2.)

$$\frac{\partial}{\partial t} u(0,t) = 0 \qquad \frac{\partial}{\partial t} u(x,0) = \sin^2(\pi x)\cos(2\pi x)$$

$$\frac{\partial}{\partial t} u(L,t) = 0$$

$$u(x,t) = X(x)\cdot T(t)$$

golfvgl: $\partial_t^2 u(x,t) = c^2 \cdot \partial_x^2 \cdot u(x,t)$

$$\Leftrightarrow T'' X = c^2 \cdot X'' T$$

$$\Leftrightarrow \frac{T''}{T} = c^2 \cdot \frac{X''}{X} \Rightarrow \begin{cases} T'' = c^2 \cdot \lambda \cdot T \\ X'' = X \cdot \lambda \end{cases}$$

homogene oplossing:

$X'' = X\cdot\lambda$, λ heeft 3 gevallen

$$X(x) = A\cos(\sqrt{\lambda}\, x) + B\sin(\sqrt{\lambda}\, x)$$

$$X'(0) = 0 \Leftrightarrow B = 0$$

$$X'(1) = 0 \Leftrightarrow -A\sin(\sqrt{\lambda}\, x) = 0$$

$$\Leftrightarrow \sqrt{\lambda} = n\pi$$

$$\Leftrightarrow \lambda = (n\pi)^2$$

$\rightarrow X_n = \cos(n\pi x)$ $\quad c=1!$

$$T'' = c^2\cdot(n^2\pi^2)\cdot T$$

$$T_n = a_n \cdot \cos(cn\pi) + b_n \cdot \sin(cn\pi)$$

$$u_h(x,t) = \sum_{n=0}^{+\infty} \cos(n\pi x)\cdot\left(a_n \cdot \cos(cn\pi t) + b_n \sin(cn\pi t)\right)$$

$$= \sum_{n=0}^{+\infty} \cos(n\pi x)\cdot\left(a_n\cdot\cos(n\pi t) + b_n\cdot\sin(n\pi t)\right)$$

find a_n and b_n.

$a_n = 0$ door de begin uitwijking.

dus: $u_h(x,t)$

$$= \sum_{n=0}^{+\infty} \cos(n\pi x)\cdot\left(b_n\cdot\sin(n\pi t)\right)$$

$$\frac{\partial}{\partial t} u(x,0) = \sin^2(\pi x)\cos(2\pi x)$$

$$\Leftrightarrow \underbrace{\sin^2(\pi x)\cdot\cos(2\pi x)}_{g(x)} = \sum_{n=0}^{\infty} \cos(n\pi x)\cdot b_n \cdot n\pi$$

$$\Leftrightarrow g(x) = \sum_{n=0}^{\infty} \cos(n\pi x)\cdot b_n \cdot n\pi$$

$$\Leftrightarrow \int_0^1 g(x)\cdot\cos(n\pi x)\cdot dx = b_n\cdot n\pi\cdot\frac{1}{2}$$

$$\Leftrightarrow \frac{2}{n\pi}\cdot\int_0^1 g(x)\cdot\cos(n\pi x)\cdot dx = b_n$$

$$\Leftrightarrow b_n = -\frac{4\cdot\sin(n\pi)\cdot(n^2+8)}{n^2\pi^2(n^4-20n^2+64)}$$