Group 7

Setting default values to get a clean output

```
knitr::opts_chunk$set(message = FALSE)
knitr::opts_chunk$set(warning = FALSE)
```

Loading the required packages

```
library(lpSolve)
library(lpSolveAPI)
```

Loading the table of students along with the factors and their values

```
stu_table <- matrix(c(1,2,3,4,5,6,7,8,9,10,11,12,
                      88,58,85,63,43,91,98,58,33,74,90,45,
                    70,58,92,89,57,61,35,95,75,91,72,68,
                    35,38,25,28,24,26,44,22,43,48,27,49,
                193,154,202,180,124,178,177,175,151,213,189,162),ncol = 5,byrow = F)

colnames(stu_table) <- c("Student","Accuracy","Analysis","Presentation","Wij")

as.table(stu_table)
```

```
##   Student Accuracy Analysis Presentation Wij
## A       1       88       70           35 193
## B       2       58       58           38 154
## C       3       85       92           25 202
## D       4       63       89           28 180
## E       5       43       57           24 124
## F       6       91       61           26 178
## G       7       98       35           44 177
## H       8       58       95           22 175
## I       9       33       75           43 151
## J      10       74       91           48 213
## K      11       90       72           27 189
## L      12       45       68           49 162
```

We can see that there are a total of 12 Students in this Class Group, who are going to work on a class project by combining into a group of 3 each.

The Three Factors that are used to assess a individual student's performance are "Accuracy" (1), "Analysis" (2), "Presentation" (3).

The weights to each of these factors are randomly generated by using the randbetween() function in Microsoft Excel, Accuracy and Analysis were weighed between 30 and 100 randomly and Presentation was weighed

between 20 and 50 randomly

Wij is defined as weights of each of the factors for an individual student,

$$\mathrm{W}_{ij} \text{ where } i = Students \text{ ranging from } 1....12 \text{ and } j \text{ is the } Factors \text{ ranging from } 1....3$$

$$Student_1 = W_{11} + W_{12} + W_{13} = 88 + 70 + 35 = 193$$
$$Student_2 = W_{21} + W_{22} + W_{23} = 58 + 58 + 38 = 154$$
$$Student_3 = W_{31} + W_{32} + W_{33} = 85 + 92 + 25 = 202$$
$$Student_4 = W_{41} + W_{42} + W_{43} = 63 + 89 + 28 = 180$$
$$Student_5 = W_{51} + W_{52} + W_{53} = 43 + 57 + 24 = 124$$
$$Student_6 = W_{61} + W_{62} + W_{63} = 91 + 61 + 26 = 178$$
$$Student_7 = W_{71} + W_{72} + W_{73} = 98 + 35 + 44 = 177$$
$$Student_8 = W_{81} + W_{82} + W_{83} = 58 + 95 + 22 = 175$$
$$Student_9 = W_{91} + W_{92} + W_{93} = 33 + 75 + 43 = 151$$
$$Student_{10} = W_{101} + W_{102} + W_{103} = 74 + 91 + 48 = 213$$
$$Student_{11} = W_{111} + W_{112} + W_{113} = 90 + 72 + 27 = 189$$
$$Student_{12} = W_{121} + W_{122} + W_{123} = 45 + 68 + 49 = 162$$

Loading the LP File

```
stu_group <- read.lp("stu_group.lp")
```

Knowing how many constraints and variables are defined in this problem

```
print(stu_group)
```

```
## Model name:
##   a linear program with 48 decision variables and 16 constraints
```

In total there are 48 decision variables i.e. 12 Students forming into a group of 3 each, so each group will have 4 students and the groups are going to be ranging from 1. . . . . 4, so 12 x 4 = 48 Decision Variables

Each student can be assigned to one and only one group so, the 16 Constraints are as follows,

$$\mathrm{X}_{ij} \text{ where } i = Students \text{ ranging from } 1....12 \text{ and } j \text{ is the } Groups \text{ ranging from } 1....4$$

$$Student_1 = X_{11} + X_{12} + X_{13} + X_{14} = 1$$
$$Student_2 = X_{21} + X_{22} + X_{23} + X_{24} = 1$$
$$Student_3 = X_{31} + X_{32} + X_{33} + X_{34} = 1$$
$$Student_4 = X_{41} + X_{42} + X_{43} + X_{44} = 1$$
$$Student_5 = X_{51} + X_{52} + X_{53} + X_{54} = 1$$
$$Student_6 = X_{61} + X_{62} + X_{63} + X_{64} = 1$$
$$Student_7 = X_{71} + X_{72} + X_{73} + X_{74} = 1$$
$$Student_8 = X_{81} + X_{82} + X_{83} + X_{84} = 1$$
$$Student_9 = X_{91} + X_{92} + X_{93} + X_{94} = 1$$

$$Student_{10} = X_{101} + X_{102} + X_{103} + X_{104} = 1$$
$$Student_{11} = X_{111} + X_{112} + X_{113} + X_{114} = 1$$
$$Student_{12} = X_{121} + X_{122} + X_{123} + X_{124} = 1$$

Each group can have only 3 students, for which the constraints are as follows,

$$Group_1 = X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} + X_{71} + X_{81} + X_{91} + X_{101} + X_{111} + X_{121} = 3$$
$$Group_2 = X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} + X_{72} + X_{82} + X_{92} + X_{102} + X_{112} + X_{122} = 3$$
$$Group_3 = X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} + X_{73} + X_{83} + X_{93} + X_{103} + X_{113} + X_{123} = 3$$
$$Group_4 = X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{64} + X_{74} + X_{84} + X_{94} + X_{104} + X_{114} + X_{124} = 3$$

Solving the LP File

```
solve(stu_group)
```

```
## [1] 0
```

Since the solution function resulted as 0, this indicates that there exists a solution for this LP Formulation.

Getting the resultant objective function value

```
get.objective(stu_group)
```

```
## [1] 2098
```

The Objective function is defined as

$Max:\ W_{ij}.X_{ij}$ *where* W_{ij} *is the weight of the factors to each student where* $i = 1---12$ *and* $j = 1---3$ *and*

$Max: 193x_{11}+154x_{21}+202x_{31}+180x_{41}+124x_{51}+178x_{61}+177x_{71}+175x_{81}+151x_{91}+213x_{101}+189x_{111}+162x_{121}+193x_{12}+154x$

The resultant objective function was to maximize the chances that all the groups perform well on a class project, for which the resultant value thus achieved was 2098.

Getting the variables - Student to Group Assignment Variables

```
get.variables(stu_group)
```

```
##  [1] 0 1 0 0 0 0 0 0 1 0 0 1 0 0 0 1 1 0 1 0 0 0 0 0 1 0 0 0 0 1 0 1 0 0 0 0 0 0
## [39] 1 0 0 0 0 0 0 0 1 1 0
```

The LP Formulation was done basing it as a "Binary Programming", where the resultant values of the constraints thus defined would be printed in the form of 0 and 1.

Here, 0 represents that a particular student won't belong to that group

and 1 represents that a particular subject belongs to that group.

Based, on the above formulation we got the binary output as

$$0.X_{11}, 1.X_{21}, 0.X_{31}, 0.X_{41}, 0.X_{51}, 0.X_{61}, 0.X_{71}, 0.X_{81}, 1.X_{91}, 0.X_{101}, 0.X_{111}, 1.X_{121}$$

The above equation indicates that the students who belong to Group 1 are Student - 2, Student - 9 and Student - 12

$$0.X_{12}, 0.X_{22}, 0.X_{32}, 1.X_{42}, 1.X_{52}, 0.X_{62}, 1.X_{72}, 0.X_{82}, 1.X_{92}, 0.X_{102}, 0.X_{112}, 1.X_{122}$$

The above equation indicates that the students who belong to Group 2 are - Student - 4, Student - 5 and Student 7

$$1.X_{13}, 0.X_{23}, 0.X_{33}, 0.X_{43}, 0.X_{53}, 1.X_{63}, 0.X_{73}, 1.X_{83}, 0.X_{93}, 0.X_{103}, 0.X_{113}, 0.X_{123}$$

The above equation indicates that the students who belong to Group 3 are - Student - 1, Student - 6 and Student 8

$$0.X_{14}, 0.X_{24}, 1.X_{34}, 0.X_{44}, 0.X_{54}, 0.X_{64}, 0.X_{74}, 0.X_{84}, 0.X_{94}, 1.X_{104}, 1.X_{114}, 0.X_{124}$$

The above equation indicates that the students who belong to Group 4 are - Student - 3, Student - 10 and Student 11

Computing the success of each of the groups with the 3 factors that was used to assess their performance

$$Group_1 = Student_2 + Student_9 + Student_{12} = 154 + 151 + 162 = 467$$

$$Group_2 = Student_4 + Student_5 + Student_7 = 180 + 124 + 177 = 481$$

$$Group_3 = Student_1 + Student_6 + Student_8 = 193 + 178 + 175 = 546$$

$$Group_4 = Student_3 + Student_{10} + Student_{11} = 202 + 213 + 189 = 604$$

The main objective was to maximize the the chance of success for each group on the class project, which refers that all the groups need to be equally maximized so that their chance to do well on the class project maximizes.

Here, we get the objective value as 2098

$$Objective = Group_1 + Group_2 + Group + 3 + Group_4 = 467 + 481 + 546 + 604 = 2098$$

But, here we get to see that all the groups have varied chances of success towards the class project, so we are creating another LP Formulation, by defining a new variable called "Z" and the objective function is to Maximize : Z

Here, Z is the smallest group score but the objective thus formulated is to maximize that so now, Z will be able to take any value >= 0

$$Maximize : Z \;\; where \;\; Z \geq 0$$

Loading the MaxMin Formulation LP File

```
max.success <- read.lp("stu_group.1.lp")
```

Knowing how many constraints and variables are defined in this new set of LP File

```
print(max.success)
```

```
## Model name:
##   a linear program with 49 decision variables and 20 constraints
```

If we observe keenly while loading the first LP File we had 48 decision variables, now we do have all those variables plus an additional variable which is nothing else other than,

$$Z \geq 0$$

In the similar way we do have all the 16 constraints previously defined as Student to Group Constraints - 12 Constraints and 4 more constraints towards the Group Assignment, additionally we have defined 4 more constraints for maximizing the success of all the groups i.e Z1—Z4, where Z >=0

$$Z1 = Z - 193x_{11} - 154x_{21} - 202x_{31} - 180x_{41} - 124x_{51} - 178x_{61} - 177x_{71} - 175x_{81} - 151x_{91} - 213x_{101} - 189x_{111} - 162x_{121} \leq 0$$

$$Z2 = Z - 193x_{12} - 154x_{22} - 202x_{32} - 180x_{42} - 124x_{52} - 178x_{62} - 177x_{72} - 175x_{82} - 151x_{92} - 213x_{102} - 189x_{112} - 162x_{122} \leq 0$$

$$Z3 = Z - 193x_{13} - 154x_{23} - 202x_{33} - 180x_{43} - 124x_{53} - 178x_{63} - 177x_{73} - 175x_{83} - 151x_{93} - 213x_{103} - 189x_{113} - 162x_{123} \leq 0$$

$$Z4 = Z - 193x_{14} - 154x_{24} - 202x_{34} - 180x_{44} - 124x_{54} - 178x_{64} - 177x_{74} - 175x_{84} - 151x_{94} - 213x_{104} - 189x_{114} - 162x_{124} \leq 0$$

Solving the LP File

```
solve(max.success)
```

```
## [1] 0
```

Since the solution function resulted as 0, this indicates that there exists a solution for this LP Formulation.

Getting the resultant objective function value

```
get.objective(max.success)
```

```
## [1] 520
```

In order to maximize the chance of success for each group on a class project, each group should score greater than or equal to 520

$$Z \geq 520$$

Getting the variables - Student to Group Assignment Variables

```
get.variables(max.success)
```

```
##  [1] 520   0   0   1   0   0   1   0   0   1   0   0   0   0   0   0   1   0   0
## [20]   1   0   0   0   0   1   0   1   0   0   1   0   0   0   1   0   0   0   0
## [39]   0   1   0   0   1   0   0   0   0   0   1
```

Here the first value which we see is nothing but the newly created variable "Z" whose value is 520.

Later, we have the binary output for the group assignments to check which student belongs to which group

Here the binary output has differed when compared to that with the first LP File Formulation, the 48 variables are then defined as,

$$X_{11} + X_{12} + X_{13} + X_{14}$$

Student 1 belonging to any one of the following Groups 1—4

$$X_{21} + X_{22} + X_{23} + X_{24}$$

Student 2 belonging to any one of the following Groups 1—4

$$X_{31} + X_{32} + X_{33} + X_{34}$$

Student 3 belonging to any one of the following Groups 1—4

$$X_{41} + X_{42} + X_{43} + X_{44}$$

Student 4 belonging to any one of the following Groups 1—4

$$X_{51} + X_{52} + X_{53} + X_{54}$$

Student 5 belonging to any one of the following Groups 1—4

$$X_{61} + X_{62} + X_{63} + X_{64}$$

Student 6 belonging to any one of the following Groups 1—4

$$X_{71} + X_{72} + X_{73} + X_{74}$$

Student 7 belonging to any one of the following Groups 1—4

$$X_{81} + X_{82} + X_{83} + X_{84}$$

Student 8 belonging to any one of the following Groups 1—4

$$X_{91} + X_{92} + X_{93} + X_{94}$$

Student 9 belonging to any one of the following Groups 1—4

$$X_{101} + X_{102} + X_{103} + X_{104}$$

Student 10 belonging to any one of the following Groups 1—4

$$X_{111} + X_{112} + X_{113} + X_{114}$$

Student 11 belonging to any one of the following Groups 1—4

$$X_{121} + X_{122} + X_{123} + X_{124}$$

Student 12 belonging to any one of the following Groups 1—4

Group Assignment of 12 students into 4 groups

$$0.X_{11}+0.X_{12}+0.X_{13}+0.X_{14}+0.X_{21}+0.X_{22}+0.X_{23}+0.X_{24}+1.X_{31}+0.X_{32}+0.X_{33}+0.X_{34}+0.X_{41}+0.X_{42}+0.X_{43}+0.X_{44}+0.X_{51}$$

The above equation indicates that the students who belong to Group 1 are - Student - 3 (X31), Student - 8 (X81) and Student 9 (X91).

$$0.X_{11}+0.X_{12}+0.X_{13}+0.X_{14}+0.X_{21}+1.X_{22}+0.X_{23}+0.X_{24}+0.X_{31}+0.X_{32}+0.X_{33}+0.X_{34}+0.X_{41}+0.X_{42}+0.X_{43}+0.X_{44}+0.X_{51}$$

The above equation indicates that the students who belong to Group 2 are - Student - 2 (X22), Student - 7 (X72) and Student 11 (X112).

$0.X_{11}+0.X_{12}+1.X_{13}+0.X_{14}+0.X_{21}+0.X_{22}+0.X_{23}+0.X_{24}+0.X_{31}+0.X_{32}+0.X_{33}+0.X_{34}+0.X_{41}+0.X_{42}+0.X_{43}+0.X_{44}+0.X_{51}$

The above equation indicates that the students who belong to Group 3 are - Student - 1 (X13), Student - 5 (X53) and Student 10 (X103).

$0.X_{11}+0.X_{12}+0.X_{13}+0.X_{14}+0.X_{21}+0.X_{22}+0.X_{23}+0.X_{24}+0.X_{31}+0.X_{32}+0.X_{33}+0.X_{34}+0.X_{41}+0.X_{42}+0.X_{43}+1.X_{44}+0.X_{51}$

The above equation indicates that the students who belong to Group 4 are - Student - 4 (X44), Student - 6 (X64) and Student 12 (X124).

Computing the success of each of the groups with the 3 factors that was used to assess their performance

$$\mathrm{G}roup_1 = Student_3 + Student_8 + Student_9 = 202 + 175 + 151 = 528 \quad i.e.\ 520 \quad (Max \quad Value) + 8$$

$$\mathrm{G}roup_2 = Student_2 + Student_7 + Student_{11} = 154 + 177 + 189 = 520 \quad i.e.\ (Max \quad Value)$$

$$\mathrm{G}roup_3 = Student_1 + Student_5 + Student_{10} = 193 + 124 + 213 = 530 \quad i.e.\ 520 \quad (Max \quad Value) + 10$$

$$\mathrm{G}roup_4 = Student_4 + Student_6 + Student_{12} = 180 + 178 + 162 = 520 \quad i.e.\ (Max \quad Value)$$

Now, here we can see that all the groups have their chances of succeeding the group maximized, all the groups have the resultant value to the project being completed as >= 520 which is the Z.

Thereby, we can call out that the objective function of maximizing the chance of success in a class project for all the groups has been satisfied.