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> restart;
with(LinearAlgebra) :

# Initial values
y_0 := 5 :
dy_0 := 1 :

# Define matrix A and function b(x)
A := Matrix([[-2, -4], [1, 0]]) :
b := x → Vector([x^2, 0]) :

# Compute MatrixExponential(-A*x) . b(x)
vec_expr := MatrixExponential(-A*x) . b(x) :

# Integrate each component of the vector separately
integral_result := Vector([
  int(vec_expr[1], x),
  int(vec_expr[2], x)
]):

# Combine the terms
sol := simplify(
  MatrixExponential(A*x) . integral_result
  + MatrixExponential(A*x) . Vector([C1, C2])
);

# Verify the solution at x = 0
vgl := simplify(subs(x = 0, sol) = Vector([dy_0, y_0]));

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$$sol := \begin{bmatrix} -\frac{\sqrt{3} e^{-x} (C1 + 4 C2) \sin(x\sqrt{3})}{3} + C1 e^{-x} \cos(x\sqrt{3}) + \frac{x}{2} - \frac{1}{4} \\ \frac{\sqrt{3} e^{-x} (C1 + C2) \sin(x\sqrt{3})}{3} + C2 e^{-x} \cos(x\sqrt{3}) + \frac{x^2}{4} - \frac{x}{4} \end{bmatrix}$$

$$vgl := \begin{bmatrix} -\frac{1}{4} + C1 \\ C2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad (1)$$

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> solve(vgl, {C1, C2})

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$$\left\{ C1 = \frac{5}{4}, C2 = 5 \right\} \quad (2)$$

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