1) golfvgl: $\partial t^2 u + \partial t u = \partial x^2 u$

$$\boxed{\begin{array}{l} u(0,t) = 0 \\ u(1,t) = 0 \\ u'(x,0) = g(x) \\ u(x,0) = f(x) \end{array}}$$

① Scheiding van variabelen

$u(x,t) = X(x) \cdot T(t)$

$\rightarrow T'' X + T' X = X'' T$

$\Leftrightarrow \frac{T''}{T} + \frac{T'}{T} = \frac{X''}{X}$

$$\begin{cases} T'' + T' = T \cdot \sigma \\ X'' = X \cdot \sigma \end{cases}$$

Space $X(x)$

Door $u(0,t), u(1,t) = 0$, weten we dat $X_m = \sin(m\pi x)$ is (voldoet met $x=0$ en $x=1$!)

Hieruitvolgt dat $\sigma = -(m\pi)^2$ (puur uit conventie)

Time $T(t)$

$T'' + T' + (m\pi)^2 T = 0$

via maple

$$T(t) = C_1 \cdot e^{\left(-\frac{1}{2} + \frac{\sqrt{-4\pi^2 m^2+1}}{2}\right)t} + C_2 \cdot e^{\left(-\frac{1}{2} - \frac{\sqrt{-4\pi^2 m^2+1}}{2}\right)t}$$

negative, no complex!

$$= e^{-\frac{1}{2}} \cdot \left(a_m \cdot \cos(\omega t) + b_m \cdot \sin(\omega t)\right), \text{ with } \omega = \sqrt{\frac{4\pi^2 m^2 + 1}{2}}$$

So, the general solution is:

$$u_h(x,t) = \sum_{m=1}^{\infty} \left(e^{-\frac{1}{2}t}\left(a_m \cos(\omega t) + b_m \cdot \sin(\omega t)\right) \cdot \sin(m\pi x)\right)$$

③ find coefficients

$u(x,0) = f(x) = \sum_{m=1}^{\infty} a_m \sin(m\pi x)$

$\Leftrightarrow a_m = 2\int_0^1 f(x) \sin(m\pi x) \cdot dx$

$u'(x,0) = \sum_{m=1}^{\infty} -\frac{1}{2} a_m \cdot \sin(m\pi x) + b_m \cdot \omega \cdot \sin(m\pi x)$

$\Leftrightarrow g(x) + \frac{1}{2} f(x) = \sum_{m=1}^{\infty} b_m \cdot \omega \cdot \sin(m\pi x)$

$\Leftrightarrow b_m = \omega \cdot \int_0^1 \left(g(x) + \frac{1}{2} f(x)\right) \cdot \sin(m\pi x) \cdot dx$