> $restart : with(inttrans) : assume(t > 0) : assume(\sigma > 0) : assume(d > 0) :$

> # initial conditions

> $pde\_k := diff(phi\_k(t), t\$2) + c\hat{\ }2 * k\hat{\ }2 * phi\_k(t) = 0;$

$$pde\_k := \frac{d^2}{dt{\sim}^2}\, phi\_k(t{\sim}) + c^2\, k^2\, phi\_k(t{\sim}) = 0 \tag{1}$$

> $sol\_k := dsolve(pde\_k, phi\_k(t));$

$$sol\_k := phi\_k(t{\sim}) = c_1 \sin(k\, c\, t{\sim}) + c_2 \cos(k\, c\, t{\sim}) \tag{2}$$

> $phi\_k\_general := c1 * \cos(k * c * t) + c2 * \sin(k * c * t);$

$$phi\_k\_general := c1 \cos(k\, c\, t{\sim}) + c2 \sin(k\, c\, t{\sim}) \tag{3}$$

> $initial\_condition\_1 := fourier\left(\exp\left(-\frac{x^2}{\sigma^2}\right), x, k\right)$

$$initial\_condition\_1 := e^{-\frac{k2\ \sigma{\sim}2}{4}} \sqrt{\pi\, \sigma{\sim}^2} \tag{4}$$

> $initial\_condition\_2 := \frac{fourier\left(\alpha \cdot x \cdot \exp\left(-\frac{x^2}{\sigma^2}\right), x, k\right)}{c \cdot k}$

$$initial\_condition\_2 := \frac{-\frac{\mathrm{I}}{2}\, \alpha\, \sigma{\sim}^3\, e^{-\frac{k2\ \sigma{\sim}2}{4}} \sqrt{\pi}}{c} \tag{5}$$

> $full\_fourrier := simplify(subs([c1 = initial\_condition\_1, c2 = initial\_condition\_2], phi\_k\_general))$

$$full\_fourrier := -\frac{e^{-\frac{k2\ \sigma{\sim}2}{4}} \sqrt{\pi}\ \sigma{\sim}\ \left(\mathrm{I}\, \alpha\, \sigma{\sim}^2 \sin(k\, c\, t{\sim}) - 2 \cos(k\, c\, t{\sim})\, c\right)}{2\, c} \tag{6}$$

> $simplify(invfourier(full\_fourrier, k, x))$

$$\frac{e^{\frac{-c2\ t{\sim}2\ -x2}{\sigma{\sim}2}} \left(\sinh\left(\frac{2\, c\, t{\sim}\, x}{\sigma{\sim}^2}\right) \alpha\, \sigma{\sim}^2 + 2 \cosh\left(\frac{2\, c\, t{\sim}\, x}{\sigma{\sim}^2}\right) c\right)}{2\, c} \tag{7}$$

> ` `