CHAPTER 22

Sampling and Filtering of Continuous Measurements

The specifications for a computer-based system to perform data acquisition and control must address several questions:

1. How often should data be acquired from a given measurement point; that is, what sampling rate should be employed?
2. Do the measurements contain a significant amount of *noise?* If so, can the data be conditioned (*filtered*) to reduce the effects of noise?
3. What digital control law should be employed?

In this chapter we will primarily be concerned with questions 1 and 2. Question 3 will be addressed in Chapter 26. The modeling and analysis of digital systems are covered in Chapters 23–25.

22.1 SAMPLING AND SIGNAL RECONSTRUCTION

As indicated in Chapter 21, when a digital computer is used for control, continuous measurements are converted into digital form by an analog to digital converter (ADC). This operation is necessary because the digital computer cannot directly process an analog signal; first the signal must be sampled at discrete points in time and then the samples must be digitized. The time interval between successive samples is referred to as the sampling period Δt. Two related terms are also used: the sampling rate $f_s = 1/\Delta t$ and the sampling frequency $\omega_s = 2\pi/\Delta t$. If Δt has units of minutes, then f_s has units of cycles per minute and ω_s has units of radians per minute.

Figure 22.1 shows an idealized periodic sampling operation in which the sampled signal $y^*(t)$ is a series of impulses that represents the measurements y_0, y_1, y_2, . . . at the sampling instants, t_0, t_1, t_2, ... The representation in Fig. 22.1 is also referred to as *impulse modulation* [1] and is used in the analysis of sampled-data systems. It is based on the assumption that the sampling operation occurs instantaneously.

In digital control applications the controller output signal must be converted from digital to analog form before being sent to the final control element. This operation is referred to as *signal reconstruction* and is performed by digital to analog converters (DACs), which were described in Chapter 21. The DAC operates as a zero-order hold as is shown schematically in Fig. 22.2. Note that the output signal

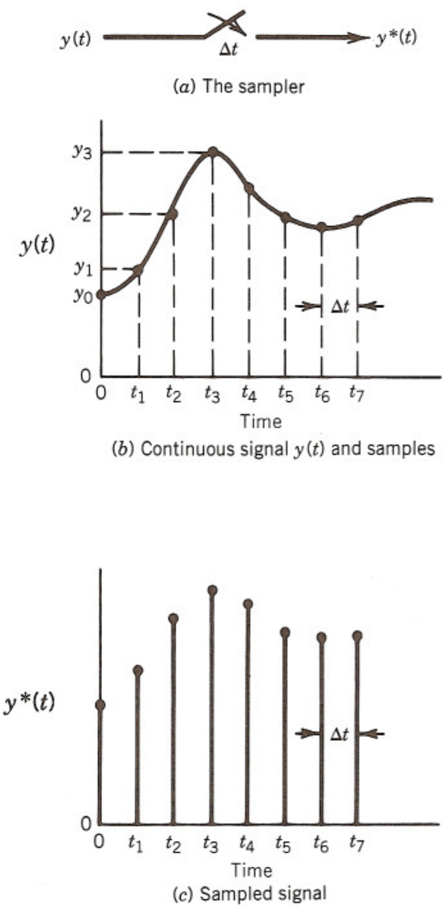


Figure 22.1. Idealized, periodic sampling.

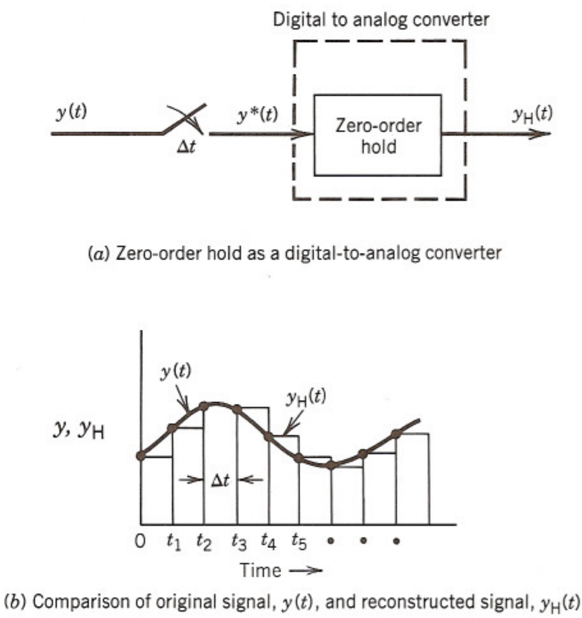


Figure 22.2. Digital-to-analog conversion using a zero-order hold.

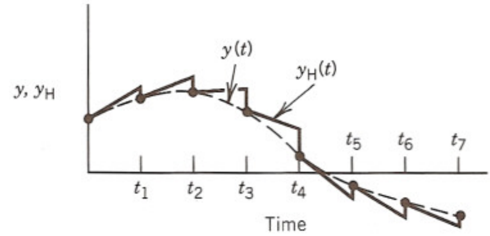


Figure 22.3. Signal reconstruction with a first-order hold.

from the zero-order hold $y_H(t)$ is held constant for one sampling period until the next sample is received. The operation of the zero-order hold can be expressed as

$$y_H(t) = y_{n-1} \qquad \text{for } t_{n-1} \leq t < t_n \tag{22-1}$$

Other types of hold devices can be employed; for example, a *first-order hold* extrapolates the digital signal linearly during the time interval from t_{n-1} to t_n based on the change during the previous interval:

$$y_H(t) = y_{n-1} + \left(\frac{t - t_{n-1}}{\Delta t}\right)(y_{n-1} - y_{n-2}) \qquad \text{for } t_{n-1} \leq t < t_n \tag{22-2}$$

Figure 22.3 illustrates the operation of a first-order hold. Although second-order and other higher-order holds can be designed and implemented as special purpose DACs [1–3], these more complicated approaches do not offer significant advantages for most process control problems. Consequently, we will emphasize the zero-order hold since it is the most widely used hold device for process control.

Figure 22.4 shows the block diagram for a typical feedback control loop with a digital controller. Note that both continuous (analog) and sampled (digital) signals appear in the block diagram. The two samplers typically operate synchronously and have the same sampling period. However, *multirate sampling* in which one

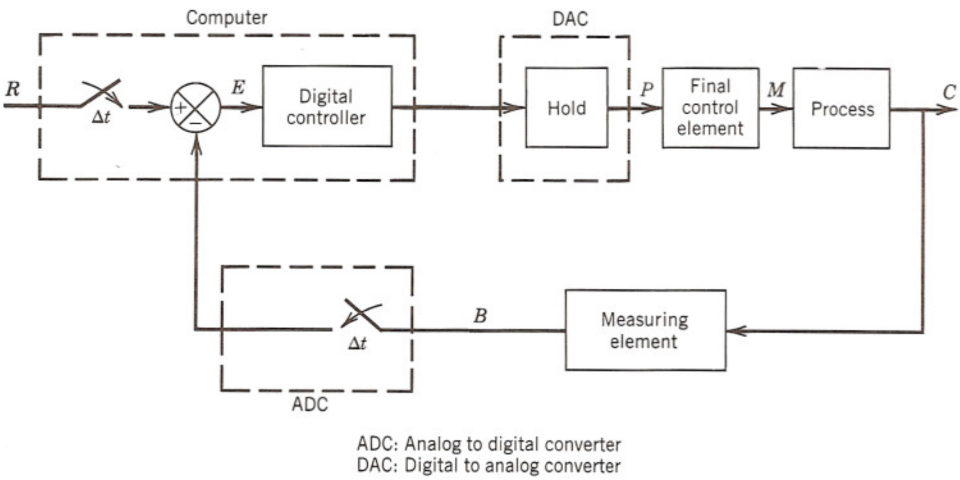


Figure 22.4. Simplified block diagram for computer control.

sampler operates at a faster rate than the other is sometimes used [1–3]. For example, we may wish to sample a process variable and filter the measurements quite frequently while performing the control calculations less often in order to avoid excessive wear in the actuator or control valve.

The block diagram in Fig. 22.4 is symbolic in that the mathematical relations between the various signals (e.g., transfer functions) are not shown. Transfer functions for sampled-data systems and the analysis of block diagrams containing samplers will be considered in Chapters 24 and 25.

22.2 SELECTION OF THE SAMPLING PERIOD

In selecting a sampling period, two questions must be considered:

1. How many measurement points does the computer monitor?
2. What is the best sampling period from a process control point of view?

If a dedicated digital control system is connected to a single measurement point, then that measurement can be sampled as often as desired, or as rapidly as the computer can perform the sampling. However, rapid sampling of a large number of measurement points may unnecessarily load the computer and restrict its ability to perform other tasks. Before introducing a number of guidelines for choosing a sampling period, we consider an important practical problem that is referred to as *aliasing*.

Aliasing

The sampling rate must be large enough so that significant process information is not lost. The loss of information that can occur during sampling is illustrated in Fig. 22.5. Suppose that a sinusoidal signal is sampled at a rate of 4/3 samples per cycle (i.e., 4/3 samples per period). This sampling rate causes the reconstructed signal to appear as a sinusoid with a much longer period than the original signal, as shown in Fig. 22.5*a*. This phenomenon is known as *aliasing*. Note that if the original sinusoidal signal were sampled only twice per period, then a constant

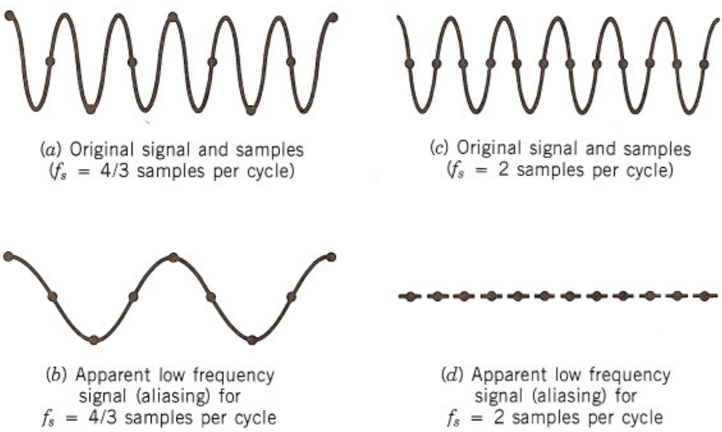


Figure 22.5. Aliasing error due to sampling too slowly.

sampled signal would result, as shown in Fig. 22.5*d*. According to Shannon's sampling theorem [1–3], a sinusoidal signal must be sampled *more* than twice each period to recover the original signal; that is, the sampling frequency must be more than twice the frequency of the sine wave.

Aliasing also occurs when a process variable that is *not* varying sinusoidally is sampled. In general, if a process measurement is sampled with a sampling frequency ω_S, high-frequency components of the process variable with a frequency greater than $\omega_S/2$ appear as low-frequency components ($\omega < \omega_S/2$) in the sampled signal. Such low-frequency components can cause control problems if they appear in the same frequency range as the normal process variations (e.g., frequencies close to the critical frequency ω_c, as discussed in Chapter 16). Aliasing can be eliminated by using an *anti-aliasing* filter, as discussed in Section 22.3.

Large Values Versus Small Values of Δt

Sampling too slowly can reduce the effectiveness of the feedback control system, especially its ability to cope with disturbances. In an extreme case, if the sampling period is longer than the process response time, then a disturbance can affect the process and the influence of the disturbance will disappear before the controller takes corrective action. In this situation the control system cannot handle transient disturbances and is capable only of steady-state control. Thus, it is important to consider the process dynamics (including disturbance characteristics) in selecting the sampling period. For composition control, the time required to complete the composition analysis (e.g., using a gas chromatograph) sets a lower limit on the sampling period.

On the other hand, there is an economic penalty associated with sampling too frequently, namely that the number of measurement points the computer can handle decreases as the sampling period Δt decreases. Since the optimum sampling period is application-specific, it is difficult to make any generalizations on this subject. However, a reasonable approach is to select a sampling period that is small enough to ensure that significant dynamic information is not lost, and then to examine whether the computer can handle the data acquisition requirements. If it cannot, then additional computing power should be considered. Commercial digital controllers which handle a small number of control loops (e.g., 8–16) typically employ a fixed sampling period of a fraction of a second. Thus, the performance of these digital controllers closely approximates continuous (analog) control.

If process conditions change significantly, then it may be necessary to change the sampling period. For example, if a feed flow rate to a processing unit is significantly increased, the residence time and hence the time constant for the unit are reduced. Consequently, it may be necessary to use a smaller sampling period in order to achieve satisfactory control. A simpler, more conservative approach would be to select the sampling period that corresponds to the worst possible conditions, that is, the smallest sampling period.

The signal-to-noise ratio (S/N) also influences the sampling period selection.[1] For low signal-to-noise ratios, rapid sampling should be avoided because changes in the measured variable from one sampling time to the next will be mainly due to high frequency noise rather than to the slower process changes. For low S/N values, a filter or filters should be used to condition the measurements in order to

[1]The signal-to-noise ratio (S/N) is usually defined as $S/N = \sigma_S^2/\sigma_N^2$, where σ_S^2 denotes the variance of the signal (or output) and σ_N^2 is the variance of a random input disturbance (i. e., "noise").

prevent the controller from acting on an *apparent* process excursion, as will be discussed in Section 22.3.

Guidelines for Selecting the Sampling Period

The selection of the sampling period remains more of an art than a science. A number of guidelines and rules of thumb have been reported for both PID controllers and model-based controllers such as the direct synthesis approach of Chapter 12 [1–4]. Representative results for PID controllers are summarized in Table 22.1.

In the early days of digital control, the suggestion was made that Δt should be selected according to the process variable being controlled (see Category 1 in

Table 22.1 Guidelines for the Selection of Sampling Periods for PID Controllers

Approach and Recommendation	*Comments*	*Reference*
1. **Type of Physical Variable**		
(a) Flow: $\Delta t = 1$ s (b) Level and pressure: $\Delta t = 5$ s (c) Temperature: $\Delta t = 20$ s	Ignore process dynamics	Williams [5]
2. **Open-Loop System**		
(a) $\Delta t < 0.1\tau_{max}$	$\tau_{max} \triangleq$ dominant time constant	Kalman and Bertram [6]
(b) $0.2 < \frac{\Delta t}{\theta} < 1.0$	For process model, $G(s) = Ke^{-\theta s}/(\tau s + 1)$	
(c) $0.01 < \frac{\Delta t}{\tau} < 0.05$	Based on (3b) and Ziegler-Nichols tuning (cf. Table 13.1)	Åström and Wittenmark [2, p. 187]
(d) $\frac{t_s}{15} < \Delta t < \frac{t_s}{6}$	t_s = settling time (95% complete)	Isermann [4]
(e) $0.25 < \frac{\Delta t}{t_r} < 0.5$	t_r = rise time for open-loop system[a]	Åström and Wittenmark [2, p. 61]
(f) $0.15\ (\Delta t)\omega_c < 0.50$	ω_c = critical frequency for continuous system (rad/s)	Åström and Wittenmark [2, p. 178]
(g) $0.050 < (\Delta t)\omega_c < 0.107$		Shinskey [7]
3. **Miscellaneous**		
(a) $\Delta t > \frac{\tau_I}{100}$	τ_I = integral time	Fertik [8]
(b) $0.1 < \frac{\Delta t}{\tau_D} < 0.5$	τ_D = derivative time	Åström and Wittenmark [2, p. 187]
(c) $0.05 < \frac{\Delta t}{\tau_D} < 0.1$		Shinskey [7]

[a] Åström and Wittenmark [2] use a rise time that differs from the definition in Section 5.4. They calculate the rise time by drawing a tangent through the inflection point of the step response; thus, it is equal to the dominant time constant. This procedure is illustrated in Fig. 7.4.

Table 22.1). According to this guideline, process variables that respond rapidly such as flow rates should be sampled more frequently than slower variables such as liquid level and temperature. However, the rules of thumb for Category 1 should be used with considerable caution since they ignore the dynamic characteristics of the individual elements in the feedback control loop. For example, if a distillation column has an open-loop response time of 4 h, a sampling period of 20 s for a temperature control loop would be much too small.

Guidelines 2a, 2b, and 2c in Table 22.1 are based on a simple transfer function model $G(s)$ which represents all of the components of the feedback control loop except the controller. The dominant time constant $\tau_{\max}$ can be determined as follows. It is set equal to the largest time constant if one time constant is much larger than the others. Alternatively, $\tau_{\max}$ can be set equal to the time constant of a first-order plus time-delay model of the process.

Guidelines 2d and 2e are based on the rise time t_r and settling time t_s of the open-loop step response of the process. Guidelines 2f and 2g relate the sampling period to the critical frequency ω_c which was defined in Section 16.2. These guidelines indicate that the recommended sampling periods are proportional to the rise time and the settling time but inversely proportional to the critical frequency.

Fertik [8] has proposed guidelines for the selection of Δt for noisy processes. He noted that the inequality in Guideline 3a should be satisfied to avoid a deadband for the integral control action, resulting from the accuracy limits of fixed-point computer calculations. Guidelines 3b and 3c relate the sampling period to the derivative time τ_D.

Next we consider a numerical example that illustrates the use of the guidelines in Table 22.1.

EXAMPLE 22.1

Consider a process with the transfer function,

$$G(s) = \frac{(2s+1)e^{-4s}}{(10s+1)(7s+1)(3s+1)}$$

where $G(s) = G_v(s)G_p(s)G_m(s)$ and the time constants and the time delay have units of minutes. Calculate recommended sampling periods for a PID controller based on Categories 2 and 3 in Table 22.1. For Category 3 assume that $\tau_I = 15$ min and $\tau_D = 4$ min.

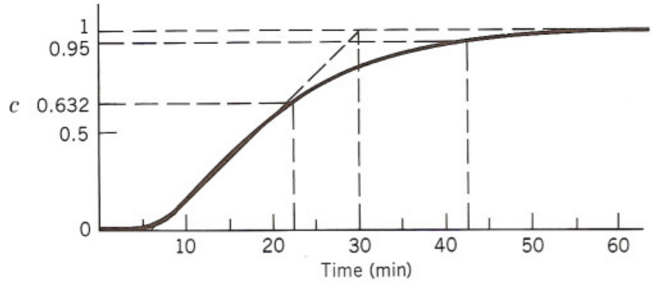
Solution

To use the guidelines in Table 22.1, we need to calculate τ, θ, t_r, $\tau_{\max}$, t_s, and ω_c. The first five of these parameters can be calculated from the open-loop response to a unit step change as shown in Fig. 22.6. The critical frequency ω_c can be determined from the following phase angle expression:

$$-180^\circ = -\tan^{-1}(10\omega_c) - \tan^{-1}(7\omega_c) - \tan^{-1}(3\omega_c) + \tan^{-1}(2\omega_c) - 4\omega_c\,(180/\pi)$$

A trial-and error solution gives $\omega_c \cong 0.21$ rad/min. The recommended sampling periods are shown in Table 22.2.

The results in Table 22.2 indicate that the recommended values of Δt vary over nearly two orders of magnitude, from 0.16 to 12 min. Guidelines 2b, 2d, and 2e produce the largest Δt values while Guidelines 2c, 2g, and 3c result in the smallest values.



Thus:
$\theta = 6$ min $\quad t_r = 30 - 6 = 24$ min
$t_s = 43$ min (95% settling time) $\tau = \tau_{max} = 22 - 6 = 16$ min

Figure 22.6. An open-loop step response and a graphical determination of rise time t_r and settling time t_s.

The effect of sampling period on control system performance for this example has been evaluated by Isermann [4]. His simulation results for set-point changes are shown in Fig. 22.7. He selected four sampling periods and used a digital PID controller with the controller settings chosen to minimize an Integral Squared Error performance index, as described in Chapter 12. Figure 22.7 indicates that the digital controller provides a good approximation to a continuous PID controller when $\Delta t = 1$ min because the manipulated variable changes in almost a continuous fashion. The response times and periods of oscillation increase as Δt increases until finally the closed-loop response is quite poor for $\Delta t = 16$ min. Based on these and other considerations, Isermann [4] concludes that an appropriate Δt for this example is in the range of 4 to 8 min.

Table 22.2 Recommended Sampling Periods for Example 22.1

Guideline	Numerical Value of Δt (min)
(2a) $\Delta t < 0.1\tau_{max}$	$\Delta t < 1.6$
(2b) $0.2 < \frac{\Delta t}{\theta} < 1.0$	$1.2 < \Delta t < 6.0$
(2c) $0.01 < \frac{\Delta t}{\tau} < 0.05$	$0.16 < \Delta t < 0.8$
(2d) $\frac{t_s}{15} < \Delta t < \frac{t_s}{6}$	$2.9 \le \Delta t \le 7.2$
(2e) $0.25 < \frac{\Delta t}{t_r} < 0.5$	$6 \le \Delta t \le 12$
(2f) $0.15 < (\Delta t)\omega_c < 0.5$	$0.71 \le \Delta t \le 2.4$
(2g) $0.05 < (\Delta t)\omega_c < 0.107$	$0.24 \le \Delta t \le 0.51$
(3a) $\Delta t > \frac{\tau_I}{100}$	$\Delta t > 0.15$ min
(3b) $0.1 < \frac{\Delta t}{\tau_D} < 0.5$	$0.4 < \Delta t < 2.0$
(3c) $0.05 < \frac{\Delta t}{\tau_D} < 0.1$	$0.2 < \Delta t < 0.4$

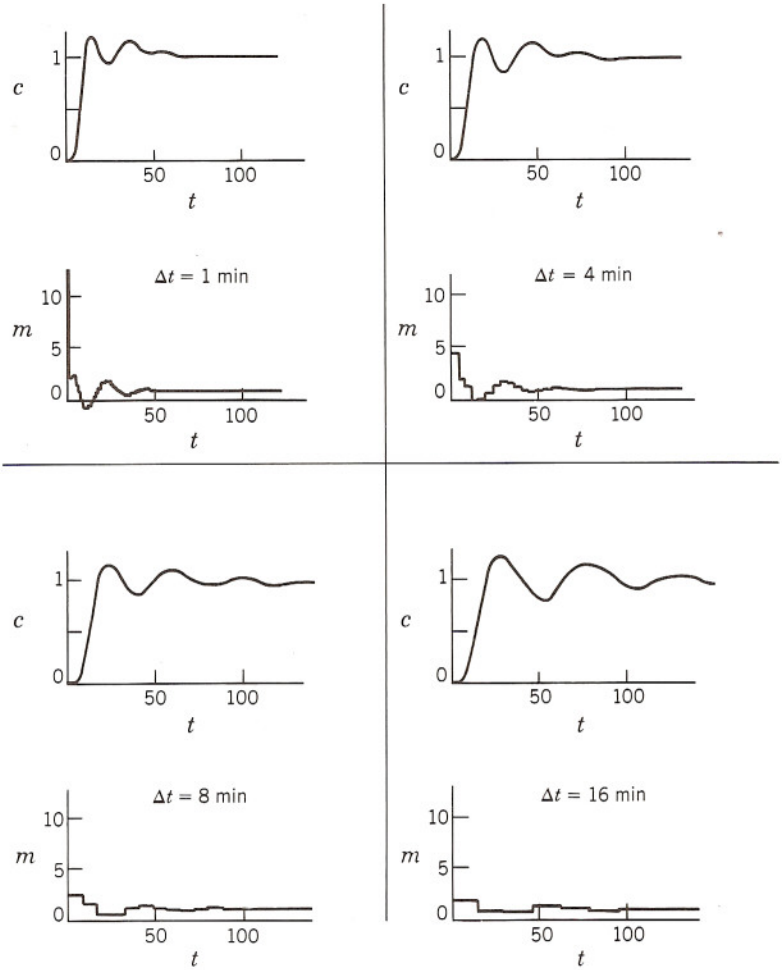


Figure 22.7. Closed-loop responses for Example 22.1 and four different sampling periods [4].

This example illustrates that published guidelines for selection of the sampling period can result in a very wide range of recommended values. Thus, although the guidelines provide useful information, a specific one should not be used blindly without some comparisons (see also Example 26.2).

22.3 SIGNAL PROCESSING AND DATA FILTERING

In process control, the noise associated with analog signals can arise from a number of sources: the measurement device, electrical equipment, or the process itself. The effects of electrically generated noise can be minimized by following established procedures concerning shielding of cables, grounding, and so forth [9]. Process-induced noise can arise from variations due to mixing, turbulence, and nonuniform multiphase flows. The effects of both process noise and measurement noise can be reduced by signal conditioning or filtering. In electrical engineering parlance, the term "filter" is synonymous with "transfer function," since a filter transforms input signals to yield output signals.

Analog Filters

Analog filters have been used for many years to smooth noisy experimental data. For example, an *exponential filter* can be used to damp out high-frequency fluctuations due to electrical noise; hence it is called a low-pass filter. Its operation can be described by a first-order transfer function or equivalently a first-order differential equation,

$$\tau_F \frac{dy(t)}{dt} + y(t) = x(t) \tag{22-3}$$

where x is the measured value (the filter input), y is the filtered value (the filter output), and τ_F is the time constant of the filter. Note that the filter has a steady-state gain of one. The exponential filter is also called an *RC filter* since it can be constructed from a simple RC electrical circuit.

As shown in Fig. 22.5, relatively slow sampling of a high-frequency signal can produce an artificial low-frequency signal. Therefore it is desirable to use an analog filter to *prefilter* process data before sampling in order to remove high-frequency noise as much as possible. For these applications, the analog filter is often referred to as an *anti-aliasing filter*. This allows the sampling period to be selected independent of signal conditioning considerations. For applications where τ_F is less than three seconds, passive analog filters constructed from resistance–capacitance components are suitable. For slowly varying dynamic signals such as *drifts* where τ_F must be greater than three seconds, active analog filters are constructed using amplifiers. However, amplifier-based filters are more expensive than equivalent digital filters implemented via computer software. Consequently, for very slowly varying signals, it may be better to perform digital filtering since the required sampling period will not be very small and the extra computational burden on the computer will be quite modest.

The filter time constant τ_F should be much smaller than the dominant time constant of the process τ_{max} to avoid introducing a significant dynamic lag in the feedback control loop. For example, choosing $\tau_F < 0.1\tau_{max}$ satisfies this requirement. On the other hand, if the noise amplitude is high, then a larger value of τ_F may be required to *smooth* the noisy measurements. The frequency range of the noise is another important consideration. Suppose that the lowest noise frequency expected is denoted by ω_N. Then τ_F should be selected so that $\omega_F < \omega_N$ where $\omega_F = 1/\tau_F$. For example, suppose we specify $\omega_F = 0.1\omega_N$ which corresponds to $\tau_F = 10/\omega_N$. Then noise at frequency ω_N will be attenuated by a factor of 10 according to Eqs 14-20a and the Bode diagram of Fig. 14.2. In summary, τ_F should be selected so that $\omega_{max} < \omega_F < \omega_N$ where $\omega_F = 1/\tau_F$ and $\omega_{max} = 1/\tau_{max}$.

Digital Filters

In this section we consider several popular digital filters. A more comprehensive treatment of digital filtering and signal processing techniques is available elsewhere [10].

Exponential Filter. First we consider a digital version of the exponential filter. We will denote the samples of the measured variable as x_{n-1}, x_n . . . and the corresponding filtered values as y_{n-1}, y_n . . . where n refers to the current sampling instant. The derivative in (22-3) at time step n can be approximated by the backward difference:

$$\frac{dy}{dt} \cong \frac{y_n - y_{n-1}}{\Delta t} \tag{22-4}$$

Substituting in (22-3) and replacing $y(t)$ by y_n and $x(t)$ by x_n yields

$$\tau_F \frac{y_n - y_{n-1}}{\Delta t} + y_n = x_n \tag{22-5}$$

Rearranging gives

$$y_n = \frac{\Delta t}{\tau_F + \Delta t} x_n + \frac{\tau_F}{\tau_F + \Delta t} y_{n-1} \tag{22-6}$$

We define

$$\alpha \triangleq \frac{1}{\tau_F/\Delta t + 1} \tag{22-7}$$

where $0 < \alpha \leq 1$. Then

$$1 - \alpha = 1 - \frac{1}{\tau_F/\Delta t + 1} = \frac{\tau_F}{\tau_F + \Delta t} \tag{22-8}$$

so that

$$y_n = \alpha x_n + (1 - \alpha) y_{n-1} \tag{22-9}$$

Equation 22-9 indicates that the filtered measurement is a weighted sum of the current measurement x_n and the filtered value at the previous sampling instant y_{n-1}. This operation is also called *single exponential smoothing*. Limiting cases for α are

$\alpha = 1$:	No filtering (the filter output is the raw measurement x_n).
$\alpha \to 0$:	The measurement is ignored.

In the above limits, note that $\tau_F = \Delta t(1 - \alpha)/\alpha$ by solving (22-7); hence, $\alpha = 1$ corresponds to a filter time constant of zero (no filtering).

Alternative expressions for α in (22-9) can be derived if the forward difference or other integration schemes for dy/dt are utilized [1].

Double Exponential Filter. Another popular digital filter is the double exponential or second-order filter, which offers some advantages for eliminating high-frequency noise. The second-order filter is equivalent to two first-order filters in series where the second filter treats the output signal from the exponential filter in Eq. 22-9. The second filter can be expressed as

$$\bar{y}_n = \gamma y_n + (1 - \gamma)\bar{y}_{n-1} \tag{22-10}$$

$$\bar{y}_n = \gamma \alpha x_n + \gamma(1 - \alpha) y_{n-1} + (1 - \gamma)\bar{y}_{n-1} \tag{22-11}$$

Writing the filter equation in Eq. 22-10 for the previous sampling instant gives

$$\bar{y}_{n-1} = \gamma y_{n-1} + (1 - \gamma)\bar{y}_{n-2} \tag{22-12}$$

Solve for y_{n-1}:

$$y_{n-1} = \frac{1}{\gamma}\,\bar{y}_{n-1} - \frac{1-\gamma}{\gamma}\,\bar{y}_{n-2} \tag{22-13}$$

Substituting (22-13) into (22-11) and rearranging gives the following expression for the double exponential filter:

$$\bar{y}_n = \gamma\alpha x_n + (2 - \gamma - \alpha)\bar{y}_{n-1} - (1 - \alpha)(1 - \gamma)\bar{y}_{n-2} \tag{22-14}$$

A common simplification is to select $\gamma = \alpha$, yielding

$$\bar{y}_n = \alpha^2 x_n + 2(1 - \alpha)\bar{y}_{n-1} - (1 - \alpha)^2\bar{y}_{n-2} \tag{22-15}$$

The advantage of the double exponential filter over the exponential filter of Eq. 22-9 is that it provides better filtering of high-frequency noise, especially if $\gamma = \alpha$. The Bode diagrams in Figs. 14.2 and 14.3 provide a frequency response interpretation of this result. Note that second-order transfer function in Fig. 14.3 provides greater attenuation of high-frequency signals than the first-order system in Fig. 14.2. Although these Bode diagrams are for continuous systems (or filters), analogous results occur for digital systems (or filters).

A disadvantage of the double exponential filter is that it is more complicated than the exponential filter. Consequently, the single exponential filter has been more widely used in process control applications.

Moving Average Filter. A third type of digital filter is the moving-average filter which averages a specified number of past data points, by giving equal weight to each data point. The moving-average filter is usually less effective than the exponential filter, which gives more weight to the most recent data.

The moving-average filter can be expressed as

$$y_n = \frac{1}{J}\sum_{i=n-J+1}^{n} x_i \tag{22-16}$$

where J is the number of past data points that are being averaged. Equation 22-16 implies that the previous filtered value, y_{n-1}, can be expressed as

$$y_{n-1} = \frac{1}{J}\sum_{i=n-J}^{n-1} x_i \tag{22-17}$$

Subtracting (22-17) from (22-16) gives the recursive form of the moving-average filter:

$$y_n = y_{n-1} + \frac{1}{J}(x_n - x_{n-J}) \tag{22-18}$$

The exponential and moving-average filters are examples of low-pass filters which are used to smooth noisy data by eliminating high-frequency noise.

Noise-Spike Filter. If a noisy measurement changes suddenly by a large amount and then returns to the original value (or close to it) at the next sampling instant, a *noise spike* is said to occur. Figure 22.8 shows two noise spikes appearing in the experimental temperature data for a fluidized sand bath. In general, noise spikes can be caused by spurious electrical signals in the environment of the sensor. If noise spikes are not removed by filtering before the noisy measurement is sent to

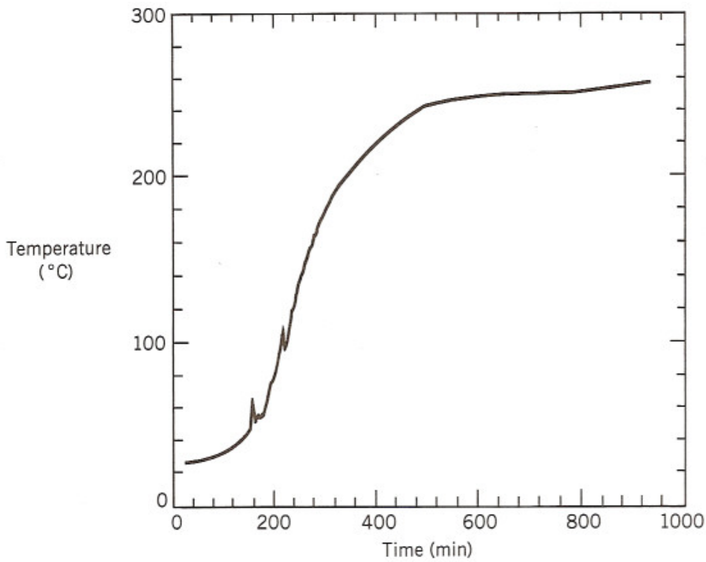


Figure 22.8. Temperature response data [11] from a fluidized sand bath contains two noise spikes.

the controller, the controller will cause large, sudden changes in the manipulated variable.

Noise-spike filters (or *rate of change* filters) are used to limit how much the filtered output is permitted to change from one sampling instant to the next. If Δx denotes the maximum allowable change, the noise-spike filter can be written as

$$y_n = \begin{cases} x_n & \text{if } |x_n - y_{n-1}| \leq \Delta x \\ y_{n-1} - \Delta x & \text{if } y_{n-1} - x_n > \Delta x \\ y_{n-1} + \Delta x & \text{if } y_{n-1} - x_n < -\Delta x \end{cases} \tag{22-19}$$

If a large change in the measurement occurs, the filter replaces the measurement by the previous filter output plus (or minus) the maximum allowable change. This filter can also be used to detect instrument malfunctions such as a power failure, a break in a thermocouple or instrument line, or an ADC "glitch."

Other types of more sophisticated digital filters are available but have not been commonly used in process control applications. These include high-pass filters and bandpass filters [4,9,10].

22.4 COMPARISON OF ANALOG AND DIGITAL FILTERS

Analog and digital filters can be compared as follows:

1. Digital filters can be easily tuned (programmed) to fit the process. They are also easily modified.
2. Digital filters require the choice of a sampling period while analog filters do not.
3. Because digital filters require computation time and computer storage, they sometimes can limit the effectiveness of the data acquisition and control system. Analog filters are separate hardware devices and do not interact with the other computational tasks of the computer.

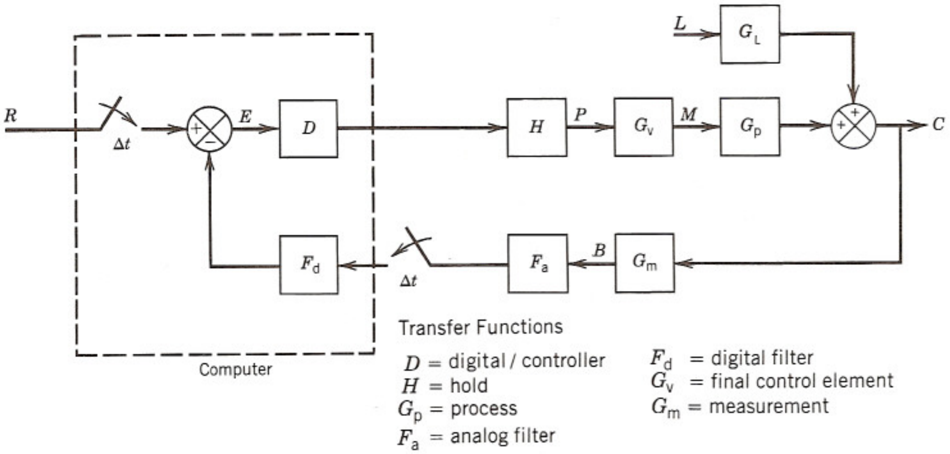


Figure 22.9. A block diagram with both analog and digital filters.

4. Analog filters are particularly effective for the elimination of high-frequency noise and aliasing.

It can be advantageous to use both analog and digital filters, as shown in the block diagram in Fig. 22.9. In this configuration high-frequency noise is filtered by the analog filter, while the digital filter suppresses lower-frequency (process) noise.

EXAMPLE 22.2

To compare the performance of alternative filters, consider a square wave signal with $f = 0.33$ cycles/min and an amplitude 0.5 corrupted by

(i) High-frequency sinusoidal noise (amplitude $= 0.25$, $f_N = 9$ cycles/min)
(ii) Random (Gaussian) noise with zero mean and a variance of 0.01.

Evaluate both analog and digital exponential filters as well as moving average filters.

Solution

(i) Sinusoidal noise

Representative results for high-frequency sinusoidal noise are shown in Fig. 22.10. The square wave with the additive noise is shown in Fig. 22.10*a*. The performance of two analog, exponential filters is shown in Fig. 22.10*b*. Choosing a relatively large filter time constant ($\tau_F = 0.4$ min) results in a filtered signal that contains less noise but is more sluggish, compared to the response for $\tau_F = 0.1$ min.

The effect of sampling period Δt on digital filter performance is illustrated in Fig. 22.10*c*. A larger sampling period ($\Delta t = 0.1$ min) results in serious aliasing because $f_s = 1/\Delta t = 10$ cycles/min, which is less than $2f_N = 18$ cycles/min. Reducing Δt by a factor of two results in much better performance. For each filter, a value of $\tau_F = 0.1$ min was chosen because this value was satisfactory for the analog filter of Fig. 22.10*b*. The smaller value of α (0.33 for $\Delta t = 0.05$ min vs. 0.5 for $\Delta t = 0.1$ min) provides more filtering.

The performance of two moving-average filters with $\Delta t = 0.05$ min is shown in Fig. 22.10*d*. Choosing $J = 7$ results in better filtering because this moving-average filter averages the sinusoidal noise over several cycles, while $J = 3$ gives a faster response but larger fluctuations.

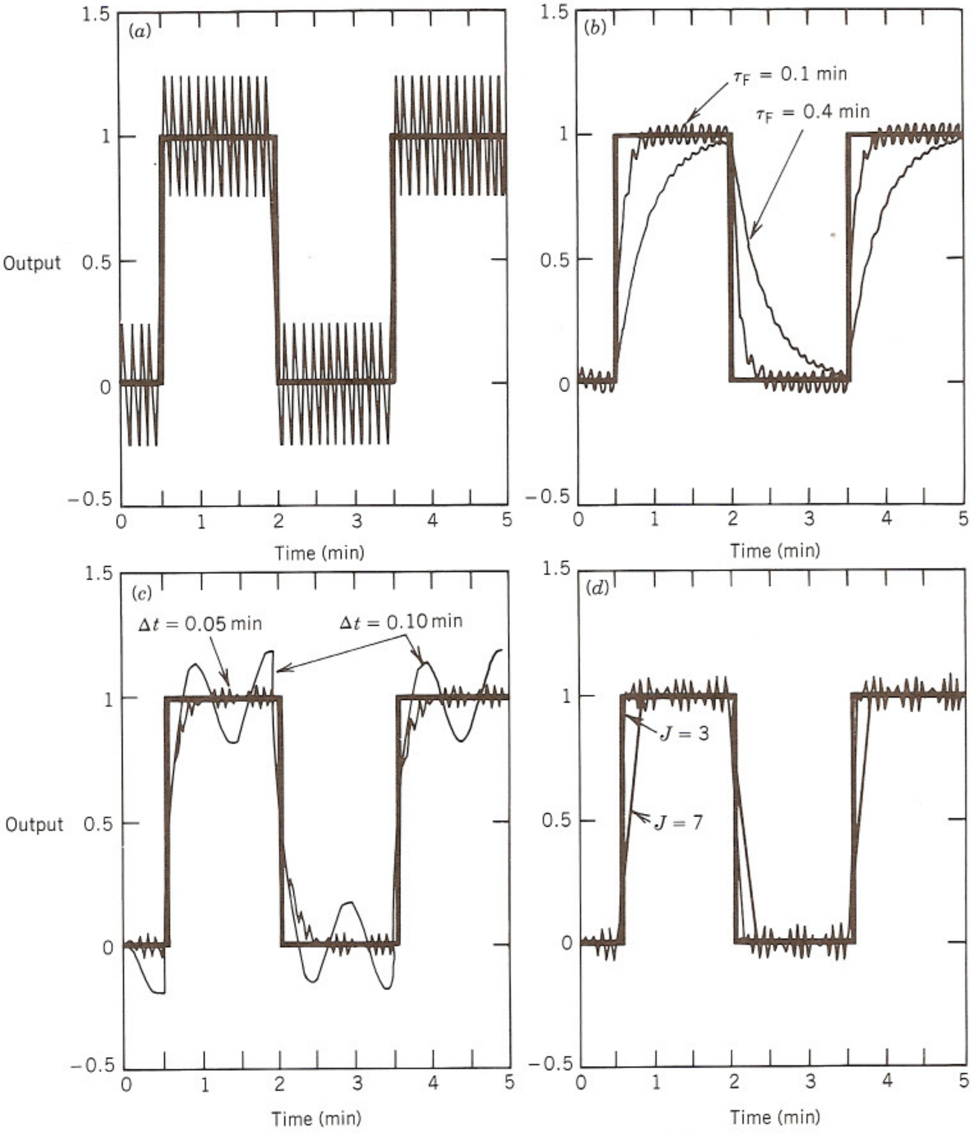


Figure 22.10. A comparison of filter performance for additive sinusoidal noise: (*a*) square wave plus noise; (*b*) analog exponential filters; (*c*) digital exponential filters; (*d*) moving-average filters.

(ii) Random noise

The filters considered in part (a) of this example were also evaluated for the situation where Gaussian noise was added to the same square wave signal. The simulations illustrating the effects of this noise level are shown in Fig. 22.11. Figure 22.11*a* shows the unfiltered signal after Gaussian noise with zero mean and a variance of 0.01 was added to the square wave signal. The analog, exponential filters in Fig. 22.11*b* provide effective filtering and again show the trade-off between degree of filtering and sluggish response that is inherent in the choice of τ_F. The digital filters in Fig. 22.11*c* and *d* are less effective even though different values of Δt and J were considered. Some aliasing occurs due to the high-frequency components of the random noise, which prevents the digital filter from performing as well as the analog filter.

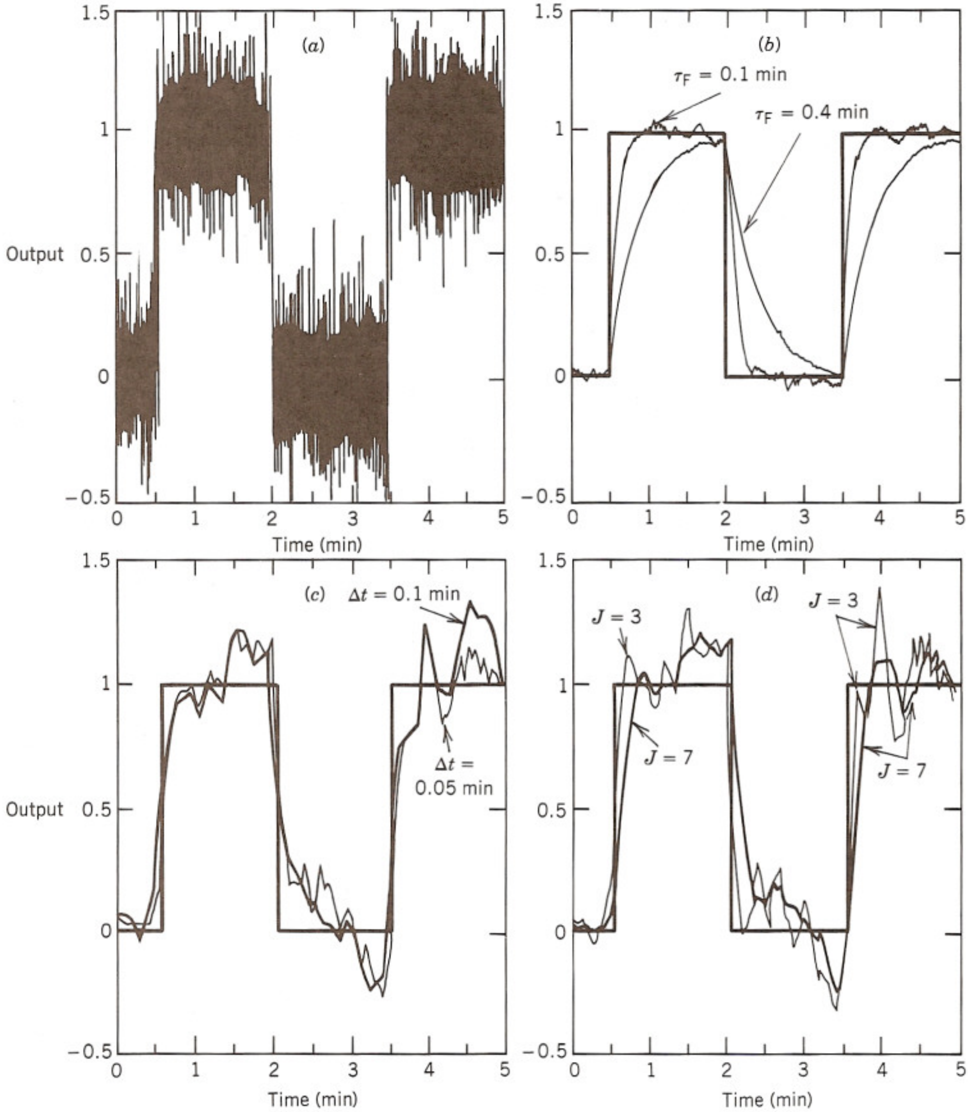


Figure 22.11. Comparison of filter performance for additive Gaussian noise: (*a*) Square wave plus noise; (*b*) analog exponential filters, (*c*) digital exponential filters; (*d*) moving-average filters.

In conclusion, both analog and digital filters can smooth noisy signals providing that the filter design parameters (including sampling period) are carefully selected.

22.5 EFFECT OF FILTER SELECTION ON CONTROL SYSTEM PERFORMANCE

Digital and analog filters are valuable for smoothing data and eliminating high-frequency noise, but they also affect control system performance. In particular a filter is an additional dynamic element in the feedback loop that causes a phase lag. Consequently, it reduces the stability margin for a feedback controller, compared to the situation where there is no filter [12]. Therefore, the controller may have to be retuned if the filter constant is changed. Derivative action can be included in the controller to provide phase lead which helps compensate for the phase lag due to the filter. However, when derivative action is used, it is important to filter noisy signals before the derivative control calculations are performed [2]. Because derivative action tends to amplify noise in the process measurement, filtering helps prevent controller saturation. Many electronic PID controllers in effect contain a high-frequency filter within their circuitry [13]. If the measurement signal is not filtered and process noise is significant, then derivative action should not be employed.

SUMMARY

When a digital computer is used for process control, measurements of the process variables are sampled and converted into digital form by an analog to digital converter (ADC). The sampling period Δt must be carefully selected. Sampling too slowly can produce aliasing and also reduce the effectiveness of the feedback control system. On the other hand, sampling too frequently tends to increase the data acquisition requirements of the computer. The choice of the sampling period should be based on the process dynamics, noise frequencies, signal-to-noise ratio, and the available computer control system.

Noisy measurements should be filtered before being sent to the controller. Analog filters are effective in removing high-frequency noise and avoiding aliasing. Digital filters are also widely used both for low-pass filters and other purposes such as the elimination of noise spikes. The choice of a filter and the filter parameters (e.g., τ_F) should be based on the process dynamics, the noise characteristics, and the sampling period. If a filter parameter is changed, it may be necessary to retune the controller since the filter is a dynamic element in the feedback control loop.

REFERENCES

1. Franklin, G. F., and J. D. Powell, *Digital Control of Dynamic Systems,* Addison-Wesley, Reading, MA, 1980.
2. Åström, K. J., and B. Wittenmark, *Computer Controlled Systems,* Prentice-Hall, Englewood Cliffs, NJ, 1984.
3. Ogata, K., *Discrete-Time Control Systems,* Prentice-Hall, Englewood Cliffs, NJ, 1987.
4. Isermann, R., *Digital Control Systems,* Springer-Verlag, New York, 1981, Chapter 27.
5. Williams, T. J., Economics and the Future of Process Control, *Automatica* **3,** 1 (1965).
6. Kalman, R. E., and J. E. Bertram, General Synthesis Procedure for Computer Control of Single-Loop and Multi-Loop Systems, *AIEE Trans.* **77,** Part 2, 602 (1958).
7. Shinskey, F. G., *Process Control Systems,* 3d ed., McGraw-Hill, New York, 1988.
8. Fertik, H. A., Tuning Controllers for Noisy Processes, *ISA Trans.* **14,** 4 (1975).
9. Wright, J. D., and T. F. Edgar, Digital Computer Control and Signal Processing Algorithms, in *Real-Time Computing,* (D. A. Mellichamp, Ed.), Van Nostrand Reinhold, New York, 1983, Chapter 22.
10. Oppenheim, A. V., and R. W. Shafer, *Digital Signal Processing,* Prentice-Hall, Englewood Cliffs, NJ, 1975.
11. Phillips, S. F., and D. E. Seborg, Adaptive Control Strategies for Achieving Desired Temperature Control Profiles During Process Startup, *IEC Res.* **27,** 1434 (1987).
12. Corripio, A. B., C. L. Smith, and P. W. Murrill, Filter Design for Digital Control Loops, *Instrum. Tech.,* **20**(1), 33 (1973).
13. Hougen, J. O., *Measurements and Control Applications,* 2d ed. ISA, Research Triangle Park, NC, 1979.

EXERCISES

22.1. A distillation column is subjected to a unit step change in feed flow rate F. Response data for the overhead product composition x_D are shown below. Previous experience has indicated that the transfer function,

$$\frac{X_D(s)}{F(s)} = \frac{5}{10s + 1}$$

provides an accurate dynamic model. Filter these data using an exponential filter with two different values of α, 0.5 and 0.8. Graphically compare the noisy data, the filtered data, and the analytical solution for the transfer function model.

Time (min)	x_D	*Time (min)*	x_D
0	0	11	3.336
1	0.495	12	3.564
2	0.815	13	3.419
3	1.374	14	3.917
4	1.681	15	3.884
5	1.889	16	3.871
6	2.078	17	3.924
7	2.668	18	4.300
8	2.533	19	4.252
9	2.908	20	4.409
10	3.351		

22.2. Show that the digital exponential filter output can be written as a function of previous measurements and the initial filter output y_0.

22.3. A signal given by

$$y(t) = t + 0.5 \sin (t^2)$$

is to be filtered with an exponential digital filter over the interval $0 \leq t \leq 20$. Using three different values of α (0.8, 0.5, 0.2), find the output of the filter at each sampling time. Do this for sampling periods of 1.0 and 0.1. Compare the three filters for each value of Δt.

22.4. The following product quality data were obtained from a reactor, based on a color evaluation of the product:

t (min)	*x (color index)*
0	0
1	1.5
2	0.3
3	1.6
4	0.4
5	1.7
6	1.5
7	2.0
8	1.5

(a) Filter the data using an exponential filter with $\Delta t = 1$ min. Use $\alpha = 0.2$ and $\alpha = 0.5$.
(b) Use a moving average filter with $J = 4$.
(c) Implement a noise-spike filter with $\Delta x = 0.5$.
(d) Plot the filtered data and the raw data for purposes of comparison.

22.5. The analog exponential filter in Eq. 22-3 is used to filter a measurement before it is sent to a proportional-only feedback controller with $K_c = 1$. The other transfer functions for the closed-loop system are $G_v = G_m = 1$, and $G_p = G_L = 1/(5s + 1)$. Compare the closed-loop responses to a sinusoidal load disturbance, $L(t) = \sin t$, for no filtering ($\tau_F = 0$) and for an exponential filter ($\tau_F = 3$ min).

22.6. Consider the first-order transfer function $Y(s)/X(s) = 1/(s + 1)$. Generate a set of data ($t = 1, 2, \ldots, 20$) by integrating this equation for $x = 1$ and randomly adding binary noise to the output, ± 0.05 units at each integer value of t. Design a digital filter for this system and compare the filtered and noise-free step responses for $\Delta t = 1$. Justify your choice of τ_F. Repeat for other noise levels, for example, ± 0.01 and ± 0.1.