2024-05-25 - Handout – Convex Hull Algorithm

**Question 1: Cow Curling**

Cow curling is a popular cold-weather sport played in the Moolympics.  Like regular curling, the sport involves two teams, each of which slides N heavy stones (3 <= N <= 50,000) across a sheet of ice. At the end of the game, there are 2N stones on the ice, each located at a distinct 2D point.

Scoring in the cow version of curling is a bit curious, however.  A stone is said to be "captured" if it is contained inside a triangle whose corners are stones owned by the opponent (a stone on the boundary of such a triangle also counts as being captured).  The score for a team is the

number of opponent stones that are captured. Please help compute the final score of a cow curling match, given the locations of all 2N stones.

INPUT FORMAT:

* Line 1: The integer N.
* Lines 2..1+N: Each line contains 2 integers specifying the x and y coordinates of a stone for team A (each coordinate lies in the range -40,000 .. +40,000).
* Lines 2+N..1+2N: Each line contains 2 integers specifying the x and y coordinates of a stone for team B (each coordinate lies in the range -40,000 .. +40,000).

SAMPLE INPUT (file curling.in):

4

0 0

0 2

2 0

2 2

1 1

1 10

-10 3

10 3

SAMPLE OUTPUT (file curling.out): 1 2 (Two space-separated integers, giving the scores for teams A and B)

**Question 2: Random Pawn** (\*Will discuss based on time constraints\*)

You are playing a game and your goal is to maximize your expected gain. At the beginning of the game, a pawn is put, uniformly at random, at a position p∈{1,2,…,N}. The N positions are arranged on a circle (so that 1 is between N and 2).

The game consists of turns. At each turn you can either end the game, and get Ap dollars (where p is the current position of the pawn), or pay Bp dollar to keep playing. If you decide to keep playing, the pawn is randomly moved to one of the two adjacent positions p−1, p+1 (with the identifications 0=N and N+1=1).

What is the expected gain of an optimal strategy? (Note: The "expected gain of an optimal strategy" shall be defined as the supremum of the expected gain among all strategies such that the game ends in a finite number of turns.)

CONSTRAINTS:

2≤N≤200,000

0≤Ap ≤1012 for any p=1,…,N

0≤Bp ≤100for any p=1,…,N

All values in input are integers.

INPUT FORMAT:

* N
* A1 A2 ⋯ AN
* B1 B2 ⋯ BN

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OUTPUT:

Print a single real number, the  .

SAMPLE INPUT:

5

4 2 6 3 5

1 1 1 1 1

SAMPLE OUTPUT: 4.700000000000 (expected gain of an optimal strategy as a single real number. Relative/absolute error should not exceed 10−10)