Dynamics and Stability of Alcoholism as a Communicable Addiction

MM Project

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Introduction

- Alcoholism: Treatable disease that includes the desire for alcohol and continuing to drink it despite its negative effect on individual's health, relationships, and social status.
- Long-term alcohol abuse will produce negative changes in the brain such as tolerance and physical dependence.
- We will premeditate two treating methods:

1) Prevention of susceptibles from alcoholism
2) Treatment on alcoholism as control variables;

- Hence, we will derive a SATQ-type model.
- Optimal Control Strategies in an Alcoholism Model-Xun-Yang Wang, 1,2 Hai-Feng Huo, 1,2 Qing-Kai Kong, 3 and Wei-Xuan Shi 2

Interesting Fact!

90% College students	Consumed alcohol at least once
40% College students	Engaged in binge drinking*

Parameter Explanation

S (Susceptibles): no. of people who never drink or drink moderately without a affecting the physical health

A (Alcoholics): no. of people who binge drink and affect the physical health seriously

T (Treatment Takers): no. of people who have been receiving treatments by taking pills or other medical interventions after alcoholism

Q (Quitters): no. of people who recover from alcoholism after treatment and stay off alcohol

N: the total number of population to be considered is a non-constant

Model Formulation

$$S' = bN - \beta \frac{SA}{N} - dS$$

$$A' = \beta \frac{SA}{N} - \xi T - (d + 0.5)A$$

$$T' = 0.5A - (d + \xi + \delta)T$$

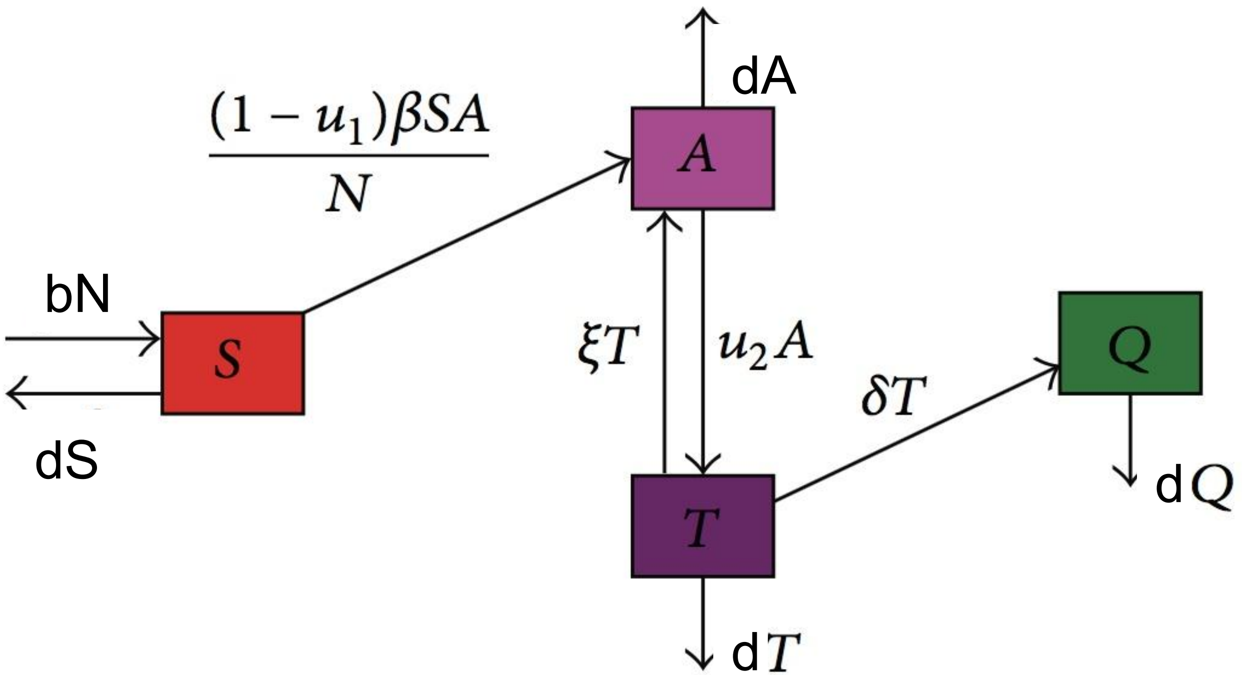
$$Q' = \delta T - dQ$$

$$N' = (b - d)N$$

Continued..

- Splitting the Q population into two seperate compartments Q1 and Q2. Q1 would not influence the rest of the equations, whereas Q2 could influence the system in the real world equivalent of reducing number of Alcoholics. Therefore increasing the accuracy and complexity of the system.

Transfer diagram for the dynamics of alcoholism model



Reproduction Number

In order to find the reproduction number we reformulate the state soace model of the SATQ model as follows:
Let $X = (A, T, Q, S)$. Then we can write the state space model as:

$$\frac{dX}{dt} = \mathcal{F}(x) - \mathcal{V}(x)$$

where we define $\mathcal{F}(x)$ and $\mathcal{V}(x)$ as follows

Continued..

$$\mathcal{F}(x) = \begin{pmatrix} \beta\frac{SA}{N} \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } \qquad \mathcal{DF}(Eo) = \begin{pmatrix} \beta\frac{b}{d} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$\mathcal{V}(x) = \begin{pmatrix} (0.5+d)A - \xi T \\ (d+\xi+\delta)T - 0.5A \\ dQ - \delta T \\ \beta\frac{SA}{N} + dS - bN \end{pmatrix} \text{ and } \qquad \mathcal{DV}(Eo) = \begin{pmatrix} 0.5+d & \xi & 0 & 0 \\ 0.5 & d+\xi+\delta & 0 & 0 \\ 0 & -\delta & d & 0 \\ \beta & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

Stability Analysis

$$\mathcal{J}(Eo) = \begin{pmatrix} \beta(d+0.5) & \xi & 0 & 0 \\ 0.5 & -(d+\xi+\delta) & 0 & 0 \\ 0 & \delta & -d & 0 \\ -\beta & 0 & 0 & -d \end{pmatrix}$$

We can easily solve for two eigenvalues $\lambda_1 = \lambda_2 = -d < 0$, while λ_3 and λ_4 satisfy the equation

$$\lambda^2 + [2d + \xi + \delta + 0.5 - \beta]\lambda + (d + \xi + \delta)(d + 0.5 - \beta) - 0.5\xi = 0$$

Manipulation of this equation proves that Real parts of both λ_3 and λ_4 are less than 0.

$\mathrm{Re}\,\lambda_3 < 0$, $\mathrm{Re}\,\lambda_4 < 0$

Equilibrium Points(Lots of solving!)

$$S^* = \frac{0.5bN^2}{0.5dN + T^*\beta(d+\xi+\delta)}$$

$$A^* = \frac{(d+\xi+\delta)T^*}{0.5}$$

$$Q^* = \frac{T^*\delta}{d}$$

$$T^* = \frac{\sigma}{\beta(d+\xi+\delta)[d\xi+(\delta+d)(d+0.5)]}$$

where,

$$\sigma = 0.5bN\beta(d+\xi+\delta) + 0.5^2 dN\xi - 0.5dN(d+0.5)(d+\xi+\delta)$$

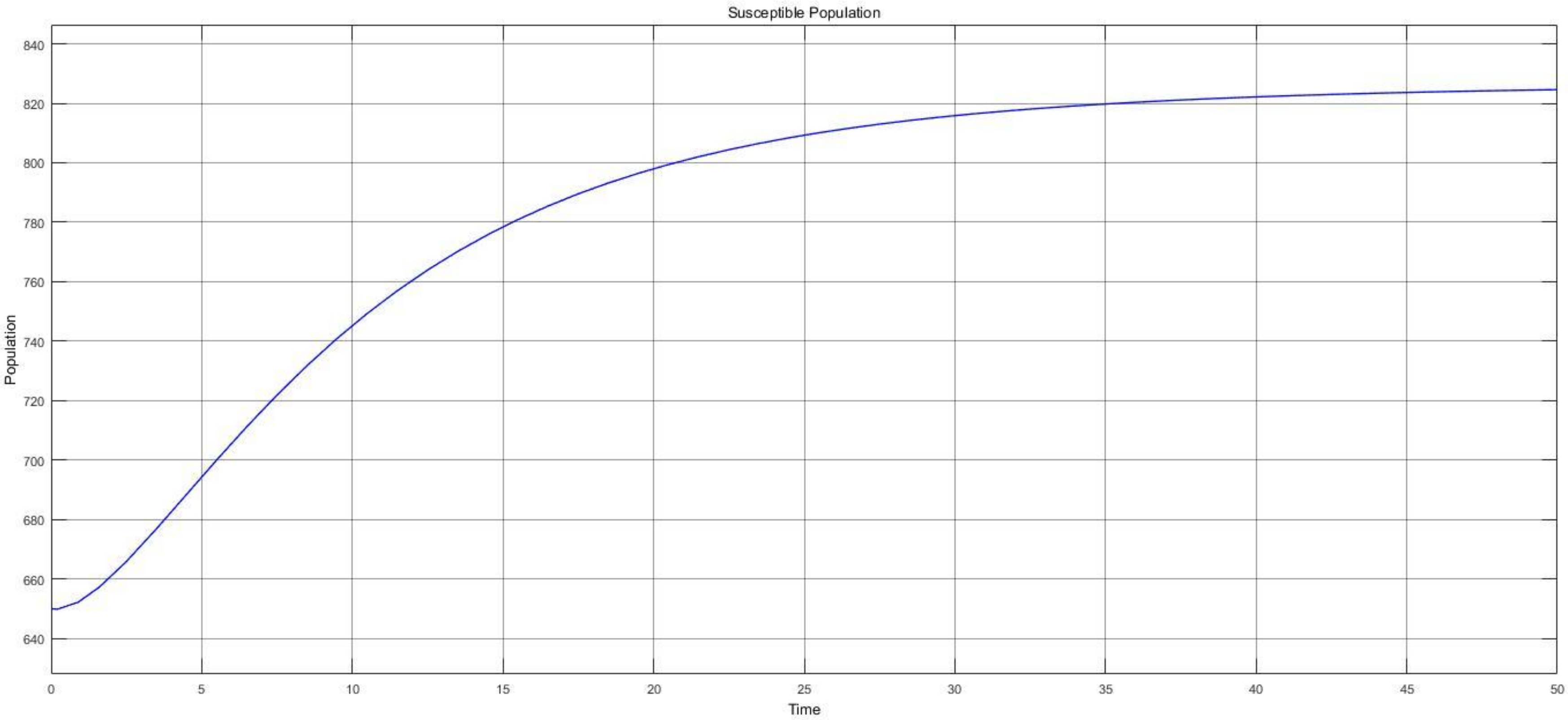
Numerical Simulation

Parameter	Description	Model
b	Natural Birth Rate	0.250025
d	Natural Death Rate	0.25
β	Transmission coefficient between alcoholism and susceptibles	0.55
No	Total Initial Population	1000
δ	Rate of population quitting from alcoholism permanently after treatment	0.3

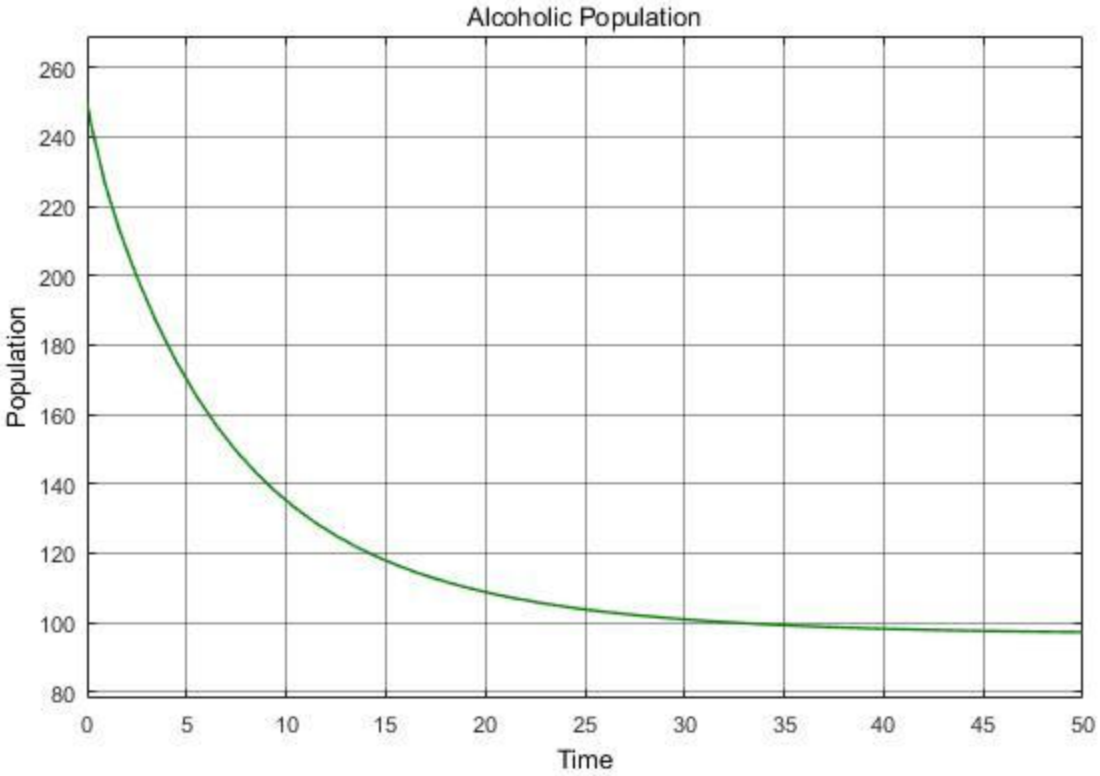
Numerical Simulation

Parameter	**Description**	**Model**
ξ	Rate of population failed in treatment and returned to be alcoholic	0.8
u1	Control Variable 1	0
u2	Control Variable 2	0.5
So	Initial Susceptibles	650
Ao	Initial Alcoholics	250
To	Initial Treatment Takers	80
Qo	Initial Quitters	20

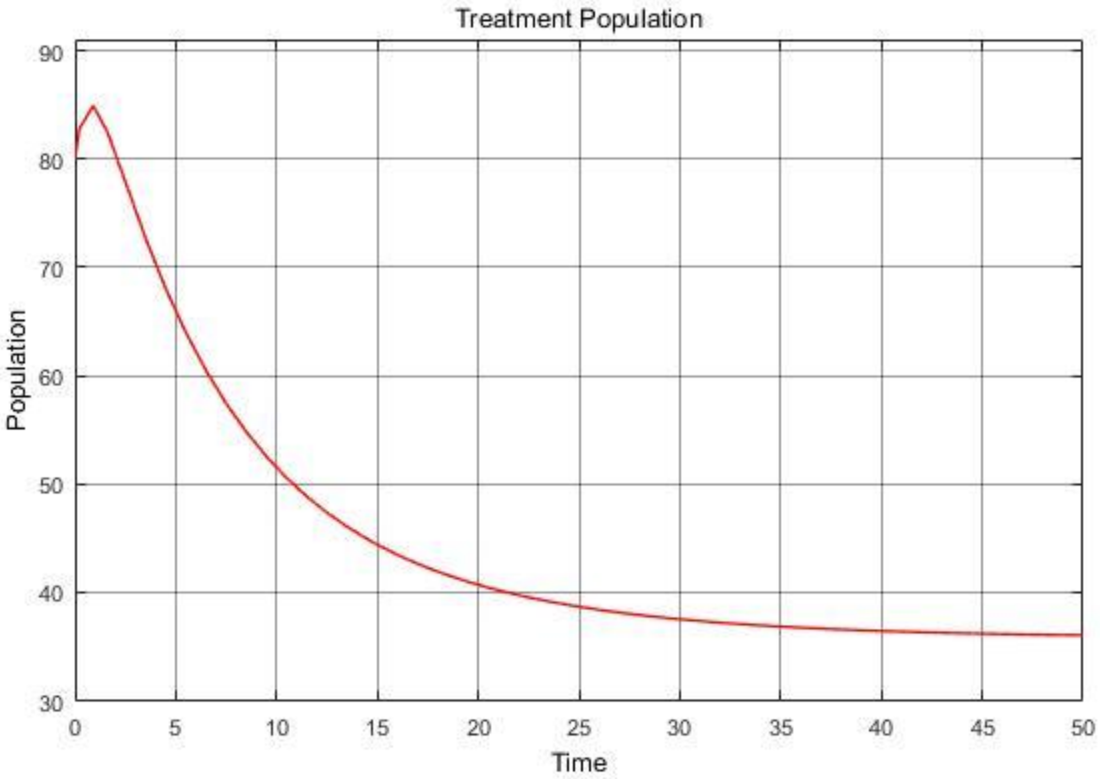
Graphs: Population of Susceptibles over time



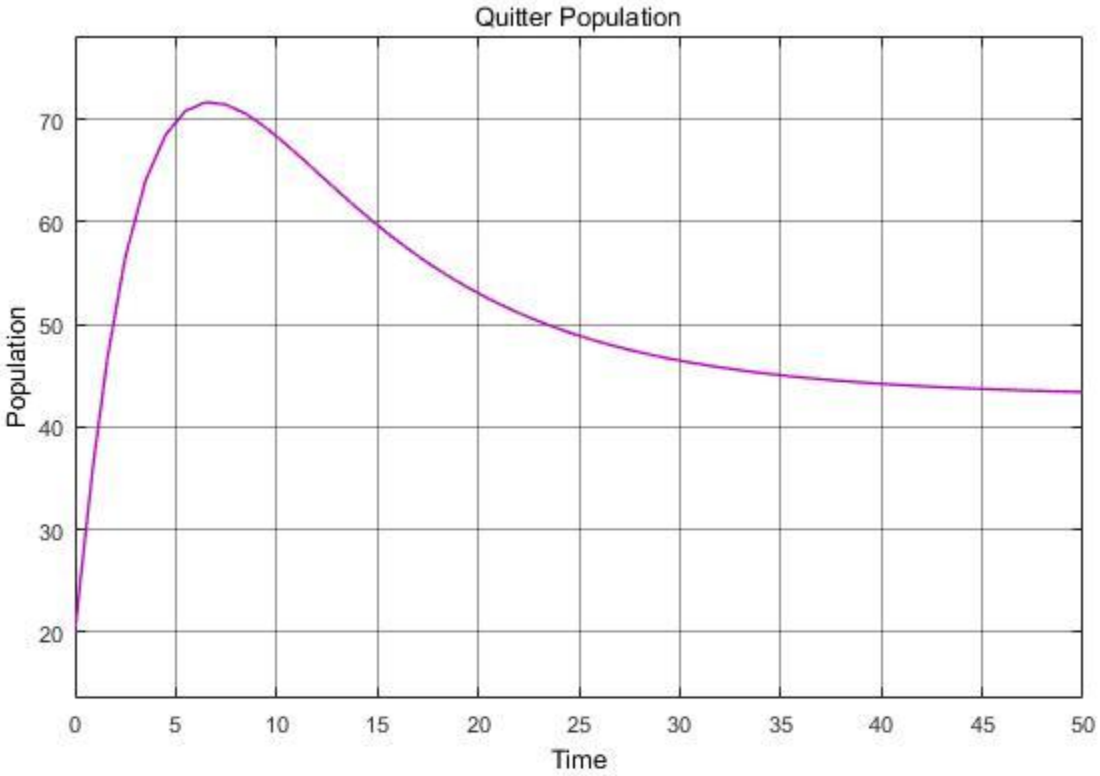
Graphs: Population of Alcoholics over time



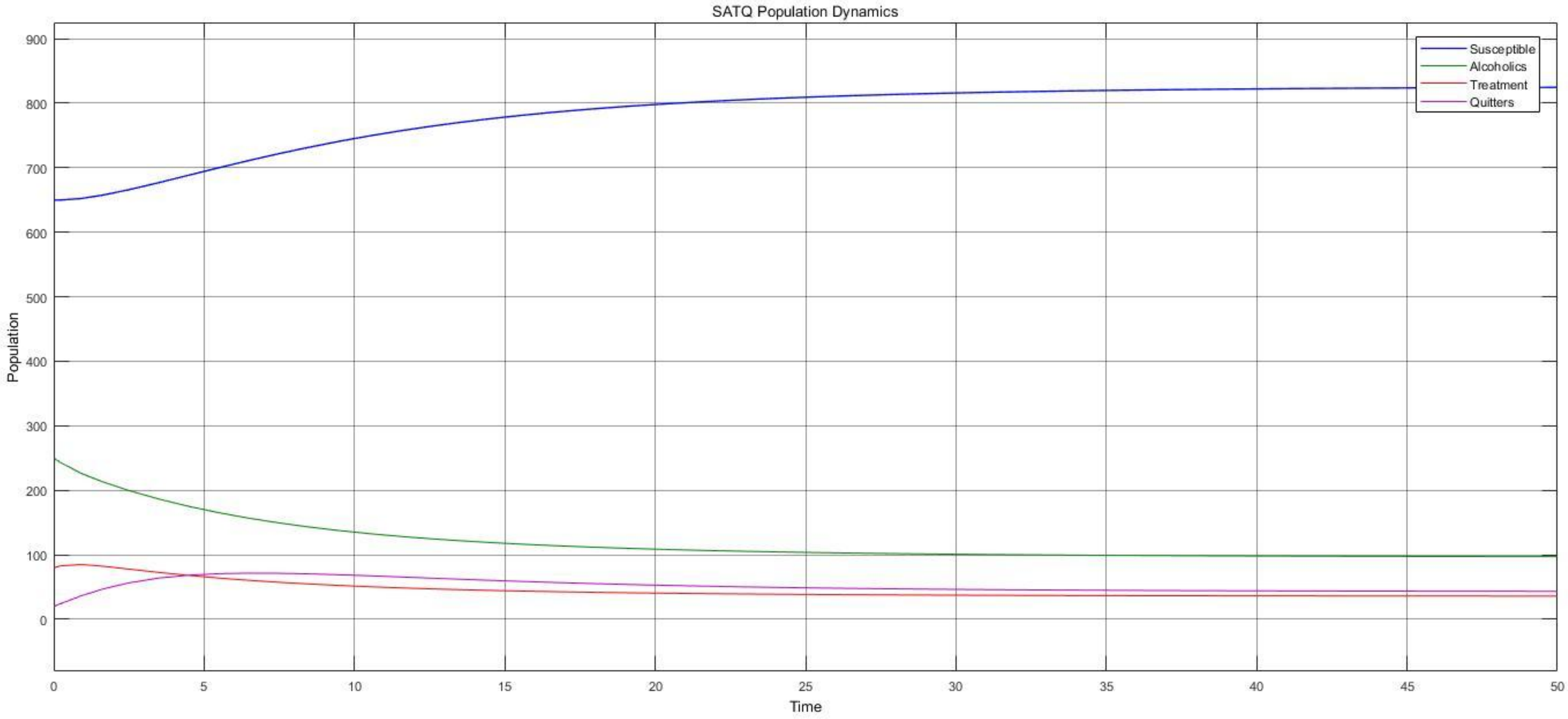
Graphs: Population of Treatment Takers over time



Graphs: Performance of Quitters over time



Graphs: Performance of SATQ over time



Model in terms of proportions(Eliminating N)

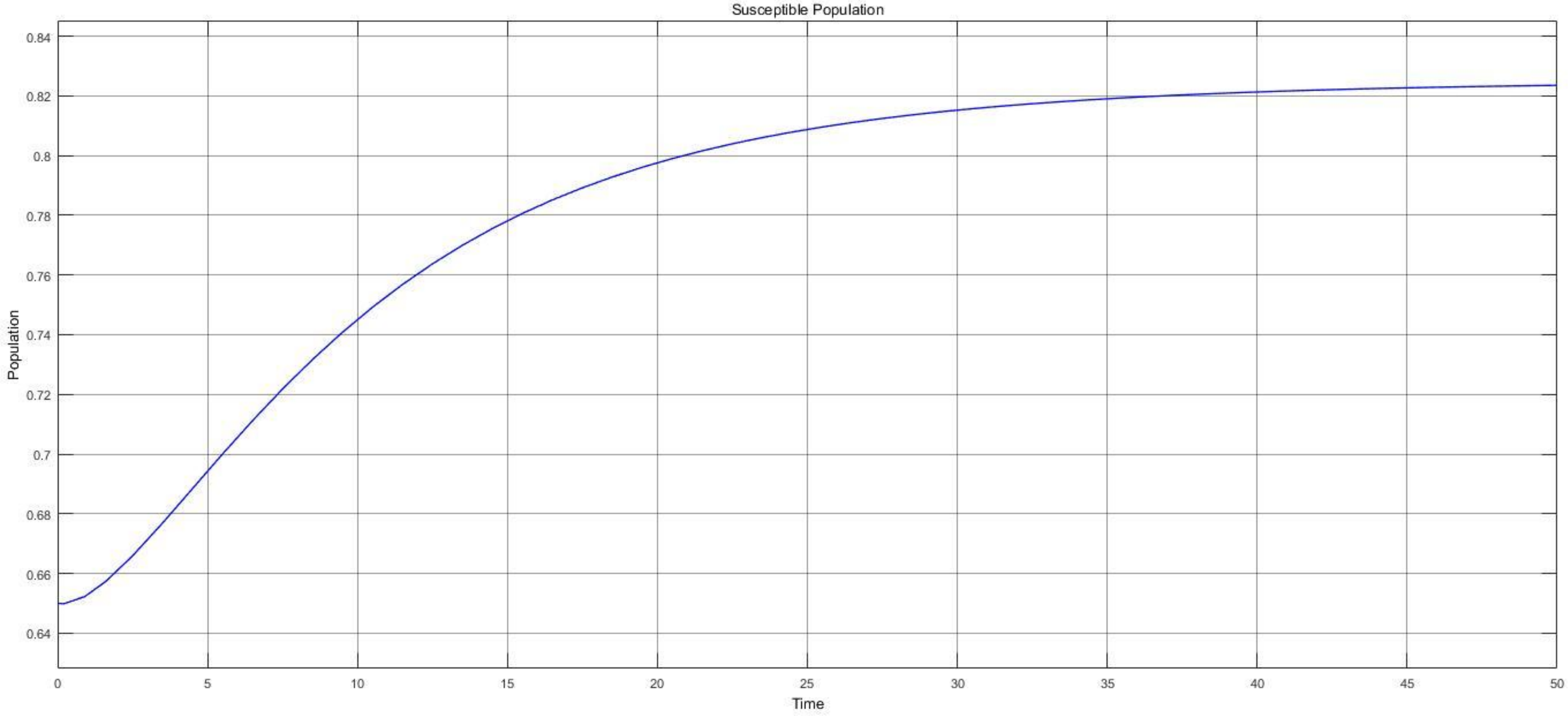
$$s' = b - \beta sa - bs$$

$$a' = \beta sa - \xi t - (b + 0.5)a$$

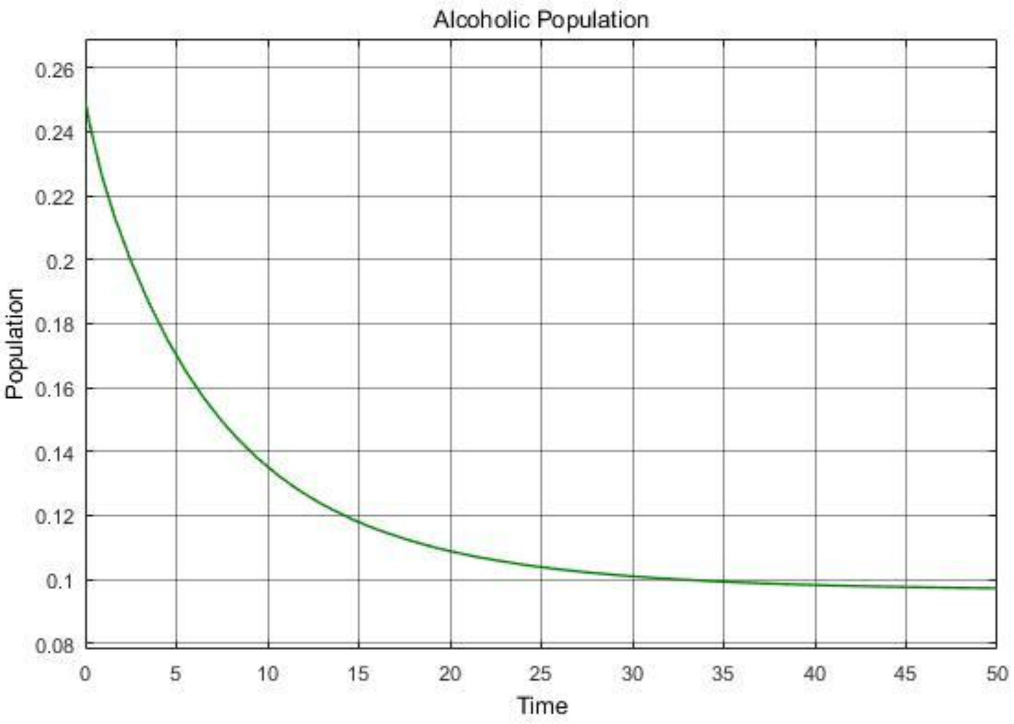
$$t' = 0.5a - (b + \xi + \delta)t$$

$$q' = \delta t - bQ$$

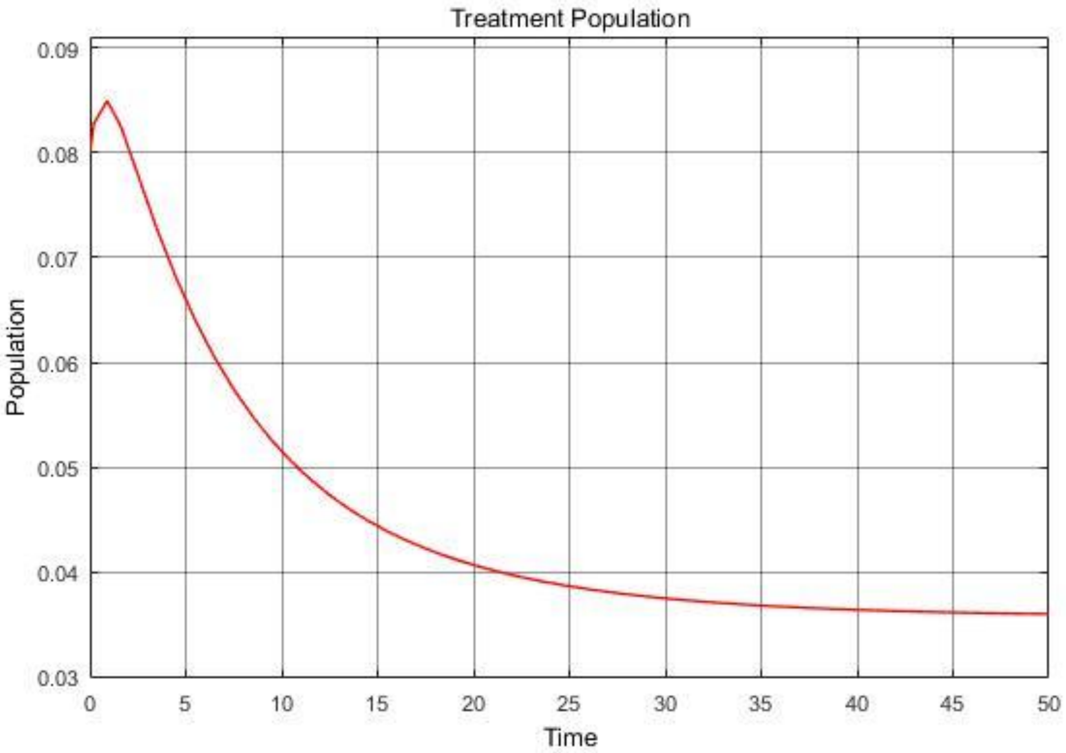
Graphs: Population of Susceptibles using proportion equations



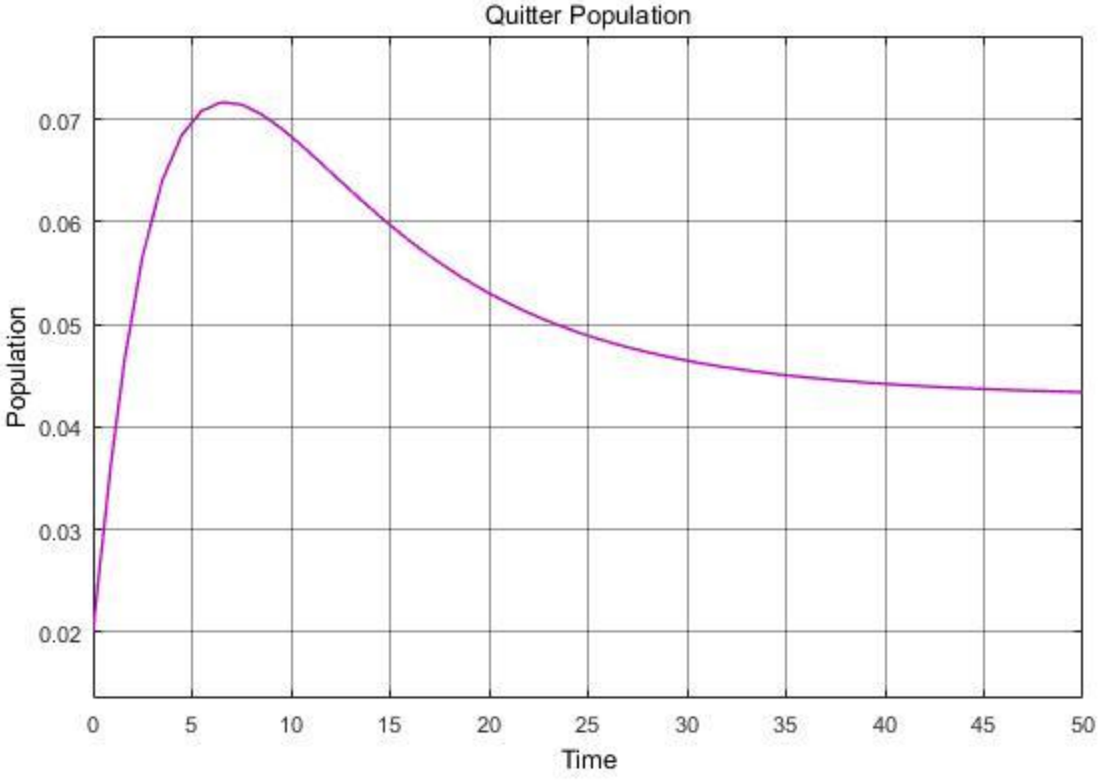
Graphs: Population of Alcoholics using Proportion equations



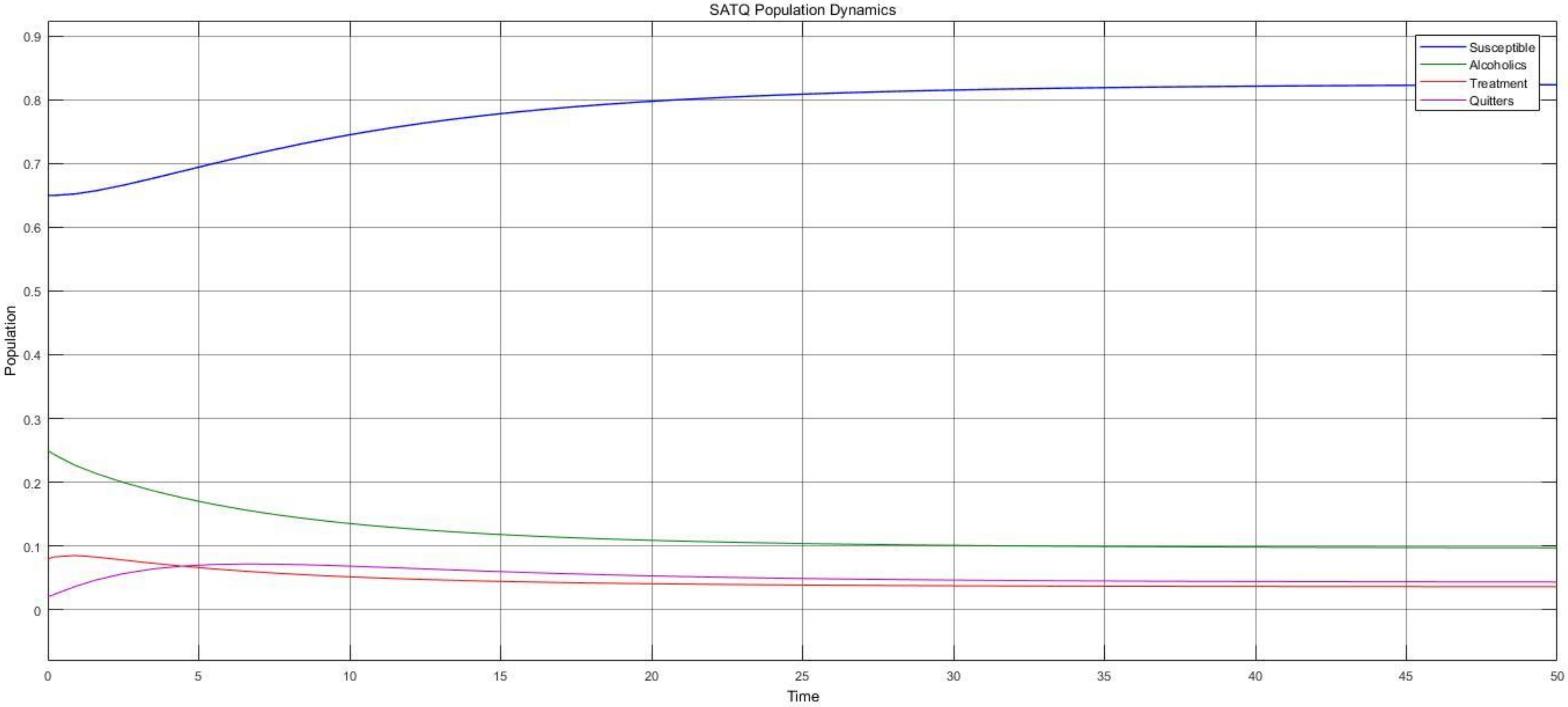
Graphs: Population of Treatment Takers using proportion equations



Graphs: Performance of Quitters using proportion equations



Graphs: Performance of SATQ using proportion equations



Future Prospects

- Adding Control Variables(u1,u2). These variables will be added to the equations of S and A. This is the real world equivalent of Holding a campaign or a Targeted effort towards the Susceptible population and the Alcoholic population to reduce the total number of Alcoholics or to prevent Susceptibles turning into Alcoholics.
- Using fractional order differential equations and accumulated data, we can minimize the error between the Defined Model and Acquired Data. This way the accuracy and prediction of our model increases.