> restart : with(LinearAlgebra) : with(VectorCalculus) :
SetCoordinates(cartesian[x, t]) :

> $area := \frac{(1 - x - x \cdot \cos(t) + 1 - x - x \cdot \cos(t) + x \cdot \sin(t)) \cdot x \cdot \cos(t)}{2}$

$$area := \frac{(2 - 2x - 2x\cos(t) + x\sin(t))\, x\cos(t)}{2} \tag{1}$$

> $gradient := simplify(Gradient(area))$

$$gradient := (-2\cos(t)^2 x + (x\sin(t) - 2x + 1)\cos(t))\bar{e}_x + \left((2x^2\cos(t) + x^2 - x)\sin(t) + x^2\left(\cos(t)^2 - \frac{1}{2}\right)\right)\bar{e}_t \tag{2}$$

> # Seems like we do not have a direction in the y direction, whatever

> $critical\_points := (solve(\{gradient[1] = 0, gradient[2] = 0\}, \{x, t\}))$

$$critical\_points := \left\{t = \frac{\pi}{2}, x = 0\right\}, \left\{t = \arctan(1, RootOf(\_Z^2 - 3)), x = \frac{4\,RootOf(\_Z^2 - 3)}{3} - 2\right\}, \left\{t = \frac{\pi}{2}, x = 2\right\}, \left\{t = -\frac{\pi}{2}, x = \frac{2}{3}\right\} \tag{3}$$

> # X needs to be]0, 1[and t]0, pi:2[

> # seems like point 2 is the only valid one

> # Hessian

> $hessian := Hessian(area) :$

> $result := evalf\left(subs\left(x = \frac{4\,RootOf(\_Z^2 - 3)}{3} - 2, t = \arctan(1, RootOf(\_Z^2 - 3)), hessian\right)\right)$

$$result := \begin{bmatrix} -2.799038106 & 0.5000000010 \\ 0.5000000010 & -0.1722201661 \end{bmatrix} \tag{4}$$

> $Eigenvalues(result)$

$$\begin{bmatrix} -2.89099141081943 + 0.\,I \\ -0.0802668612805653 + 0.\,I \end{bmatrix} \tag{5}$$

> # These two values are < 0, so they define the maximum, so the second critical point isthe maximum area.

> $max\_area := evalf\left(subs\left(t = \arctan(1, RootOf(\_Z^2 - 3)), x = \frac{4\,RootOf(\_Z^2 - 3)}{3} - 2, area\right)\right)$

$$max\_area := 0.1339745961 \tag{6}$$