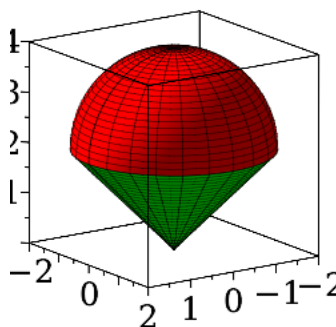


```

> restart: with(plots):
> # first draw the plots
> pkegel:=plot3d([r,phi,Pi/4],r=0..2*sqrt(2),phi=0..2*Pi,coords=
spherical,color=green):
pbol:=plot3d([4*cos(theta),phi,theta],phi=0..2*Pi,theta=0..Pi/4,
coords=spherical,color=red):
> display(pkegel,pbol)

```



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> # Eerst sferische coördinaten

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> x := r*sin(theta)*cos(phi)

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$$x := r \sin(\theta) \cos(\phi) \quad (1)$$

```

> y := r*sin(theta)*sin(phi)

```

$$y := r \sin(\theta) \sin(\phi) \quad (2)$$

```

> z := r*cos(theta)

```

$$z := r \cos(\theta) \quad (3)$$

```

> j := r^2*sin(theta)

```

$$j := r^2 \sin(\theta) \quad (4)$$

```

> # hoe is onze r? just fill in into sphere eq

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> sphere_eq := x^2 + y^2 + (z - a)^2 = a^2

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$$\text{sphere_eq} := r^2 \sin^2(\theta) \cos^2(\phi) + r^2 \sin^2(\theta) \sin^2(\phi) + (r \cos(\theta) - a)^2 = a^2 \quad (5)$$

```

> result := solve(sphere_eq, r)

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$$\text{result} := 0, \frac{2 \cos(\theta) a}{\sin^2(\theta) \cos^2(\phi) + \sin^2(\theta) \sin^2(\phi) + \cos^2(\theta)} \quad (6)$$

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> simplify(result[2 ])

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$$2 \cos(\theta) a \quad (7)$$

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> # so r goes from 0 to 2*cos(theta)*a

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> # theta goes from zero to 45 degrees

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> # phi is 360 degrees

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> # so the mass is:

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> M := rho*int(int(int(j, r = 0..2*cos(theta)*a), theta = 0..Pi/4), phi = 0..2*Pi)

```

$$M := \rho a^3 \pi \quad (8)$$

```

> # de massa middelpunten zijn dan (zie formules)
> x_ :=  $\frac{1}{M} \cdot \text{int}\left(\text{int}\left(\text{int}(\rho \cdot j \cdot x, r = 0..2 \cdot a \cdot \cos(\theta)), \theta = 0..\frac{\text{Pi}}{4}\right), \phi = 0..2\right. \\ \left. \cdot \text{Pi}\right)$ 
<
x_ := 0 (9)
> y_ :=  $\frac{1}{M} \cdot \text{int}\left(\text{int}\left(\text{int}(\rho \cdot j \cdot y, r = 0..2 \cdot a \cdot \cos(\theta)), \theta = 0..\frac{\text{Pi}}{4}\right), \phi = 0..2\right. \\ \left. \cdot \text{Pi}\right)$ 
<
y_ := 0 (10)
> z_ :=  $\frac{1}{M} \cdot \text{int}\left(\text{int}\left(\text{int}(\rho \cdot j \cdot z, r = 0..2 \cdot a \cdot \cos(\theta)), \theta = 0..\frac{\text{Pi}}{4}\right), \phi = 0..2\right. \\ \left. \cdot \text{Pi}\right)$ 
<
z_ :=  $\frac{7a}{6}$  (11)
> # Nu met cilinder coords, basically hetzelfde.
> restart:
> x := r*cos(theta)
<
x := r*cos(theta) (12)
> y := r*sin(theta)
<
y := r*sin(theta) (13)
> z := z
<
z := z (14)
> j := r
<
j := r (15)
> # De theta doet 360
> # De r is van 0 -> a (simply see formula)
> # voor z, steken we de variabelen in de sphere equation
> sphere_eq := x^2 + y^2 + (z - a)^2 = a^2
<
sphere_eq := r^2*cos(theta)^2 + r^2*sin(theta)^2 + (z - a)^2 = a^2 (16)
>
> result := solve(sphere_eq, z)
<
result := a + sqrt(a^2 - r^2), a - sqrt(a^2 - r^2) (17)
> upperbound_z := result[1]
<
upperbound_z := a + sqrt(a^2 - r^2) (18)
> # dus z is van a tot upperbound_z
> # vul in in de vorige equations
>

```