

$$\frac{f^{(5)}(t)}{5!} + \frac{f^{(4)}(t)}{4!} + f(t) = 0$$

$$\frac{d}{dt} \begin{pmatrix} f^3(t) \\ f^2(t) \\ f'(t) \\ f(t) \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f^3(t) \\ f^2(t) \\ f'(t) \\ f(t) \end{pmatrix}$$

we write the 4th matrix.

$$f = \frac{2 \left(e^{-t/2} + \frac{3}{2} e^{t/2} \right) \sqrt{3} \sin\left(\frac{\sqrt{3}}{2} t\right) + e^{-t/2} \cos\left(\frac{\sqrt{3}}{2} t\right)}{3}$$

i.i) we get an homogeneous eqn:

$$\frac{d}{dt} \begin{pmatrix} f^3(t) \\ f^2(t) \\ f'(t) \\ f(t) \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & -1 & 2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} f^3(t) \\ f^2(t) \\ f'(t) \\ f(t) \\ 1 \end{pmatrix}$$