$\langle f, g \rangle = \int_{-\pi}^{\pi} f(t) \cdot g(t) \cdot dt$

span $\langle 1, \cos(kt), \sin(kt) \rangle$ met $k \in [1, +\infty[$

dus de componenten zijn 1, $\cos(kt)$ en $\sin(kt)$

we normeren :

$\langle 1, 1 \rangle = \sqrt{2\pi}$

$\langle \cos(kt), \cos(kt) \rangle = \sqrt{\pi}$

$\langle \sin(kt), \sin(kt) \rangle = \sqrt{\pi}$

dus onze vectoren die de basis vormen zijn:

$$\left\{ c_0 = \frac{1}{\sqrt{2\pi}} \ , \ c_k = \frac{\cos(kt)}{\sqrt{\pi}} \ , \ s_k = \frac{\sin(kt)}{\sqrt{\pi}} \right\}$$

2) de projectie f_0 is calculated with:

$$f_0 = \langle c_0, t^2 \rangle c_0 + \sum \langle c_k, t^2 \rangle \cdot c_k + \sum \langle s_k, t^2 \rangle \cdot s_k$$

$\langle c_0, t^2 \rangle = \dfrac{\sqrt{2}\,\pi^{5/2}}{3}$

$\langle c_k, t^2 \rangle = \dfrac{4\sqrt{\pi}\,(-1)^k}{k^2}$ $\rightarrow f_0 = \dfrac{\sqrt{2}\,\pi^{5/2}}{3} + \dfrac{4\sqrt{\pi}\cdot(-1)^k}{k^2}$

$\langle s_k, t^2 \rangle = 0$

3) de afstand is defined door : $\sqrt{\langle f_0 - f, f_0 - f \rangle}$

$$= \| f(t) - f_0 \|^2 = \langle f(t), f(t) \rangle^2 - 2 \cdot \langle f(t), f_0 \rangle + \langle f_0, f_0 \rangle^2$$