

5) $f(t)$ is odd, so $a_n = 0$
 $\hookrightarrow g(t) = t(2-t) \rightarrow$ there is symmetry around $t=1$, so $b_{2m} = 0$.
 $T = 1, L = 2$

So:

$$f(t) = \sum_{n=0}^{+\infty} b_{2n+1} \cdot \sin\left(\frac{(2n+1) \cdot \pi t}{L}\right)$$

and $t=1$

$$\rightarrow f(1) = \sum_{n=0}^{+\infty} b_{2n+1} \cdot \sin\left(\frac{(2n+1) \pi}{2}\right)$$

$$= \sum_{n=0}^{+\infty} b_{2n+1} \cdot (-1)^n, \text{ so}$$

$$\sum_{n=0}^{+\infty} (-1)^n b_{2n+1} = 1$$