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> restart : with(LinearAlgebra) : with(plots) : with(plottools) : assume(k,
  'integer') :
> inproduct := (f, g) → int(conjugate(f) · g, t = -Pi..Pi) :
> NORM := f → sqrt(inproduct(f, f)) :
> v1 := 1 :
> v2 := cos(k · t) :
> v3 := sin(k · t) :
> c0 :=  $\frac{1}{\text{NORM}(1)}$ 

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$$c0 := \frac{\sqrt{2}}{2\sqrt{\pi}} \quad (1)$$

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> sk :=  $\frac{\sin(k \cdot t)}{\text{NORM}(\sin(k \cdot t))}$ 

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$$sk := \frac{\sin(k \sim t)}{\sqrt{\pi}} \quad (2)$$

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> ck :=  $\frac{\cos(k \cdot t)}{\text{NORM}(\cos(k \cdot t))}$ 

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$$ck := \frac{\cos(k \sim t)}{\sqrt{\pi}} \quad (3)$$

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> #ii

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> part_1 := inproduct(c0, t^2)

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$$part\_1 := \frac{\sqrt{2} \pi^{5/2}}{3} \quad (4)$$

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> part_2 := inproduct(ck, t^2)

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$$part\_2 := \frac{4\sqrt{\pi} (-1)^{k \sim}}{k \sim^2} \quad (5)$$

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> part_3 := inproduct(sk, t^2)

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$$part\_3 := 0 \quad (6)$$

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> f0 := part_1 + part_2 + part_3 :

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> #iii

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> p1 := inproduct(t^2, t^2)^2 :

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> p2 := 2 · inproduct(t^2, f0)

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$$p2 := \frac{4\pi^{7/2} (\sqrt{2} \pi^2 k \sim^2 + 12 (-1)^{k \sim})}{9 k \sim^2} \quad (7)$$

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> p3 := simplify(inproduct(f0, f0)^2)

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$$p3 := \frac{4\pi^4 (\sqrt{2} \pi^2 k \sim^2 + 12 (-1)^{k \sim})^4}{81 k \sim^8} \quad (8)$$

$$\begin{aligned}
 & \text{res} := evala(simplify(p1 - p2 + p3)) \\
 & \text{res} := -\frac{1}{2025 k^8} \left( 4 \pi^3 \left( -100 \pi^9 k^8 - 2400 \pi^7 (-1)^{k^{\sim}} \sqrt{2} k^6 - 81 \pi^7 k^8 \right. \right. \\
 & \quad \left. \left. + 225 \pi^{5/2} \sqrt{2} k^8 - 43200 \pi^5 k^4 + 2700 (-1)^{k^{\sim}} k^6 \sqrt{\pi} \right. \right. \\
 & \quad \left. \left. - 172800 \pi^3 (-1)^{k^{\sim}} \sqrt{2} k^2 - 518400 \pi \right) \right)
 \end{aligned}
 \tag{9}$$