

$$\begin{aligned}
& \text{restart: with(LinearAlgebra):} \\
& \text{\# First we define the relationship as matrix} \\
& A := \text{Matrix}([[a, b], [1, 0]]) \\
& A := \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \tag{1}
\end{aligned}$$

$$\begin{aligned}
& J, Q := \text{JordanForm}(A, \text{output} = ['J', 'Q']) \\
& J, Q := \begin{bmatrix} \frac{a}{2} - \frac{\sqrt{a^2 + 4b}}{2} & 0 \\ 0 & \frac{a}{2} + \frac{\sqrt{a^2 + 4b}}{2} \end{bmatrix}, \\
& \begin{bmatrix} \frac{\sqrt{a^2 + 4b} - a}{2\sqrt{a^2 + 4b}} & \frac{a + \sqrt{a^2 + 4b}}{2\sqrt{a^2 + 4b}} \\ -\frac{1}{\sqrt{a^2 + 4b}} & \frac{1}{\sqrt{a^2 + 4b}} \end{bmatrix} \\
& \tag{2}
\end{aligned}$$

$$\begin{aligned}
& \text{result} := Q \cdot \text{MatrixPower}(J, n) \cdot \text{MatrixInverse}(Q) \cdot \text{Vector}([1, 1]): \\
& \text{limit_} := \text{simplify}\left(\frac{\text{result}[1]}{\text{result}[2]}\right) \\
& \text{limit_} := \left((\sqrt{a^2 + 4b} - a - 2b) \left(\frac{a}{2} - \frac{\sqrt{a^2 + 4b}}{2} \right)^n + (\sqrt{a^2 + 4b} + a \right. \\
& \quad \left. + 2b) \left(\frac{a}{2} + \frac{\sqrt{a^2 + 4b}}{2} \right)^n \right) / \left((\sqrt{a^2 + 4b} + a - 2) \left(\frac{a}{2} - \frac{\sqrt{a^2 + 4b}}{2} \right)^n \right. \\
& \quad \left. - \left(\frac{a}{2} + \frac{\sqrt{a^2 + 4b}}{2} \right)^n (a - \sqrt{a^2 + 4b} - 2) \right) \\
& \tag{3}
\end{aligned}$$

$$\begin{aligned}
& \text{limit}(\text{limit_}, n = \text{infinity}) \\
& \lim_{n \rightarrow \infty} \left((\sqrt{a^2 + 4b} - a - 2b) \left(\frac{a}{2} - \frac{\sqrt{a^2 + 4b}}{2} \right)^n + (\sqrt{a^2 + 4b} + a + 2b) \left(\frac{a}{2} \right. \right. \\
& \quad \left. \left. + \frac{\sqrt{a^2 + 4b}}{2} \right)^n \right) / \left((\sqrt{a^2 + 4b} + a - 2) \left(\frac{a}{2} - \frac{\sqrt{a^2 + 4b}}{2} \right)^n - \left(\frac{a}{2} \right. \right. \\
& \quad \left. \left. + \frac{\sqrt{a^2 + 4b}}{2} \right)^n (a - \sqrt{a^2 + 4b} - 2) \right) \\
& \tag{4}
\end{aligned}$$

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