> restart: with(LinearAlgebra): with(plots): with(VectorCalculus):

>
$$f := (x, y) \rightarrow x \cdot y \cdot (x^2 - y^2 - 1)$$

 $f := (x, y) \mapsto x \cdot y \cdot (x^2 + (-y^2) - 1)$ (1)

- > SetCoordinates(cartesian[x, y]):
- > grad := Gradient(f(x, y)) $grad := (y(x^2 - y^2 - 1) + 2x^2y)\bar{e}_x + (x(x^2 - y^2 - 1) - 2xy^2)\bar{e}_y$ (2)
- > $stationary_points := solve(\{grad[1] = 0, grad[2] = 0\}, \{x, y\})$ $stationary_points := \{x = 0, y = 0\}, \{x = 0, y = RootOf(_Z^2 + 1)\}, \{x = 1, y = 0\},$ (3)

$$\{x = -1, y = 0\}, \left\{x = -\frac{1}{2}, y = \frac{RootOf(Z^2 + 1)}{2}\right\}, \left\{x = \frac{1}{2}, y = \frac{RootOf(Z^2 + 1)}{2}\right\}$$

 \rightarrow hessian := Hessian(f(x, y))

hessian :=
$$\begin{bmatrix} 6xy & 3x^2 - 3y^2 - 1 \\ 3x^2 - 3y^2 - 1 & -6xy \end{bmatrix}$$
 (4)

- # So now for the first critical point for example:
- \rightarrow Eigenvalues(subs(x = 0, y = 0, hessian))

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \tag{5}$$

- -Bo this is a sadle
 - # And so forth...