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> restart: with(VectorCalculus) : with(LinearAlgebra) :
> LL := diff(y(x), x$2) + 2*diff(y(x), x) + 4*y(x);
```

$$LL := \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} y(x) + 4 y(x) \quad (1)$$

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> g := x→x^2 :
> y_0 := 5 : dy_0 := 1 :
> dsolve(LL = 0, y(x))
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$$y(x) = c_1 e^{-x} \sin(\sqrt{3} x) + c_2 e^{-x} \cos(\sqrt{3} x) \quad (2)$$

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> # Homogene oplossingen
> y1 := x→exp(-x)*sin(sqrt(3)*x) :
> y2 := x→exp(-x)*cos(sqrt(3)*x) :
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> # Wronskiaan
> simplify(Determinant(Wronskian([y1(x), y2(x)], x)));
Wr := x→-exp(-2*x)*sqrt(3);
```

$$Wr := x \mapsto -e^{-2x} \sqrt{3} \quad (3)$$

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> # We can now solve u1 and u2
> u1 := simplify(int(-y2(x)*g(x)/Wr(x), x)) + C1;
> u2 := simplify(int(y1(x)*g(x)/Wr(x), x)) + C2;
```

$$u1 := \frac{e^x ((x^2 + x - 1) \cos(\sqrt{3} x) + \sin(\sqrt{3} x) \sqrt{3} x (x - 1)) \sqrt{3}}{12} + C1$$

$$u2 := \frac{e^x \sqrt{3} ((-x^2 - x + 1) \sin(\sqrt{3} x) + \cos(\sqrt{3} x) x \sqrt{3} (x - 1))}{12} + C2 \quad (4)$$

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> # The particular solution is
> yp := u1*y1(x) + u2*y2(x);
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$$yp := \left( \frac{e^x ((x^2 + x - 1) \cos(\sqrt{3} x) + \sin(\sqrt{3} x) \sqrt{3} x (x - 1)) \sqrt{3}}{12} + C1 \right) e^{-x} \sin(\sqrt{3} x)$$

$$+ \left( \frac{e^x \sqrt{3} ((-x^2 - x + 1) \sin(\sqrt{3} x) + \cos(\sqrt{3} x) x \sqrt{3} (x - 1))}{12} + C2 \right) e^{-x} \cos(\sqrt{3} x) \quad (5)$$

```
> y0 := simplify(subs(x = 0, yp));
> dy0 := simplify(subs(x = 0, diff(yp, x)));
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$$\begin{aligned} y_0 &:= C_2 \\ dy_0 &:= -\frac{1}{4} + \sqrt{3} C_1 - C_2 \end{aligned} \quad (6)$$

```
> sol := solve({y_0 = y_0, dy_0 = dy_0}, {C_1, C_2});
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$$sol := \left\{ C_1 = \frac{25\sqrt{3}}{12}, C_2 = 5 \right\} \quad (7)$$

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> solution := simplify(subs(sol, yp));
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$$solution := \frac{x^2}{4} + \frac{25 e^{-x} \sin(\sqrt{3} x) \sqrt{3}}{12} - \frac{x}{4} + 5 e^{-x} \cos(\sqrt{3} x) \quad (8)$$