Wiskundige modellering in de ingenieurswetenschappen: bordoefeningenles 9

Theorie Fourierreeksen

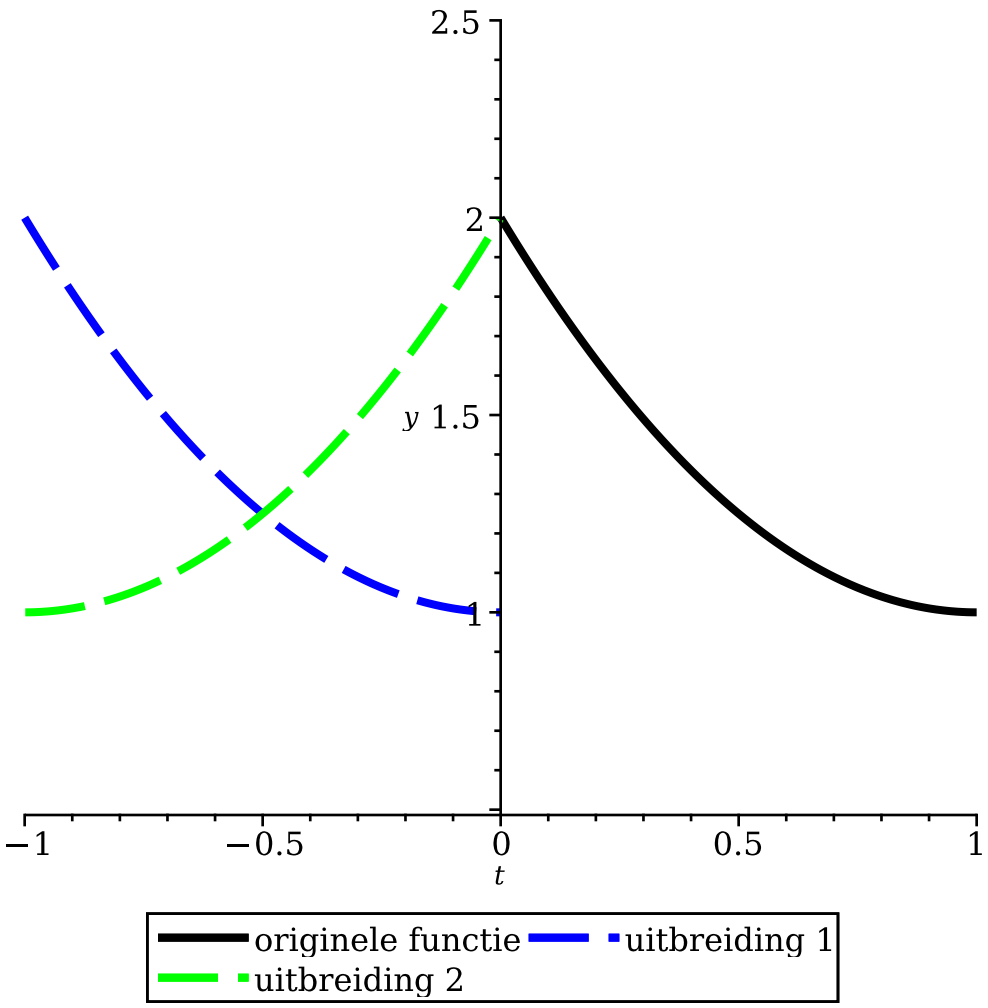
Oefening 1:

Oefening 2:

```
> restart:with(plots):
> L := 1:
> f := t-> (t-1)**2+1;
```

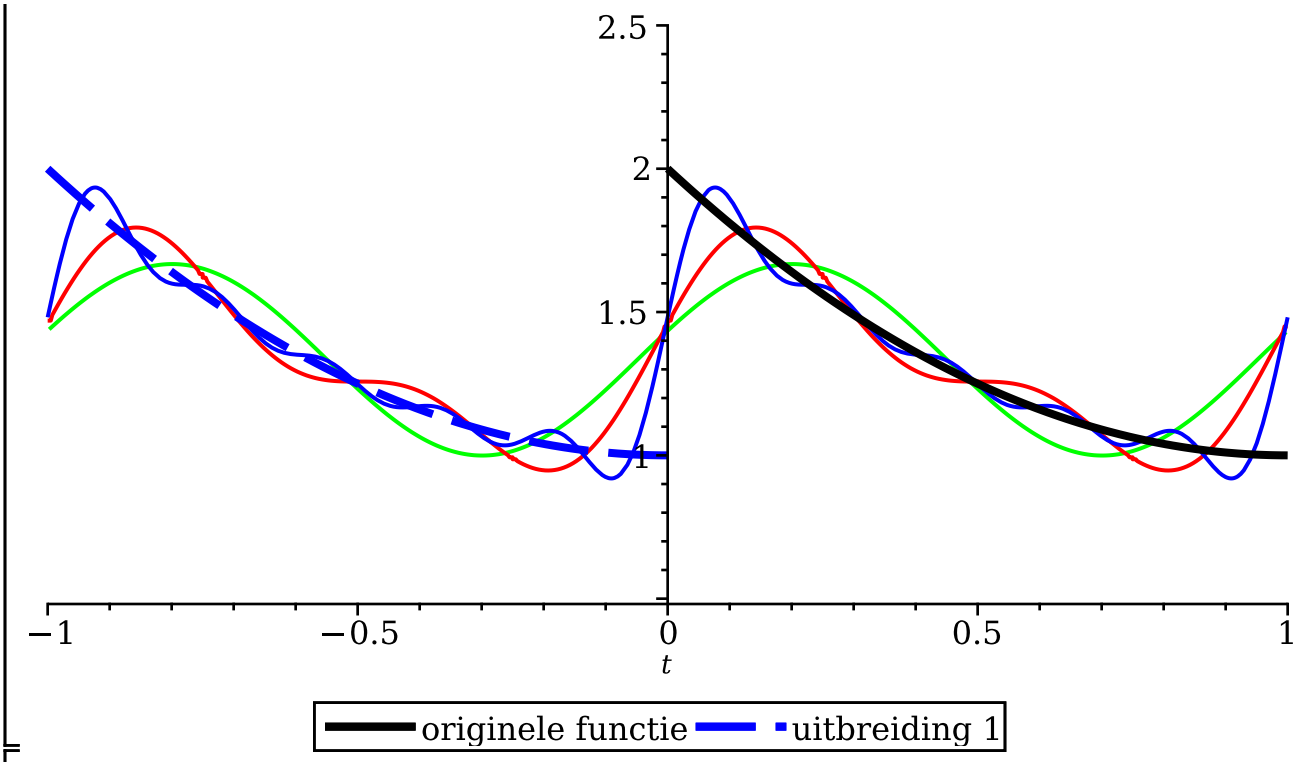
$$f := t \mapsto (t-1)^2 + 1 \qquad (3.1)$$

```
> fplot  := plot(f(t),t=0..1, y=0.5..2.5, legend="originele
  functie", color="black", thickness=3):
  f1plot := plot(f(t+1), t=-1..0,color=blue, legend="uitbreiding
  1", linestyle="dash", thickness=3):
  f2plot := plot(f(abs(t)), t=-1..0,color=green, legend=
  "uitbreiding 2", linestyle="dash", thickness=3):
  display(fplot, f1plot, f2plot);
```



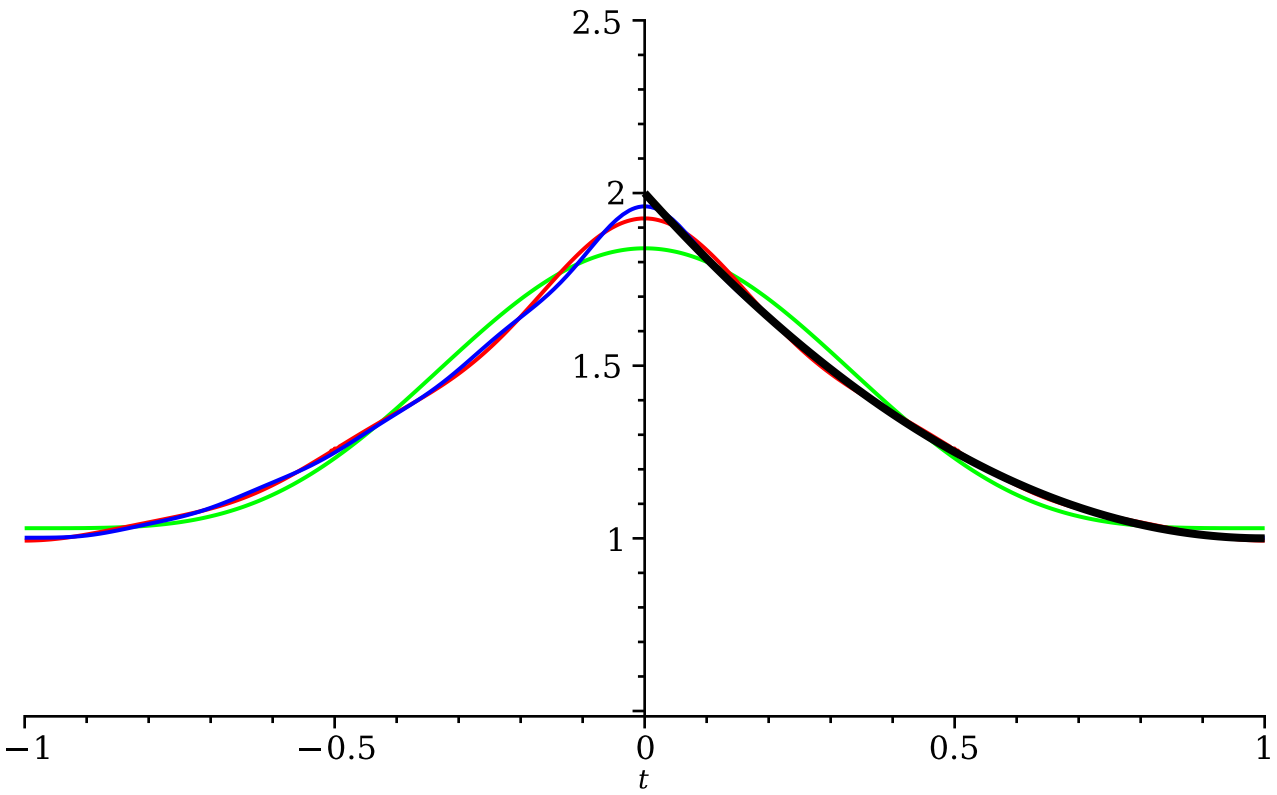
Fourier uitbreiding 1

```
> a01 := simplify(int(f(t+1), t=-L..0) + int(f(t), t=0..L)):
> a1  := k->int(f(t+1)*cos(k*t*Pi/L), t = -L .. 0)/L+int(f(t)*cos
  (k*t*Pi/L), t = 0..L)/L:
  b1  := k->int(f(t+1)*sin(k*t*Pi/L), t=-L..0)/L+int(f(t)*sin(k*
  t*Pi/L), t = 0..L)/L:
> F1  := (t, N) -> a01/2+sum(a1(k)*cos(k*Pi*t/L)+b1(k)*sin(k*Pi*
  t/L), k = 1 .. N):
> p1  := plot({F1(t,2),F1(t,5),F1(t,10)}, t=-L..L, color=[green,
  red, blue]):
  display(p1, fplot, f1plot);
```



Fourier uitbreiding 2

```
> a02 := simplify(int(f(abs(t)), t=-L..0) + int(f(t), t=0..L)):
> a2  := k->int(f(abs(t))*cos(k*t*Pi/L), t = -L .. 0)/L+int(f(t)*
  cos(k*t*Pi/L), t = 0..L)/L:
  b2  := k->int(f(abs(t))*sin(k*t*Pi/L), t=-L..0)/L+int(f(t)*sin
  (k*t*Pi/L), t = 0..L)/L:
> F2  := (t, N) -> a02/2+sum(a2(k)*cos(k*Pi*t/L)+b2(k)*sin(k*Pi*
  t/L), k = 1 .. N):
> p2  := plot({F2(t,2),F2(t,5),F2(t,10)}, t=-L..L, color=[green,
  red, blue]):
  display(p2, fplot);
```



Fourier uitbreiding 3

Bepaling g(t):

```
> g := t-> a*t**3+b*t**2+c*t+d;
```

$$g := t \mapsto a \cdot t^3 + b \cdot t^2 + c \cdot t + d \qquad (3.2)$$

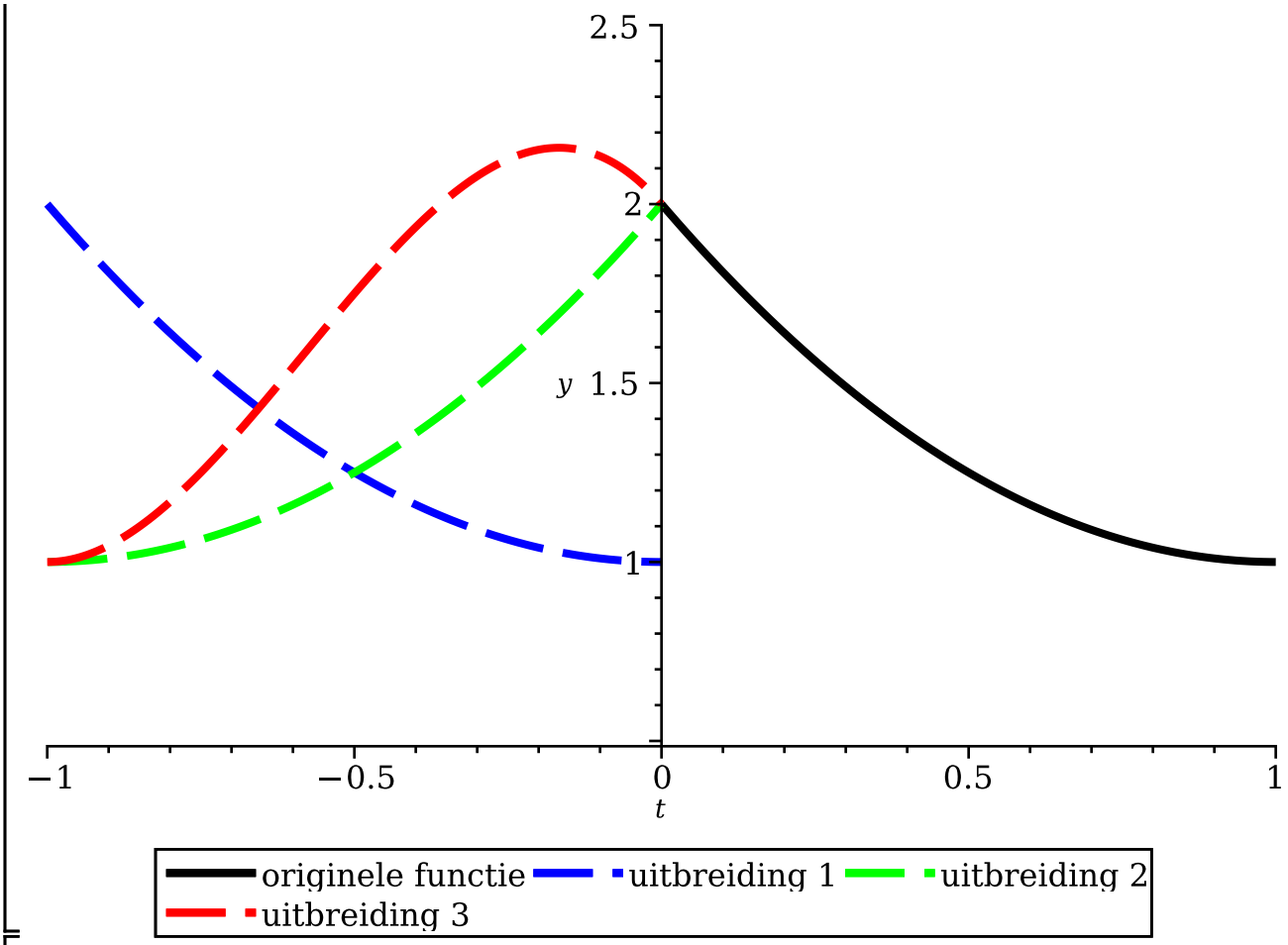
```
> sols:=solve({g(0)=f(0), g(-1)=f(1), D(g)(0)=D(f)(0), D(g)(-1)=
  D(f)(1)},{a,b,c,d});
```

$$sols := \{a = -4, b = -7, c = -2, d = 2\} \qquad (3.3)$$

```
> g_sol := t-> subs(sols, g(t));
```

$$g\_sol := t \mapsto subs(sols, g(t)) \qquad (3.4)$$

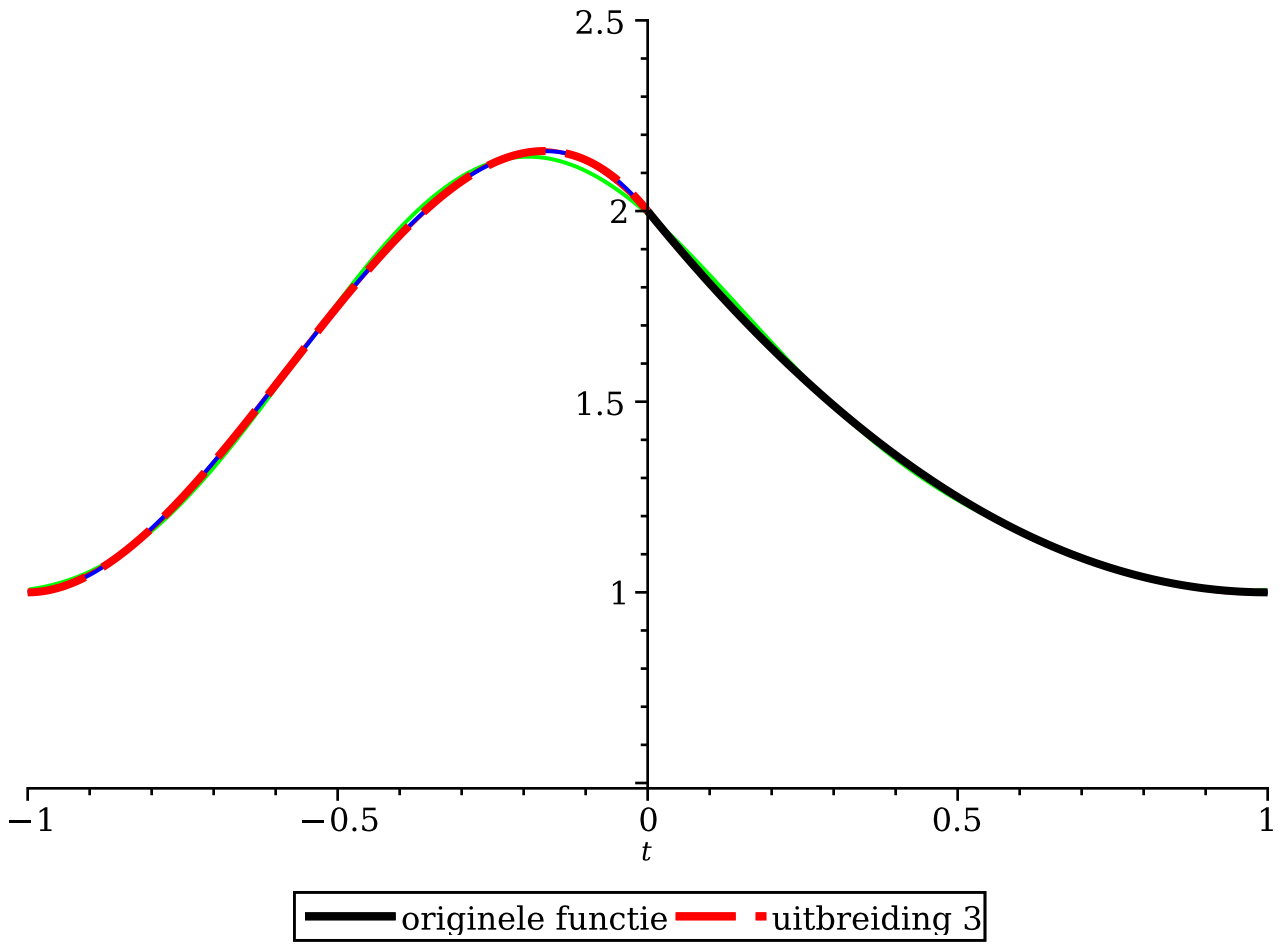
```
> gplot := plot(g_sol(t), t=-1..0, color=red, legend="uitbreiding
  3", linestyle="dash",thickness=3):
  display(fplot, f1plot, f2plot, gplot);
```




```
> a03 := simplify(int(g_sol(t), t=-L..0) + int(f(t), t=0..L)):
> a3  := k ->int(g_sol(t)*cos(k*t*Pi/L), t = -L .. 0)/L+int(f(t)*
  cos(k*t*Pi/L), t = 0..L)/L:
  b3  := k->int(g_sol(t)*sin(k*t*Pi/L), t=-L..0)/L+int(f(t)*sin
  (k*t*Pi/L), t = 0..L)/L:
> F3  := (t, N) -> a03/2+sum(a3(k)*cos(k*Pi*t/L)+b3(k)*sin(k*Pi*
  t/L), k = 1 .. N);
```

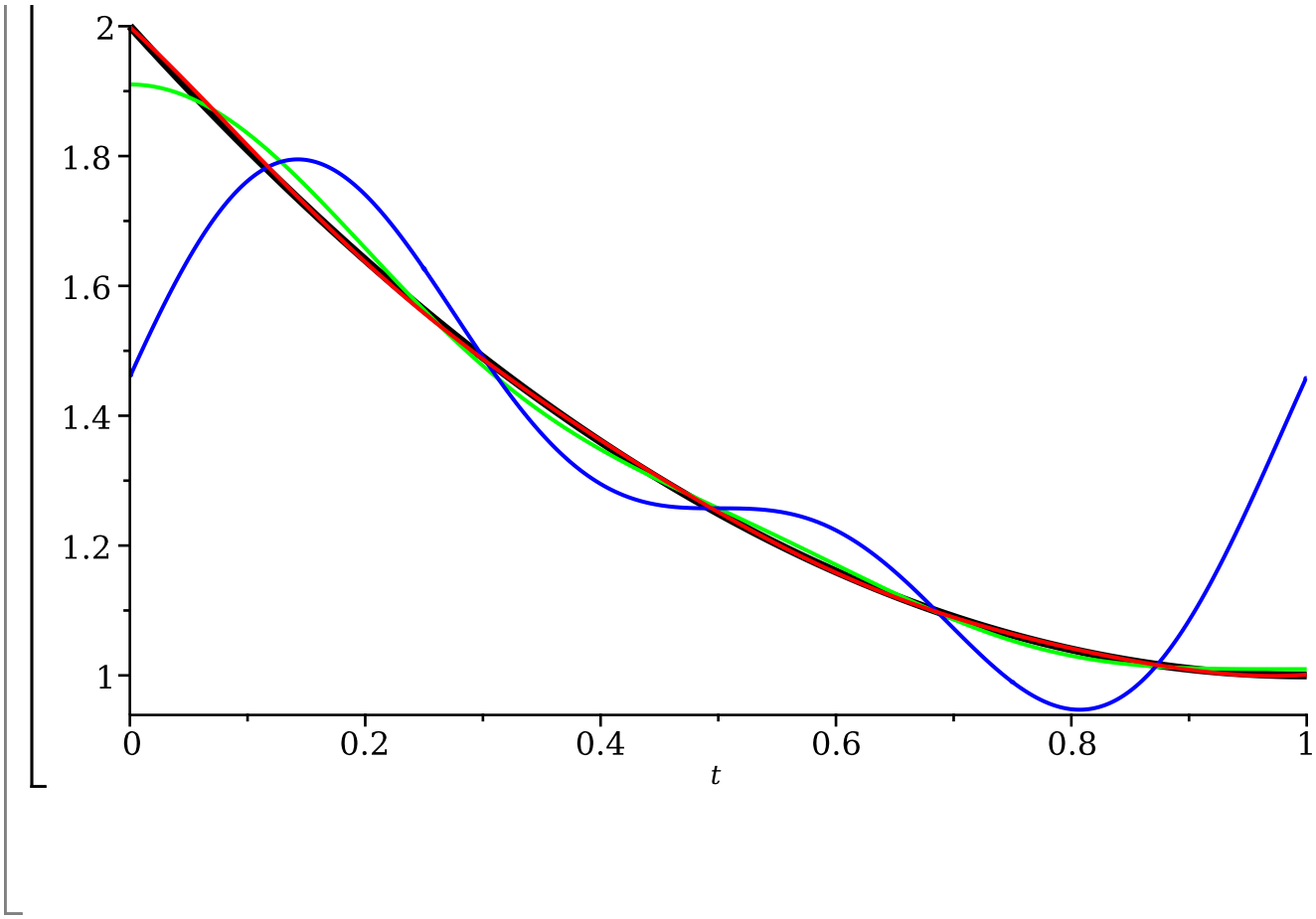
$$F3 := (t, N) \mapsto \frac{a03}{2} + \sum_{k=1}^{N} \left(a3(k) \cdot \cos\left(\frac{k \cdot \pi \cdot t}{L} \right) + b3(k) \cdot \sin\left(\frac{k \cdot \pi \cdot t}{L} \right) \right) \qquad \textbf{(3.5)}$$

```
> p3  := plot({F3(t,2),F3(t,5),F3(t,10)}, t=-L..L, color=[green,
  red, blue]):
  display(p3, fplot, gplot);
```



Vergelijking

```
> plot({f(t), F1(t,4), F2(t,4), F3(t,4)}, t=0..L, color=[black,
  green, blue, red],thickness=[3.5,1,1,1])
```



▶ **Oefening 3:**

▶ **Oefening 5:**