2024-05-25 - Handout – Convex Hull Algorithm

Question 1: Cow Curling

Cow curling is a popular cold-weather sport played in the Moolympics. Like regular curling, the sport involves two teams, each of which slides N heavy stones (3 <= N <= 50,000) across a sheet of ice. At the end of the game, there are 2N stones on the ice, each located at a distinct 2D point.

Scoring in the cow version of curling is a bit curious, however. A stone is said to be "captured" if it is contained inside a triangle whose corners are stones owned by the opponent (a stone on the boundary of such a triangle also counts as being captured). The score for a team is the number of opponent stones that are captured. Please help compute the final score of a cow curling match, given the locations of all 2N stones.

INPUT FORMAT:

- Line 1: The integer N.
- Lines 2..1+N: Each line contains 2 integers specifying the x and y coordinates of a stone for team A (each coordinate lies in the range -40,000 .. +40,000).
- Lines 2+N..1+2N: Each line contains 2 integers specifying the x and y coordinates of a stone for team B (each coordinate lies in the range -40,000 .. +40,000).

SAMPLE INPUT (file curling.in):

4

0 0

0 2

2 0

2 2

1 1

1 10

-10 3

10 3

SAMPLE OUTPUT (file curling.out): 1 2 (Two space-separated integers, giving the scores for teams A and B)

Question 2: Random Pawn (*Will discuss based on time constraints*)

You are playing a game and your goal is to maximize your expected gain. At the beginning of the game, a pawn is put, uniformly at random, at a position $p \in \{1,2,\ldots,N\}$. The N positions are arranged on a circle (so that 1 is between N and 2).

The game consists of turns. At each turn you can either end the game, and get A_p dollars (where p is the current position of the pawn), or pay B_p dollar to keep playing. If you decide to keep playing, the pawn is randomly moved to one of the two adjacent positions $p-1$, $p+1$ (with the identifications $0=N$ and $N+1=1$).

What is the expected gain of an optimal strategy? (Note: The "expected gain of an optimal strategy" shall be defined as the supremum of the expected gain among all strategies such that the game ends in a finite number of turns.)

CONSTRAINTS:

$2 \le N \le 200{,}000$

$0 \le A_p \le 10^{12}$ for any $p=1,\ldots,N$

$0 \le B_p \le 100$ for any $p=1,\ldots,N$

All values in input are integers.

INPUT FORMAT:

- N
- $A_1\ A_2 \cdots A_N$
- $B_1\ B_2 \cdots B_N$

OUTPUT:

Print a single real number, the .

SAMPLE INPUT:

5

4 2 6 3 5

1 1 1 1 1

SAMPLE OUTPUT: 4.700000000000 (expected gain of an optimal strategy as a single real number. Relative/absolute error should not exceed 10^{-10})