

2024-04-20 - Handout – Kruskals Algorithm

Q1. Longest Consecutive Sequence

Link: <https://leetcode.com/problems/longest-consecutive-sequence/>

Given an unsorted array of integers nums, return the length of the longest consecutive elements sequence.

You must write an algorithm that runs in $O(n)$ time.

Example 1:

Input: nums = [100,4,200,1,3,2]

Output: 4

Explanation: The longest consecutive elements sequence is [1, 2, 3, 4]. Therefore its length is 4.

Example 2:

Input: nums = [0,3,7,2,5,8,4,6,0,1]

Output: 9

Q2. Min Cost to Connect All Points

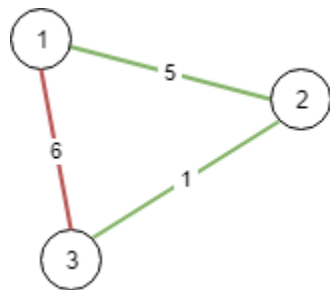
Link: <https://leetcode.com/problems/connecting-cities-with-minimum-cost/description/>

There are n cities labeled from 1 to n . You are given the integer n and an array connections where $\text{connections}[i] = [x_i, y_i, \text{cost}_i]$ indicates that the cost of connecting city x_i and city y_i (bidirectional connection) is cost_i .

Return the minimum cost to connect all the n cities such that there is at least one path between each pair of cities. If it is impossible to connect all the n cities, return -1,

The cost is the sum of the connections' costs used

Example 1:



Input: $n = 3$, connections = $[[1,2,5],[1,3,6],[2,3,1]]$

Output: 6

Explanation: Choosing any 2 edges will connect all cities so we choose the minimum 2.

Q3. Find Critical and Pseudo-Critical Edges in Minimum Spanning Tree

Link: <https://leetcode.com/problems/find-critical-and-pseudo-critical-edges-in-minimum-spanning-tree/description/>

Given a weighted undirected connected graph with n vertices numbered from 0 to $n - 1$, and an array edges where $\text{edges}[i] = [a_i, b_i, \text{weight}_i]$ represents a bidirectional and weighted edge between nodes a_i and b_i . A minimum spanning tree (MST) is a subset of the graph's edges that connects all vertices without cycles and with the minimum possible total edge weight.

Find all the critical and pseudo-critical edges in the given graph's minimum spanning tree (MST). An MST edge whose deletion from the graph would cause the MST weight to increase is called a *critical edge*. On the other hand, a pseudo-critical edge is that which can appear in some MSTs but not all.

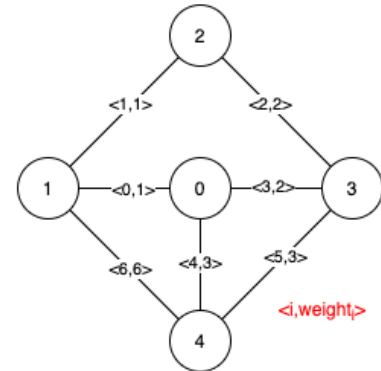
Note that you can return the indices of the edges in any order.

Example 1:

Input: $n = 5$, edges = $[[0,1,1],[1,2,1],[2,3,2],[0,3,2],[0,4,3],[3,4,3],[1,4,6]]$

Output: $[[0,1],[2,3,4,5]]$

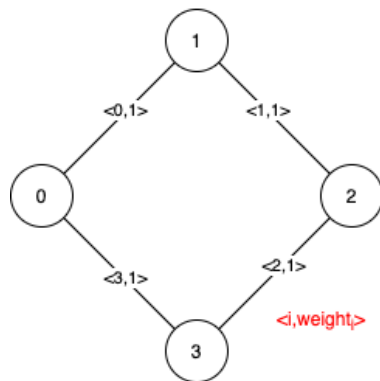
Explanation: The figure beside describes the graph.



The following figure shows all the possible MSTs:

Notice that the two edges 0 and 1 appear in all MSTs, therefore they are critical edges, so we return them in the first list of the output. The edges 2, 3, 4, and 5 are only part of some MSTs, therefore they are considered pseudo-critical edges. We add them to the second list of the output.

Example 2:



Input: $n = 4$,
edges =

$[[0,1,1],[1,2,1],[2,3,1],[0,3,1]]$

Output: $[[],[0,1,2,3]]$

Explanation: We can observe that since all 4 edges have equal weight, choosing any 3 edges from the given 4 will yield an MST. Therefore all 4 edges are pseudo-critical

