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[> #  $x^3 + \text{epsilon} \cdot x = 1$ 
[=
[>
[>  $f := x \mapsto x^3 + \text{epsilon} \cdot x = 1$ 
[=
[>  $f := x \mapsto x^3 + e \cdot x = 1$  (1)
[=
[> # When epsilon is null, we get  $x = 1$ 
[>  $g := 1 + u \cdot \text{epsilon} + v \cdot \text{epsilon}^2$ 
[=
[>  $g := v e^2 + u e + 1$  (2)
[=
[>  $f\_subs := \text{subs}(x = g, f(x))$ 
[=
[>  $f\_subs := (v e^2 + u e + 1)^3 + e(v e^2 + u e + 1) = 1$  (3)
[=
[>  $f\_expand := \text{expand}(f\_subs)$ 
[>  $f\_expand := e^6 v^3 + 3 e^5 u v^2 + 3 e^4 u^2 v + 3 e^4 v^2 + e^3 u^3 + 6 e^3 u v + e^3 v + 3 e^2 u^2$  (4)
[>  $+ e^2 u + 3 v e^2 + 3 u e + e + 1 = 1$ 
[=
[> # First keep the left hand side
[>  $\text{left\_hand\_side} := \text{lhs}(f\_expand)$ 
[>  $\text{left\_hand\_side} := e^6 v^3 + 3 e^5 u v^2 + 3 e^4 u^2 v + 3 e^4 v^2 + e^3 u^3 + 6 e^3 u v + e^3 v$  (5)
[>  $+ 3 e^2 u^2 + e^2 u + 3 v e^2 + 3 u e + e + 1$ 
[=
[> # Extract coeff 1 and 2
[>  $\text{coeff}_1 := \text{coeff}(\text{left\_hand\_side}, \text{epsilon}, 1)$ 
[=
[>  $\text{coeff}_1 := 3 u + 1$  (6)
[=
[>  $\text{coeff}_2 := \text{coeff}(\text{left\_hand\_side}, \text{epsilon}, 2)$ 
[=
[>  $\text{coeff}_2 := 3 u^2 + u + 3 v$  (7)
[=
[>  $\text{solve}(\{\text{coeff}_1 = 0, \text{coeff}_2 = 0\}, \{u, v\})$ 
[=
[>  $\left\{u = -\frac{1}{3}, v = 0\right\}$  (8)
[=
[> # Final result
[>  $\text{result} := \text{subs}\left(\left\{u = -\frac{1}{3}, v = 0\right\}, g\right)$ 
[=
[>  $\text{result} := -\frac{e}{3} + 1$  (9)
[=
[>

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