17) $\partial_t^2 u(\alpha,t) = c^2 \cdot \partial_\alpha^2 u(\alpha,t)$ → golfvgl.

— Condities

$$\begin{cases} u'(0,t) = 0 \\ u(L,t) = h_0 \\ u(\alpha,0) = f(\alpha) \\ u'(\alpha,0) = g(\alpha) \end{cases}$$

— Scheiding der veranderlijken.

$u(\alpha,t) = X(\alpha)\cdot T(t)$

$\Leftrightarrow T''X = c^2 X'' T$

$\Leftrightarrow \frac{T''}{T} = c^2 \cdot \frac{X''}{X} \Rightarrow \begin{cases} T'' = c^2 \cdot T \cdot \sigma \\ \alpha'' = X \cdot \sigma \end{cases}$

— Bepaling normale modes

• Homogene randvoorwaarden

$X'' = X\cdot\sigma$, we hebben Neumann en dirichlet vw'en. dus :

1) $\sigma > 0$: $c_1 e^{\sqrt{\sigma}\alpha} + c_2 e^{-\sqrt{\sigma}\alpha} = 0$

2) $\sigma = 0$: $c_1 + c_2 \cdot x = 0$

3) $\sigma < 0$: $c_1 \cos(k\alpha) + c_2 \sin(k\alpha)$, $k^2 = -\sigma$

1) $X'(0) = c_1 \cdot \sqrt{\sigma} - c_2 \cdot \sqrt{\sigma} = 0$ → $c_1 = c_2 = 0$!
$X(L) = c_1 \cdot e^{\sqrt{\sigma}L} + c_2 \cdot e^{-\sqrt{\sigma}\cdot L} = 0$

2) $- c_1 = 0$
$- c_1 + c_2 \cdot L = 0$ → $c_1 = 0, c_2 = 0$

3) $X'(0) = k \cdot c_2 = 0 \Leftrightarrow c_2 = 0$

$X(L) = c_1 \cos(k \cdot L) = 0$

$k = \frac{\pi}{2L} \cdot (2n+1)$

dus $\alpha_n = \cos\left(\frac{\pi}{2L}\cdot(2n+1)\alpha\right)$ met

$\sigma = -\left(\frac{\pi}{2L}\cdot(2n+1)\right)^2$

$T_n = c^2 \cdot T \cdot \sigma$

$\hookrightarrow \sigma \cdot c^2 < 0$

dus: $T_n = s_n \cdot \cos\left(c \cdot \frac{\pi}{2L} \cdot (2n+1)\, t\right)$

$+ t_n \cdot \sin\left(c \cdot \frac{\pi}{2L} \cdot (2n+1)\, t\right)$

Algemeen:

$u_H(\alpha,t) = \sum_{n=0}^{+\infty} \left(s_n \cdot \cos(\,'') + t_n \cdot \sin(\,'')\right) \cdot \cos\left(\frac{\pi}{2L}\cdot(2n+1)\alpha\right)$

• inhomogene randvoorwaarden :

$$\begin{cases} u'(0,t) = 0 \\ u(L,t) = h_0 \end{cases}$$

$A + B\alpha$

→ 1) $B = 0$

$\Leftrightarrow A + B(L) = h_0 \Leftrightarrow A = h_0 = u_P$

totale oplossing

$u(\alpha,t) = \sum_{n=0}^{+\infty} \left(s_n \cdot \cos(\,'') + t_n \cdot \sin(\,'')\right) \cdot \cos\left(\frac{\pi}{2L}\cdot(2n+1)\alpha\right) + h_0$

— verwerking van begin vw.

$u(\alpha,0) = f(\alpha) = \sum_{n=0}^{+\infty} s_n \cdot \cos\left(\frac{\pi}{2L}\cdot(2n+1)\alpha\right) + h_0$

$\Leftrightarrow f(\alpha) - h_0 = \sum_{n=0}^{+\infty} s_n \cdot \cos(\,'')$

$\Leftrightarrow s_n = \frac{2}{L} \cdot \int_0^L (f(\alpha) - h_0) \cdot \cos(\,'') \cdot d\alpha$

nu nog t_n :

$u'(x,0) = \sum_{n=0}^{+\infty} t_n \cdot \left(\frac{\pi}{2L} \cdot c \cdot (2n+1)\right) \cdot \cos(\,'') = g(\alpha)$

$\cos\left(\frac{\pi}{2L}\cdot(2n+1)\, x\right)$

$\int_0^L g(\alpha)\cdot\cos(\,'') = t_n(\,'') \cdot \frac{L}{2}$

$\Leftrightarrow \frac{4L}{\pi \cdot c \cdot (2n+1)} \cdot \int_0^L g(\alpha)\cdot\cos\left(\frac{\pi}{2L}\cdot(2n+1)\alpha\right) = t_n$

— limiet.

Blijft oscilleren (geen demping) en de grootste periode

is wanneer $(n=0)$, dus

$T = \frac{2\pi}{\omega c} = \frac{2\pi}{\frac{\pi}{2L} c \cdot (2n+1)} = \frac{2\pi}{\frac{\pi}{2L} c} = \frac{4L}{c}$

$n = 0$