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PES1201800389

ASSIGNMENT

1. Find the equation of the parabola $y = A + Bx + Cx^2$ that passes through 3 points $(1,1), (2,-1), (3,1)$, using Gaussian Elimination.

a) $(1,1) \Rightarrow 1 = A + B + C$

b) $(2,-1) \Rightarrow -1 = A + 2B + 4C$

c) $(3,1) \Rightarrow 1 = A + 3B + 9C$

$$A = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{array}\right]$$

$$A = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{array}\right] \qquad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$A = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{array}\right] \qquad R_3 \to R_3 - 2R_2$$

$2C = 4 \Rightarrow C = 2$

$B + 3C = -2 \Rightarrow B = -8$

$A + B + C = 1 \Rightarrow A = 6$

$\Rightarrow$ Equation of parabola $\Rightarrow \boxed{y = 2x^2 - 8x + 6}$

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2. Find the LU decomposition for the matrix

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_4 \to R_4 - 5R_1$
$R_3 \to R_3 + 5R_1$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$
$R_4 \to R_4 + 2R_2$

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$= \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

$$A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

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3. Let $T: R^3 \to R^3$ defined by
$T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$

(a) Find the matrix T relative to the standard basis of R^3

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1+0-0\\0+0\\1+0-0\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0+2-0\\1+0\\0+1+0\end{bmatrix} = \begin{bmatrix}2\\1\\1\end{bmatrix}$$

$$T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0+0-1\\0+1\\0+0-2\end{bmatrix} = \begin{bmatrix}-1\\1\\-2\end{bmatrix}$$

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = 1\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$T\begin{bmatrix}0\\1\\0\end{bmatrix} = 2\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$T\begin{bmatrix}0\\0\\1\end{bmatrix} = -1\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + -2\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix}1 & 2 & -1\\0 & 1 & -1\\1 & 1 & -2\end{bmatrix}$$

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5) Find the basis for 4 fundamental subspaces of T

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

1) C(A)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$R_3 \to R_3 + R_2$

$$= \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C(A) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

2) N(A)

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x = 3z$

$y = -z$

$$\Rightarrow N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

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3) $C(A^T)$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + R_1$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C(A^T) = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

4) $N(A^T) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$x + z = 0$
$y - z = 0$

$$N \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

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c) Find the eigen values and vectors of T

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$[A - \lambda I] = 0$$

$$= \begin{bmatrix} 1-\lambda(1) & 2-\lambda(0) & -1-\lambda(0) \\ 0-\lambda(0) & 1-\lambda(1) & 1-\lambda(0) \\ 1-\lambda(0) & 1-\lambda(0) & -2-\lambda(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix}$$

$\Rightarrow \lambda^3 - (\text{sum of ele of diagonals})\lambda^2 + (\text{Sum of minors of principal diagonal})\lambda - |A| = 0$

$$\lambda^3 - (1+1-2)\lambda^2 + (-3)\lambda - (0) = 0$$

$$\lambda^3 - 3\lambda + 1 = 0$$

$\lambda =$

$$\lambda(\lambda^2 - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}.$$

Case 1 : $\lambda = \sqrt{3}$

$$A = \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & 2/(1-\sqrt{3}) & -1/(1-\sqrt{3}) \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-2/(1-\sqrt{3}) & (-2-\sqrt{3})+1/(1-\sqrt{3}) \end{bmatrix} \quad \begin{matrix} R_1 \leftarrow R_1/(1-\sqrt{3}) \\ R_3 \leftarrow R_3 - R_1 \end{matrix}$$

$$= \begin{bmatrix} 1 & -1-\sqrt{3} & 1/2(1+\sqrt{3}) \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 2+\sqrt{3} & 1/2(-5-3\sqrt{3}) \end{bmatrix} \quad R_3 \rightarrow R_3 - \left(\frac{2+\sqrt{3}}{1-\sqrt{3}}\right)R_2$$

$$A = \begin{bmatrix} 1 & -1-\sqrt{3} & 1/2(1+\sqrt{3}) \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$N(A) =$

$$(1-\sqrt{3})y + z = 0$$
$$x + (-1-\sqrt{3})y + 1/2(1+\sqrt{3})z = 0$$

$$\Rightarrow y = \frac{(1+\sqrt{3})}{2} z$$

$$x = \left(\frac{3+\sqrt{3}}{2}\right) z$$

$$\text{Eigen vector} = N(A) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (3+\sqrt{3})/2 \\ (1+\sqrt{3})/2 \\ 1 \end{bmatrix}$$

Case 2, $\lambda = -\sqrt{3}$

$$\text{Eigenvector} = N(A) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (3-\sqrt{3})/2 \\ (1-\sqrt{3})/2 \\ 1 \end{bmatrix}$$

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Case 3, $\lambda = 0$

Eigen vector. $N(A) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

d) Decompose $T = QR$

$$T = \begin{array}{ccc} a & b & c \\ \end{array}$$
$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$q_1 = \frac{a}{||a||} = \frac{a}{\sqrt{2}} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$q_2 = \frac{b - (q_1^T b) q_1}{||(b - q_1^T b) q_1||}$$

$$= \frac{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}}{\sqrt{\frac{3}{2}}}$$

$$q_2 = \begin{bmatrix} 0.408 \\ 0.8164 \\ -0.408 \end{bmatrix}$$

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$$q_2' = c - (q_1^T c)q_1 - (q_2^T c)q_2$$

$$\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$- \begin{bmatrix} 0.408 & 0.816 & -0.408 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \begin{bmatrix} 0.408 \\ 0.816 \\ -0.408 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \left(-3/\sqrt{2}\right) \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} - (1.224) \begin{bmatrix} 0.408 \\ 0.816 \\ -0.408 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} -1.5 \\ 0 \\ -1.5 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 1 \\ -0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$T = QR = \begin{bmatrix} 1/\sqrt{2} & 0.408 & 0 \\ 0 & 0.816 & 0 \\ 1/\sqrt{2} & -0.408 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 1.224 & 1.224 \\ 0 & 0 & 0 \end{bmatrix}$$

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4. Find the best fit line $y = c + dx$ for the data using least squares fit

x	-4	1	2	3
y	4	6	10	8

a) $4 = c + (-4)d$

b) $6 = c + (1)d$

c) $10 = c + (2)d$

d) $8 = c + (3)d$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

Assume $x = \hat{x}$, $\hat{x}$ can be found by $A^T A \hat{x} = A^T y$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 6.665 \\ 0.6936 \end{bmatrix}$$

$y = c + dx$

$y = 6.665 + 0.6936x$

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5). Find the projection matrices P and Q onto the plane $x_1 + x_2 + 3x_3 + 4x_5 = 0$, and its orthogonal complement respectively.

$A = [1\ 1\ 3\ 0\ 4]$

$$N(A) = \begin{bmatrix} -1 & -3 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Projection matrix $= A(A^TA)^{-1}A^T$.

$$= \begin{bmatrix} 26/27 & -1/27 & -1/9 & 0 & 4/27 \\ -1/27 & 26/27 & -1/9 & 0 & 4/27 \\ -1/9 & -1/9 & 2/3 & 0 & 4/27 \\ 0 & 0 & 0 & 1 & 0 \\ 4/27 & 4/27 & 4/9 & 0 & 11/27 \end{bmatrix}$$

$$e = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

$C(A^T) \perp$ to $N(A)$

$$Q = \frac{ee^T}{e^Te} = \frac{1}{15}\begin{bmatrix} 1 & 1 & 3 & 0 & 4 \\ 1 & 1 & 3 & 0 & 4 \\ 3 & 3 & 9 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 12 & 0 & 16 \end{bmatrix}$$

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6 for which range of numbers 'a' is the matrix A positive definite?

$$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix} \quad \begin{aligned} R_2 &\leftarrow R_2 - \frac{2}{a}R_1 \\ R_3 &\leftarrow R_3 - \frac{2}{a}R_1 \end{aligned}$$

$$= \begin{bmatrix} a & 2 & 2 \\ 0 & (a^2-4)/a & (2a-4)/a \\ 0 & (2a-4)/a & (a^2-4)/a \end{bmatrix}$$

$$(a^2-4)/a > 0$$

$$\begin{bmatrix} a & 2 & 2 \\ 0 & (a^2-4)/a & (2a-4)/a \\ 0 & 0 & x \end{bmatrix} \quad R_3 \leftarrow R_3 - \frac{(2a-4)}{(a^2-4)}R_2$$

$$\rightarrow \left[\frac{(a^2-4)}{a} - \frac{(2a-4)^2}{(a^2-4)a}\right] > 0$$

$$(a^2-4)^2 - (2a-4)^2 > 0$$

$$a^4 + 16 - 2(a^2)(4) - (4a^2 + 16 - 2(2a)(4)) > 0$$

$$a^4 + \cancel{16} - 8a^2 - 4a^2 - \cancel{16} + 16a > 0$$

$$a^4 - 12a^2 - 16a > 0$$

$$a^2(a^2-12) - 16a > 0$$

$$\Rightarrow a > 2$$

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Which 3x3 matrix B produces the function: $f = x^T A x$?

where $f = 2(x_1^2 + x_2^2 + x_3^2 - x_1 x_2 - x_2 x_3)$

$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3.$

$$(x_1\ x_2\ x_3)\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$\Rightarrow a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3$

On comparision with $f \Rightarrow$ $a_{11} = 2$ $a_{12} = -1$

$a_{22} = 2$ $a_{23} = -1$

$a_{33} = 2$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

7) Find the SVD of A, $U \Sigma V^T$ when

$$A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$$

a) $A^T A = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ 0 & 0 \end{bmatrix} = U$ $R_2 \leftarrow R_2 + 3R_1$

b) $\lambda^2 - (90)\lambda + 0 = 0$

$\lambda = 90, \quad \lambda = 0$

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Case 1 : $\lambda_2 = 0$

Eigen vectors $= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

Case 2 $\lambda_1 = 90$ $\lambda_1 > \lambda_2$.

Eigen vectors: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

$$V = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix} = \frac{1}{\sqrt{10}} \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$

$\sigma_1 = \sqrt{90}$ $\sigma_2 = 0$

c) $$\Sigma = U_1 \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

d) $$AA^T = \begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ $R_2 \leftarrow R_2 + 2R_1$ $R_3 \leftarrow R_3 + 2R_1$

$$U_1 = \frac{Au_1}{\sigma_1} = \begin{pmatrix} 1.05 \\ -2.108 \\ -2.108 \end{pmatrix} \Rightarrow \begin{pmatrix} 0.33 \\ -0.67 \\ -0.67 \end{pmatrix}$$

e) Using Grahm's process find U_2 and U_3.

$$\lambda^3 - 90\lambda^2 - 0\lambda = 0$$

$$\lambda = 90, 0, 0$$

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⇒ Case 1, $\lambda = 90$

Eigen vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1 \\ 1 \end{pmatrix}$

Case 2, $\lambda = 0$

Eigen vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

Case 3, $\lambda = 0$

Eigen vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$$V_1 = \frac{\begin{pmatrix} -1/2 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{1/4+1+1}} = \begin{pmatrix} -1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$$

$$V_2 = \frac{\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}}{\sqrt{4+1}} = \begin{pmatrix} 2/\sqrt{5} \\ 0 \\ 1/\sqrt{5} \end{pmatrix}$$

$$V_3 = \frac{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{4+1}} = \begin{pmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{pmatrix}$$

$$U = \begin{bmatrix} -1/3 & 2/\sqrt{5} & 2/\sqrt{5} \\ 2/3 & 0 & 1/\sqrt{5} \\ 2/3 & 1/\sqrt{5} & 0 \end{bmatrix}$$

$$\Rightarrow U \Sigma V^T = \begin{bmatrix} -1/3 & 2/\sqrt{5} & 2/\sqrt{5} \\ 2/3 & 0 & 1/\sqrt{5} \\ 2/3 & 1/\sqrt{5} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{10} & 3/\sqrt{10} \\ -3/\sqrt{10} & 1/\sqrt{10} \end{bmatrix}$$