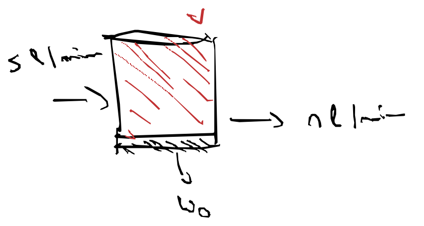
2) eenvoudige fase:



, $s > n$ → vult op over tijd

1

$V' = s - n$, $V(0) = w$ ⟹ $V(t) = (s-n)t + w$

Op t_1 is de tank vol : $V(t_1) = (s-n)t_1 + w$

⟹ $t_1 = \dfrac{V(t_1) - w}{s-n}$

Hoeveelheid zout:

constant zout.

1 $z'(t) = -n \cdot \dfrac{z(t)}{V(t)}$, $z_1(0) = z$

(1) $\dfrac{z'(t)}{z(t)} = -\dfrac{n}{V(t)}$ ⟺ $\displaystyle\int \frac{1}{z_1(t)}\, dz_1 = -n \cdot \int \frac{1}{V(t)} \cdot dt$

⟺ $\displaystyle\int \frac{1}{z_1(t)} \cdot dz_1 = -n \cdot \int \frac{1}{(s-n)t + w} \cdot dt$

(using maple:)

$z_1(t) = e^c \cdot ((s-n)t + w)^{-\frac{n}{s-n}}$

using $z(0) = z$ ⟹ $z = e^c \cdot w^{-\frac{n}{s-n}}$

⟺ $e^c = z \cdot w^{\frac{n}{s-n}}$

thus: $z_1(t) = z \cdot w^{k} \cdot ((s-n)t + w)^{-k}$ met $k = \dfrac{n}{s-n}$

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2 DV: $z_2'(t) = -\dfrac{n \cdot z_2(t)}{V_2(t)} + k \cdot n$ zout lemtoevoeging

zout leaving

$z_2(t_1) = z_1(t_1)$

we herschrijven DV:

$k n = z_2'(t) + \dfrac{n z_2(t)}{V_2(t)}$

homogeneous solution

$z_2'(t) + \dfrac{n z_2(t)}{V_2(t)} = 0$ ⟹ $z_2 = C \cdot e^{-\frac{n}{V} \cdot t}$

particulier

met $z_2(t) = A$

→ $0 + \dfrac{n \cdot A}{V} = k \cdot n$ ⟺ $A = k \cdot V$

Algemeen:

$z_2(t_1) = C \cdot e^{-\frac{n}{V} t_1} + V \cdot k$ → initial condition.

⟺ $C = (z_2(t_1) - V \cdot k)\, e^{\frac{n t}{V}}$

dus: $z_2(t) = \left[z_1(t_1) - V \cdot k \cdot e^{\frac{n}{V} \cdot t_1}\right] e^{-\frac{n}{V} \cdot t} + V \cdot k$

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3 $V : [0, \infty[\; \to \begin{cases} V_1(t) : 0 \leq t \leq t_1 \\ V_2(t) : t_1 \leq t \end{cases}$

$z : [0, \infty[\; \to \begin{cases} z_1(t) : 0 \leq t \leq t_1 \\ z_2(t) : t_1 \leq t \end{cases}$

4 $\displaystyle\lim_{t \to +\infty} z(t) = k \cdot V$

5 we zoeken $t_2 > t_1$, waarvoor $z(t_2) = N \cdot z_1(t_1)$, met $N = 3$

$N \cdot z_1(t_1) = \left[z_1(t_1) - V \cdot k \cdot e^{\frac{n}{V} \cdot t_1}\right] e^{-\frac{n}{V}(t_2 - t_1)} + V \cdot k$

⟺ $e^{-\frac{n}{V}(t_2 - t_1)} = \dfrac{z V^{-k} - k V w^{-k}}{N z V^{-k} - k \cdot V \cdot w^{-k}}$ met $k = \dfrac{n}{s-n}$

$t_2 - t_1 = -\dfrac{V}{n} \ln \left(\quad \right)$

needs to be positive;

- $z V^{-k} - k V w^{-k} \geq 0$
- $N z V^{-k} - k \cdot V \cdot w^{-k} > 0$

so for $N z > k \cdot V$, $t_2 - t_1$ wordt negatief, dus niet mogelijk.

6) zie maple.