Project Report

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Adaptive Manipulator Control:
A Case Study
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Abstract

This paper discusses the adaptive control technique on a high-speed two degree-of-freedom semi-direct-drive robot. An approach is presented to control the trajectory of initially unknown dynamics of a robot manipulator which is a nonlinear, time-varying,multi-input-multi-output. The experiment setup estimates the values of unknown dynamics in the first half second of the run and uses them to enhance the performance of the manipulator.

1 Introduction to Adaptive Control

The basic problem in designing a robot manipulator is the variability of the loads to be manipulated. The design parameters if once fixed incorrectly in the starting may lead to faults later in real time. More-so the uncertainties associated with the dynamic properties such as the moment of inertia of links and the manipulation system on the whole and the center of mass transition once the load is grasped are also few contributing factors which limit robots ability. The accuracy of computed-torque method in direct-drive robots where gear-reduction is not available; suffer because of these uncertainties. The two methods to deal with this are Robust Control and Adaptive Control.

The basis of adaptive control is to apply a mechanism which extracts information from the tracking errors are and improves the accuracy of the system with time for varied load. Due to the uncertainties of the model a lot of assumptions like decoupled-dynamics, local linearization have to be made to converge the error. In[3] the author makes an approach which does not requires a lot of assumptions but it required a lot of computations related to estimating the joint accelerations and inertia matrices. In [7] a different approach is used in which the unknown parameters are being estimated in the real time based on the a PD feedback and adaptive compensation. In this paper the calculations of motion tracking error is conducted.

Due to the fact that in the analysis of unmodeled high-frequency dynamics and measurement noise are neglected in the stability analysis, the theoretical analysis and computer simulations are not sufficient.

In this paper an experiment is conducted using a joint-space adaptive tracking controller on a high-speed two degree-of-freedom semi-direct-drive robot. The results show that within first half second of running the manipulator on large load of unknown mass; the initially undefined mass properties can be estimated even with its various error sources.

In section 2 theoretical background of the experiment with the modeling and design of the adaptive trajectory controller, the implementation aspects and how this is combined with respect to more robust control and other disturbances. In section 3 the experimental background with the equipment are discussed. Section 4 the results of the experiments are being discussed. Section 5 states the conclusion.

2 Discussions

2.1 Manipulator Model

As we know the general equation for manipulator dynamics can be written as

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where q is the n x 1 vector of joint displacements, τ is the n x 1 vector of applied joint torques (or forces), H (q) is the n x n symmetric positive definite manipulator inertia matrix, C(q, $\dot{q}$)$\dot{q}$ is the n x 1 vector of centripetal and Coriolis torques, and g (q) is the n x 1 vector of gravitational torques.

The 2 properties for the dynamics 1 defined to be used further in the paper are:

- The two n x n matrices H and C are not independent. The matrix

$$(\dot{H} - 2C) \tag{2}$$

  is skew- symmetric, a property which is derived from the Lagrangian formulation of the manipulator dynamics as:

$$L = KE - PE = \frac{1}{2}\dot{q}^T H(q)\dot{q} - G(q)$$

  Lagrange's equation,

$$\frac{d}{dt}\frac{L}{\dot{q}} - \frac{L}{q} = \tau$$

  then yields the dynamic equation of manipulator in form,

$$H\ddot{q} + C(q,\dot{q}) + g(q) = \tau$$

  when we define the C component wise, we get

$$C_{ij} = \frac{1}{2}\sum_{k=1}^{n}\frac{\partial H_{ij}}{\partial q_k}\dot{q}_k + \frac{1}{2}\sum_{k=1}^{n}(\frac{\partial H_{ik}}{\partial q_j} - \frac{\partial H_{jk}}{\partial q_i})\dot{q}_k$$

  The above equations explain that $(\dot{H} - 2C)$ is indeed a skew-symmetric matrix, a fact which also reflects conservation of energy [6]. This property can also be written as

$$\dot{H} = C + C^T \tag{3}$$

- Also the dynamics are linear for a appropriately selected set of manipulator and load parameters as the terms on left side of are (1) linear.

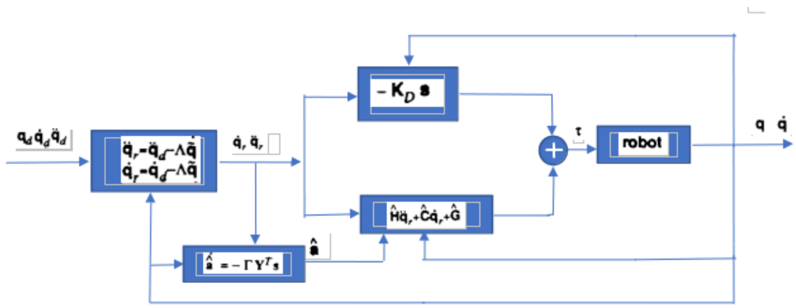

Figure 1: The structure of adaptive controller.

2.2 Controller Design

For the adaptive controller design problem in this paper;
Given: desired joint position $q_d(t)$
To Find: a control law for the actuator torques and an estimation law for the unknown parameters in a way that manipulator joint position q(t) precisely tracks $q_d(t)$ after an initial adaptation process.

We design our law in a way that it is linear as;

$$\tilde{H}(q)\ddot{q} + \tilde{C}(q, \dot{q})\dot{q} + \tilde{g}(q) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{a} \tag{4}$$

where a is a constant m-dimensional vector containing the undefined values, $\hat{a}$ is (time-varying) estimate, and $\hat{H}$, $\hat{C}$ and $\hat{g}$ are matrices of H , C , and g we get after substituting the estimated $\hat{a}$ for the actual a.
The terms in 4 are $\tilde{a} = \hat{a} - a$(estimation error), Y is an n x m matrix, and $\dot{q}_r$ is:

$$\dot{q}_r = \dot{q}_d - \wedge\tilde{q} \tag{5}$$

with $\wedge$ being a positive definite matrix, and $\tilde{q}(t) = q(t) - q_d(t)$; the position tracking error.

As the reference velocity $\dot{q}_r$ is the difference of desired velocity $\dot{q}_d$ and the position tracking error $\tilde{q}$. It is used for the convergence of tracking error. It is observed that the reference velocity $\dot{q}_r$ increases if the actual trajectory q lags behind the desired $\dot{q}_d$.

The following are the suggested control and adaptation law:

$$\tau = \hat{H}\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{g}(q) - K_D s \tag{6}$$

$$\dot{\hat{a}} = -\mathcal{T} Y^T s \tag{7}$$

where $\mathcal{T}$ is a constant positive definite matrix, $K_D(t)$ is a uniformly positive definite matrix, and the vector s, which can be thought of as a measure of tracking accuracy, is defined as

$$s = \dot{q} - \dot{q}_r = \dot{\tilde{q}} + \wedge\tilde{q} \tag{8}$$

In this law when the desired trajectories $q_{d,d}$, and $\ddot{q}_d$ are bounded, we get convergence of errors. When we consider the Lyapunov function and differentiate we can show as:

$$V(t) = \frac{1}{2}[s^T H s + \tilde{a}^T \mathcal{T} \tilde{a}] \tag{9}$$

The differentiation of V(t) leads to

$$\begin{aligned} \dot{V}(t) &= s^T(\mathbf{H}\ddot{\mathbf{q}} - \mathbf{H}\ddot{\mathbf{q}}_\mathbf{r}) + \tilde{\mathbf{a}}^T \mathcal{T}^{-1} \dot{\tilde{\mathbf{a}}} + \frac{1}{2}) s^T \dot{\mathbf{H}} \mathbf{s} \\ &= s^T(\tau - H\ddot{q}_r - C\dot{q}_r - g) + \tilde{a}^T \mathcal{T}^{-1} \dot{\tilde{\mathbf{a}}} \end{aligned} \tag{10}$$

in this the skew-symmetry of $(\dot{H} - 2C)$ has been used to eliminate the term $\frac{1}{2} s^T \dot{H} s$ so as to avoid the time-varying nature of the inertia matrix. Substituting the control law (6) into the above expression, and using the linearity property (4), we get,

$$\begin{aligned} \dot{V}(t) &= s^T(\tilde{\mathbf{H}}\ddot{\mathbf{q}}_\mathbf{r} + \tilde{C}\dot{q}_r + \tilde{g} - K_D s) + \tilde{\mathbf{a}}^T \mathcal{T}^{-1} \dot{\tilde{\mathbf{a}}} \\ &= s^T(Y\tilde{a} - K_D s) + \tilde{a}^T \mathcal{T}^{-1} \dot{\tilde{\mathbf{a}}} \end{aligned} \tag{11}$$

$$= -s^T K_D s + \tilde{a}^T [\mathcal{T}^{-1} \dot{\hat{a}} + Y^T s]$$

In order to remove the second term from the last expression (7) and substituting (7) we get;

$$\dot{V}(t) = -s^T K_D s <= 0 \tag{12}$$

As from above the V(t) is lower bounded by zero and is decreasing for any nonzero s, the $\dot{V}(t)$ and hence tracking error measure s, must converge to zero. This convergence of s to zero implies $\tilde{q}$ and $\dot{\tilde{q}}$ also converge to zero.

Hence it can be interpreted that for a stable linear filter, if the input converges to zero the output also converges to zero. Hence by this adaptive approach we get stability as well as convergence of the tracking errors.

The **structure of the adaptive controller** consists of two parts. First is is based on estimated parameters with three terms of inertial, centripetal and Coriolis, and gravitational torques. This in turn provides the joint dynamic torques necessary to make the desired motions. The second part has two terms representing PD feedback which is used to keep the real trajectories close to desired trajectories.

$$-K_D s = -K_D \dot{\tilde{q}} - K_D \wedge \tilde{q} \tag{13}$$

Hence the inputs will be joint position $q_d, velocity_d$, and acceleration $\ddot{q}_d$ and the measured outputs are the joint position q and velocity $\dot{q}$.

2.3 Remarks and Discussion for the proposed Model

Remark 1 : Using (6) and (1) we obtain closed loop dynamics. This can be proved by writing (6) as:

$$\tau = Y\hat{a} - K_D s \tag{14}$$

so closed-loop dynamics are;

$$H\dot{s} + (K_D + C)s = Y\tilde{a} \tag{15}$$

also as $\tilde{a}$ determined by the adaptation law 7 can be expressed as,

$$\dot{\hat{a}} = -\mathcal{T}\frac{\partial \tau}{\partial \hat{a}} s$$

matrix $\mathcal{T}$ represents the adaptation gain which intuitively means speed of adaptation.

Remark 2: For nonlinear dynamics; similar to linear dynamics the estimated parameters asymptotically converge to true values if the matrix Y in (4) is continuous.

Remark 3 : To represent the control law (6) similar to computed-torque-controller, we can represent K_D as $K_D(t) = \lambda\hat{H}$, with λ a positive constant. So if we replace $\wedge =$ we get the new control, law with critical damping error dynamics as;

$$\tau = \hat{H}[\ddot{q}_d - 2\lambda\dot{\tilde{q}} - \lambda^2\tilde{q}] + \hat{C}\dot{q}_r + \hat{g}$$

Now as H also varies with adaptation so the adaptation law (7) can be rewritten as:

$$\tilde{H}(q)(\ddot{q}_r - \lambda s) + \tilde{C}(q,\dot{q})\dot{q}_r + \tilde{g}(q) = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{a} \tag{16}$$

Now to obtain the global tracking convergence using Lyapunov equation:

$$\dot{V}(t) = -s^T Hs <= 0$$

From the above equation we can say H is uniformly positive definite.

Remark 4: In some tasks where the manipulator ,motion is already planned, we can use the adaptive controller rather than model-reference method as in this we can enjoy a desired joint trajectory rather than planned one constructed by a reference model. Also other important perk of adaptive is it need not require assumptions and approximations and measurements of joint and inertia parameters.

Implementation Aspects

- We can consider the load as a part of the last link. On doing so the parameters we need to estimate in real time are the equivalent dynamic parameters of the load being held. Hence for example we have a 6 DOF arm, so to implement a adaptive technique we will need to estimate 10 parameters on the go. These are 3 denoting the center of mass, 6 denoting moment of inertia and a friction.
- For achieving convergence of tracking errors we can use a simple technique of stopping the measurement of undefined parameters and resuming as soon as the signs of their derivatives changes.

- For x and x_d are the actual and desired positions in Cartesian space and J being the Jacobian matrix of the manipulator. We define $\dot{q}_r = J^{-1}[\dot{x}_d + \wedge(x_d - x)]$ to use in Cartesian model.
- Due to the difference in the rate of update in the measuring parameters $\hat{H}, \hat{C}$ than the terms $\dot{q}_r$ which happens as the tracking error updates faster than the dynamic matrices [4].
- When a actuator torque used during adaptation is physically saturated so we won't be able to implement our algorithm. As this saturation occurs when the load is greater than the capacity, so in order to rectify the situation we can reduce the speed of the desired trajectories which in turn reduces the magnitude of the torques.

Combining Adaptation with Robust Control
Some parameters can be used to make the system robust as these have less importance in the robot dynamics for example the frictions coefficients in direct-drive robot joints, cross moments of inertia of loads, centre of mass of the load which can be estimated beforehand using CAD, and also the disturbances such as torque ripple need not affect the system in the real-time. By adding a sliding control term in the control law (6) we get,

$$\dot{V}(t) <= -s^T[K_D s + \eta_i sgn(s_i)^T_{i=1,..,n}]$$
$$= -\sum_{i=1} \eta_i |S_i| - s^T K_D s <= 0$$

here k_i are variables and η_i are constants. Now we have surety on the trajectories reaching zero and hence convergence to zero will occur.

3 Experimental Background

The equipment, dynamic model and design of the adaptive controller are discussed below.

3.1 Equipment

The equipment used for experiment in this paper is defined as:
A two degree-of-freedom semi- direct-drive robot arm [2] developed at the Whitaker College of Health Sciences at M.I.T., with "semi" indicating that the second link is indirectly driven by a motor located at the base through a four-bar mechanism. The system consists of a two-link arm motors with amplifiers, two optical encoders, two tachometers, and a microcomputer PDP 11/73. The two links are made of aluminium, with lengths of 0.37 m and 0.34 m, and masses of 0.9 kg and 0.6 kg, respectively. The arm lies in the horizontal plane, and therefore, the effects of gravity are absent. The motors are driven by PMI SSA 40-10-20 pulse-width- modulated switching servo amplifiers. The two JR16M4CH motors, mounted on a rigid supporting frame, are rather large and heavy (16 kg

each). A four-bar linkage mechanism is used to transmit the torque from the upper motor to the outer link. The motion of the relative angle between the inner and outer links ranges from 39" to 139". The maximum torque each motor can generate is 9 N-m. The motors together with the amplifiers are regarded as having a constant of 1.12 N.m/V as their transfer function. The joint positions are measured by incremental optical encoders attached to the output shaft of each torque motor, with a resolution of 12 bits/l80", i.e., 0.045'. The joint velocities are directly measured by tachometers. Position and velocity measurements are sent to the PDP 11/73 for torque computation, with the control programs written in the C language. The resulting sampling frequency is 200 Hz.

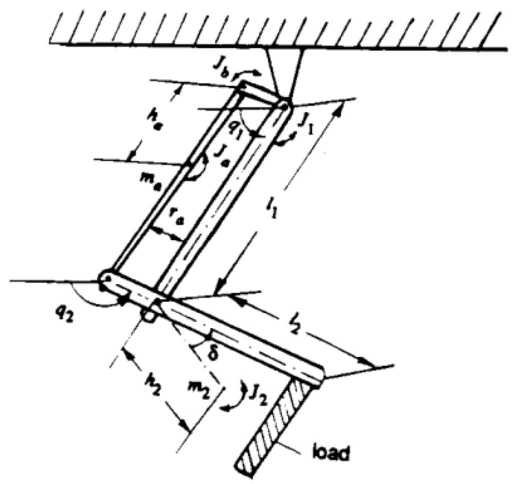

Figure 2: The schematic structure of manipulator

3.2 Dynamic Model

The dynamic law is derived from Lagrange's equation as:

$$a_1\ddot{q}_1 + (a_3c_{21} + a_4s_{21})\ddot{q}_2 - a_3s_{21}\dot{q}_2^2 + a_4c_{21}\dot{q}_2^2 = \tau$$
$$(a_3c_{21} + a_4s_{21})\ddot{q}_1 + a_2\ddot{q}_2 + a_3s_{21}\dot{q}_1^2 + a_4c_{21}\dot{q}_1^2 = \tau$$

where

$$c_{21} = cos(q_2 - q_1)$$
$$s_{21} = sin(q_2 - q_1)$$
$$a_1 = J_1 + J_a + m_2l_1^2 + m_ah_a^2$$
$$a_2 = J_2 + J_b + m_2h_2^2 + m_ar_a^2$$
$$a_3 = m_2h_2l_1cos\delta - m_ah_ar_a$$
$$a_4 = m_2h_2l_1sin\delta$$

The load is considered as part of the second link and hence we define matrix C as:

$$
\begin{aligned}
C(1,1) &= C(2,2) = 0 \\
C(1,2) &= (a_4 c_{21} - a_3 s_{21})\dot{q_2} \\
C(2,1) &= (a_3 s_{21} - a_4 c_{21})\dot{q_1}
\end{aligned}
$$

The feedback gain matrix and the adaptation gain matrix in controller design are chosen as diagonal:

$$
\begin{aligned}
K_D &= diag(k_{d1}, k_{d2}) \\
\mathcal{T} &= diag(\gamma_1, \gamma_2, \gamma_3, \gamma_4)
\end{aligned}
$$

Hence we get the control law combining the above values into (6) as:

$$
\begin{aligned}
\tau_1 &= Y_{11} a_1 + Y_{13} a_3 + Y_{14} a_4 - k_{d1} s_1 \\
\tau_2 &= Y_{22} a_2 + Y_{23} a_3 + Y_{24} a_4 - k_{d2} s_2
\end{aligned}
$$

where the values are

$$
\begin{aligned}
Y_{11} &= \ddot{q_{r1}} \\
Y_{22} &= \ddot{q_{r2}} \\
Y_{13} &= c_{21}\ddot{q_{r2}} - s_{21}\dot{q_2}\dot{q_{r2}} \\
Y_{23} &= c_{21}\ddot{q_{r1}} - s_{21}\dot{q_1}\dot{q_{r1}} \\
Y_{14} &= s_{21}\ddot{q_{r2}} - c_{21}\dot{q_2}\dot{q_{r2}} \\
Y_{24} &= s_{21}\ddot{q_{r1}} - c_{21}\dot{q_1}\dot{q_{r1}}
\end{aligned}
$$

3.3 Adaptation Law

Similar to control law the adaptation law can be defined by combining the above results of adaptation gain matrix and (7) as:

$$
\begin{aligned}
\dot{\hat{a_1}} &= -\gamma_1 Y_{11} s_1 \\
\dot{\hat{a_2}} &= -\gamma_2 Y_{22} s_2 \\
\dot{\hat{a_3}} &= -\gamma_3 (Y_{13} s_1 + Y_{23} s_2) \\
\dot{\hat{a_4}} &= -\gamma_4 (Y_{14} s_1 + Y_{24} s_2)
\end{aligned}
$$

In the dynamics calculation the Coulomb and viscous frictions at the motor shafts and at the joints between links are neglected in the adaptive controller design and regarded as disturbances.

3.4 Physical Interpretation of the Adaptation Mechanism

- The rate of adaptation is linearly proportional to the tracking error measurement

- The estimates $\hat{a_1} and \hat{a_2}$ are driven only by the tracking error of the first joint.
- $\hat{a_3} and \hat{a_4}$ are driven by a combination of the first joint error and the second joint error.

4 Experimental Results and Discussions

The results of the experiments demonstrate the stability and performance characteristics which are inferred from the above theoretical study and comparison of the the adaptive technique with that of PD and computed-torque. The PD controller considered is represented as:

$$\tau = -K_D\dot{\tilde{q}} - K_P\tilde{q}$$

The results are accounted for 1 seconds of the run. The first half-second for tracking motion between initial position q_s and end position q_e, and the second half-second for regulation of the residual tracking errors to zero. The desired joint trajectories to be tracked are two fifth-order polynomials interpolated between $q_s = [20100]^T degrees$ and $q_e = [70100]^T degrees$, with zero desired velocities and accelerations at t = 0 and t = 0.5 s.

4.1 Comparison of PD and Adaptive Controllers

In first case no load is attached to the manipulator. Initially all the parameters are assumed to be unknown. The feedforward by the PD controller helps in the adaptation as we have the tracking error. The adaptation gains are chosen as 0.2. On increasing K_d and K_p both controllers were found unstable. This proved that adaptive controller has same level of robustness to noises and high-frequency unmodeled dynamics as the PD controller. The values chose are $k_{d1} = k_{d2} = 2.0Nms/rad, k_{p1} = 40Nm/rad; k_{p2} = 30Nm/rad; \lambda 1 = 20; \lambda 2 = 15$. These values yield the better accuracy. At the end of first half second the joint errors of adaptive controller were found less than that of PD controller.Small steady-state joint errors are observed by adaptive controller due to the static friction of motor shafts of manipulator.

4.2 Comparison of Computed Torque Control and Adaptive Control

Based on some prior knowledge we can fix few parameters in model-based controllers. In absence of gravity, input torque is:

$$\tau = \hat{H}(\ddot{q}_d - k_1\dot{\tilde{q}} - k_2\tilde{q}) + \hat{C}\dot{q}$$

where k1 and k2 are diagonal matrices. The parameters are set to $a_1 = 0.11kg.m^2; a_2 = 0.0285kg.m^2; a_3 = 0.033kg.m^2; a4 = 0$. These values were calculated from the engineering drawings of arm links as mentioned in the paper. From the results in the paper it is told that tracking error for the first joint comes to be smaller than second due to the fact that the second link which is attached to the 4 bar mechanism has greater uncertainties.

4.3 Comparisons in the Presence of a Large Load

To check the versatility of the adaptive controller, large loads were tested to the end effector. All the three types of controller were tested for this large load. For the PD controller the errors do not settle down easily. For computed torque control a increase in second joint is observed due to the uncertainty of its joint dynamics while there is not much increase in for the first joint. For adaptive controller after the run when the estimated parameters were used the maximum errors for both the joints stayed within 1 degrees. In long term for different trajectories were small.

4.4 Error Analysis

It is expected that tracking errors of the adaptive controller globally converge to zero but the actual experiment demonstrate otherwise. This is because of disturbances and unmodeled dynamics in experiments but were ignored in the theoretical analysis. The tracking errors observed arise from many hardware or software sources such as

- Arm Modeling Errors: The neglected Coulomb and viscous frictions at the linkage joints in the experiment give rise to tracking error. Also the vibrational dynamics of the links may also have contributed a certain amount of error.

- Actuation Errors: The motors and the amplifiers may also possess their own gains which have been neglected in our calculations but may play an effect in the half-second run.

- Measurement Errors: The tachometer signals are small hence sensitive to noise so the Joint velocity measurements have error. Also the A/D converters for the velocity signals also generate error.

- Real- Time Computing Limitations: At higher speeds the error approximations and limitation of sample size also contribute a significant bit.

5 Conclusions

The performance of adaptive controller in high seed operations with uncertain dynamic factors and undefined errors; comes out to be better or at least similar to PD controller and computed-torque schemes. With respect to the standard rigid body model of robot arm dynamics the adaptive algorithm enjoys better convergence of tracking error over the desired trajectories, a global asymptotic stability, easier computation plus it requires less approximations on our part and provides same level of robustness. Hence, adaptation is the easiest, fastest, and most accurate method for obtaining parameter values for use in fixed-parameter controllers.

The adaptive control is still in its early development stages. The extent to which we can improve the performance of conventional industrial robot manipulators seems limited by the validity of the controller's actuator models. Thus the development of such models is of considerable practical importance. Also research in cases without measurements of joint acceleration also has lot of potential [1] [5].

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