

```

> # i)
>
> restart: with(LinearAlgebra):
> A := Matrix([[0,-1,0,-1], [1,0,0,0], [0,1,0,0], [0,0,1,0]])

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$$A := \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (1)$$

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> # ii)
J, Q := JordanForm(A, output = ['J', 'Q'])

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$$J, Q := \begin{bmatrix} -\frac{1}{2} - \frac{I\sqrt{3}}{2} & 0 & 0 & 0 \\ 0 & -\frac{1}{2} + \frac{I\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} - \frac{I\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} + \frac{I\sqrt{3}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{1}{-3+I\sqrt{3}}, \frac{\frac{1}{3}\sqrt{3}}{-1+I\sqrt{3}}, -\frac{1+I\sqrt{3}}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, -\frac{1}{-3+I\sqrt{3}} \\ \left[ -\frac{-1+I\sqrt{3}}{2(-3+I\sqrt{3})}, \frac{1}{-3+I\sqrt{3}}, -\frac{1}{-3+I\sqrt{3}}, \frac{-1+I\sqrt{3}}{2(-3+I\sqrt{3})} \right] \\ \left[ -\frac{2}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, \frac{2}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, \right. \\ \left. \frac{2}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, \frac{1+I\sqrt{3}}{2(-3+I\sqrt{3})} \right] \\ \left[ -\frac{1}{-3+I\sqrt{3}}, -\frac{1+I\sqrt{3}}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, \frac{1+I\sqrt{3}}{(-3+I\sqrt{3})(-1+I\sqrt{3})}, \right. \\ \left. \frac{1}{-3+I\sqrt{3}} \right] \end{bmatrix} \quad (2)$$

$$\begin{aligned}
& \triangleright J\_exp := \text{Matrix}\left(\left[\left[\exp\left(\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right) \cdot t\right), 0, 0, 0\right], \left[0, \exp\left(\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right) \cdot t\right), \right.\right.\right. \\
& \quad \left.\left.\left.0, 0\right], \left[0, 0, \exp\left(\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right) \cdot t\right), 0\right], \left[0, 0, 0, \exp\left(\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right) \cdot t\right)\right]\right]\right) \\
& J\_exp := \begin{bmatrix} e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t} & 0 & 0 & 0 \\ 0 & e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t} & 0 & 0 \\ 0 & 0 & e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t} & 0 \\ 0 & 0 & 0 & e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t} \end{bmatrix} \tag{3}
\end{aligned}$$

$$\begin{aligned}
& \triangleright f := Q \cdot J\_exp \cdot \text{MatrixInverse}(Q) \cdot \text{Vector}([1, 0, 2, 1]) : \\
& \left[ \left[ \frac{1}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \left( (-10I\sqrt{3} + 6) e^{-\frac{(1+I\sqrt{3})t}{2}} + (-2I\sqrt{3} \right. \right. \right. \tag{4} \\
& \quad \left. \left. - 18) e^{\frac{(-1+I\sqrt{3})t}{2}} + 6(-3 + I\sqrt{3}) \left( e^{-\frac{(-1+I\sqrt{3})t}{2}} - e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right], \\
& \left[ \frac{1}{3(1 + I\sqrt{3})^2} \left( -4I\sqrt{3} e^{\frac{(-1+I\sqrt{3})t}{2}} + 3I e^{\frac{(1+I\sqrt{3})t}{2}} \sqrt{3} + 6 e^{\frac{(-1+I\sqrt{3})t}{2}} \right. \right. \\
& \quad \left. \left. - 9 e^{\frac{(1+I\sqrt{3})t}{2}} - 5I\sqrt{3} e^{-\frac{(1+I\sqrt{3})t}{2}} + 6I\sqrt{3} e^{-\frac{(-1+I\sqrt{3})t}{2}} + 3 e^{-\frac{(1+I\sqrt{3})t}{2}} \right) \right], \\
& \left[ \frac{1}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \left( 2 \left( I e^{-\frac{(1+I\sqrt{3})t}{2}} \sqrt{3} - 4I\sqrt{3} e^{\frac{(-1+I\sqrt{3})t}{2}} \right. \right. \right. \\
& \quad \left. \left. - 6I e^{-\frac{(-1+I\sqrt{3})t}{2}} \sqrt{3} - 3I e^{\frac{(1+I\sqrt{3})t}{2}} \sqrt{3} - 9 e^{-\frac{(1+I\sqrt{3})t}{2}} + 6 e^{\frac{(-1+I\sqrt{3})t}{2}} \right. \right. \\
& \quad \left. \left. - 9 e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right], \\
& \left[ \frac{1}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \left( (-10I\sqrt{3} + 6) e^{-\frac{(1+I\sqrt{3})t}{2}} + (-2I\sqrt{3} \right. \right. \\
& \quad \left. \left. - 18) e^{\frac{(-1+I\sqrt{3})t}{2}} - 6(-3 + I\sqrt{3}) \left( e^{-\frac{(-1+I\sqrt{3})t}{2}} - e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right] \right] \\
& f\_trig := \left[ \left[ -\frac{e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} + \frac{I\sqrt{3} e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{3(-1 + I\sqrt{3})} - \frac{(1 + I\sqrt{3}) e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} \right. \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} + \frac{8e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})^2} + \frac{4e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}(-3 + I\sqrt{3})}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \\
& - \frac{4e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} - \frac{2e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}(-1 + I\sqrt{3})}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \\
& - \frac{2e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} + \frac{2I\sqrt{3}e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}(-3 + I\sqrt{3})}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})(I\sqrt{3} + 3)} \\
& + \frac{2e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-1 + I\sqrt{3})(I\sqrt{3} + 3)} + \frac{2e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \Bigg], \\
& \left[ - \frac{(-1 + I\sqrt{3})e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{2(-3 + I\sqrt{3})} + \frac{e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} - \frac{e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} \right. \\
& + \frac{(-1 + I\sqrt{3})e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{2(-3 + I\sqrt{3})} + \frac{4(-1 + I\sqrt{3})e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})^2} \\
& - \frac{4Ie^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}\sqrt{3}}{3(1 + I\sqrt{3})^2} - \frac{4e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \\
& + \frac{(-1 + I\sqrt{3})^2e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} - \frac{e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}(-1 + I\sqrt{3})}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \\
& + \frac{2e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(1 + I\sqrt{3})(I\sqrt{3} + 3)} + \frac{2e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(1 + I\sqrt{3})(I\sqrt{3} + 3)} \\
& \left. - \frac{e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}(-1 + I\sqrt{3})}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \right], \\
& \left[ - \frac{2e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} + \frac{2e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} \right. \\
& \left. + \frac{2e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} + \frac{(1 + I\sqrt{3})e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{2(-3 + I\sqrt{3})} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{16 e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} - \frac{8 I e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t} \sqrt{3}}{3(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \\
& + \frac{8 e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})(1 + I\sqrt{3})} + \frac{(-1 + I\sqrt{3}) e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} \\
& - \frac{4 e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})(1 + I\sqrt{3})} \\
& + \frac{4 e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-1 + I\sqrt{3})(1 + I\sqrt{3})(I\sqrt{3} + 3)} \\
& - \frac{4 e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-1 + I\sqrt{3})(1 + I\sqrt{3})(I\sqrt{3} + 3)} - \frac{e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} \Bigg], \\
& \left[ -\frac{e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} - \frac{(1 + I\sqrt{3}) e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} + \frac{(1 + I\sqrt{3}) e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} \right. \\
& + \frac{e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{-3 + I\sqrt{3}} + \frac{8 e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})^2} + \frac{4 I e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t} \sqrt{3}}{3(1 + I\sqrt{3})(-1 + I\sqrt{3})} \\
& + \frac{4 e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(-1 + I\sqrt{3})} + \frac{2 e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t} (-1 + I\sqrt{3})}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \\
& - \frac{2 e^{\left(-\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} - \frac{2 e^{\left(-\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-1 + I\sqrt{3})(I\sqrt{3} + 3)} \\
& \left. - \frac{2 e^{\left(\frac{1}{2} - \frac{I\sqrt{3}}{2}\right)t}}{(-1 + I\sqrt{3})(I\sqrt{3} + 3)} - \frac{2 e^{\left(\frac{1}{2} + \frac{I\sqrt{3}}{2}\right)t}}{(-3 + I\sqrt{3})(1 + I\sqrt{3})} \right] \Bigg] \\
f_{real} := & \left[ \left[ -\frac{1}{3} \left( 2 \Re \left( \frac{1}{(-1 + I\sqrt{3})(1 + I\sqrt{3})^2} \left( (5I\sqrt{3} - 3) e^{-\frac{(1 + I\sqrt{3})t}{2}} - 3 \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. -3 + I\sqrt{3} \right) \left( e^{-\frac{(-1 + I\sqrt{3})t}{2}} + \frac{2 e^{\frac{(-1 + I\sqrt{3})t}{2}}}{3} - e^{\frac{(1 + I\sqrt{3})t}{2}} \right) \right) \right) \right] \right] \quad (5)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\Re\left(e^{\frac{(-1+I\sqrt{3})t}{2}}\right)}{2} \Bigg], \\
& \left[ -\frac{1}{3} \left( \Re\left( \frac{1}{(1+I\sqrt{3})^2} \left( (5I\sqrt{3}-3) e^{-\frac{(1+I\sqrt{3})t}{2}} - 6I e^{-\frac{(-1+I\sqrt{3})t}{2}} \sqrt{3} \right. \right. \right. \right. \\
& \left. \left. \left. - 3I e^{\frac{(1+I\sqrt{3})t}{2}} \sqrt{3} - 6 e^{\frac{(-1+I\sqrt{3})t}{2}} + 9 e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right) \right. \\
& \left. + \frac{4\sqrt{3} \Im\left( \frac{e^{\frac{(-1+I\sqrt{3})t}{2}}}{(1+I\sqrt{3})^2} \right)}{3} \right], \\
& \left[ \frac{1}{3} \left( 2 \Re\left( \frac{1}{(-1+I\sqrt{3})(1+I\sqrt{3})^2} \left( (I\sqrt{3}-9) e^{-\frac{(1+I\sqrt{3})t}{2}} \right. \right. \right. \right. \\
& \left. \left. \left. - 6I e^{-\frac{(-1+I\sqrt{3})t}{2}} \sqrt{3} - 3I e^{\frac{(1+I\sqrt{3})t}{2}} \sqrt{3} + 6 e^{\frac{(-1+I\sqrt{3})t}{2}} - 9 e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right) \right) \\
& \left. + \frac{8\sqrt{3} \Im\left( \frac{e^{\frac{(-1+I\sqrt{3})t}{2}}}{(-1+I\sqrt{3})(1+I\sqrt{3})^2} \right)}{3} \right], \\
& \left[ -\frac{1}{3} \left( 2 \Re\left( \frac{1}{(-1+I\sqrt{3})(1+I\sqrt{3})^2} \left( (5I\sqrt{3}-3) e^{-\frac{(1+I\sqrt{3})t}{2}} \right. \right. \right. \right. \\
& \left. \left. \left. + (3I\sqrt{3}+3) e^{\frac{(-1+I\sqrt{3})t}{2}} + 3(-3+I\sqrt{3}) \left( e^{-\frac{(-1+I\sqrt{3})t}{2}} - e^{\frac{(1+I\sqrt{3})t}{2}} \right) \right) \right) \right) \right) \\
& \left. + \frac{\sqrt{3} \Im\left( e^{\frac{(-1+I\sqrt{3})t}{2}} \right)}{3} \right] \Bigg]
\end{aligned}$$

> # This is how it should be done. Sadly Maple sucks and can't simplify this

└ *for me. But this is the structure of the exercise!*