```
> restart : with(LinearAlgebra) : with(VectorCalculus) :
> #ii)
> SetCoordinates(cartesian[x, y, z]) :
  r := sqrt(x^2 + y^2) :
```

$$> \ vector\_field\_outside := VectorField\left(\left[\left(-\frac{\text{alpha}\cdot y}{r^2}\right), \frac{\text{alpha}\cdot x}{r^2}, 0\right]\right) :$$

```
> simplify(Curl(vector_field_outside))
```

$$(0)\bar{e}_x + (0)\bar{e}_y + (0)\bar{e}_z \tag{1}$$

```
> # Here we see its not = 0 !
```

$$> \ vector\_field\_inside := VectorField\left(\left[-\frac{\text{alpha}\cdot x}{x^2+y^2}, \frac{\text{alpha}\cdot x}{x^2+y^2}, 0\right]\right) :$$

```
> Curl(vector_field_inside)
```

$$(0)\bar{e}_x + (0)\bar{e}_y + \left(\frac{\alpha}{x^2+y^2} - \frac{2\,\alpha\,x^2}{(x^2+y^2)^2} - \frac{2\,\alpha\,x\,y}{(x^2+y^2)^2}\right)\bar{e}_z \tag{2}$$

```
> # Not zero!
> # IV
> restart : with(LinearAlgebra) : with(VectorCalculus) :
    SetCoordinates(cartesian[x, y, z]) :
> # Define the components of the velocity field symbolically
  vx := -alpha * y / (x^2 + y^2) :
  vy := alpha * x / (x^2 + y^2) :
  vz := 0 :

  # Parameterize x and y for the circular path in terms of r and theta
  x_expr := r * cos(theta) :
  y_expr := r * sin(theta) :

  # Substitute x and y expressions into the vector field components
  vx_param := subs(x = x_expr, y = y_expr, vx) :
  vy_param := subs(x = x_expr, y = y_expr, vy) :

  # Compute dr as the derivative of the parameterized curve
  dr := diff([x_expr, y_expr, 0], theta) :


    # Calculate the integrand as the dot product of [vx_param, vy_param, vz]
    and dr
  integrandum := simplify(vx_param * dr[1] + vy_param * dr[2] + vz * dr[3]) :

  # Evaluate the integral over theta from 0 to 2*Pi
  result := int(integrandum, theta = 0 .. 2 * Pi);
```

$$result := 2\,\alpha\,\pi \tag{3}$$

```
>
```