restart: with(LinearAlgebra):

- [inproduct := $(f, g) \rightarrow int(conjugate(f) \cdot g, t = -Pi..Pi)$:
- \triangleright *NORM* := $f \rightarrow \text{sqrt}(inproduct(f, f))$:
- > simplify(NORM(1))

$$\sqrt{2}\sqrt{\pi}$$
 (1)

> assume(k, 'integer');
simplify(NORM(cos(k*t)), symbolic);

$$\sqrt{\pi}$$
 (2)

> simplify(NORM($\sin(k \cdot t)$), symbolic);

$$\sqrt{\pi}$$
 (3)

-> # ii

$$t1 := \frac{\sqrt{2} \pi^{5/2}}{3} \tag{4}$$

 $ightharpoonup t2 := simplify \Big(inproduct \Big(rac{\cos(k \cdot t)}{\operatorname{sqrt}(\operatorname{Pi})}, t^2 \Big), symbolic \Big)$

$$t2 := \frac{4\sqrt{\pi} (-1)^{k^{\sim}}}{k^{\sim^2}}$$
 (5)

> $simplify \left(inproduct \left(\frac{\sin(k \cdot t)}{\operatorname{sqrt}(\operatorname{Pi})}, t^2 \right), symbolic \right)$

(6)

 $\rightarrow f0 := simplify(t1 + t2)$

$$f0 := \frac{\sqrt{\pi} \left(\sqrt{2} \pi^2 k^{2} + 12 (-1)^{k^{2}} \right)}{3 k^{2}}$$
 (7)

⇒ distance := simplify(inproduct(t^2 , t^2)² −2·inproduct(t^2 , t^2) + inproduct(t^2 , t^2) + inproduct(t^2 , t^2) + inproduct($t^$

distance :=
$$\frac{1}{81 \, k^{-8}} \left(16 \left(-\frac{9 \, \pi^{5/2} \sqrt{2} \, k^{-8}}{4} - 27 \, (-1)^{k^{-}} k^{-6} \sqrt{\pi} \right) \right)$$
 (8)

$$+ \left(24 \pi^{2} (-1)^{k} k^{2} (\pi^{4} k^{4} + 72) \sqrt{2} + 5184 + \left(\pi^{8} + \frac{81}{100} \pi^{6}\right) k^{8} + 432 4 \mu^{4} (\pi^{8} + \pi^{4})^{3}\right)$$

 $+432 \pi^4 k \sim^4 \pi^3$

limit(distance, k = infinity)

$$\frac{16\,\pi^{12}}{81} + \frac{4\,\pi^{10}}{25} - \frac{4\,\sqrt{2}\,\pi^{11/2}}{9} \tag{9}$$

> eval(distance, k = 100000); # we can see it going to null