Assignment -2 LP Problems

Solution – 1

a). Clearly define the decision variables.

Collegiate (C) & Mini (M)

(Back Savers is a company producing two different types of backpacks i.e., Collegiate and Mini).

b). Define Objective Function

If converting the theoretical statement into the base objective function it is basically to know the maximum quantity of two different types of backpacks which Back Savers can manufacture in a week. Generally, this can be identified by knowing the maximum profit Back Savers can make in a week by manufacturing these two different types of backpacks.

Profit (P)

P (C, M) = 32C + 45M

This is the objective function (Z) – To find out the Maximum Profit which Back Savers can earn by manufacturing two different types of backpacks within the constraints given.

c) What are the constraints?

Material is the first constraint; Back Savers receives a max shipment of 5000 sft. Of nylon fabric in a week. Each Collegiate requires 3 sft. to manufacture and Mini requires 2 sft. to manufacture.

Material: 3 C + 2 M <= 5000 (<= Less than or equal to) → ***First Constraint***

Labor: Number of employees = 35

Maximum Hours of employability per worker per week = 40 Hrs.

Total working hours = 35*40 = 1400 Hrs. in a week

Each Collegiate requires 45 minutes of labor = 45/60 = 3/4

Each Mini requires 40 minutes of labor = 40/60 = 2/3

Labor: 3/4 C + 2/3 M <= 1400 (<= Less than or equal to) → ***Second Constraint***

On the other hand, we also have sales limits per week for these two backpacks i.e.;

C <= 1000, M <= 1200 (<= Less than or equal to) → ***Third Constraint***

Non – Negativity Constraints:

C >= 0, M >= 0 → ***Fourth Constraint***

d). Write down the full mathematical formulation of this problem

All the above steps put together make the mathematical formulation complete for this given problem by defining the variables, objectives and constraints.

Objective (Z): 32 C + 45 M

Decision Variables: Collegiate (C) & Mini (M)

Constraints: 1). Material: 3 C + 2 M <=5000

2). Labor: 3/4 C + 2/3 M <=1400

3). Sales Limit (per week): C <= 1000, M <= 1200

4). Non – Negativity Constraints: C >= 0, M >= 0

Solution – 2

a). Clearly define the decision variables

Manufacturing Plants:

Plant -1 – P1, Plant – 2 – P2, Plant -3 – P3

Sizes of the product:

Large – L, Medium – M, Small – S

Let the number of large products being manufactured at Plant 1 be P1L

Let the number of medium products being manufactured at Plant 1 be P1M

Let the number of small products being manufactured at Plant 1 be P1S

Let the number of large products being manufactured at Plant 2 be P2L

Let the number of medium products being manufactured at Plant 2 be P2M

Let the number of small products being manufactured at Plant 2 be P2S

Let the number of large products being manufactured at Plant 3 be P3L

Let the number of medium products being manufactured at Plant 3 be P3M

Let the number of small products being manufactured at Plant 3 be P3S

P1L, P1M, P1S, P2L, P2M, P2S, P3L, P3M and P3S are the decision variables.

b). Define Objective Function

Maximization of profit is the objective function. This can be defined by identifying how much of each of the sizes should be manufactured by each of the plants (P1, P2 and P3).

Net Unit Profit by manufacturing each of the sized products are:

Large (L) = $420, Medium (M) = $360, Small (S) = $300

Z = Objective Function

Z = 420.P1L + 360.P1M + 300.P1S + 420.P2L + 360.P2M + 300.P2S + 420.P3L + 360.P3M + 300.P3S. (or)

Z = 420(P1L + P2L + P3L) + 360(P1M + P2M + P3M) + 300(P1S + P2S +P3S)

c) What are the constraints?

1. Excess production capacities of each plant → ***First Constraint***

   1.1. P1L + P1M + P1S <= 750 Units

   1.2. P2L + P2M + P2S <= 900 Units

   1.3. P3L + P3M + P3S <= 450 Units

   (Plant 1 has excess capacity to manufacture 750 units, Plant 2 has excess capacity to manufacture 900 units and Plant 3 has excess capacity to manufacture 450 units regardless of the size or the combinations of sizes involved).

   There's a further constraint to the employees being laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs, if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. → ***Second Constraint***

   1.1.1. 750(P1L + P1M + P1S) − 900(P2L + P2M + P2S) = 0

1.1.2. 900(P2L + P2M + P2S) − 450(P3L + P3M + P3S) = 0

1.1.3. 450(P3L + P3M + P3S) – 750(P1L + P1M + P1S) = 0

2. Sales forecast per day basis the size of the product → ***Third Constraint***

2.1. P1L + P2L + P3L <= 900 Units

2.2. P1M + P2M + P3M <= 1200 Units

2.3. P1S + P2S + P3S <= 750 Units

(Forecast of large-sized products supposedly being sold in a day is 900 units, forecast of medium-sized products supposedly being sold in a day is 1200 units and forecast of small-sized products supposedly being sold in a day is 750 units).

3. In–process storage space limitation per plant per day,

Plant – 1 – 13000 Sft., Plant – 2 – 12000 Sft., Plant – 3 – 5000 Sft.

Storage requirement per product per day,

Large – 20 Sft., Medium – 15 Sft., Small – 12 Sft.

Combining both the constraints to form a better understanding, → ***Fourth Constraint***

3.1. 20.P1L + 15.P1M + 12.P1S <= 13000 Sft.

3.2. 20.P2L + 15.P2M + 12.P2S <= 12000 Sft.

3.3. 20.P3L + 15.P3M + 12.P3S <= 5000 Sft.

4. Non-Negativity Constraints: → ***Fifth Constraint***

P1L, P1M, P1S, P2L, P2M, P2S, P3L, P3M and P3S >= 0

d). Write down the full mathematical formulation of this problem

All the above steps put together make the mathematical formulation complete for this given problem by defining the variables, objectives and constraints.

Objective (Z): 420.P1L + 360.P1M + 300.P1S + 420.P2L + 360.P2M + 300.P2S + 420.P3L + 360.P3M + 300.P3S or 420(P1L + P2L + P3L) + 360(P1M + P2M + P3M) + 300(P1S + P2S +P3S)

Decision Variables: P1L, P1M, P1S, P2L, P2M, P2S, P3L, P3M and P3S.

Constraints:

1. Excess capacity production:
   1.1. P1L + P1M + P1S <= 750 Units
   1.2. P2L + P2M + P2S <= 900 Units
   1.3. P3L + P3M + P3S <= 450 Units

2. Percentage of excess production to produce a new product to avoid layoffs:
   2.1.1. 750(P1L + P1M + P1S) − 900(P2L + P2M + P2S) = 0
   2.1.2. 900(P2L + P2M + P2S) − 450(P3L + P3M + P3S) = 0
   2.1.3. 450(P3L + P3M + P3S) – 750(P1L + P1M + P1S) = 0

3. Sales forecast per day:
   3.1. P1L + P2L + P3L <= 900 Units
   3.2. P1M + P2M + P3M <= 1200 Units
   3.3. P1S + P2S + P3S <= 750 Units

4. Storage space limitation:
   4.1. 20.P1L + 15.P1M + 12.P1S <= 13000 Sft.
   4.2. 20.P2L + 15.P2M + 12.P2S <= 12000 Sft.
   4.3. 20.P3L + 15.P3M + 12.P3S <= 5000 Sft.

5. Non-Negativity: P1L, P1M, P1S, P2L, P2M, P2S, P3L, P3M and P3S >= 0