> # De recursie relatie is: $x_k = x_{k-1} + x_{k-2}$

> # Als we dit schrijven in matrix vorm krijgen we:

> $restart : with(LinearAlgebra) :$

> $A := Matrix([[1, 1], [1, 0]])$

$$A := \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \tag{1}$$

> $J, Q := JordanForm(A, output = ['J', 'Q'])$

$$J, Q := \begin{bmatrix} -\frac{\sqrt{5}}{2} + \frac{1}{2} & 0 \\ 0 & \frac{\sqrt{5}}{2} + \frac{1}{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{2} - \frac{\sqrt{5}}{10} & \frac{\left(\sqrt{5}+1\right)\sqrt{5}}{10} \\ -\frac{\sqrt{5}}{5} & \frac{\sqrt{5}}{5} \end{bmatrix} \tag{2}$$

> $JK := Matrix\left(\left[\left[\left(-\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^k, 0\right], \left[0, \left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^k\right]\right]\right)$

$$JK := \begin{bmatrix} \left(-\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^k & 0 \\ 0 & \left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)^k \end{bmatrix} \tag{3}$$

> $Q\_inverse := MatrixInverse(Q)$

$$Q\_inverse := \begin{bmatrix} 1 & -\frac{\sqrt{5}}{2} - \frac{1}{2} \\ 1 & -\frac{\left(-5+\sqrt{5}\right)\sqrt{5}}{10} \end{bmatrix} \tag{4}$$

> $x\_0 := 1$

$$x\_0 := 1 \tag{5}$$

> $x\_1 := \frac{(1 - \mathrm{sqrt}(5))}{2}$

$$x\_1 := -\frac{\sqrt{5}}{2} + \frac{1}{2} \tag{6}$$

> $v\_1 := Vector([x\_1, x\_0])$

$$v\_1 := \begin{bmatrix} -\frac{\sqrt{5}}{2} + \frac{1}{2} \\ 1 \end{bmatrix} \tag{7}$$

> $result := Q \cdot JK \cdot Q\_inverse \cdot v\_1$

(8)

$$result := \left[\left[\left(\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k+\frac{(\sqrt{5}+1)\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)-\frac{(\sqrt{5}+1)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k(-5+\sqrt{5})}{20}\right],\left[\left(-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k}{5}+\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k}{5}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k(-5+\sqrt{5})}{10}\right]\right] \quad (8)$$

> *# calculate the limit*

> $limit\left(\frac{result[1]}{result[2]}, k = \text{infinity}\right)$

$$-\frac{\sqrt{5}}{2}+\frac{1}{2} \quad (9)$$

> *# Et voila ;)*

>

> *# Ok but for b) we need to add a small epsilon to this shit*

> epsilon :=

> $x\_0 := 1 + e$

$$x\_0 := 1 + e \quad (10)$$

> $x\_1 := \frac{(1-\text{sqrt}(5))}{2} + e$

$$x\_1 := \frac{1}{2}-\frac{\sqrt{5}}{2}+e \quad (11)$$

> $v\_1 := Vector([x\_1, x\_0])$

$$v\_1 := \begin{bmatrix} \frac{1}{2}-\frac{\sqrt{5}}{2}+e \\ 1+e \end{bmatrix} \quad (12)$$

> $result := Q \cdot JK \cdot Q\_inverse \cdot v\_1$

$$result := \left[\left[\left(\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k+\frac{(\sqrt{5}+1)\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^k}{10}\right)\left(\frac{1}{2}\right.\right.\right. \quad (13)$$

$$-\frac{\sqrt{5}}{2}+e\Big)+\Bigg(\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)$$

$$-\frac{\left(\sqrt{5}+1\right)\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}\left(-5+\sqrt{5}\right)}{20}\Bigg)\left(1+e\right)\Bigg],$$

$$\Bigg[\Bigg(-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}}{5}+\frac{\sqrt{5}\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}}{5}\Bigg)\left(\frac{1}{2}-\frac{\sqrt{5}}{2}+e\right)+\Bigg($$

$$-\frac{\sqrt{5}\left(-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}-\frac{\left(\frac{\sqrt{5}}{2}+\frac{1}{2}\right)^{k}\left(-5+\sqrt{5}\right)}{10}\Bigg)\big(1$$

$$+e\big)\Bigg]\Bigg]$$

> *# calculate the limit*

> $result := limit\left(\frac{result[1]}{result[2]}, k = \text{infinity}\right)$

$$result := \frac{3\sqrt{5}+5}{5+\sqrt{5}} \tag{14}$$

> $evala(result)$

$$\frac{\sqrt{5}}{2}+\frac{1}{2} \tag{15}$$

> *# Hupa, correct bitches*

>