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search algorithm for solving the container  
loading problem

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## Abstract

*Abstract* This paper presents a parallel tabu search algorithm for the container loading problem with a single container to be loaded. The emphasis is on the case of a weakly heterogeneous load. The distributed-parallel approach is based on the concept of multi-search threads according to Toulouse et al. [Issues in designing parallel and distributed search algorithms for discrete optimization problems, Publication CRT-96-36, Centre de recherche sur les transports, Université de Montréal, Canada, 1996] i.e., several search paths are investigated concurrently. The parallel searches are carried out by differently configured instances of a tabu search algorithm, which cooperate by the exchange of (best) solutions at the end of defined search phases. The parallel search processes are executed on a corresponding number of LAN workstations. The efficiency of the parallel tabu search algorithm is demonstrated by an extensive comparative test including well-known reference problems and loading procedures from other authors. 2003 Elsevier Science B.V. All rights reserved.

In the interest of stability of the load, both horizontal dimensions of each box are to be supported according to a predefined percentage. In any case the centre of gravity of each box must be supported in order to avoid boxes tipping over. It is assumed that the centre of gravity and the geometric centre of each box coincide. Box types are defined as follows. Two boxes are the same type if they coincide in all three side dimensions. On the basis of this concept of box types, the following three categories of box sets can be distinguished. A homogeneous box set is given if all boxes are of the same type. A box set is called weakly heterogeneous if there exist a few box types and many items per type. Finally, a strongly heterogeneous box set is characterized by a greater number of box types and only few items per type. Here, a weakly heterogeneous set of boxes is assumed. In the recent years, many (sequential) solution methods for the container loading problem have been developed. It is well known that the container loading problem is NP-hard (cf. [18]). Hence, the methods developed are heuristic approaches.

The only block of a 1-arrangement is always placed in the reference corner of the packing space. From the two blocks of a 2-arrangement, one is arranged in the reference corner. The second block can alternatively be placed as a neighbour in  $x$ -direction (arrangement type “in front of”), as a neighbour in  $y$ -direction (arrangement type “beside”) or as a neighbour in  $z$ -direction (arrangement type “above”). In the case of a placement according to arrangement type “in front of”, the block with the larger  $y$ -dimension is positioned in the reference corner, while for the arrangement type “beside” the block with the larger  $x$ -dimension is positioned in the reference corner. The arrangement type “above” is only used if both horizontal dimensions of a block are not smaller than the corresponding dimensions of the other block. The block with the larger horizontal dimensions is positioned in the reference corner and the other above. Fig. 2 illustrates a 1-arrangement and two 2-arrangements of the arrangement types “in front of” and “beside”.

**Palabras clave:** Tabu searchin: is one of the searching method.

# Chapter 1

## Heuristic Algorithm

By means of the basic heuristic a given container is loaded in several iterations. Within an iteration a so-called packing space is filled with one or more boxes. A packing space is an empty rectangular space within the container with defined side dimensions. In the first iteration the complete interior of the container is used as the packing space. For the loading of a packing space only box arrangements with a predefined simple structure are considered. These are called local arrangements. The feasible local arrangements for a packing space are generated and evaluated by means of certain criteria. The unused part of the packing space is completely subdivided into several residual packing spaces. These are filled later. A rough description of the algorithm of the basic heuristic is given in Fig. 1. The overall algorithm presented in Fig. 1 requires some comments.

- In order to enhance the chances of loading small packing spaces, the packing space with the smallest volume is always processed first.

- The container is embedded in a three-dimensional coordinates system. The bottom left-hand rear corner of a packing space is used as the reference corner. The coordinates of the reference corner are stored together with the dimensions of the packing space. The position of a box results from the coordinates of the reference corner of the respective packing space and its placement within the respective local arrangement (see below).

- In this section, the basic heuristic is presented as a greedy heuristic. In step (5) the best evaluated first local arrangement of ArrList is selected. In Section 3 the basic heuristic is extended in such a way that the best arrangement is not necessarily used for a packing space with packing space index  $i_{pr}$ . Only with this extension can the basic heuristic be used for the generation of different solutions to a problem instance. It should be mentioned that an index  $i_{pr}$  is only assigned to fillable packing spaces in which at least one box can be placed.

- At the same time as a local arrangement is generated and evaluated (step 2), the residual packing spaces that would occur if this local arrangement was used are generated. In step (6) these residual packing spaces are possibly inserted into the packing space list PrList. From the last comment it can be concluded that a more detailed description of the basic heuristic requires merely a refinement of step (3), which is subsequently described.

# The basic heuristic

## - The algorithm

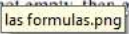
- (1) Initialize:  
the set of residual boxes  $BRes :=$  set of all boxes;  
the packing space list  $PrList := \{\text{Container}\}$ ;  
the packing space index  $iPr := 0$ ;  
the stowing list  $StList := \{ \}$ .
- (2) Determine the current packing space  $pcurr$  as the packing space from  $PrList$  with minimum volume and delete  $pcurr$  from  $PrList$ .
- (3) For packing space  $pcurr$ , initialize the arrangement list as empty list  $ArrList := \{ \}$ . Generate and evaluate all local arrangements for  $pcurr$ . Insert the local arrangements in descending order with respect to the evaluation into the arrangement list  $ArrList$ .
- (4) If  $ArrList$  is empty, go to step (8).
- (5) Update the packing space index  $iPr := iPr + 1$ . Insert the pair  $(pcurr, ArrList(1))$  as  $iPr$ -th element into the stowing list  $StList$ .
- (6) Insert the residual packing spaces for the packing space  $pcurr$  and the local arrangement  $ArrList(1)$  into the packing space list  $PrList$ .
- (7) Update the set of residual boxes  $BRes$ .
- (8) If the packing space list  $PrList$  is  then go to step (2).
- (9) Stop.

Fig. 1. Overall algorithm for the basic heuristic.

## Chapter 2

# Designing and analyzing

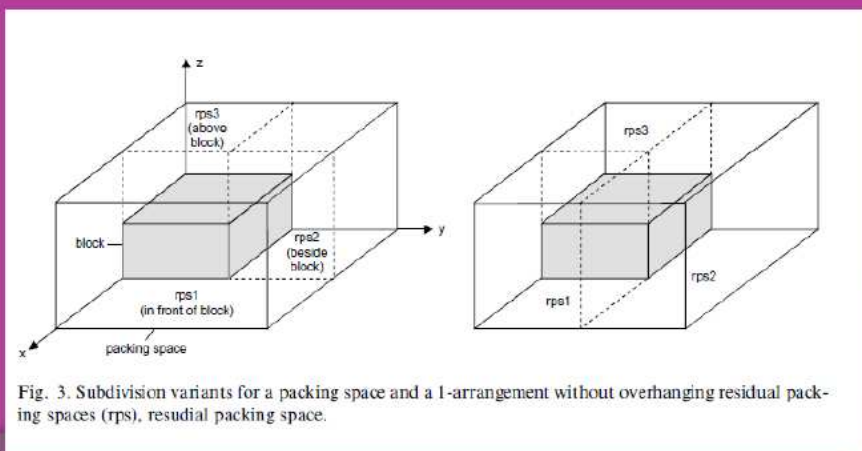
The structure of local arrangements for a packing space is defined as follows. A local arrangement consists of one or two so-called blocks and is therefore referred to as a 1- or 2-arrangement (see Fig. 2). A block is formed from boxes of the same type. Furthermore, all boxes of a block are arranged in an identical spatial orientation variant. In each of the three dimensions (x-, y- and z-direction) a block consists of one or more boxes. The only block of a 1-arrangement is always placed in the reference corner of the packing space. From the two blocks of a 2-arrangement, one is arranged in the reference corner. The second block can alternatively be placed as a neighbour in x-direction (arrangement type “in front of”), as a neighbour in y-direction (arrangement type “beside”) or as a neighbour in z-direction (arrangement type “above”). In the case of a placement according to arrangement type “in front of”, the block with the larger y-dimension is positioned in the reference corner, while for the arrangement type “beside” the block with the larger x-dimension is positioned in the reference corner. The arrangement type “above” is only used if both horizontal dimensions of a block are not smaller than the corresponding dimensions of the other block. The block with the larger horizontal dimensions is positioned in the reference corner and the other above. Fig. 2 illustrates a 1-arrangement and two 2-arrangements of the arrangement types “in front of” and “beside”. For the blocks of an arrangement, the box numbers in all three dimensions are first determined in such a way that the concerned dimensions of the packing space are utilized as fully as possible. If the number of boxes of a given type required for a block exceeds the number of still available items of this type, then the numbers of boxes are reduced appropriately. With the selection of a box type and an orientation variant for its block, an 1-arrangement is defined unambiguously. Analogously a 2-arrangement is completely defined by the selection of two box types, two orientation variants, and an arrangement type. All 1-arrangements and all 2-arrangements, which can occur if the box types, the orientation variants and—in the case of 2-arrangements—the arrangement type are varied, are generated. However, only those box types for which at least one item is still available are considered here. Furthermore, the variation of the orientation variants has to take the orientation constraint (C1) into consideration.

# The basic heuristic

## - Generation of residual packing spaces

Immediately after the generation of a local arrangement for a packing space, the unused part of the packing space is subdivided into residual packing spaces.

In order to enable an evaluation of a local arrangement (see below), different Subdivisions into residual packing spaces are experimentally generated.



## Chapter 3

# Technics of parallelization

For the evaluation of the local arrangements generated for a packing space, two modes are available which are applied alternatively. The selection of the relevant mode is also controlled by a parameter, named `arrEvalMode`. The first mode, encoded by the parameter value 0, applies a single evaluation criterion: the total volume of the boxes stowed in the packing space which should be as large as possible. The second mode, encoded by the parameter value 1, additionally applies two further evaluation criteria. These are the already introduced quantities loss volume and maximum effective volume. Both criteria refer to the residual packing spaces of a local arrangement. Analogous to the evaluation of subdivisions, the loss volume should be as small as possible and the maximum effective volume as large as possible. Since the three evaluation criteria applied are weighted equally, the evaluation procedure is organized as a series of comparisons of the local arrangements for a packing space in pairs. Finally, two additional parameters of the basic heuristic are introduced and briefly discussed. The parameter `maxArr` defines the maximum length of the arrangement list `ArrList` for a packing space. A local arrangement is only considered in tabu search process if it occurs in `ArrList`, i.e. belongs to the `maxArr` best arrangements. The parameter `aboveArr` determines whether 2-arrangements of type “above” are generated (parameter value 1) or not (parameter value 0). Like the different modes for subdivisions and arrangements, the parameter `aboveArr` serves the diversification of the tabu search

# The sequential tabu search algorithm

## - Encoding of feasible solutions

In order to define neighbourhoods for the tabu search that can be easily manipulated, an encoding of feasible solutions to the container loading problem is chosen.

```

Initialize:
  generate an initial solution  $s$ ;
  set best solution  $s_{best} := s$ ;
  set  $Tabulist := \{ \}$ ;

Perform a neighbourhood search:
  while (termination criterion is not met) do
    generate a neighbourhood  $N(s)$ ;
    initialize the value of the objective function  $f(s_{best}) := -\infty$ ;
    for all  $s' \in N(s)$  do
      if  $f(s') > f(s_{best})$  and solution  $s'$  is not tabu then
         $s_{best} := s'$ ;
      endif
    endfor
    if  $f(s_{best}) > f(s)$  then
       $s := s_{best}$ ;
    endif
    update  $Tabulist$ ;
  endwhile
  
```

algorithmo\_tsa.png

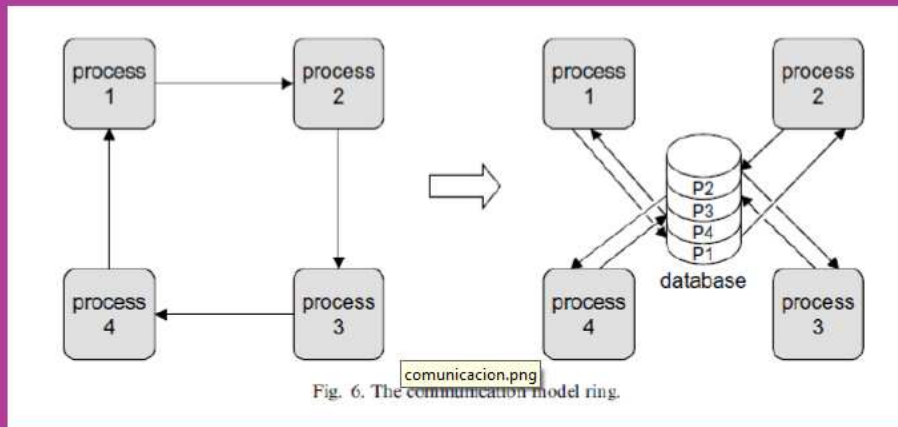
Define the best solution  $s_{best}$  as solution to the problem.

Fig. 5. Generic algorithm of a tabu search for solving a maximization problem.



## The parallel tabu search algorithm

- Communication model



## The parallel tabu search algorithm

- Exchange of solutions

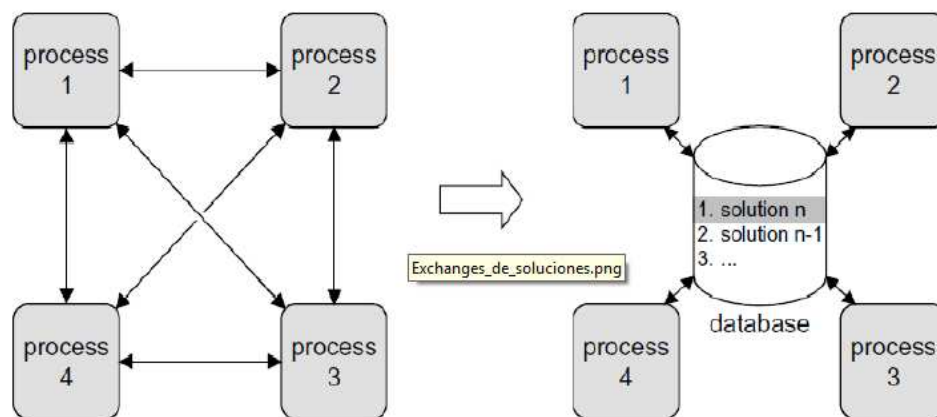


Fig. 7. The communication model blackboard.

## The parallel tabu search algorithm

### - Further details

One of the processes performing the concurrent search is excluded from the communication.

Operating as sequential TSA this process carries out an isolated search. Its best generated solution is, however, finally included in the determination of the solution of the parallel method. Hence, the solution quality of the sequential method will be achieved in any case.

The termination of the parallel TSA is controlled by an additional process –the so-called master. The parallel method is terminated by the master, if either all processes have carried out all their search phases, or if the computing time consumed by the distributed-parallel system has exceeded a predefined time limit `maxTime`.

After the end of the concurrent search, the solution of the parallel method is determined as the best solution found by the whole process group.

## Chapter 4

# Conclusiones

According to Toulouse et al. [20], three types of parallelization strategies seem to be appropriate for the methods most often used in combinatorial optimization: (1) parallelization of operations within an iteration of the solution method, (2) decomposition of problem domain or search space, and (3) multi-search threads with various degrees of synchronization and cooperation. Which of these types is suited for the parallelization of an optimization method depends mainly on the goal pursued by the parallelization. Since the enhancement of the solution quality is in the foreground here, an approach of type 3 is chosen for the parallelization. An instance of a container loading problem is treated by several processes. Each process is an instance of the sequential TSA and solves the complete problem. However, the individual instances are configured differently. Furthermore, the processes cooperate through the exchange of calculated solutions. A transmitted solution is possibly used by the receiving process as a starting point for further search. While the varying configuration of the processes causes a diversification of the search, the exchange of solutions serves the intensification of the search within the regions of best solutions. Each of the autonomous processes is assigned to a workstation of a local network (LAN). Hence, more precisely expressed, the parallel TSA is a distributed-parallel method. It is described more closely in the following. According to the diversification concept of the sequential TSA, the search in each process is structured into several phases. In order to determine favourable parameter settings with respect to the definition of the phases of all processes, a series of experiments was carried out by means of the sequential TSA (cf. Section 5). The best parameter settings are distributed approximately evenly over the processes and per process the most promising parameter settings are, as far as possible, applied in early phases. In this way the intended intensified exploration of regions that contain solutions of high quality, is supported to a greater extent. As to the communication frequency or the number of communications, the type of the underlying sequential method (here a TSA) is to be considered. The consequence of a very high communication frequency is that the individual processes are prevented from intensively exploring limited regions of the search space. Therefore, a lower communication frequency is to be chosen in advance. Here, an exchange of best solutions is only carried out at the transition from one phase to the next phase of each of the processes.

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