Math 358
Final Report
TNNT

Bison Fresh Sauté Station Queue Analysis

Executive Summary:

We analyzed the Bison Fresh Sauté station from 6-8pm on Tuesday April 5th. We modeled this food station by using 3 queues in series: one to explain the cashier service, one to explain the ingredients service, and one to explain the sautee service. We broke the analysis into these three sections because it explains how customers move through the entire food station. We found that on average, there are 4.01 people in the food station at a time. This is useful to know given that many students are ordering food for dinner between 6 and 8pm during the week. On average there are 1.89 people waiting online to be served in the system. This is with one server at the cashier, one server at the ingredients station, and 4 burners available at the saute station. Customers spend approximately 386.19 seconds or 6.4 minutes at the Bison Fresh Sauté station. Additionally, they spend on average 58.65 seconds, a little under a minute, waiting online to get their food. This is an interesting statistic as it could help Bucknell better understand what the service time is at a busier hour during the day. That is, operating under the assumption that dinner is one of the busiest times on a Tuesday. We also tested some alternative outcomes that were a result of changing the number of workers in the system using a simulation application called FlexSim. If there are 2 workers at the ingredients station instead of 1 then people will get served approximately 20 seconds faster. Another scenario we tested was to see if increasing the amount of time between when a customer got online increased and what the outcome would look like. Using FlexSim we can see that the number of people at the station at a time decreases by 2 people, on average, and the amount of time a customer spends in the system decreases by a minute, on average.

PART I - Data Collection and raw data values:

Our queueing model is truly a series of 3 separate queues. This means that we had seven checkpoints for our entire model. Those being the entering of the system, entering of the cashier line, leaving the cashier, entering the ingredients line, leaving the ingredients line, entering the saute station, and leaving the saute station. The purpose of the first checkpoint, entering the system, is distinct from entering the cashier line in our model (despite not being markedly different in practice) is to allow us to create a queue for the first service. This isn't necessary for the third and final queue, and also explains why we don't have an additional checkpoint to denote leaving the system; leaving the saute station and the system are equivalent in both practice and theory.

Also worth noting is the fact that the first two queues in series, the cashier and ingredients line, leave no room for customers to "jump" each other since there is only one server: they will leave the second queue in the order of which they arrived at the first queue. However, due to the fact that the saute station has 4 servers and we assume them to have an exponentially distributed service time, it is possible for customers to leave the entire system in a different order from which they arrived, which is reflected in the data.

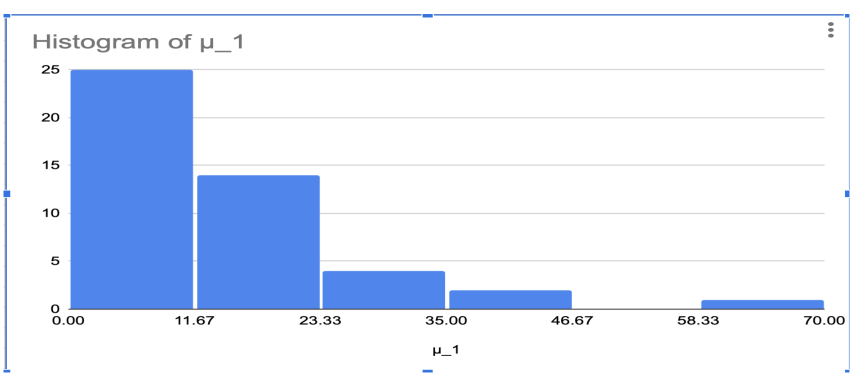

Figure showing: Histogram of cahier queue service time.

The first queue consisted of one server (a cashier) and the average service time of this queue followed an exponential distribution with 0.071 customers per second. A customer would simply pay money for his order by swiping his BUID or credit card. As such, the service was comparatively faster here, rarely taking very long due to some unforeseen predicament. A peculiarity of this queue was that roughly 8/46 people left the queueing system after going through this queue service, the cashier, to go to another independent system, the smoothie bar.

However, our ingredient service, although had a single server again, had a normally distributed service time with a mean of 60.22 and a standard deviation of 27.42 (these statistics were calculated after removing the outlier but the histograms still contain the outliers). On average, the service times were seemingly equal between customers. However, a few customers would take longer than usual or would go through this queue instantly. We observed the variety and size of the ingredients order placed to be a key reason as to why this phenomenon occurred.

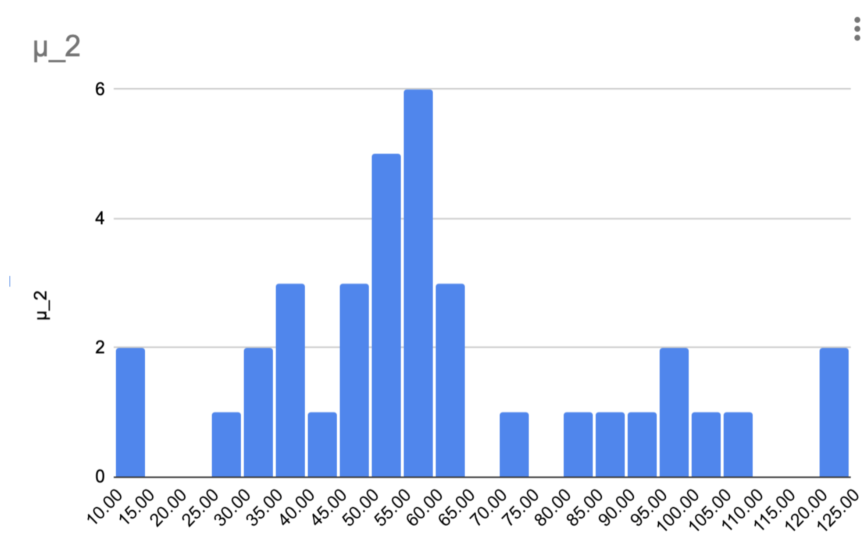

Figure showing: Histogram of the Ingredients queue service time.

Finally, our final queue service, the sautee service, was a four server system whose average service time also followed a normal distribution with a mean of 357.242 and a standard deviation of 99.251. The average service time was comparatively longer than any other queue in our system and served as a bottleneck to the overall performance of our system.

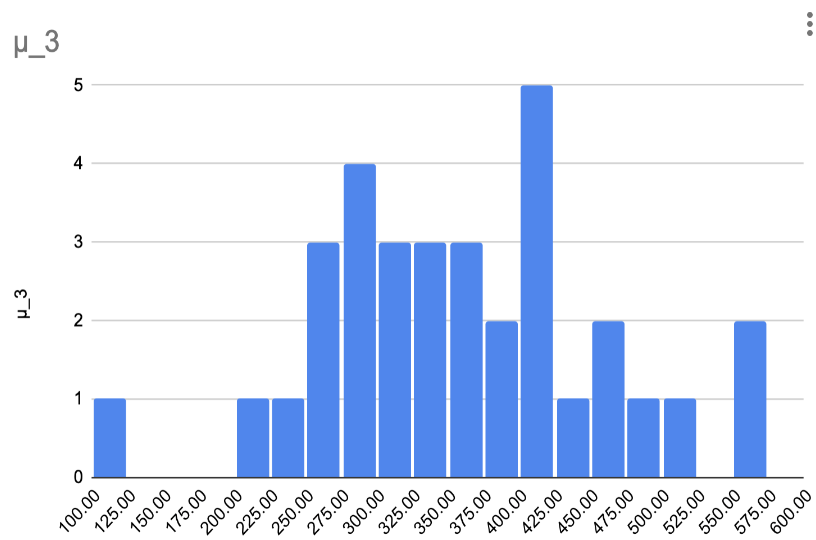

Figure showing: Histogram of the Sautee queue service time.

Apart from this, we were also able to observe customers arriving at the queueing system at an exponentially distributed rate of 0.0077 customers per second (Inter-Arrival time). More often than not, customers would arrive in pairs or groups.We used these statistics to model our simulation model.

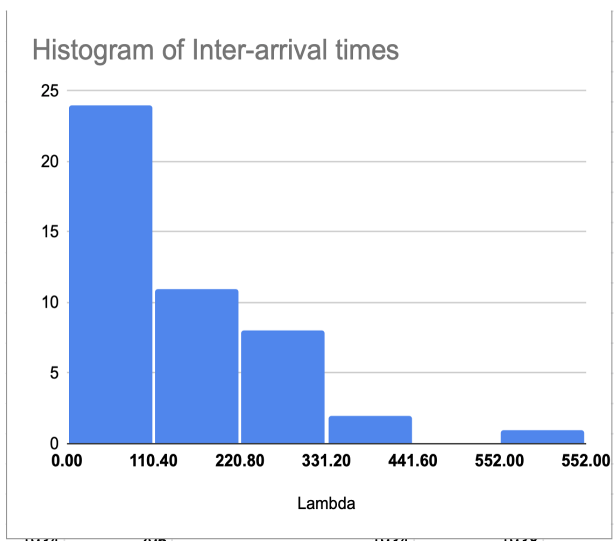

Figure showing: Histogram of the Inter-arrival times.

PART II - Models:

Raw-Data Value Model:

Moving on, the first model we were able to create using our data was simply the raw data model. The relevant queue indicators, such as the average waiting time in the queue system, were retrieved through basic calculations and probability formulas. In particular, a step function was used to calculate the average number of customers in the queueing system. The step function helped us track the different number of customers present in our system at given time intervals. The y-axis of the step function kept a record of the number of people in our system while the x-axis had time points when customers either entered the system or left the system. Please note, we chose to not include customers who chose to enter the smoothie station in our step function, as we didn't track their departure. We were able to discover that, at most, eight customers were present in our system at a given time.

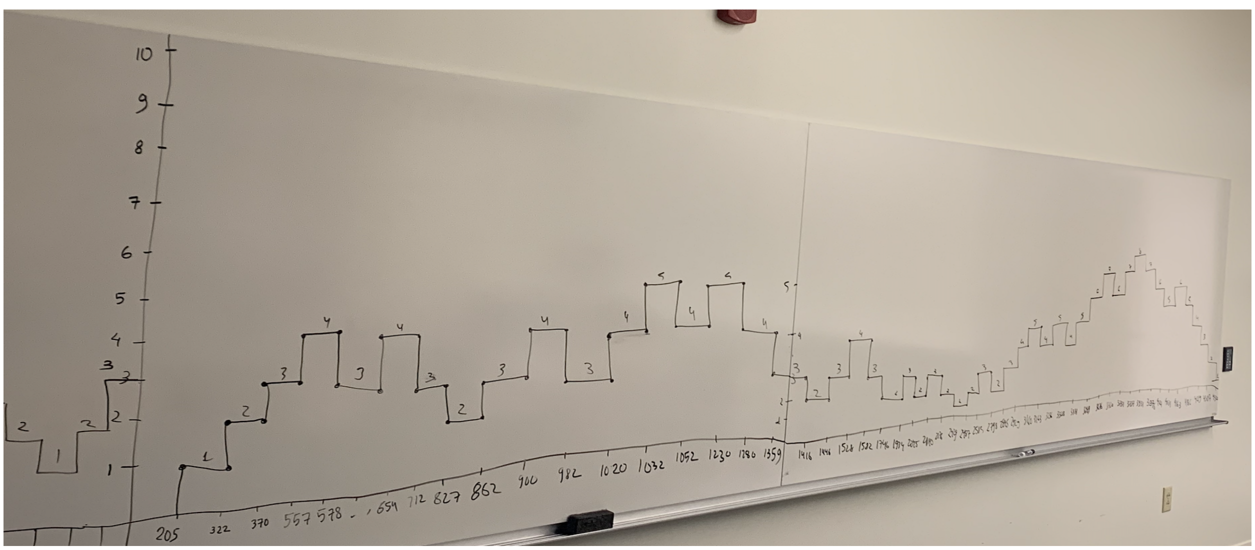

Figure showing: the step function created using our data.

The average number of customers in our system was observed to be 4.43 customers. Surprisingly, the average number of customers in our queue lines was only 0.42 customers.

Again, the average waiting time in our system was 495.43 seconds but customers only spent 54.88 seconds waiting in lines in between our queuing services or in the beginning before entering the first queue. This means that customers of Bison Fresh spent little time in lines, only about 11% of the total time they spent in the system. Most of their time is spent being served by the servers, mainly in the sautee service.

FlexSim Model:

For the FlexSim model, we created three separate queueing systems. Based on the observed distributions of each queue, we used that information to run our simulations. The Cash Register Queue was modeled as an exponential distribution with a rate of 0.0077, while the Ingredients Queue and Saute Queue were both modeled as normal distributions with averages 60.22 and 357.242 respectively. In order to obtain appropriate L, Lq, W, and Wq values for our FlexSim Model, we stopped our simulation at 144,000 seconds. This comes out to be that the Bison Fresh System operates 8 hours a day, 5 days a week, totaling 144,000 seconds. We decided to do this in order to obtain statistics for a week's operation. Since each queue was created as their own separate system, we simulated values for L, Lq, W, and Wq for all three denoting each, for example, as either L1, L2, or L3. The Cash Register queue represents 1, the Ingredients queue represents 2, and the Saute queue represents 3. In order to calculate the statistics for the entire Bison Fresh System, we simply summed all three L's, Lq's, W's, and Wq's. For the model, the average number of customers in the system was 3.19 and the average amount of customers waiting in the queue was 0.42. As for the stay times, the average amount of time a customer spends in the system was 493.99 seconds and the average amount of time a customer spends waiting in line is 65.60 seconds.

As we watched our simulation run, we considered different scenarios that would change our statistical values. We thought, what if there were two servers at the Ingredients Service instead of one? We ran this simulation and found that all the values slightly decreased. We were expecting a small change considering customers don't spend most of their time at the Ingredients Station, they spend the most at the

Saute Service. We also wondered what would happen if the interarrival times of the customers doubled. All of our statistics decreased significantly, and in order to return to our baseline values we believe we would need to increase service times. Lastly, we wondered what would happen if the service time of the Saute Station increased due to cooking ingredients that took a lot longer. The statistics increased significantly, which makes sense since customers have to wait longer for their food to be prepared.

Mathematical Model:

Development of our Mathematical model involved calculating the inter-arrival time, and average service time for all of our queueing services using our raw data. These statistics were then plugged into theoretical formulas for queueing systems. Our cashier queue was modeled based on an M/M/1 queue with a service rate of 0.071 customers per second . Likewise, our ingredients queue service was also an M/M/1 queue with a service rate of 0.016 customers per second, and our Sautee queue was an M/M/4 queue with a service rate of 0.0025 customers per second. Results of this model have been discussed in the executive summary.

PART III - Appendix:

Raw Data Value model:

$\mathbf{L} = \Sigma n \times P(n) \qquad for\ n = 0,\ldots, 8.$

$$0 + 1 \times \frac{6}{69} + 2 \times \frac{16}{69} + 3 \times \frac{18}{69} + 4 \times \frac{13}{69} + 5 \times \frac{8}{69} + 6 \times \frac{4}{69} + 7 \times \frac{3}{69} + 8 \times \frac{1}{69} = 4.43$$

Wq1 = 14.02 (average of every customer's time waiting to enter the cashier queue after arrival)
Wq2 = 9.51 (average of every customer's time waiting to enter the ingredient's queue after leaving the cashier queue)
Wq3 = 31.35 (average of every customer's time waiting to enter the sautee queue after leaving the ingredient's queue)

Wq = 54.88 (sum of Wq1, Wq2, and Wq3)

Lq = λWq = 0.42

W = 495.43 (average of every individual customer's time spent in the system)

Queueing System Model(Mathematical model):

Formulas Used:

$$L = \frac{\lambda}{\mu - \lambda}$$

$$Lq = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W = \frac{1}{\mu - \lambda}$$

$$Wq = \frac{\lambda}{\mu(\mu - \lambda)}$$

Using the exponential distribution.

λ = 0.007702612188
μ1 = 0.07131782946
μ2 = 0.01610169492
μ3 = 0.002590673575

L = L1 + L2 + L3
L1 =0.007702612188/(0.07131782946 -0.007702612188) = 0.12108
L2 =0.007702612188/(0.01610169492 -0.007702612188) = 0.917078
L3 = 2.97301
L = 4.011168

W = W1 + W2 + W3
W1 = 1/(14.03478261 - 0.027226) = 0.07139
W2 = 1/(62.10526316 - 0.027226) = 0.016108
W3 = 386.105
W = 386.192498

Lq
<u>For queue 3:</u>
Use Little's formula to get Wq from Lq3
Rou = .743243
1/(1 + 2.9732083 + 4.4199838+ 4.38051085 + 12.6814299)
P0 = 0.03928
Lq3 = 1.4414856
Lq1 = $\frac{\lambda^2}{\mu(\mu - \lambda)}$
= (0.007702612188)^2/(0.07131782946(0.07131782946-0.007702612188))
=.013077

Lq2 = (0.007702612188)^2/(0.01610169492(0.01610169492-0.007702612188))
= 0.4387

Lq= Lq1 + Lq2 + Lq3
= 1.4414856 + .013077 + 0.4387
Lq = 1.8932

Wq3
= lambda/Lq3
.0077/1.4414856
= .005341711

W = Wq + average service time (1/mu3)
W3 = .005341711 + (386.100386)
= 386.105

L3 = lambda(W)
= 2.97301

Wq = Wq1 + Wq2 + Wq3
Wq1 = (0.007702612188)/(0.07131782946(0.07131782946-0.007702612188))
= 1.69777
Wq2 = (0.007702612188)/(0.01610169492(0.01610169492-0.007702612188))
= 56.95357
Wq3 = .0053
Wq = 1.69777 + 56.95357 + .0053
Wq = 58.65664

L = 4.011168
Lq = 1.8932
W = 386.192498
Wq = 58.65664

Simulation (Flex Sim) model:
L = **3.19**

L1 = 0.12
L2 = 0.55
L3 = 2.52

Lq = **0.42**

Lq1 = 0.01
Lq2 = 0.16

Lq3 = 0.25

W = **493.99**

W1 = 15.92
W2 = 85.25
W3 = 392.82

Wq = **65.50**

Wq1 = 1.84
Wq2 = 24.95
Wq3 = 38.71

Part III
What-If Scenarios

What if there were 2 servers at the Ingredients Service instead of 1? (structural)

a. How does this impact L, Lq, W, Wq?
    i. L = 0.12 + 0.40 + 2.57 = **3.09, decreases**
    ii. Lq = 0.01 + 0.01 + 0.30 = **0.32, decreases**
    iii. W = 15.94 + 61.75 + 401.31 = **479.00, decreases**
    iv. Wq = 1.84 + 1.42 + 47.09 = **50.35, decreases**
b. What could be done to mitigate these effects?
    i. If the customer inter-arrival time increases, we could return to our baseline statistics. We expected this to occur since customers spend most of their time in the saute station.

What if the inter-arrival time of the customers doubled?

a. How does this impact L, Lq, W, Wq?
    i. L = 0.06 + 0.21 + 1.06 = **1.33**
    ii. Lq = 0.00 + 0.02 + 0.00 = **0.02**
    iii. W = 15.13 + 68.24 + 350.29 = **433.66**
    iv. Wq = 0.72 + 6.56 + 0.27 = **7.55**
b. What could be done to mitigate these effects?
    i. All of our statistics decrease significantly, the service times could increase in order for us to return to the baseline.

What if the service time of the Saute Service increased, say the ingredients being cooked needed a lot more time on the stove than other ingredients (Increased to mean of 500 sec)?

a. How does this impact L, Lq, W, Wq?
    i. L = 0.12 + 0.55 + 4.74 = **5.41**
    ii. Lq = 0.01 + 0.16 + 1.57 = **1.74**
    iii. W = 15.96 + 85.32 + 742.62 = **843.90**
    iv. Wq = 1.83 + 25.01 + 244.82 = **271.66**
b. What could be done to mitigate these effects?
    i. All of our statistics increase significantly, which makes sense because more customers are in service waiting for their food to be prepared.