

# A (first?) foray into Linear Algebra

## Day 1 – Fundamentals of vectors, matrices and their operations

### Motivated by searching and ranking

**vector** vector multiplication (inner product), cosine rule, vector norms

**matrix** matrix-vector multiplication, matrix-vector systems, matrix-matrix multiplication, matrices as operators, orthogonal matrices, inverses and determinants, and a bit about eigenvalues and eigenvectors

## Day 2 – Diving deeper with as ultimate goal the SVD and PCA

### With compression and regression

**matrix** Singular Value Decomposition, least squares, normal equations, QR decomposition

# Approximate Schedule

## Day 1

8-8:15	Intros, breakouts
8:15-9	New material with Margot
9-9:30	Practice with Nadim
9:30-9:45	Break
9:45-10:30	New material with Margot
10:30-11	Practice with Nadim

## Day 2

8-9	New material with Margot
9-9:30	Practice with Nadim
9:30-9:45	Break
9:45-10:30	New material with Nadim
10:30-10:50	Practice with Nadim
10:50-11	Closing

Slides after this are used  
throughout the short course



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en.wikipedia.org › wiki › List\_of\_women\_in\_mathema... ⋮

### List of women in mathematics - Wikipedia

Vera Faddeeva (1906–1983), Russian expert on numerical **linear algebra** · Fariba Fahroo, Persian-American expert in pseudospectral optimal control, winner of ...

mathsci2.appstate.edu › ncctm › activities › browne PDF ⋮

### Women and Minorities in Mathematics - Appalachian State ...

algebra, and **linear algebra**. While a college level student could explore more of her work, this worksheet is aimed at high school students. Solutions can be ...

math.

Nati

Mar 1

conce

Google returns **exact and related matches**

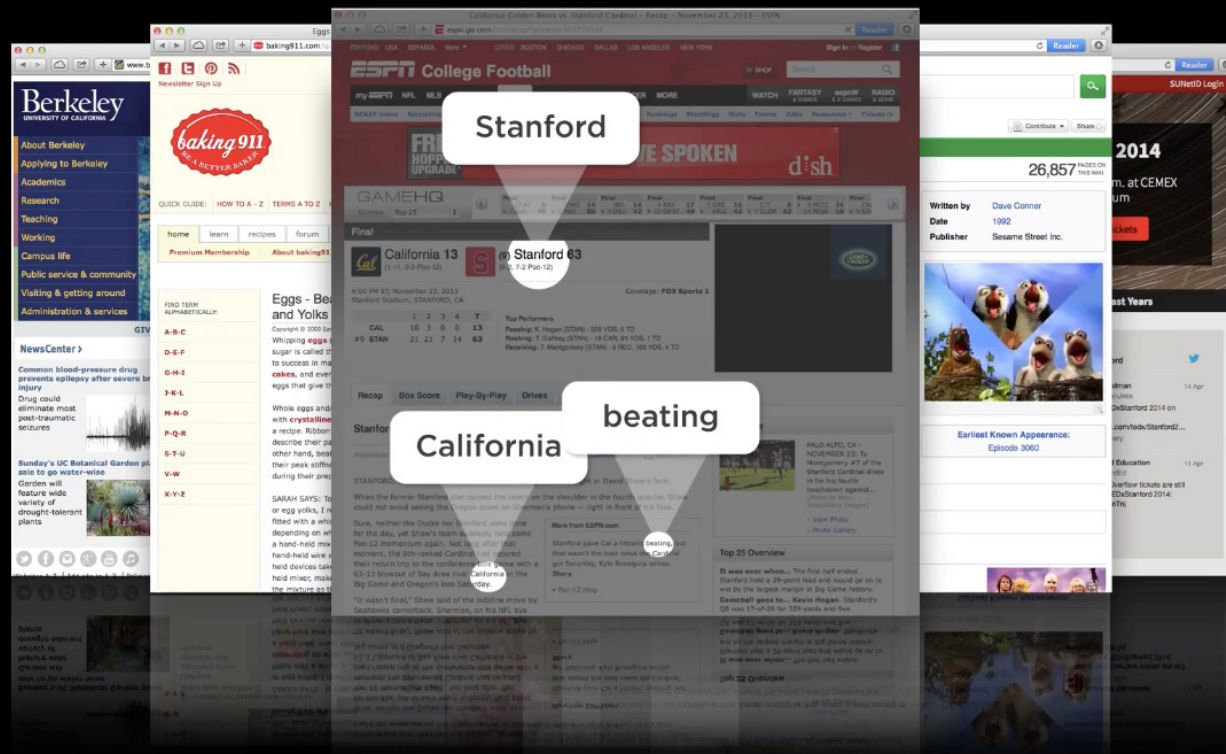
The resulting web pages are **ranked in some order**

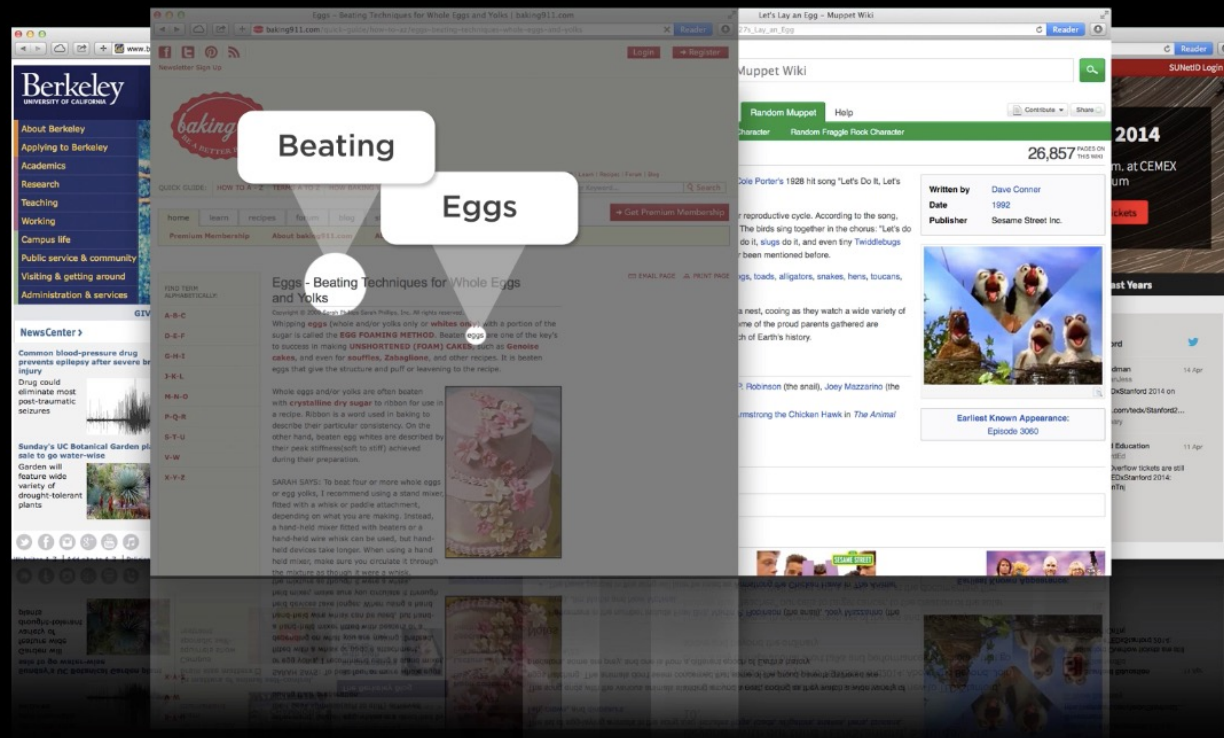
www.math.wisc.edu › women-mathematicians ⋮

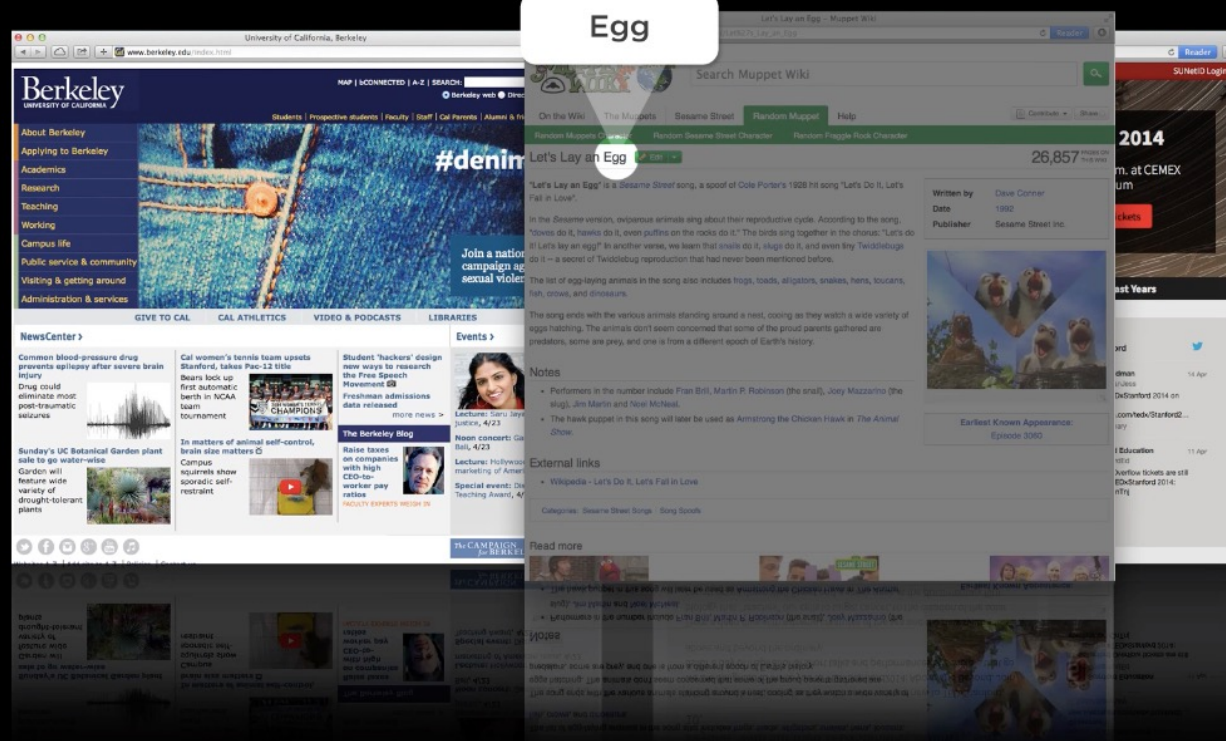
### History of Women Mathematicians | Department of Mathematics

1886: The first American **Woman** to receive a PhD. in **math** (from Columbia ... The title of Dr.

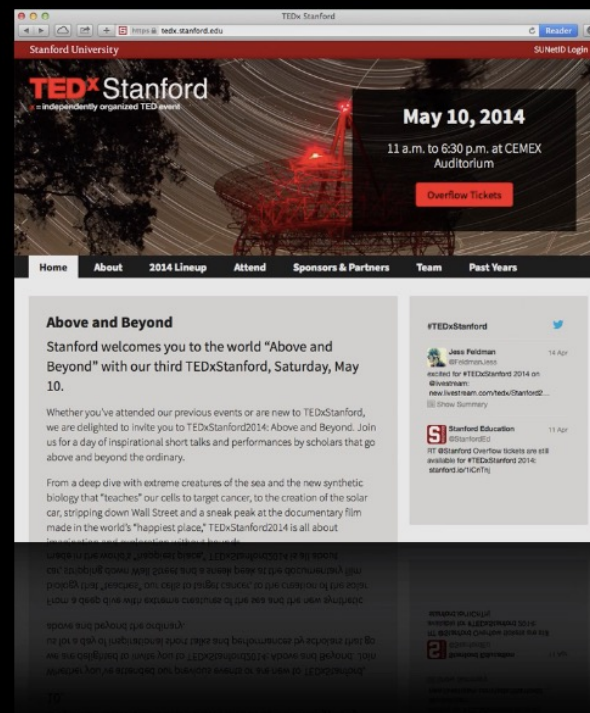
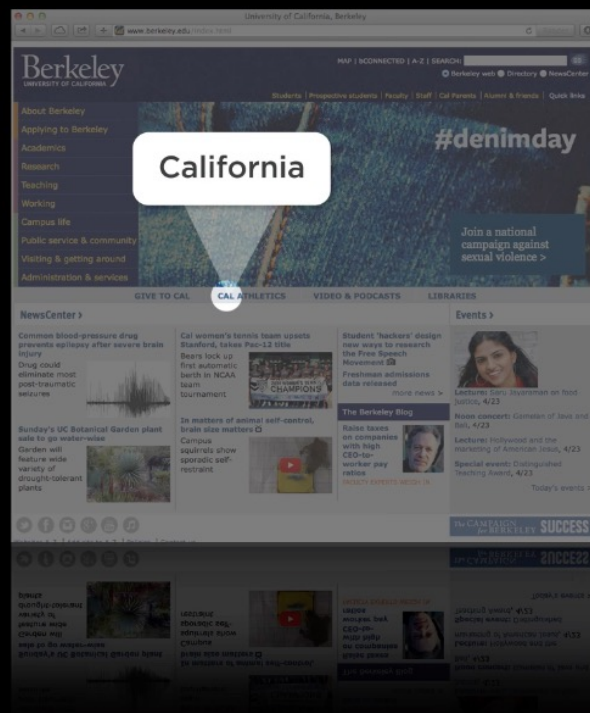
SERP: SEARCH ENGINE RESULTS PAGE



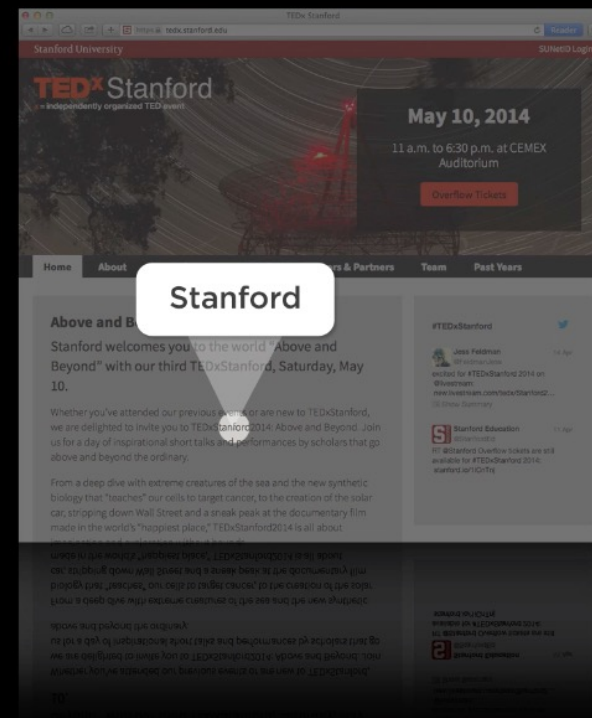












words \ website	Page 1	Page 2	Page 3	Page 4	Page 5
stanford	1	1	0	0	0
Beating	1	0	0	1	0
CAL	1	0	1	0	0
Eggs	0	0	0	1	1

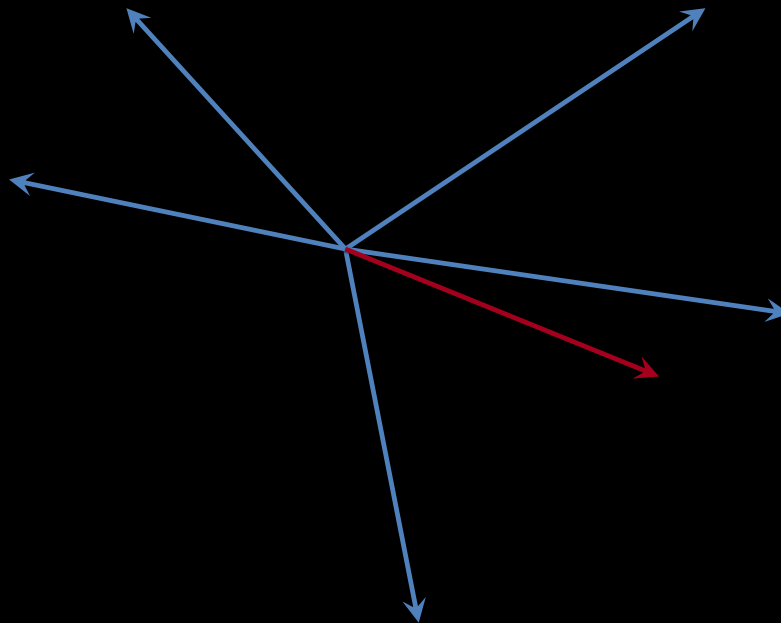
## How to search for the phrase "Stanford eggs"

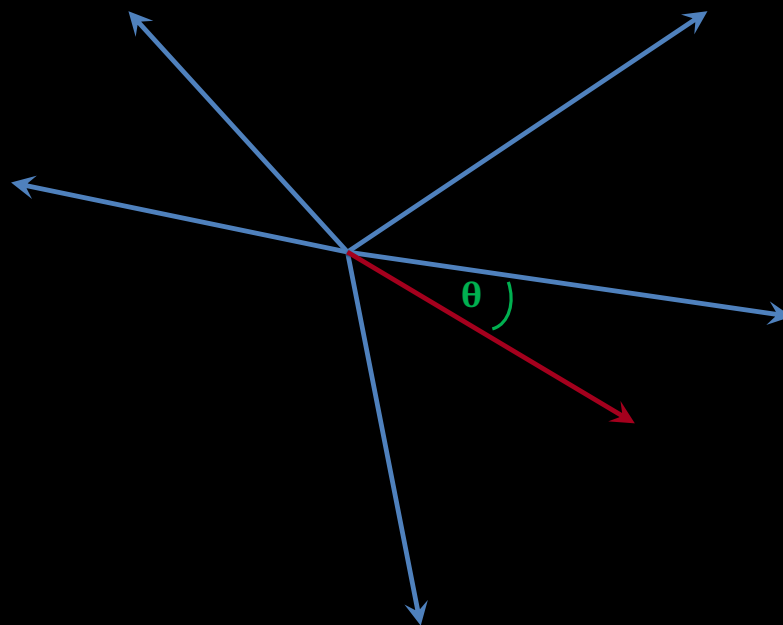
words \ website	Page 1	Page 2	Page 3	Page 4	Page 5
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CAL	1	0	1	0	0
Eggs	0	0	0	1	1



1
0
0
1

**How would you determine which of the page vectors are closest?**



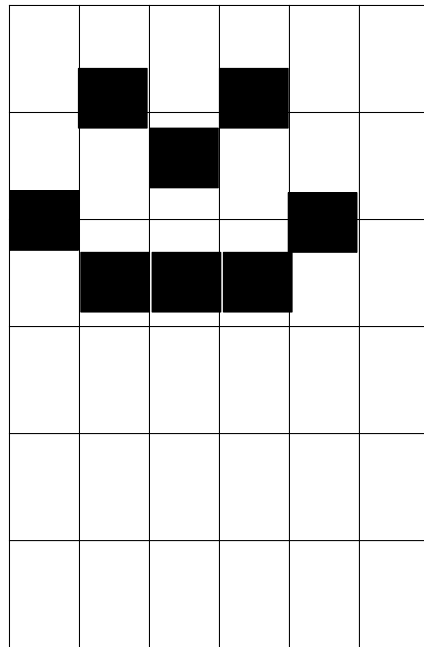


words \ website	Page 1	Page 2	Page 3	Page 4	Page 5
stanford	1	1	0	0	0
Beating	1	0	0	1	0
CAL	1	0	1	0	0
Eggs	0	0	0	1	1

1	1	0	0	0
1	0	0	1	0
1	0	1	0	0
0	0	0	1	1



$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



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### Women and Minorit

algebra, and linear algebra.

worksheet is aimed at high s

math.mit.edu › wim › 2019/03/10 › national-mathemati...

### National Mathematics Survey – Women In Math - MIT Math

Mar 10, 2019 — The above table gives the percentage of undergraduate mathematics majors/ concentrators that took calculus, multivariable and linear algebra ...

www.math.wisc.edu › women-mathematicians

### History of Women Mathematicians | Department of Mathematics

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SERP: SEARCH ENGINE RESULTS PAGE

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**How would YOU do it?**

# **RANKING DEPENDS ON MANY FACTORS**

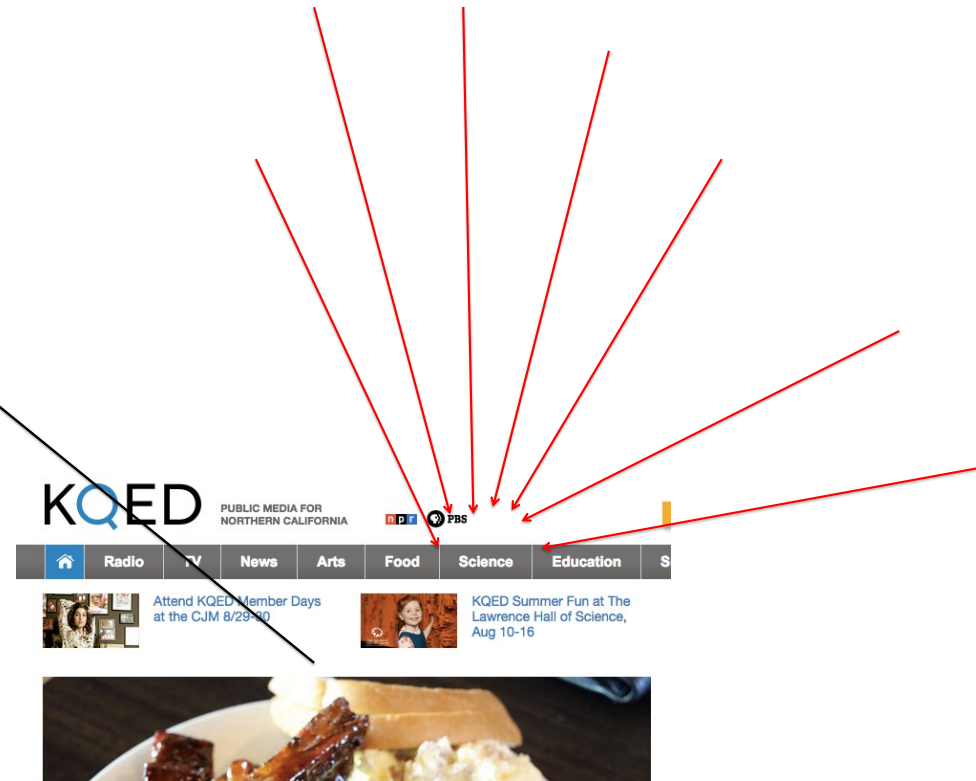
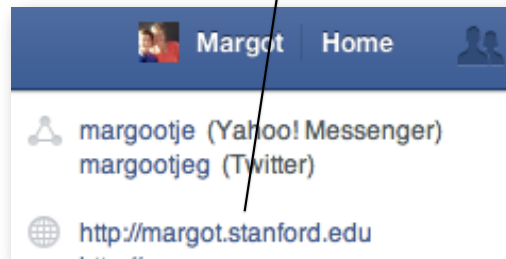
## **DIFFERENT SEARCH ENGINES USE DIFFERENT FACTOR/WEIGHTINGS**

- The traditional **page rank**  
(measure of importance of a page)
- Number of visits
- Age
- Recent edits

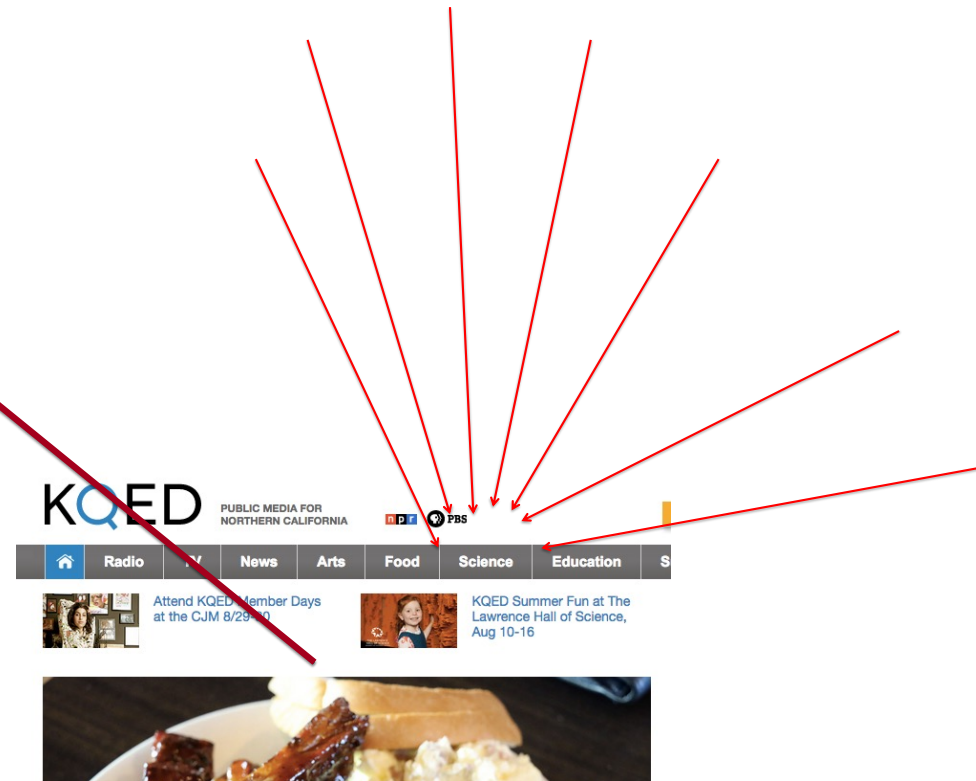
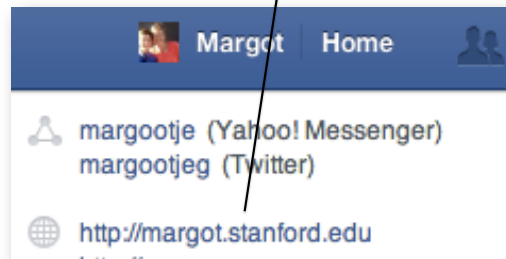
.....

## PAGE RANKING A LA 1990s

Lawrence Page, Sergey Brin, Rajeev Motwani, Terry Winograd  
“The PageRank Citation Ranking: Bringing Order to the Web”  
Technical Report, Stanford InfoLab, 1999

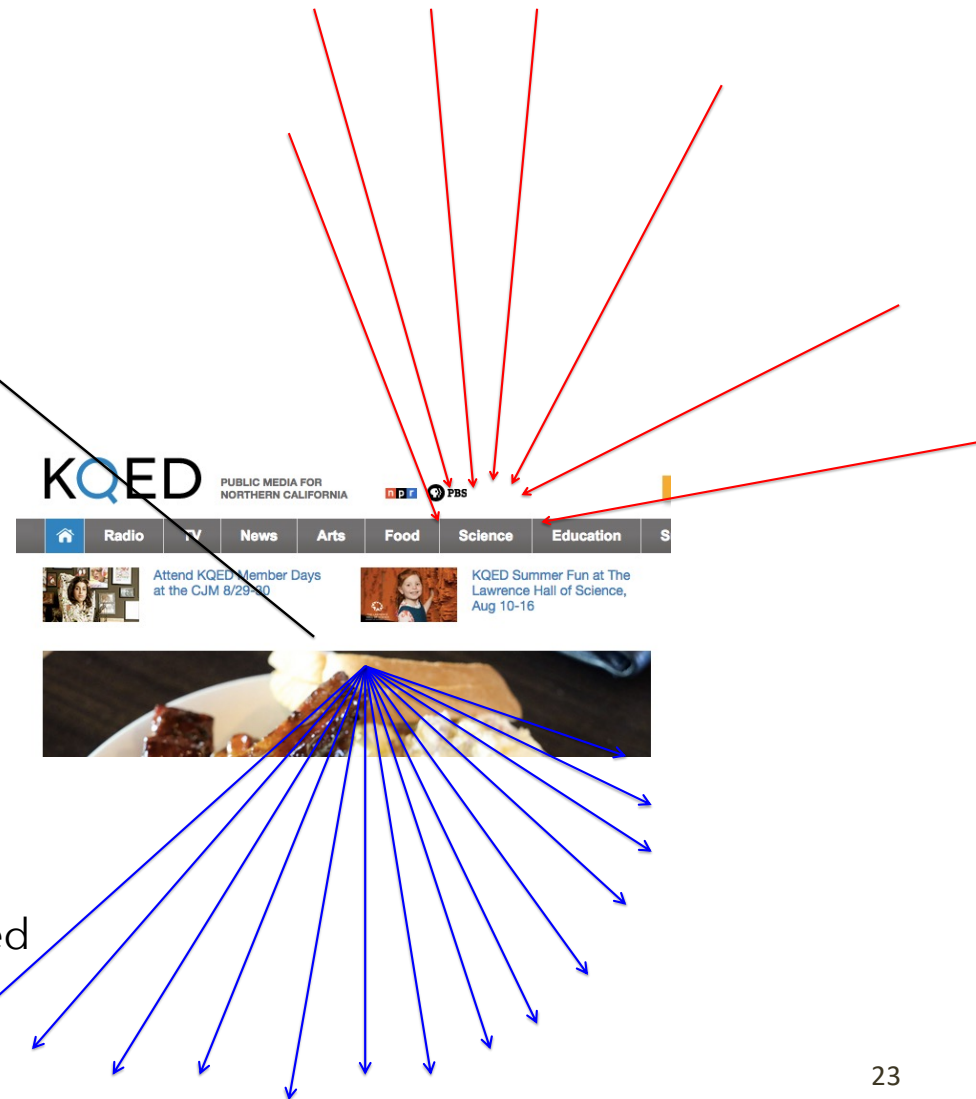
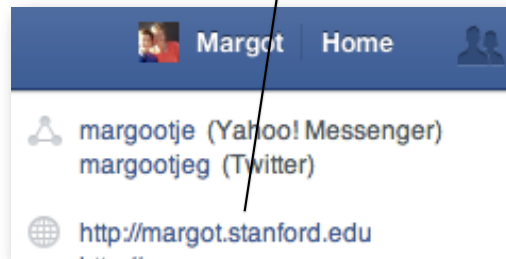


Idea: The more incoming links, the higher the page rank



Idea: links coming in from an important page have more weight

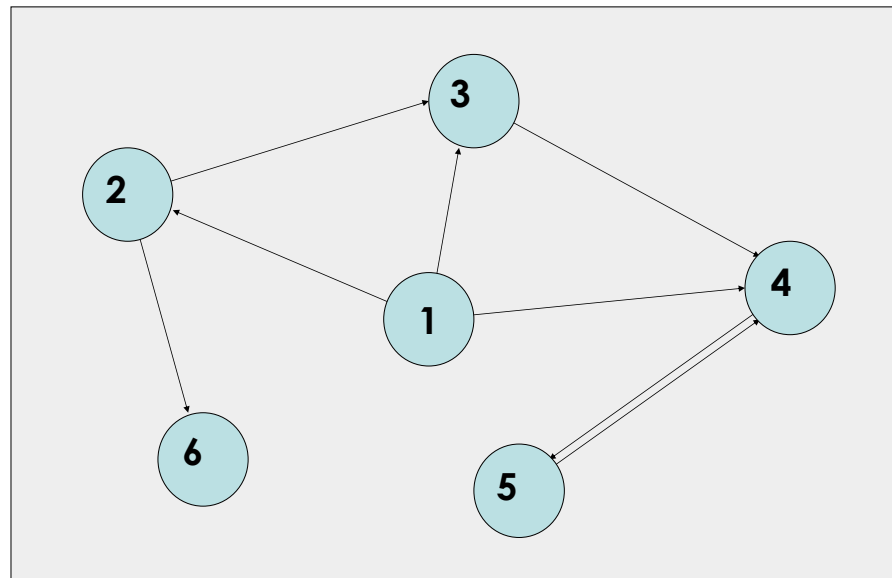




# RANK PROPAGATION

Rank of page  $j$  denoted by  $x_j$

$$x_1 = 0$$



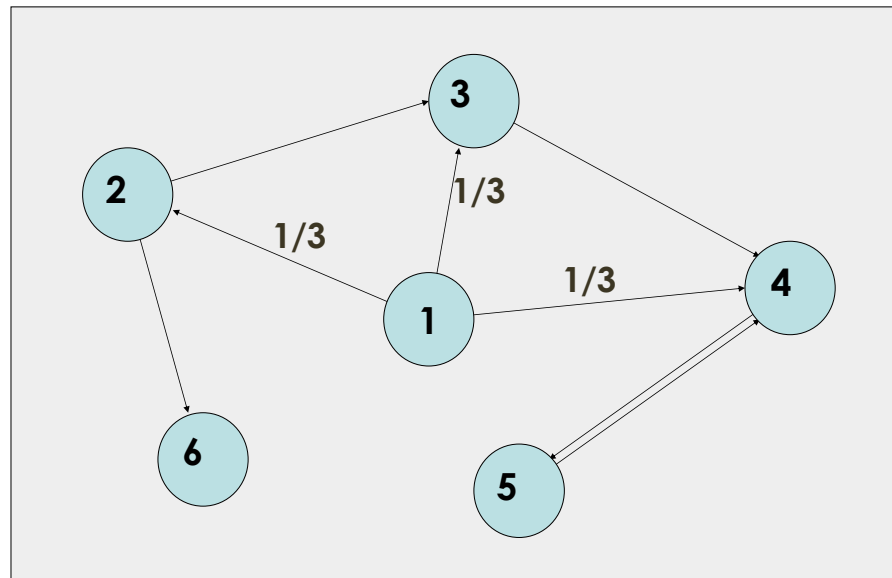
A very small internet "graph"

# RANK PROPAGATION

Rank of page  $j$  denoted by  $x_j$

$$x_1 = 0$$

$$x_2 = \frac{1}{3} x_1$$



A very small internet "graph"

# RANK PROPAGATION

Rank of page  $j$  denoted by  $x_j$

$$x_1 = 0$$

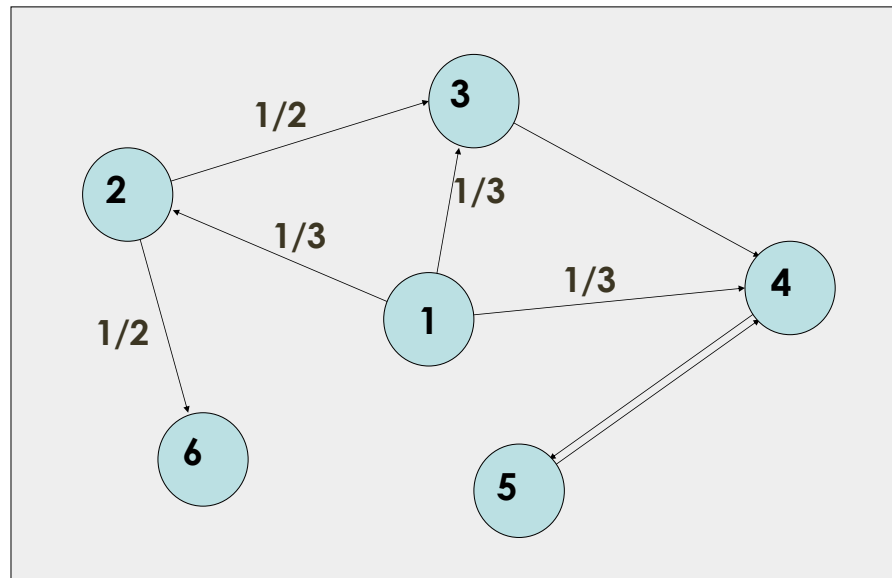
$$x_2 = \frac{1}{3}x_1$$

$$x_3 = \frac{1}{3}x_1 + \frac{1}{2}x_2$$

$$x_4 = \frac{1}{3}x_1 + x_3 + x_5$$

$$x_5 = x_4$$

$$x_6 = \frac{1}{2}x_2$$



A very small internet "graph"

$$x_1 = 0$$

**A COUPLED AND LINEAR  
SYSTEM OF EQUATIONS**

$$x_2 = \frac{1}{3} x_1$$

$$x_3 = \frac{1}{3} x_1 + \frac{1}{2} x_2$$

$$x_4 = \frac{1}{3} x_1 + x_3 + x_5$$

$$x_5 = x_4$$

$$x_6 = \frac{1}{2} x_2$$

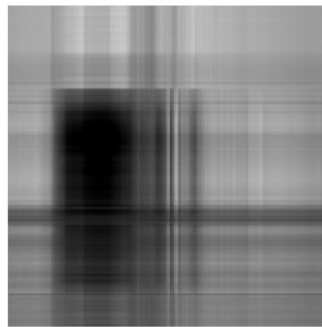
## IN MATRIX-VECTOR NOTATION

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \quad \text{or} \quad \begin{matrix} x = Px \\ Px = x \end{matrix}$$

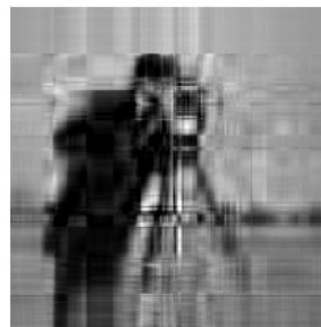
# SVD FOR COMPRESSION ON MORE THAN A 5x5 MATRIX



256x256



keep 2 terms



keep 8 terms



keep 32 terms