1D ydf vgl:

$$\partial_t^2 \phi(x,t) - c^2 \cdot \partial_x^2 \phi(x,t) = 0$$

met als begrenswaarde: $\phi(x,0) = e^{-x^2/\sigma^2}$

$$\partial_t \phi(x,t)\Big|_{t=0} = \alpha x \cdot e^{-x^2/\sigma^2}$$

fourier $\Rightarrow$ $\phi(x,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{\phi}(k,t) e^{ikx} \cdot dk$

dit geeft ons: $\partial_t^2 \hat{\phi}(k,t) + c^2 k^2 \hat{\phi}(k,t) = 0$

$$\hat{\phi}(k,t) = a(k) \cdot \cos(ckt) + b(k) \sin(ckt)$$

voor de eerste randvw krijgen we dan dat:

$$\hat{\phi}(k,0) = a(k) = \int_{-\infty}^{+\infty} \phi(x,0) e^{-ikx} dx \overset{!}{=} \sigma\sqrt{\pi}\, e^{-\frac{\sigma^2 \cdot k^2}{4}}$$

voor de tweede vinden we:

$$\partial_t \hat{\phi}(k,t)\Big|_{t=0} = c \cdot k b(k) = \int_{-\infty}^{+\infty} \partial_t \phi(x,t)\Big|_{t=0} e^{-ikx} \cdot dx$$

$$= \alpha \cdot \int_{-\infty}^{+\infty} \ldots$$

$$= -\frac{1}{2} \sigma^3 k \alpha i \sqrt{\pi}\, e^{-\frac{\sigma^2 k^2}{4}}$$

$$a(k) = \sigma\sqrt{\pi}\, e^{-\frac{\sigma^2 k^2}{4}}$$

$$b(k) = -\sigma^3 \sqrt{\pi} \frac{i\alpha}{2c} e^{-\frac{\sigma^2 k^2}{4}}$$

we hebben dat:

$$\hat{\phi}(k,t) = \sqrt{\pi}\, \sigma e^{-\frac{k^2 \sigma^2}{4}} \left(\cos(ckt) - \frac{i\alpha\sigma^2}{2c} \sin(ckt) \right)$$

$\hookrightarrow$ neem de inverse fourier.