# 1 Model

$S' = bN - \beta\frac{SA}{N} - dS$

$A' = \beta\frac{SA}{N} - \xi T - (d + 0.5)A$

$T' = 0.5A - (d + \xi + \delta)T$

$Q' = \delta T - dQ$

$N' = (b - d)N$

# 2 Reproduction Number

In order to find the reproduction number we reformulate the state space model of the SATQ model as follows:
Let $X = (A, T, Q, S)$. Then we can write the state space model as:

$$\frac{dX}{dt} = \mathcal{F}(x) - \mathcal{V}(x)$$

where we define $\mathcal{F}(x)$ and $\mathcal{V}(x)$ as follows

$\mathcal{F}(x) = \begin{pmatrix} \beta\frac{SA}{N} \\ 0 \\ 0 \\ 0 \end{pmatrix}$ and

$\mathcal{V}(x) = \begin{pmatrix} (0.5 + d)A - \xi T \\ (d + \xi + \delta)T - 0.5A \\ dQ - \delta T \\ \beta\frac{SA}{N} + dS - bN \end{pmatrix}$ and

$\mathcal{D}\mathcal{F}(Eo) = \begin{pmatrix} \beta\frac{b}{d} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and

$\mathcal{D}\mathcal{V}(Eo) = \begin{pmatrix} 0.5 + d & \xi & 0 & 0 \\ 0.5 & d + \xi + \delta & 0 & 0 \\ 0 & -\delta & d & 0 \\ \beta & 0 & 0 & 0 \end{pmatrix}$ and

$$R_0 = \rho(FV^{-1}) = \frac{b\beta(d+\xi+\delta)}{0.5d(d+\delta+d(d+\xi+\delta)}$$

# 3 Stability Analysis

## 3.1 Method 1 - Dealing with Populations

$$\mathcal{J}(Eo) = \begin{pmatrix} \beta(d+0.5) & \xi & 0 & 0 \\ 0.5 & -(d+\xi+\delta) & 0 & 0 \\ 0 & \delta & -d & 0 \\ -\beta & 0 & 0 & -d \end{pmatrix}$$

We can easily solve for two eigenvalues $\lambda_1 = \lambda_2 = -d < 0$, while $\lambda_3$ and $\lambda_4$ satisfy the equation
$\lambda^2 + [2d + \xi + \delta + 0.5 - \beta]\lambda + (d + \xi + \delta)(d + 0.5 - \beta) - 0.5\xi = 0$

Manipulation of this equation proves that Real parts of both $\lambda_3$ and $\lambda_4$ are less than 0.
Re $\lambda_3 < 0$, Re $\lambda_4 < 0$

## 3.2 Method 2 - Dealing with Proportions

# 4 Equlibibrium Points

$S^* = \frac{0.5bN^2}{0.5dN+T^*\beta(d+\xi+\delta)}$

$A^* = \frac{(d+\xi+\delta)T^*}{0.5}$

$Q^* = \frac{T^*\delta}{d}$

$T^* = \frac{\sigma}{\beta(d+\xi+\delta)[d\xi+(\delta+d)(d+0.5)]}$
where,
$\sigma = 0.5bN\beta(d + \xi + \delta) + 0.5^2dN\xi - 0.5dN(d + 0.5)(d + \xi + \delta)$

## 5 Proportion Model

$s' = b - \beta sa - bs$

$a' = \beta sa - \xi t - (b + 0.5)a$

$t' = 0.5a - (b + \xi + \delta)t$

$q' = \delta t - bQ$