golfvgl: $\partial t^2 u + \partial t \cdot u = \partial x^2 u$

i) $u = X(x) \cdot T(t)$ (scheiding der veranderlijken)

$\Rightarrow \partial t^2 X(x) \cdot T(t) + \partial t \cdot (X(x) T(t)) = \partial x^2 \cdot X(x) \cdot T(t)$

$\Leftrightarrow T''(t) \cdot X(x) + T'(t) \cdot X(x) = X''(x) \cdot T(t)$

$\Leftrightarrow \dfrac{T''(t) + T'(t)}{T(t)} = \dfrac{X''(x)}{X(x)}$

$\Rightarrow \begin{cases} X''(x) = \sigma \cdot X(x) \\ T''(t) + T'(t) = \sigma \cdot T(t) \end{cases}$

again, bestuderen σ. enkel niet triviaal als $\sigma < 0$

$X(x) = c_1 \cdot \cos(kx) + c_2 \sin(kx)$

$\left| \begin{array}{l} u(0,t) = 0 \Rightarrow c_1 = 0 \\ u(L,t) = 0 \Rightarrow k = n \cdot \pi ! \end{array} \right.$

$X_n(x) = \sin(n\pi \cdot x)$ en $\sigma = -k^2$ dus $\sigma = -(n\pi)^2$

$T''(t) + T'(t) + n^2\pi^2 \cdot T(t) = 0$

$\Rightarrow z^2 + z + \sigma = 0$

$\Rightarrow z \pm \dfrac{-1}{2} \pm i \cdot \dfrac{\sqrt{4\sigma - 1}}{2}$

$T_n(t) = a_n \cdot e^{-\frac{t}{2}} \cos\left(\dfrac{\sqrt{4\sigma-1}}{2} t\right) + b_n e^{-t/2} \cdot \sin\left(\dfrac{\sqrt{4\sigma-1}}{2} t\right)$

som sinus $u(x,t) =$

$\sum_{n=0}^{+\infty} \left(a_n \cdot e^{-\frac{t}{2}} \cdot \cos\left(\dfrac{\sqrt{4\sigma-1}}{2} t\right) + b_n e^{-t/2} \cdot \sin\left(\dfrac{\sqrt{4\sigma-1}}{2} \cdot t\right)\right) \sin(n\pi \cdot x)$

3) beginvw:

$u(x,0) = f(x) = \sum_{n=0}^{+\infty} a_n \cdot \sin(n\pi x)$

dus $a_n = 2 \cdot \int_0^1 f(x) \cdot \sin(n\pi x)\, dx$

$\partial t\, u(x,0) = g(x) = \ldots$

just check solutions

2) superpositie tot bord oplossing