Oliver Heaviside's Operational Calculus: The Foundations of Electrical Engineering

By Marcelo Godoy Simões

TOWARD THE END OF THE 19th century, Oliver Heaviside developed a formal calculus of differential operators to solve various physical problems. Oliver Heaviside was the man who wrote, for the first time in the history of science, the now-so-called *Maxwell's Four Laws of Electricity and Magnetism*. Maxwell published his two-volume work, *A Treatise on Electricity and Magnetism*, in 1873.

Origin of Operational Calculus

Heaviside first came across this seminal work as part of his "self-study" work while he was living in Newcastle upon Tyne in late 1873. He was working for the Great Northern Telegraph Company, which was his only paid work; he left in 1873, at the age of 23, to concentrate on his studies. He rearranged James Clerk Maxwell's 22 quaternionic equations into the four coupled partial differential equations that we know today. From those four Maxwell's equations—written by Heaviside—it is easy to derive a simple wave equation that works for both the electric and the magnetic fields, the basis for modern wireless communications. Such work made by Heaviside has electromagnetic wave equations that allow the description of both light rays as well as radio waves. This work made Guglielmo Marconi work on the radio, but the reality is that another genius and giant, Nikola Tesla, was the inventor of the radio; the U.S. patent office did not want to recognize Tesla's invention—there was a convoluted layer of influences to have the United States keep in good terms with the Italians, but this is another record of our history.

Operational calculus converts derivatives and integrals to operators that act on functions, and by doing so, ordinary and partial linear differential equations can be reduced to purely algebraic equations that are much easier to solve. There have been a number of operator methods created as far back as Leibniz, and some operators, such as the Dirac delta function, created controversy at the time among mathematicians, but no one wielded operators with a sense of practical use and relaxing the constraints of flair and abandon over the objections of mathematicians as Oliver Heaviside did. He was an applied mathematician, physicist, and pioneer of electromagnetic theory, laying out the foundations to establish the early existence of electrical engineering (EE) as an independent field.

During the first decades of the 20th century, several attempts were made to rigorize Heaviside's operational calculus; such as that time the nascent EE field was having some divergence in approaches. Some industrials wanted EE to become technician oriented, giving skills to people in practicing technology as it was necessary to have enough workforce individuals as resources of the major electricity- and radio-oriented companies that were developing in the United States and the United Kingdom. Therefore, many EE courses and studies were initially established during the transition of the 19th to the 20th century and were constrained as a track—or as a path—inside of the Physics Departments of most notable universities. Heaviside made operational calculus so simple and easy that other intellectuals at that time favored the birth of EE as a scientific and independent endeavor in a more complicated and more mathematically tedious way. There were two paths, one going along for the support and explanation of operational calculus in terms of integral transformations; some mathematicians were more interested in having a two-way solution, from the time to the operator domains (p or s) and the inverse, so a lot of efforts happened to formalize using the Bromwich contour and Cauchy theorem for the inverse domain, based on complex analysis.

Some more EE-oriented theoreticians wanted to have a system domain analysis; the relationship with convolution was considered to

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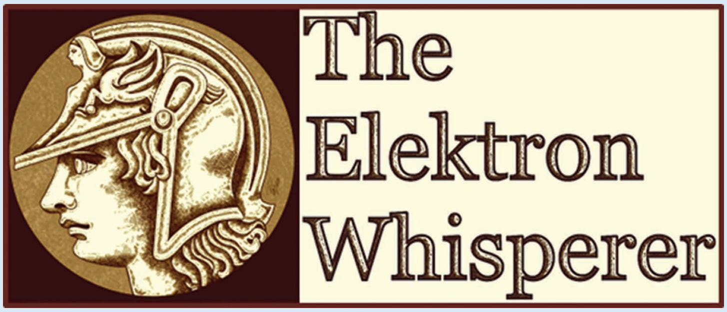
A Recipe for Learning



What happened to you to make you decide to become an electrical engineer? I could fill a whole hundred pages (which would be the threshold of a diary to an essay) out of my memory. I was still a child and observed the fiancé of my cousin changing the fuse of my house, which suddenly did not have lights, with neither the TV nor the radio working. Suddenly, after he changed the fuse, they started working again. My mother was just taking her shower, and my cousin and her fiancé showed up for an impromptu visit to our home. In Brazil, the showerheads are real-time water heaters connected to the wall in the bathroom with an internal 2-kW resistance immersed in the water flowing through the device, which heats, as the person uses the shower, the cold water coming from the pipe. Therefore, at the same time the water heats, it accumulates inside the shower in a small reservoir and then flows down through gravity for the individual who is showering. Those electric showers are connected to 220 V, while the rest of the house is connected to 110 V (at least in São Paulo; some other cities are only 220 V in the local distribution). In all my travels around the world, I never saw electric showers as they are in Brazil. People are afraid of electricity and water in the bath; I myself had several shocks from wires that were touching the metallic pipes. Now, everything is safer than when I was a child.

When I got older, I started to blow transistors and diodes, and even capacitors, but that is another story. I observed since very young how people would cook, and I paid attention to the fact that beans were very hard seeds, but somehow, they became soft and edible on my plate. Have you ever wondered about the word *recipe*? It usually follows a format, with ingredients, their measurements, sometimes a prep with initial steps, and then a method to transform all the ingredients perhaps into casseroles, stovetop, or oven food or other ways of cooking. Cornbread, cakes, and chocolate chip cookies—how to make all of them may be described through a recipe. A receipt is something different, but long ago, both the words *recipe* and *receipt* were derived from *recipere*, the Latin verb meaning “to receive or take.” Receipt is the older word, dating at least back to the 14th century, when it appeared in reference to a medicinal preparation; now, such a receipt that was once a recipe appears in place of what we now call a *prescription*, commonly abbreviated in the present day as “Rx.” Recipes and methods are related; receipts might be good as well (purchasing things and having a receipt for payment is always a blessing), but as far as prescriptions go, we try to be far away from them.

The general idea is that a person willing to learn is called a *student*. They will initially start on rote learning and memorization of basic facts and rules; the old paradigm that a student would be a vessel is no longer a good one, but it was very common in my teenage years when I was going through my middle and high school years. Then, through a teacher, a student would get to do some critical thinking and logical reasoning, and probably after a lot of refinements, time, work, and dedication, one would get to the rhetorical stage, capable of persuasive communication. Then, I found later in my life that there was a final degree conferred for that and that a student would eventually be certified with a diploma as capable of philosophizing in a certain domain—that is, a Ph.D. or an equivalent doctoral degree for in-depth expertise. All of this is possible if we keep working hard, preparing elements for our transformation, keeping things in our memory, and using imagination to desire and create new forms. The recipe is applied to us; we become something else, or we think we do. In reality, we get older and wiser.

In this “The Elektron Whisperer” (TEW) column, I thought it would be important to tell you that electromagnetism, Laplace transforms, and even Maxwell equations, as well as much of the terminology that we learn in the EE bachelor’s degree, would not be possible without one who is more than a hero—he is a giant to us—Oliver Heaviside.

—Marcelo Godoy Simões

be important. There was also the algebraic formulation developed by Mikusinski’s *Operational Calculus* as well as Schwartz’s theory of distributions. The mathematicians Carson and Bromwich demonstrated that Heaviside’s operators were analogous to well-developed integral equations and contour integrals in the complex plane. Integral-based transformations have been mostly adopted by professors and educators of the new 20th century, and there are at least four generations of EE departments that used the rebranded Heaviside’s operational calculus as a Laplace transform, with all the theory supporting it, the incorporation of convolution in the whole theoretical framework, and further mathematical depths in complex variable analysis with contour and line integration.

During the Victorian age, there was resistance in society to suppressing technological advancements. The United Kingdom had its Luddites active and breaking machinery, disrupting industrial activities. It was in such an industry, academia, erudite, and real-life social fabric when, in 1893, Heaviside published the first of a three-part series describing his operator calculus in the *Proceedings of the Royal Society*. Later in the year, the second part appeared, but this "was the last straw for mathematicians," and his third part was rejected. What killed the third part was Heaviside's unconcerned use of divergent series, dismissing their tendencies to infinity while producing accurate results by manipulating them. Some historians say that Heaviside also produced an article on similar ideas of convolution to complete his initial series of articles, and one of the most important "contributions" of the early scientists in the 20th century, in addition to connecting his procedures to Laplace, was to develop what they believe was "missing," i.e., the convolution approach, where the time-domain response for an impulse would be the transfer function, in the s-domain, of that system.

Heaviside was more concerned about time-domain solutions instead of theorizing about a frequency domain, which became a whole paradigm of design in control systems and analysis of linear systems throughout the 20th century. During the middle of the 20th century, the Polish mathematician Jan Mikusinski (1913–1987) developed a direct algebraic approach to Heaviside's operational calculus and changed the viewpoint of many mathematicians on it. His calculus is known as *Mikusinski's operational calculus*; people who know both ways state that in practice, there is absolutely no difference in Mikusinski's approach when adopting the original Heaviside's operational calculus for solving electrical circuits.

How Heaviside Developed Operational Calculus

Just after completely rebranding Maxwell's equations, Heaviside developed multivariable calculus definitions to what we call today *curl* and *Laplacian* operators. The curl is a vector operator which defines the infinitesimal circulation of a vector field in the 3D Euclidean space, divergence is a differential operator, which is applied to the 3D vector-valued function, and Laplacian is a scalar value that represents the divergence of the gradient used to identify a maximum curvature associated as well as the gradient which is a vector that indicates the direction of steepest ascent or descent. Definitions for Maxwell's equation reformulation became a motivational precursor for his operational calculus for ordinary differential equations. Heaviside was a self-taught mathematician and scientist; his main approach was to solve some practical problems (which today we identify as an engineering personality). At that time, long-distance telegraphy with good transmission capability was a technical challenge. Companies involved in the commercialization of this technology were aiming for faster communication over longer distances; the investments were gigantic for wiring that could reach all the areas of services. It was very common to have squashed signaling at the receiving end, reducing the signal rate, and it was not only a matter of a bigger copper cross-section area for the wires. There were some unexpected situations, such as when cabling for telegraphy submerged in the ocean, that would have asymmetrical features, making the communication not completely half-duplex. Full-duplex communication was still unimaginable at that time.

Lord Kelvin, who was a famous scientist, made some strides in obtaining a theoretical analysis in the form of a "heat diffusion equation," where the voltage V would be defined along x as a space coordinate, also involving R and C for the distributed electrical resistance and capacitance. Heaviside revolutionized the theory of signaling along wires, using his approach of assuming "electromagnetic waves." Those early days would be rather a breakthrough given the theoretical eruditeness of that period—to think about electromagnetic waves traveling in a copper cable. Lord Kelvin missed a crucial component in the physical model, his "telegraph equation," and that was—*electromagnetic inductance*. Limiting the model to electrostatic capacitance and electrical resistance missed such an element of Maxwell's theory—consequently, the falling apart of a "wave" description. Heaviside, in his article on "the extra current," derived the now familiar differential equations, which formulate voltage V and current I on an electrical transmission line as a function of distance x- and time t-damped wave equation.

In that equation, a second-order partial derivative with respect to the distance was equated to the components of a second-order ordinary differential equation on the right-end side. Heaviside understood the nature of this problem and defined that the speed and characteristics of the signaling would depend on the circuit parameters (currently described as the propagation/attenuation constants determined by line parameters R, L, and C). In the second stage, he applied the telegrapher's equations to derive closed-form solutions for numerous electric circuit configurations. Those problems and solutions were written in his volumes of *Electrical Papers,* including further studies of faults in cables and the impact of signaling through telegraph circuits; all of these were worked and done during his daily job as a telegraph clerk at the Great Northern Telegraph Company in Denmark and Newcastle upon Tyne. He conceptualized the fundamentals of telegraphic propagation of signals.

When Heaviside made a whole reinterpretation of Maxwell's theory, he started to have further deep insights for a comprehensive understanding of the practical engineering problems of his time. Heaviside conducted several studies on physics and found out that the ionosphere was a layer of charged ions in the upper atmosphere, and for many years in the 20th century, the schoolbooks used to call that the *Heaviside layer*. His geophysics studies would then establish surprisingly long distances for radio waves to travel through the air. He paved the way for the discovery and technological application of radio transmission as well as the tale regarding Tesla versus Marconi. Heaviside did not socialize with others as a result, also spending little time in his writing to justify his methods for a social elite. The absence of rigor is historically very often a peculiarity in many innovative ideas; on the other hand, mathematical science progresses mostly on abstractions, and often, the proofs of those mathematical procedures lag many years in their applications (for example, Boolean algebra and the invention of the electronic computer 100 years later).

Heaviside was ready (after all that was mentioned previously) to make his contribution to developing a simpler way to solve electrical circuits, using his proposal of operational calculus. Nowadays, people have no idea how difficult and cumbersome it was to solve transients of electrical circuits; there were many different voltage and current sources, sometimes switching or assuming a sinusoidal nature. Today, we can realize that all of this can be considered with algebraic differential equations and multivariable calculus. We need expanded infinite time series, with homogeneous solutions, plus forcing functions, assuming linearity to aggregate the homogeneous plus input excitation conditions, and we need to take care of initial conditions. However, such a thematic understanding was not clear. I learned a methodology of time series in my courses on "Physics III" and then on "Electromagnetism and Electrical Circuits" in 1982 and 1983. I was probably among one of the last generations to learn based on Heaviside's approach to be prepared for another course where parallel courses in circuits as well as in linear systems taught to us (in 1983) the Laplace transform methodology. It was a very heavy scientific preparation at the Escola Politécnica da Universidade de São Paulo (in Brazil); my professors used to comment that "you will learn in 'Linear Systems' as well as in the third course of 'Electrical Circuits' that you can adopt Laplace transforms, and everything in the time domain, complex domain, and *s*-domain will make sense."

In fact, it still took me many years to figure out the relationships of Laplace transforms, intertwined with the Fourier series, as well as their relationships to Fourier transforms and the Z-transform. I comprehended this only after my course in "Advanced Mathematics for Electrical Engineering" in my Ph.D. program at the University of Tennessee in Knoxville in 1991 and 1992. This was taught by Prof. M. O. Pace in his mandatory course for the qualifying exam, based on the book by Oppenheim and Schafer, *Discrete-Time Signal Processing*, with extensive sets of handouts and notes. This was also taught in "Applied Math," which was taught on the basis of integral equations. This happened a long time ago, and students currently have a more simulation-based approach, which facilitates their understanding. Notwithstanding, I was educated under the approach established by Oliver Heaviside, and to me, there are many other champions for our modern 20th-century EE approach.

Step-by-Step Procedural Conversion

Heaviside proposed taking the basic differential equations for voltage $v(t)$ and current $i(t)$ for a resistance R, a capacitance C, and an inductance L, rewriting them using an operator $\boldsymbol{p}$, which performed the derivative with respect to time on the function to the right of it. Effectively, he made the derivative d/dt of a function such as $df(t)/dt$ to be $\boldsymbol{p}f(t)$; then, he also assumed the inverse of $\boldsymbol{p}$, i.e., $1/\boldsymbol{p}$, as the operator performing the integral of a function so that $\boldsymbol{p}(1/\boldsymbol{p}) = (1/\boldsymbol{p})\boldsymbol{p} = 1$ because, in practice, the integral should be the inverse operation of a derivative. There are cases where this is not completely true—for example, for nonlinear cases or with parameters dependent on time. A simple and common case is when there is a constant term that might be considered, and the derivative of any constant would be zero. This is regarded as initial conditions in differential equations. The proposed methodology is for problems where $f(0) = 0$. In practical electrical circuits, if the function $f(t)$ represents a variable that is associated with energy in a particular form, $f(0) = 0$ means the energy stored at the chosen instant $t = 0$ is zero.

For example, for the current in an inductor, we have this voltage and current relationship, $v = L(di/dt)$ with energy $E = (1/2)Li^2$, as well as the relationships for the current and voltage of a capacitor, where $i = C(dv/dt)$ with energy $E = (1/2)Cv^2$. It is elementary to validate, by just assuming an adequate time shift, $t = t_0$ to comply with $f(t_0) = 0$, particularly for periodic systems. For nonperiodic systems, an inspection must be made in the evolution of the response to define a new starting point with such a time shift. Then, the energy stored in that element would be zero, and the input-forcing function (or circuit excitation, such as a voltage source or a current source) would be shifted to that initial chosen time. Therefore, with such assumptions, we have for resistors, capacitors, and inductors that

$$v = iR$$
$$i = C\boldsymbol{p}v \text{ from } v = \frac{1}{C}\int i dt$$
$$v = L\boldsymbol{p}i \text{ from } v = L\frac{di}{dt}.$$

The overall description of all circuit relationships will allow the separation (algebraically) of the operators from their functions, such as if they could have an independent existence; then, Heaviside proposed an impedance defined as v/i that he called Z (a terminology we still use today).

$$Z = R$$
$$Z = \frac{1}{C\boldsymbol{p}}$$
$$Z = L\boldsymbol{p}.$$

In the Heaviside treatment, it was assumed that a constant voltage was applied to a circuit at the time $t = 0$ because his initial interest was in the impulse (or step function), such as the ones encountered as transient signals on telegraphic cables. In telegraphy, these transient effects limit the signaling speed, while in telephony, the transient effects limit the line length. In EE, it is mostly assumed that for a particular initial time, the differential equations for transient currents have an applied voltage of 0 for $t < 0$. Heaviside wrote this step function as the bold symbol **1.** Such a step function has been defined in the areas of controls, signals, and systems as a Heaviside function, usually denoted as **H(t)** or **U(t)**, and also has the property that $f(0) = 0$, a condition for the commutativity of the inverse operation. For example, to solve the differential equation $(d^2y/dt) + y = 1$ for $t > 0$, with zero initial conditions, which means that $y(0) = 0$ and $dy(0)/dt = 0$, applying a step function, Heaviside would rewrite this as

$$\boldsymbol{p}^2 y + y = \mathbf{1}.$$

The "**1**" at the right-hand side is a step function for $t > 0$. The solution of y is $1 - \cos(t)$. Then, isolating y at the left side to solve for the solution, we have

$$y = \frac{\mathbf{1}}{(\boldsymbol{p}^2 + 1)}$$

So far, we kept $\boldsymbol{p}$ in ***bold italic*** and **1** in **bold** to emphasize that there is an *operator* and a forced function or an excitation from an input source; nonetheless, from this point on, we can basically treat all of them algebraically, as simple numbers and variables.

$$y = \frac{p^{-2}}{(p^{-2}+1)}\mathbf{1} = p^{-2}(1 - p^{-2} + p^{-4} - \cdots)\mathbf{1}$$
$$= \frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} - \cdots = -\cos(t) + 1$$

The expression of y could be expanded by using the binomial series and then inverting to the time domain, as shown previously.

Suppose a telegraph wiring section, made of a resistance R in series with an inductance L, to which a step voltage would be applied; the current response would be

$$i = \frac{v}{Z} = \frac{1}{(R + Lp)}\mathbf{1} = \frac{1}{R} \times \frac{R}{Lp}\frac{1}{\left(1 + \frac{R}{Lp}\right)}\mathbf{1}$$
$$= \frac{1}{R} \times \frac{1}{\tau p}\frac{1}{\left(1 + \frac{1}{\tau p}\right)}\mathbf{1}$$

where $\tau = L/R$.

This term $(1 + (1/\tau p))^{-1}$ can be expanded

$$\left(1 + \frac{1}{\tau p}\right)^{-1} = 1 - \frac{1}{\tau p} + \left(\frac{1}{\tau p}\right)^2 - \left(\frac{1}{\tau p}\right)^3 + \cdots$$

by the binomial series

$$i = \frac{1}{R} \times \frac{1}{\tau p}\left[1 - \frac{1}{\tau p} + \left(\frac{1}{\tau p}\right)^2 - \left(\frac{1}{\tau p}\right)^3 + \cdots\right]\mathbf{1}$$

distributing the outside term with the p into the fractions inside the brackets

$$i = \frac{1}{R} \times \left[\frac{1}{\tau p} - \left(\frac{1}{\tau p}\right)^2 + \left(\frac{1}{\tau p}\right)^3 - \cdots \cdots\right].$$

However, the step function, when integrated (since we are now doing the inverse), means that the "integral"

$$\frac{\mathbf{1}}{\boldsymbol{p}}\mathbf{1} = t$$

and in general

$$\frac{\mathbf{1}}{\boldsymbol{p}^n}\mathbf{1} = \frac{t^n}{n!}.$$

Therefore

$$i = \frac{1}{R} \times \left[\frac{t}{\tau} - \frac{1}{2!}\left(\frac{t}{\tau}\right)^2 + \frac{1}{3!}\left(\frac{t}{\tau}\right)^3 - \cdots\right].$$

With ad hoc knowledge, it is possible to compare the power series expansion of e^{-t}

$$e^{-t} = 1 - t + \frac{t^2}{2} - \frac{t^3}{6} + \cdots + \frac{t^k}{k!}$$

Finally, the current can be rewritten in the time domain as

$$i = \frac{1}{R}[1 - e^{\left(-\frac{t}{\tau}\right)}].$$

Then, using the superposition property on the original problem for other waveforms, making the solution per the linear combination of waveforms, and then adding the solutions allowed Heaviside to analyze distortion at all signal frequencies. Also, making recommendations on how to match segments of telegraphic wiring with extra capacitances, he was solving the problem of long-distance telegraphic losses in the line communication through an ad hoc solution of his just invented operational calculus.

Let us do this by solving a second-order differential equation, where it happens to have a minus on the coefficient, as opposed to the previously positive coefficient exemplified earlier

$$\frac{d^2y}{dt} - y = 1.$$

For $t>0$, with zero initial conditions, that means $y(0)=0$ and $dy(0)/dt=0$. We would simply apply the operator p to have the algebraic solution in y, expanding through a binomial series as well, still assuming that $((1/p^n)1=t^n/n!)$ then as before, rearranging the y terms in rising powers to eventually find out the time-domain solution for such a second-order differential equation with ad hoc resemblance of exponential series expansion as

$$y=\frac{t^2}{2!}+\frac{t^4}{4!}+\frac{t^6}{6!}+\cdots=\frac{1}{2}(e^t-e^{-t})-1.$$

It is easy to just get in the habit of dropping the 1 altogether, working with p as a simple variable once a routine is established. For circuits with continuously distributed impedance, such as telegraph lines, and nowadays for distribution power lines, sometimes having an impulse or a particular theoretical infinite current appear instantaneously at $t=0$, it is possible to expand with the Kron or Dirac delta functions. Also, we have the possibility of fractional powers of p, such as the $p(1/2)$ cases that have been extensively studied in the past 50 years.

Heaviside proposed the following operator function:

$$\left[\frac{p}{p+B}\right]^{\frac{1}{2}}$$

Acting on a step function, he divided through by p to get

$$\left[\frac{1}{1+\frac{B}{p}}\right]^{-\frac{1}{2}}$$

and expanded into a power series, arriving at a solution in terms of modified Bessel functions; supporting such a methodology provides viable and right solutions. He also found a technique in using Fourier series methods (for the diffusion equation) and applied it to a problem for which $p^{\frac{1}{2}}$ showed in the formulation; then, he made an operator formulation for current from a step voltage in an infinitely long cable. Heaviside equated the form of his solution to the Fourier solution to deduce the fractional p-operator term, and he declared the results to be true for all practical purposes. Later, Heaviside presented a direct derivation based on the gamma function, which can also be deduced using Carson integrals and other methods. Heaviside often produced two versions of his power series solution: a convergent one that was useful for small t but was too slow to converge for large t and a divergent one that was useful for large t when it was taken to a small number of terms. His treatise works on all these and other possible cases, but the people of the 20th-century newly organized cohort of the school of EE did not take his methodologies further.

Other Techniques Derived by Heaviside

Heaviside developed the Heaviside expansion theorem to convert Z into partial fractions to simplify his work. For $f=1/Z$ and Z, a polynomial in p, the roots of Z can be found, and f can be expressed as a sum of terms consisting of constants divided by the simpler factors. A similar procedure is done when using Laplace transforms. Heaviside developed this method of calculating those coefficients for the expansion in partial fractions; it is called the *Heaviside cover-up method*. There are some discussions in the literature for some classes of problems for which this methodology does not work, particularly for nonlinear and time-varying coefficients, or systems that do not allow the approach of convolution, but those are not often found in regular EE problems, so they are not discussed here.

Heaviside had ingenious solutions using his operators for the time-varying current in circuits with additional continuously distributed parameters, such as found in actual telegraph lines. He added a section of cable to the beginning of an infinite line, found the current as a function of time for that configuration, and then "removed" the initial section to end up with the solution for the original cable. In fact, Heaviside used his operator calculus to design a transmission line with zero distortion (but with exponential attenuation over distance), and this is a possible system decoupling that would require some theoretical explanations. When Z is a polynomial of degree greater than four, its roots are difficult or impossible to find directly. Also, the expansion theorem does not work for a Z with a root of zero or with repeated roots, a situation not encountered in passive networks but one that can occur when, for example, an active amplifier (with transistors) will source energy from a different path of the regular input.

Heaviside treated equal roots as being unequal and solved for the transient current, letting those roots approach equality as a limit, and then applied calculus-based theorems of limit approaching zero or infinity. Heaviside removed these difficulties by expanding Z in their inverse powers of p and then replacing p^{-n} by $t^n/n!$ exactly as discussed before. This approach is called *Heaviside's extended expansion theorem*. Probably if such a method had a fancier name, it would have been preserved in the history of EE. However, we are forever fortunate that Heaviside coined many terms; they were disliked in his day, but now, they are adopted and frequent, such as impedance, inductance, conductance, admittance, and reluctance. He designated as *algebraizing* a differential equation with his operators and *logarizing* when taking a logarithm, and he called the e-ph the *spotting function* because it isolates, or spots, a certain value of the function. In addition, he also designated the *cover-up method* taught in Laplace transforms. All those new terms invented by him became memorable names.

Heaviside used physical intuition to guide him in handling these series, and he was unparalleled in his electromagnetic intuition. Because Heaviside's work was results oriented, he sometimes provided ad hoc arguments to support his derivations. He often suggested in rather abrupt prose that mathematicians should provide rigorous proofs for what he did. EE education as a whole has been influenced by operational calculus based on Laplace—the original ideas of Heaviside have been deemed as simply historical—but he is indeed the father of operational calculus and of many techniques used in EE up to now.

The Laplace transform has the advantage of incorporating initial conditions in the formulation. The Heaviside operational calculus and the Laplace transform formulations are pretty much identical for vanishing initial and boundary value problems. The constructions for Heaviside in (i) and for Laplace in (ii) are expressions in $\boldsymbol{p}$ and in s.

1) $v(t) = G(\boldsymbol{p})u(t)$ for all t (expanding in time series and then associating those time series with recognized exponential functions).
2) $V(s) = G(s)U(s)$ when $Re[s] > \sigma$ (theoretically, it would be possible to perform the inverse-Laplace operation; in reality, most engineers simply look to a table of Laplace inverses).

Conceptually, there is a difference between $G(p)$ as an operator and $G(s)$ as a complex number, but we can mathematically formulate, on an easy assumption, that $G(s)$ is the evaluation of $G(p)$ at $p = s$.

Growth of EE Based on Operational Calculus

There was a tremendous growth of EE from 1930 to 1960, establishing the basis of the curriculum that eventually educated many people in the second half of the century. Under the leadership of Vannevar Bush, a modernization of the EE curriculum at MIT was launched in the late 1930s and 1940s, based more on sciences like physics and mathematics than on craftmanship. Ernst Guillemin moved temporarily from MIT to the University of München, Germany, for a doctorate in mathematical physics in 1926 with Arnold Sommerfeld. Ernst Guillemin is generally considered as the founding father of circuit theory education; upon returning to MIT, he was invited to assist in the development of a communications option for undergraduate students. Then, he worked on revised and expanded communication transmission lines, telephone repeaters, balancing networks, and filter theory.

Guillemin had a radical approach to the teaching of his approach. He published many important books, and in 1953, his *Introductory Circuit Theory* begins with the concepts of graphs, networks and trees, cut sets, duality, and so on before even mentioning Kirchhoff and loops and nodes. Then, he showed what became the most common way to teach electrical circuits throughout the whole 20th century with impulse, the step response, and the sinusoidal steady-state response. Next, he published *The Synthesis of Passive Networks* (1957) and *The Theory of Linear Physical Systems* (1963), in many ways the first coherent and cohesive presentation of the new discipline of network synthesis, with realization theory and methods as well as a presentation of Butterworth, Chebyshev, and elliptic approximation techniques for filter design. The book again offers us insights into Guillemin's teaching style, filling some of the gaps not covered by the earlier books, and partly offers alternative approaches and theoretical consolidation. The book *Operational Calculus Based on the Two-Sided Laplace Integral*, by Balthasar van der Pol and H. Bremmer, was published by the University of Michigan in 1955, with a unique treatment with applications to mathematics and physics as well as circuit theory.

The development of inexpensive computers and computational facilities, together with strong circuit simulation programs, like SPICE, and system analysis programs, like MATLAB, offered new opportunities for designers and the education of design. In the last 30 years, several universities dropped or reduced their circuit analysis in favor of SPICE simulations, but that made some professionals go into their careers thinking that design is merely a trial-and-error ad hoc approach with SPICE simulations for some random values of components until they go through a more complex or real-life circuit. Nowadays, with the advent of MATLAB-based books in EE, students learn directly using toolboxes and functions in MATLAB how to expand partial fractions. It is not even necessary to memorize a long table of Laplace transforms; we can use computational-based software such as MATLAB, Mathematica, Maple, and many others because those tools became embedded and ready to use.

With the interest in digital signal processing (DSP), there has been a move to reverse the traditional order in EE education of analog circuits and signal processing first and then DSP. Today, most engineering computational software will have some sort of blockset or a connection to embed a microcontroller, a DSP, with promotional videos and toolboxes ready-made by the company that developed the simulation system. But there are still some educational paths in which EE should deal with discrete time and continuous time in an interlaced way, at the same time with a push toward "coding" as necessary to prepare the workforce. Hardware in the loop and the possibilities of digital twins have become emphasized in the past few years. However, a good EE education should approach building in their minds the concepts of circuits and systems to integrate continuous-time and discrete-time models, based on the fact that we will have controls, communications, energy conversion, and recently, enhanced artificial intelligence-based requirements for huge data systems. We have to define today what is EE education for the next generation, and energy transformation and sustainability should be taken into account.

Heaviside was a mathematical pioneer, but mostly, he was an electrical engineer ahead of his time. He was an early adopter of vector analysis and calculus and developed many of his contributions to suit engineering and physics problems. In 1922, three years before his death, he was awarded the very first Faraday Medal by the Institution of Electrical Engineers. He was an outstanding polymath, self-taught, with the humble origins of a lower middle-class son of a wood engraver from Stockton-on-Tees. Heaviside was a remarkable man, an original thinker with brilliant mathematical and physics insights. We owe him the foundations of modern EE and the contemporary horizons that we have today.

Thank you for reading "The Elektron Whisperer" (TEW) column by Marcelo Godoy Simões. See you next time.

For Further Reading

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Biography

Marcelo Godoy Simões (marcelo.godoy.simoes@uwasa.fi) is a professor at the Electrical Engineering Department, School of Technology and Innovations, University of Vaasa, 65200 Vaasa, Finland.

From the Editor

(continued from page 3)

Pierre-Simon Laplace (1749–1827) | Joseph Fourier (1768–1830) | Oliver Heaviside (1850–1925)

Figure 1. Laplace, Fourier, and Heaviside.

physical insight, and he shaped the electrical engineering discipline.

We hope that you enjoy this issue. We would like to acknowledge the contributing authors, the magazine's senior publications administrator, Randi E. Scholnick-Philippidis, and the production manager, Christie Inman, for their efforts in producing this issue.

Appendix: Related Articles

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