

5) a) find the vectors :

$$\begin{aligned}
 v_1 &= 1 & \text{norm} & \rightarrow & c_0 &= \frac{\sqrt{2}}{2\sqrt{\pi}} \\
 v_2 &= \cos(kx) & & & c_k &= \sin(kx)/\sqrt{\pi} \\
 v_3 &= \sin(kx) & & & n_k &= \cos(kx)/\sqrt{\pi}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f_0 &= (c_0 \cdot t^2) \cdot c_0 + \sum_{k=-\infty}^{\infty} (c_k \cdot t^2) \cdot c_k + \sum_{k=0}^{\infty} (n_k \cdot t^2) \cdot n_k \\
 &= \frac{\sqrt{2}}{3} \pi^{5/2} + \frac{4\sqrt{\pi} (-1)^k}{k^2} + 10
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \text{distance} &= \|f - f_0\|^2 = \langle f, f \rangle^2 - 2 \cdot \langle f, f_0 \rangle \\
 &\quad + \langle f_0, f_0 \rangle^2
 \end{aligned}$$

$$= \frac{6\pi^{10}}{25} - 2 \cdot \frac{4\pi^{7/2} (\sqrt{2} \pi^2 k^2 + 12 (-1)^k)}{8k^2} + \frac{4\pi^4 (\sqrt{2} \pi^2 k^2 + 12 (-1)^k)^2}{81k^8}$$