3) $f(t) = \sum_{k=0}^{+\infty} a_k \cos(kt)$

$$g(t) = \frac{f(\frac{t}{2}) + f(\frac{t}{2} + \pi)}{2}$$

$$f(\frac{t}{2}) = \sum_{k=0}^{+\infty} a_k \cdot \cos(k \cdot \frac{t}{2})$$

$$f(\frac{t}{2} + \pi) = \sum_{k=0}^{+\infty} a_k \cdot \cos(k \cdot (\frac{t}{2} + \pi))$$

$$\rightarrow g(t) = \frac{1}{2} \left(\sum_{k=0}^{+\infty} a_k \cdot \cos(k \cdot \frac{t}{2}) + \underbrace{\sum_{k=0}^{\infty} a_k \cdot \cos(k \cdot (\frac{t}{2} + \pi))}_{(1)} \right)$$

$$(1) = \sum_{k=0}^{\infty} a_k \cdot \cos(k \cdot \frac{t}{2}) \cdot (-1)^k$$

$$\rightarrow g(t) = \frac{1}{2} \cdot \left(\sum_{k=0}^{+\infty} a_k \cdot \cos(k \frac{t}{2}) \cdot \left(1 + (-1)^k \right) \right)$$

When k is even we get 2

When k is uneven, we get 0

$$\rightarrow g(t) = \sum_{k=0}^{+\infty} a_{2k} \cos\left(\frac{2k \cdot t}{2}\right)$$

$$= \sum_{k=0}^{+\infty} a_{2k} \cos(k \cdot t)$$