1 $\partial_t^2 u + \partial_t u = \partial_x^2 u$

condities

$u(0,t) = u(1,t) = 0$

$u(x,0) = f(x)$

$\frac{\partial u}{\partial t}(x,0) = g(x)$

Scheiding van veranderlijken.

$u(x,t) = X(x)\cdot T(t)$

$\Rightarrow T''X + T'X = X''T$ } deel door $\frac{1}{TX}$

$\Leftrightarrow \frac{T''}{T} + \frac{T'}{T} = \frac{X''}{X}$ → hieruit volgt: $\begin{cases} T''+T' = \sigma\cdot T \\ X'' = X\cdot\sigma \end{cases}$

EWP $X(x)$

x in 0 bij $x=0$, $x=L$ ($=1$), dus we kunnen $X_m = \sin(m\cdot\pi\cdot x)$

→ als $x=0$ → $\sin(0)=0$, en $x=1$ → $\sin(0)=0$

dan: $\iint \sin(m\pi x) = \sin(m\pi x)\cdot\sigma$

$\Leftrightarrow -(m\cdot\pi)^2\cdot\sin(m\pi x) = \sin(m\cdot\pi\cdot x)\cdot\sigma$

$\Leftrightarrow \sigma = -(m\cdot\pi)^2$

Nu $T(t)$

$T''+T' = \sigma\cdot T$

$\Leftrightarrow T''+T'+(m\pi)^2\cdot T = 0$

$\Rightarrow z^2 + z + (m\pi)^2 = 0$ → via Maple (=) $z = -\frac{1}{2} \pm i\cdot\frac{\sqrt{4m^2\pi^2+1}}{2}$

→ $T(m) = s_m e^{-\frac{1}{2}t}\cdot\cos\left(\frac{\sqrt{4\pi^2m^2+1}}{2}t\right) + t_m\cdot e^{-\frac{1}{2}t}\cdot\sin\left(\frac{\sqrt{4m^2\pi^2+1}}{2}t\right)$

Algemene oplossing:

$u_h(x,t) = \sum_{m=1}^{\infty}\left(s_m\cdot e^{-t/2}\cdot\cos(\,''\,t) + t_m\cdot e^{-t/2}\cdot\sin(\,''\,t)\right)\sin(m\pi x)$

Vanuit een algemene oplossing vinden we s_m en t_m

→ we kijken naar beginvoorwaarden:

1) $u(x,0) = f(x) = \sum_{m=1}^{\infty} s_m\cdot\sin(m\pi x)$. Gebruik orthogonale relaties om dit eruit te halen.

$\frac{1}{2}s_m = \int_0^1 f(x)\cdot\sin(m\pi x)\,dx \Leftrightarrow s_m = 2\cdot\int_0^1 f(x)\cdot\sin(m\pi x)\,dx$

2) $\frac{\partial}{\partial t}u(x,0) = g(x) = \sum_{m=1}^{\infty}\left(-\frac{1}{2}s_m + t_m\cdot\frac{\sqrt{4\pi^2m^2+1}}{2}\right)\sin(m\pi x)$

$\Leftrightarrow g(x) + \frac{1}{2}f(x) = \sum_{m=1}^{\infty} t_m\cdot\,''\,\cdot\sin(m\pi x)$

$\Leftrightarrow$ integreer en vermenigvuldig met $\sin(m\pi x)$

$\Leftrightarrow \int_0^1\left(g(x)+\frac{1}{2}f(x)\right)\sin\pi x\,dx = t_m\cdot\,''\,\cdot\frac{1}{2}$

$\sin\pi m\cdot\sin n\pi = \frac{1}{2}$ (zie orthogonale relaties)

$\Leftrightarrow \frac{4}{\sqrt{4\pi^2m^2+1}}\cdot\int_0^1\left(g(x)+\frac{1}{2}f(x)\right)\sin(\pi m x)\,dx = t_m$