Transportation Model

Formulating Transportation Problem Using R

Activating all the required packages

```
library("lpSolveAPI")
library("lpSolve")
library("tinytex")
```

Creating a table for better understanding of the data

```
cost <- matrix(c(22,14,30,600,100,
                 16,20,24,625,120,
                 80,60,70,"-","-"), ncol=5,byrow=T)

colnames(cost) <- c("Warehouse1", "Warehouse2", "Warehouse3", "ProductionCost", "ProductionCapacity")

rownames(cost) <- c("PlantA", "PlantB", "Demand")

cost <- as.table(cost)
cost
```

```
##        Warehouse1 Warehouse2 Warehouse3 ProductionCost ProductionCapacity
## PlantA 22         14         30         600            100
## PlantB 16         20         24         625            120
## Demand 80         60         70         -              -
```

The Objective Function is to Minimize the Transportation cost

$$Z = 622X_{11} + 614X_{12} + 630X_{13} + 0X_{14} + 641X_{21} + 645X_{22} + 649X_{23} + 0X_{24}$$

Subject to the following constraints

$$Supply\ \ Constraints$$

$$X_{11} + X_{12} + X_{13} + X_{14} <= 100$$

$$X_{21} + X_{22} + X_{23} + X_{24} <= 120$$

$$Demand\ \ Constraints$$

$$X_{11} + X_{21} >= 80$$

$$X_{12} + X_{22} >= 60$$

$$X_{13} + X_{23} >= 70$$

$$X_{14} + X_{24} >= 10$$

$$\text{Non} - \textit{Negativity Constraints}$$

$$X_{ij} >= 0 \quad \text{Where i = 1,2 and j = 1,2,3,4}$$

```r
#Since demand is not equal to supply creating dummy variables

#Creating a matrix for the given objective function
trans.cost <- matrix(c(622,614,630,0,
                  641,645,649,0), ncol=4, byrow=T)
trans.cost
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  622  614  630    0
## [2,]  641  645  649    0
```

```r
#Defining the column names and row names
colnames(trans.cost) <- c("Warehouse1", "Warehouse2", "Warehouse3", "Dummy")

rownames(trans.cost) <- c("PlantA", "PlantB")

trans.cost
```

```
##        Warehouse1 Warehouse2 Warehouse3 Dummy
## PlantA        622        614        630     0
## PlantB        641        645        649     0
```

```r
#Defining the row signs and row values
row.signs <- rep("<=",2)
row.rhs <- c(100,120)
#Since it's supply function it cannot be greater than the specified units.

#Defining the column signs and column values
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
#Since it's demand function it can be greater than the specified units.

#Running the lp.transport function
lptrans.cost <- lp.transport(trans.cost,"min", row.signs,row.rhs,col.signs,col.rhs)
```

```r
#Getting the objective value
lptrans.cost$objval
```

```
## [1] 132790
```

*The minimization value so obtained is **$132,790** which is the minimal combined cost thereby attained from the combined cost of production and shipping the defibrilators.*

```
#Getting the constraints value
lptrans.cost$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

***80 AEDs** in Plant B - Warehouse1*

***60 AEDs** in Plant A - Warehouse2*

***40 AEDs** in Plant A - Warehouse3*

***30 AEDs** in Plant B - Warehouse3 should be produced in each plant and then distributed to each of the three wholesaler warehouses in order to minimize the overall cost of production as well as shipping.*

Formulate the dual of the above transportation problem

Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

$$\text{Maximize} \quad VA = \quad 80W_1 + 60W_2 + 70W_3 - 100P_A - 120P_B$$

Subject to the following constraints

$$\text{Total} \;\; Payments \;\; Constraints$$
$$W_1 - P_A >= 622$$
$$W_2 - P_A >= 614$$
$$W_3 - P_A >= 630$$
$$W_1 - P_B >= 641$$
$$W_2 - P_B >= 645$$
$$W_3 - P_B >= 649$$

$$\text{Where} \;\; W_1 = Warehouse \;\; 1$$
$$W_2 = Warehouse \;\; 2$$
$$W_3 = Warehouse \;\; 3$$
$$P_1 = Plant \;\; 1$$
$$P_2 = Plant \;\; 2$$

Economic Interpretation of the dual

$$W_1 <= 622 + P_A$$

$$W_2 <= 614 + P_A$$
$$W_3 <= 630 + P_A$$
$$W_1 <= 641 + P_B$$
$$W_2 <= 645 + P_B$$
$$W_3 <= 649 + P_B$$

$$\text{From} \quad the \quad above \quad we \quad get \quad to \quad see \quad that \quad W_1 - P_A >= 622$$

$$that \quad can \quad be \quad exponented \quad as \quad W_1 >= 622 + P_A$$

$$\text{Here} \quad W_1 \quad is \quad considered \quad as \quad the \quad price \quad payments \quad being \quad received \quad at \quad the \quad origin \quad which \quad is \quad nothing \quad else,$$
$$\text{but} \quad the \quad revenue, \quad whereas \quad P_A + 622 \quad is \quad the \quad money \quad paid \quad at \quad the \quad origin \quad at \quad Plant_A$$

$$\text{Therefore} \quad the \quad equation \quad turns, \quad out \quad to \quad be \quad MR_1 >= MC_1.$$

$$\text{For} \quad a \quad profit \quad maximization, \quad The \quad Marginal \quad Revenue(MR) \quad should \quad be \quad equal \quad to \quad Marginal \quad Costs(MC)$$

$$\text{Therefore}, \quad MR_1 = MC_1$$

$$\text{Based} \quad on \quad above \quad interpretation, \quad we \quad can \quad conclude \quad that,$$
$$\text{Profit} \quad maximization \quad takes \quad place \quad if \quad MC \quad is \quad equal \quad to \quad MR.$$

If MR $<$ MC, in order to meet the Marginal Revenue (MR), we need to decrease the costs at the plants.

If MR $>$ MC, in order to meet the Marginal Revenue (MR), we need to increase the production supply.