```
> restart: with(LinearAlgebra):
> inproduct := (f, g) -> integrate(f*g, t=-infinity..infinity);
  NORM      := f    -> sqrt(inproduct(f, f)):
```

$$inproduct := (f, g) \mapsto integrate(g \cdot f, t = -\infty .. \infty) \tag{1}$$

```
> f0 := exp(-t^2/2)/NORM(exp(-t^2/2)):
> w1 := t*exp(-t^2/2)-f0*inproduct(f0, t*exp(-t^2/2));
  f1 := w1/NORM(w1):
```

$$w1 := t\, e^{-\frac{t2}{2}} \tag{2}$$

```
> w2 := t^2*exp(-t^2/2) - f1*inproduct(t^2*exp(-t^2/2), f1) - f0
       *inproduct(t^2*exp(-t^2/2), f0);
  f2 := w2/NORM(w2):
```

$$w2 := t^2 e^{-\frac{t2}{2}} - \frac{e^{-\frac{t2}{2}}}{2} \tag{3}$$

```
> w3 := t^3*exp(-t^2/2)- f2*inproduct(t^3*exp(-t^2/2), f2)- f1
       *inproduct(t^3*exp(-t^2/2), f1)-f0*inproduct(t^3*exp(-t^2/2),
       f0);
  f3 := w3/NORM(w3):
```

$$w3 := t^3 e^{-\frac{t2}{2}} - \frac{3\, t\, e^{-\frac{t2}{2}}}{2} \tag{4}$$

```
> w4 := t^4*exp(-t^2/2)- f3*inproduct(t^4*exp(-t^2/2), f3)- f2
       *inproduct(t^4*exp(-t^2/2), f2)- f1*inproduct(t^4*exp(-t^2/2),
       f1)-f0*inproduct(t^4*exp(-t^2/2), f0);
  f4 := w4/NORM(w4):
```

$$w4 := t^4 e^{-\frac{t2}{2}} - 3\, t^2 e^{-\frac{t2}{2}} + \frac{3\, e^{-\frac{t2}{2}}}{4} \tag{5}$$

```
> # i: de orthonormale basis is dus de span van <f_0, w_1, w_2, w_3, w_4, ...>
> # ii: zie notes
>
```