$u_t(x,0) = \sin^2(\pi x)\cdot\cos(2\pi x)$

- the wave equation: $u_{xx} = u_{tt}$

Boundaries: $u_x(0,t) = 0$, $u_x(1,t) = 0$

The initial values

- $u_x(x,0) = 0$ (no displacement)
- $u_t(x,0) = \sin^2(\pi x)\cdot\cos(2\pi x)$

$u(x,t) = X(x)\cdot T(t)$

$\Rightarrow \dfrac{T''(t)}{T(t)} = \dfrac{X''(x)}{X(x)} \Leftrightarrow \boxed{\dfrac{T''(t)}{T(t)} = -\lambda,\ \dfrac{X''(x)}{X(x)} = -\lambda}$

Spatial eq.

$$X''(x) + \lambda\cdot X(x) = 0$$

→ Apply boundary: $X'(0) = 0$, $X'(1) = 0$

→ general solution:

$X(x) = A\cos(\sqrt{\lambda}\,x) + B\sin(\sqrt{\lambda}\,x)$

$\Rightarrow X'(x) = -A\sqrt{\lambda}\cdot\sin(\sqrt{\lambda}\,x) + B\sqrt{\lambda}\cdot\cos(\sqrt{\lambda}\,x)$

at $x=0 \Rightarrow B\sqrt{\lambda} = 0 \Leftrightarrow \boxed{B=0}$

thus $X(x) = A\cos(\sqrt{\lambda}\,x)$ and $\lambda = n^2\pi^2$

Temporal: $T''(t) + \lambda T(t) = 0$

$\Leftrightarrow T''(t) + n^2\pi^2\cdot T(t) = 0$

$\Rightarrow T(t) = C\cdot\cos(n\pi t) + D\sin(n\pi t)$

thus the general solution is:

$$u(x,t) = \sum_{t=0}^{\infty}\left(D_n\cdot\cos(n\pi\cdot t) + t_n\cdot\sin(n\cdot\pi\cdot t)\right)\cos(n\pi t)$$

beginning: $s_n = 0$

$$\rightarrow u(x,t) = \sum_{t=0}^{\infty} t_n\cdot\sin(n\pi t)\cdot\cos(n\pi t)$$

at $t=0 \rightarrow u(x,0) = \sum_{n=1}^{\infty} n\pi t_n\cos(n\pi x)$

$$t_n = \frac{2}{n\cdot\pi}\cdot\int_0^1 g(x)\cdot\cos(n\pi x)\cdot dx$$