3) $\frac{\partial^2}{\partial t^2} u(x,t) + \partial t \cdot u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t)$

conditions

$$\begin{cases} u(0,t) = 0 \\ \partial_x u(L,t) = 0 \\ u(x,0) = 0 \\ \frac{\partial}{\partial t} u(x,0) = \left(\sin\left(\frac{\pi}{4}x\right)\right)^3 \end{cases}$$

Scheiding der veranderlijken:

$$T''X + T'X = X''T$$

$$\Leftrightarrow \frac{T''}{T} + \frac{T'}{T} = \frac{X''}{X} \Rightarrow \begin{cases} T'' + T' = T \cdot \sigma \\ X'' = \sigma \cdot X \end{cases}$$

eerst $X(x)$:

$$X'' = \sigma X \qquad \begin{cases} u(0,t) = 0 & (1) \\ \frac{\partial u}{\partial x}(L,t) = 0 & (2) \end{cases}$$

De enige niet triviale oplossing is:

$$X(x) = c_1 \cdot \cos(kx) + c_2 \cdot \sin(kx) \text{ met } k^2 = -\sigma$$

(1) $c_1 = 0 \rightarrow$ triviaal

(2) $c_2 \cdot k \cdot \cos(k \cdot L) = 0$

$\Leftrightarrow \cos(kL) = 0$ via maple $k = \frac{\pi}{4} \cdot (2m+1)$

$$\sigma = -\left(\frac{\pi(2m+1)}{4}\right)^2 \qquad X_m(x) = \sin\left(\frac{\pi}{4}(2m+1)x\right)$$

$T_m(t) \Leftrightarrow T'' + T' - T \cdot \overset{=k^2}{\sigma} = 0$

$$T_m(t) = e^{-t/2}\left(s_m \cdot e^{\left(\frac{\sqrt{-4k^2+1}}{2}t\right)} + t_m e^{\left(\frac{\sqrt{-4k^2+1}}{2}t\right)}\right)$$

$$= e^{-t/2} \cdot \left(s_m \sin\left(\frac{\sqrt{4k^2+1}}{2}t\right) + t_m \cos\left(\frac{\sqrt{-4k^2+1}}{2}t\right)\right)$$

$$u_k(x,t) = \sum_{m=0}^{\infty} \left(e^{-t/2} \cdot \left(s_m \cdot \sin\left(\frac{\sqrt{4k^2+1}}{2}t\right) + t_m \cos\left(\frac{\sqrt{4k^2+1}}{2}t\right)\right) \cdot \sin\left(\frac{\pi}{4}(2m+1)x\right)\right)$$

vind s_m, t_m

$$u(x,0) = \sum_{m=0}^{\infty} t_m \cdot \sin\left(\frac{\pi}{4}(2m+1)x\right) = 0$$

$\rightarrow t_m = 0$

$$\frac{\partial}{\partial t} u(x,0) \Leftrightarrow \left(\sin\left(\frac{\pi}{4}x\right)\right)^3 = \sum_{m=0}^{\infty}\left(-\frac{1}{2}\right) \cdot s_m \cdot \frac{\sqrt{-4k^2+1}}{2} \cdot \sin\left(\frac{\pi}{4}(2m+1)x\right)$$

$$\Rightarrow \frac{-4 \cdot \left(\sin\left(\frac{\pi}{4}x\right)\right)^3}{\sqrt{-4k^2+1}} = \sum_{m=0}^{\infty} s_m \cdot \sin\left(\frac{\pi}{4}(2m+1)x\right)$$

$$\Leftrightarrow \int_0^2 \frac{-4}{\sqrt{-4k^2+1}} \cdot \sin\left(\frac{\pi}{4}x\right)^3 \cdot \sin\left(\frac{\pi}{4}(2m+1)x\right) \cdot \frac{2}{2} = s_m$$

maple

geen effect.

$\Rightarrow s_m = \ldots$

als $t \rightarrow \infty \Rightarrow$ damping dominant, ansolf naar 0.