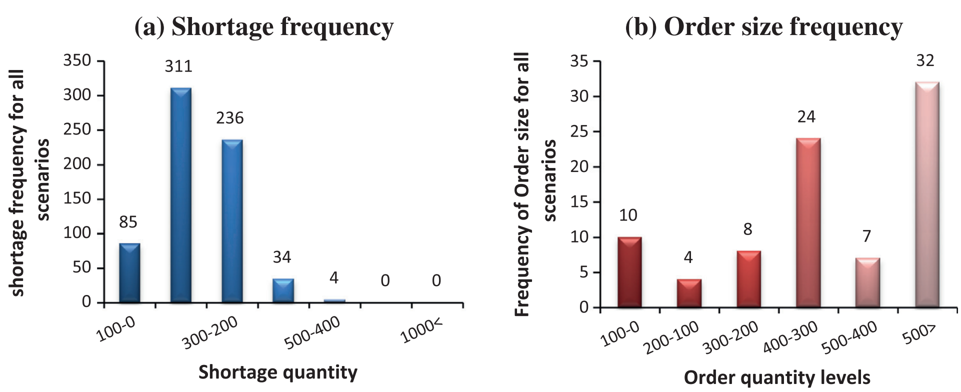


Fig. 9. Shortage and order size frequencies under different scenarios.

Using Eqs. (69)–(72), and considering the desired confidence interval as $0.05 \times E(Z)$, the required number of scenarios for confidence level 95%, will be roughly equal to 60. Note that, the lower the level of maximum possible error (*er*), the larger the number of required scenarios will be. For example, for desired confidence interval as $0.04 \times E(Z)$, $0.03 \times E(Z)$ and $0.02 \times E(Z)$, the required number of scenarios will be equal approximately to 91, 162 and 365. In this paper we consider the confidence level and maximum possible error as 95% and $0.05 \times E(Z)$, respectively. Consequently, the number of scenarios is set at 60. Hereafter, the reports are based on 60 scenarios generated, based on the normal distribution.

5.3. Statistical results

Due to the multiple breakpoint form of the shortage cost function, the shortage cost decreases to approximately 1.3% of the average total cost in comparison with the linear form of the shortage cost function (3%). The number of shortages mostly falls into the second interval ($100 < B_{nit} < 500$) of the cost function (see Fig. 9a). The rationale behind this observation is that the nonlinear form of the shortage cost prevents the model from cancelling all orders when it is under pressure of the GHGs limitation. According to Fig. 9a, in 311 cases (46%) the number of shortages falls in the range of 100–200, and in 236 cases (35%) it falls in the range of 200–300. In 85 cases (13%), the number of shortages is less than 100 units, in 38 cases (6%) it is more than 300, and it never exceeds 500. On the other hand, the multiple breakpoint discount function causes the order quantity, released by factories, to shift up (>500). As shown in Fig. 9b, the orders larger than 500 are the most frequent ones. For instance, in 10 cases (12%) the order size is less than 100 units, in 24 cases (28%) it is less than 400 units, and in 63 cases (72%) it is more than 400 units. Note that the order size (XQ) is a first-stage decision variable and does not vary within scenarios. Therefore, it could be concluded that due to the quantity discount function considered in the proposed model, there is a trend to merge the succeeding orders to take advantage of lower purchasing costs. Note also that Fig. 9 counts the number of periods for all product types in which shortages (orders) occurred, rather than the number of scenarios.

In order to manage risk, the variance management model is usually applied, which is a straightforward approach to reduce both expected value and variance of cost, but it includes quadratic terms in the objective function (Goha et al., 2007). To manage the variability of the solutions among different scenarios, we add a new term (*semivar*$^+$) as a 1-norm variability index (semi-variance) to the original proposed two stage stochastic programming.

$semivar_s^+$ is also called as the 'upper partial mean', that is, if $SS_s$is less than the $E(SS_s)$, the $semivar_s^+$ will be zero; if SS_s is greater than the $E(SS_s)$, the $semivar_s^+$ will be equal to their positive difference ($SS_s - E(SS_s)$) (See Fig. 10). Therefore, the aim of the new model is to minimize the weighted sum between the expected value of the total loss and the expected variability index as follows:

$$\text{Min } Z = FS + E(SS_s) + \lambda \cdot E(semivar_s^+) \tag{73}$$

s.t.

$$semivar_s^+ \geqslant SS_s - E(SS_s), \quad \forall s \tag{74}$$

$$semivar_s^+ \geqslant 0, \quad \forall s \tag{75}$$

Constraints 10, (11), (12), (15), (16), (18), (21), (23), (29)–(31), (43), (51)–(53), (58)–(68)

Where, λ is the relative weight for the variability index usually determined by the decision maker.

Fig. 11 shows the result of the sensitivity analysis which is done for the value of λ. As expected, for a higher value of λ, the expected value of the total losses increases (profit decreases) while the value of the variability index decreases. Conversely, for a lower value of λ, the expected total losses of the supply chain decreases (profit increases) while its variability increases. In other words, Fig. 11 is a Pareto curve, and the decision maker can select the desirable amount of λ according to his/her preferences. For instance, for $\lambda = 1$, the expected profit is approximately equal to 117,134, while the variability index is 7848. Conversely, for $\lambda = 7$, the expected profit is 108,043, whereas the variability index is 1939. In other words, the expected profit decreases by nearly 7.76%, while the variability index decreases by around 75%. This means that a 7.76% increase in expected profits is accessible once the decision maker can accept its higher risk.

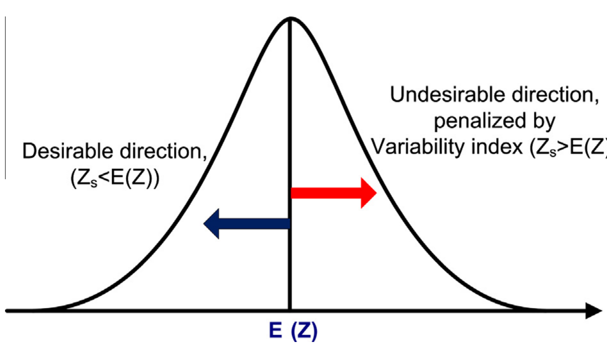


Fig. 10. Variability index.

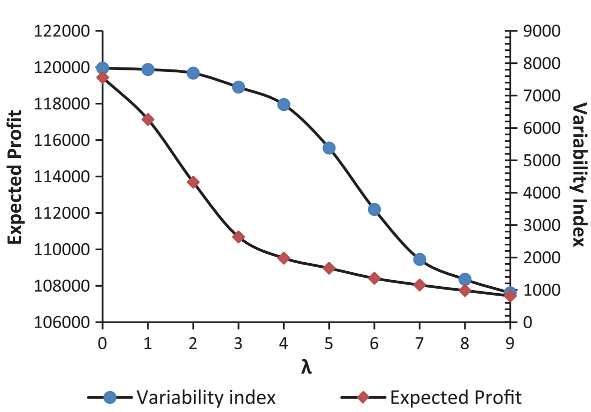


Fig. 11. Expected profit against variability index.

6. Conclusion

In this paper a two-stage stochastic programming model is developed to deal with aggregate production–distribution planning in a green supply chain under the assumption of demand fluctuation. The proposed model considers not only most of the supply chain cost parameters, but also some more realistic cost functions such as multiple breakpoint discount function and multiple breakpoint shortage penalty function. These assumptions make the model more complicated because of their nonlinear structure. Therefore by applying some numerical and theoretical techniques, the original nonlinear model is converted to a linear one. We also consider the interrelationship between lead time and transportation costs, by considering the vehicle type selection decision variables. The model could therefore vary the vehicle type to gain more profit. On the other hand, we incorporate the green concept in transportation issues. The sensitivity analysis demonstrates that considering green logistics has a considerable effect on the configuration of the total cost components as well as the profit margin. Managing waste in the production process is moreover considered to control the production rates of the product with a higher level of waste. The proposed model is firstly solved deterministically, and then solved again under a two-stage stochastic programming scheme. The result shows that this research is a good step to develop a more comprehensive aggregate planning model for dealing with some challenging issues such as nonlinearity and uncertainty, simultaneously. Effort to implement the proposed model in real cases, considering green indicators as an additional objective function in a multi-objective scheme and also modeling the relationship between price and end customer demand are the most promising areas for future research.

Appendix A

Let us assume a general multiple breakpoint function of type:

$$f(X) = \begin{cases} a_1X + b_1 & \text{if } C_0 \leqslant X \leqslant C_1 \\ a_2X + b_2 & \text{if } C_1 \leqslant X \leqslant C_2 \\ a_3X + b_3 & \text{if } C_2 \leqslant X \end{cases} \tag{A.1}$$

This nonlinear function could be converted to a linear form with the help of the following equations:

$$f(X) = a_1X_1 + b_1t_1 + a_2X_2 + b_2t_2 + a_3X_3 + b_3t_3 \tag{A.2}$$

$s.t.$

$$C_0t_1 \leqslant X_1 \leqslant C_1t_1 \tag{A.3}$$

$$C_1t_2 \leqslant X_2 \leqslant C_2t_2 \tag{A.4}$$

$$C_2t_3 \leqslant X_3 \leqslant Mt_3 \tag{A.5}$$

$$\sum_{i=1} t_i = 1, t_i \in \{0, 1\} \tag{A.6}$$

$$X = \sum_i X_i, \text{ where } M \text{ is an arbitrary large number.} \tag{A.7}$$

Proof. If $t_1 = 1$ then $t_2 = 0$, $t_3 = 0$ and according to (A.4) and (A.5), $0 \leqslant X_2 \leqslant 0$, $0 \leqslant X_3 \leqslant 0$ therefore $f(X)$ will be equal to $f(X) = a_1X_1 + b_1 \times 1 + a_2 \times 0 + b_2 \times 0 + a_3 \times 0 + b_3 \times 0$, $\Rightarrow f(X) = a_1X_1 + b_1$, where X is limited by constraint (A.3), $C_0 \times 1 \leqslant X_1 \leqslant C_1 \times 1 \Rightarrow C_0 \leqslant X_1 \leqslant C_1$

If $t_2 = 1$, in the same way as above, we have $\Rightarrow f(X) = a_2X_2 + b_2$, $C_1 \leqslant X_2 \leqslant C_2$

Finally, if $t_3 = 1 \Rightarrow f(X) = a_3X_3 + b_3$, $C_2 \leqslant X_3 \leqslant M$,

We see that these Equations, A.2, (A.3)–(A.7) are equivalent to the original nonlinear structure (A.1).

Appendix B

According to the method proposed by Tsai (2007), we have:

$$CM_{nk}(XQ_{nkjgt}) = \sum_{m'=1}^{m-1} t_{nkjgtm'}(CM_{nkm'} + r_{nkm'}(XQ_{nkjgt} - XQ_{nkm'})) \quad (B.1)$$

s.t.

$$\sum_{m'=1}^{m-1} XQ_{nkm'}t_{nkjgtm'} \leqslant XQ_{nkjgt} \leqslant \sum_{m'=1}^{m-1} XQ_{nk(m'+1)}t_{nkjgtm'} \quad (B.2)$$

$$\sum_{m'=1}^{m-1} t_{nkjgtm'} = 1 \quad (B.3)$$

By substitution of Eq. (B.1) in the purchasing cost, it could be rewritten as follows:

$$\sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt})XQ_{nkjgt} = \sum_{n,k,j,g,t} \sum_{m'=1}^{m-1} t_{nkm'}\left[(CM_{nkm'} - r_{nkjgtm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt}\right] \quad (B.4)$$

s.t.

$$\sum_{m'=1}^{m-1} XQ_{nkm'}{}_{t_{nkjgtm'}} \leqslant XQ_{nkjgt} \leqslant \sum_{m'=1}^{m-1} XQ_{nk(m'+1)}t_{nkjgtm'} \quad (B.5)$$

$$\sum_{m'=1}^{m-1} t_{nkjgtm'} = 1 \quad (B.6)$$

As can be seen, it has some explicit nonlinear terms ($t \cdot XQ_{nkjgt}$). As Tsai (2007) proves, $z = t \cdot f(x); t \in \{0,1\}$ could be converted to an equivalent linear equation as follows:

$$f(x) - (1-t)M \leqslant z \leqslant f(x) + (1-t)M \quad (B.7)$$

$$-tM \leqslant z \leqslant tM, M \text{ is an arbitrary big number} \quad (B.8)$$

Therefore, by considering $[(CM_{nkm'} - r_{nkm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt}]$ as $f(x)$ we have:

$$\sum_{n,k,j,g,t} CM_{nk}(XQ_{nkjgt})XQ_{nkjgt} = \sum_{n,k,j,g,t} \sum_{m'=1}^{m-1} \overset{(m')}{Z}_{nkjgt} \quad (B.9)$$

s.t.

$$\sum_{m'=1}^{m-1} XQ_{nkm'}t_{nkjgtm'} \leqslant XQ_{nkjgt} \leqslant \sum_{m'=1}^{m-1} XQ_{nk(m'+1)}t_{nkjgtm'} \quad (B.10)$$

$$\sum_{m'=1}^{m-1} t_{nkjgtm'} = 1 \quad (B.11)$$

$$(CM_{nkm'} - r_{nkm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt} - (1 - t_{nkjgtm'})M \leqslant z_{nkjgtm'} \quad (B.12)$$

$$z_{nkjgtm'} \leqslant (CM_{nkm'} - r_{nkm'}XQ_{nkm'})XQ_{nkjgt} + r_{nkm'}XQ^2_{nkjgt} + (1 - t_{nkjgtm'})M \quad (B.13)$$

$$-t_{nkjgtm'}M \leqslant z_{nkjgm'} \leqslant t_{nkjgtm'}M \quad (B.14)$$

References

Bai, C., Sarkis, J., 2010. Green supplier development: analytical evaluation using rough set theory. Journal of Cleaner Production 18 (12), 1200–1210.

Baykasoglu, A., 2001. MOAPPS 1.0: aggregate production planning using the multiple-objective tabu search.. International Journal of Production Research 39 (16), 3685–3702.

Buxey, G., 1993. Production planning and scheduling for seasonal demand. International Journal of Operations & Production Management 13 (7), 4–21.

Carter, J.R., Ferrin, B., Carter, C., 1995. The effect of less-than truckload rates on the purchase order lot size decision. Transportation Journal 34 (3), 35–44.

Chang, C.T., 2002. A modified goal programming model for piecewise linear functions. European Journal of Operational Research 139, 62–67.

Dekker, R., Bloemhof, J., Mallidis, I., 2012. Operations research for green logistics – an overview of aspects, issues, contributions and challenges. European Journal of Operational Research 219, 671–679.

Diabat, A., Govindan, K., 2011. An analysis of the drivers affecting the implementation of green supply chain management. Resources, Conservation and Recycling 55 (6), 659–667.

Eltayeb, K., Zailani, S., Ramayah, T., 2011. Green supply chain initiatives among certified companies in Malaysia and environmental sustainability. Investigating the outcomes Resources, Conservation and Recycling 55, 495–506.

Ertogral, K., Darwish, M., Ben-Daya, M., 2007. Production and shipment lot sizing in a vendor–buyer supply chain with transportation cost. European Journal of Operational Research 176, 1592–1606.

Foote, B.L., Ravindran, A., Lashine, S., 1988. Production planning & scheduling: computational feasibility of multi-criteria models of production, planning and scheduling. Computers & Industrial Engineering 15 (1), 129–138.

Gallego, G., 2001. Lecture note, IEOR 4000: Production Management, Lecture 5 <http://columbia.edu/~gmg2/4000/pdf/lect_05.pdf>.

Goha, M., Limb, J.Y., Mengc, F., 2007. A stochastic model for risk management in global supply chain networks. European Journal of Operational Research 182 (1,1), 164–173.

Goodman, D.A., 1974. A goal programming approach to aggregate planning of production and work force. Management Science 20 (12), 1569–1575.

Haehling, L.C., 1970. Production and employment scheduling in multi-stage production systems. Naval Research Logistics Quarterly 17 (2), 193–198.

Hanssman, F., Hess, S., 1960. A linear programming approach to production and employment scheduling. Management Technology 1 (1), 46–51.

Hill, J., Galbreth, M., 2008. A heuristic for single-warehouse multi-retailer supply chains with all-unit transportation cost discounts. European Journal of Operational Research 187, 473–482.

Holt, C.C., Modigliani, F., Simon, H.A., 1955. A linear decision rule for production and employment scheduling. Management Science 2 (1), 1–30.

Huppes, G., Ishikawa, M., 2005. A framework for quantified eco-efficiency analysis. Journal of Industrial Ecology 9 (4), 25–41.

Jamalnia, A., Soukhakian, M.A., 2009. A hybrid fuzzy goal programming approach with different goal priorities to aggregate production planning. Computers & Industrial Engineering 56 (4), 1474–1486.

Kumar, S., Teichman, S., Timpernagel, T., 2012. A green supply chain is a requirement for profitability. International Journal of Production Research 50 (5), 1278–1296.

Leblanc, L., Hill, J., Greenwell, G., Czesnat, A., 2004. Nu-Kote's spreadsheet linear programming models for optimizing transportation. Interfaces 34 (2), 139–149.

Leenders, M., Fearon, H., Flynn, A., Johnson, P., 2002. Purchasing and Supply Management, 12th ed. McGraw-Hill, New York, NY.

Li, H.L., Yu, C.S., 1999. A global optimization method for nonconvex separable programming problems. European Journal of Operational Research 117 (2), 275–292.

Markovitz-Somogyi, R., Nagy, Z., Torok, A., 2009. Greening supply chain management. Acta Technica Jaurinensis Series Logistica. 2 (3), 367–374.

Masoud, A.S.M., Hwang, C.L., 1980. An aggregate production planning model and application of three multiple objective decision methods. International Journal of Production Research 18, 741–752.

Mazzola, J.B., Neebe, A.W., Rump, C.M., 1998. Multi product production planning in the presence of work-force learning. European Journal of Operational Research 106 (2), 336–356.

Min, H., Kim, I., 2012. Green supply chain research: past, present, and future. Logistics Research 4, 39–47.

Mirzapour Al-e-hashem, S.M.J., Aryanezhad, M.B., Sadjadi, S.J., 2012. An efficient algorithm to solve a multi-objective robust aggregate production planning in an uncertain environment. International Journal of Advanced Manufacturing Technology 58 (5–8), 765–782. http://dx.doi.org/10.1007/s00170-011-3396-1.

Mirzapour Al-e-hashem, S.M.J., Baboli, A., Sadjadi, S.J., Aryanezhad, M.B., 2011a. A multi-objective stochastic production-distribution planning problem in an uncertain environment considering risk and workers productivity. Mathematical problems in engineering. http://dx.doi.org/10.1155/2011/406398.

Mirzapour Al-e-hashem, S.M.J., Malekly, H., Aryanezhad, M.B., 2011b. A multi-objective robust optimization model for multi-product multi-site aggregate production planning in a supply chain under uncertainty. Journal of Production Economics 134 (1), 28–42. http://dx.doi.org/10.1016/j.ijpe.2011.01.027.

Mirzapour Al-e-hashem, S.M.J., Baboli, A., Aryanezhad, M.B., Sazvar, Z., 2011c. Aggregate production planning in a green supply chain by considering flexible lead times and multi breakpoint purchase and shortage cost functions. In: 41st International Conference on Computers & Industrial Engineering. Los Angeles, CA USA.

Nam, S.J., Logendran, R., 1992. Aggregate production planning—a survey of models and methodologies. European Journal of Operational Research 61 (3), 255–272.

Ng, C.T., Li, Y.O., Chakhlevitch, K., 2001. Coordinated replenishments with alternative supply sources in two-level supply chains. International Journal of Production Economics 73, 227–240.

Network for Transport and Environment, (NTM). <http://www.ntmcalc.se/index.html>.

Porkka, P., Vepsäläinen, A.P.J., Kuula, M., 2003. Multi period production planning carrying over set-up time. International Journal of Production Research 41 (6), 1133–1148.

Quariguasi Frota Neto, J., Walther, G., Bloemhof, J., van Nunen, J.A.E.E., Spengler, T., 2009. A methodology for assessing eco-efficiency in logistics networks. European Journal of Operational Research 193, 670–682.

Ramezanian, R., Rahmani, D., Barzinpour, F., 2012. An aggregate production planning model for two phase production systems: solving with genetic algorithm and tabu search. Expert Systems with Applications 39 (1), 1256–1263.

Sheu, J.B., 2008. Green supply chain management, reverse logistics and nuclear power generation. Transportation Research Part E: Logistics and Transportation Review 44 (1), 19–46.

Shi, Y., Haase, C., 1996. Optimal trade-offs of aggregate production planning with multiple objective and multi-capacity demand levels. International Journal of Operations and Quantitative Management 2 (2), 127–143.

Srivastava, S., 2007. Green supply-chain management: a state-of-the-art literature review. International Journal of Management Reviews 9 (1), 53–80.

Thomas, D.J., Griffin, P.M., 1996. Coordinated supply chain management. European Journal of Operational Research 94, 1–15.

Tsai, F., 2007. An optimization approach for supply chain management models with quantity discount policy. European Journal of Operational Research 177, 982–994.

Wang, F., Lai, X., Shi, N., 2011. A multi-objective optimization for green supply chain network design. Decision Support Systems 51 (2), 262–269.

Xue, G., Offodile, O.F., Zhou, H., Troutt, M.D., 2011. Integrated production planning with sequence-dependent family setup times. International Journal of Production Economics 131 (2), 674–681.

Yeh, W.C., Chuang, M.C., 2011. Using multi-objective genetic algorithm for partner selection in green supply chain problems. Expert Systems with Applications 38 (4), 4244–4253.

You, F., Wassick, J.M., Grossmann, I.E., 2009. Risk management for a global supply chain planning under uncertainty: models and algorithms. American Institute of Chemical Engineers Journal 55 (4), 931–946.

Yu, C.S., Li, H.L., 2000. A robust optimization model for stochastic logistic problems. International Journal of Production Economics 64 (1–3), 385–397.

Zhu, Q., Sarkis, J., 2011. Relationships between operational practices and performance among early adopters of green supply chain management practices in Chinese manufacturing enterprises. Journal of Operations Management 22 (3), 265–289.