Wiskundige modellering in de ingenieurswetenschappen: bordoefeningenles 10

Oefening 2:

```
> restart:with(plots):
```

Gegevens

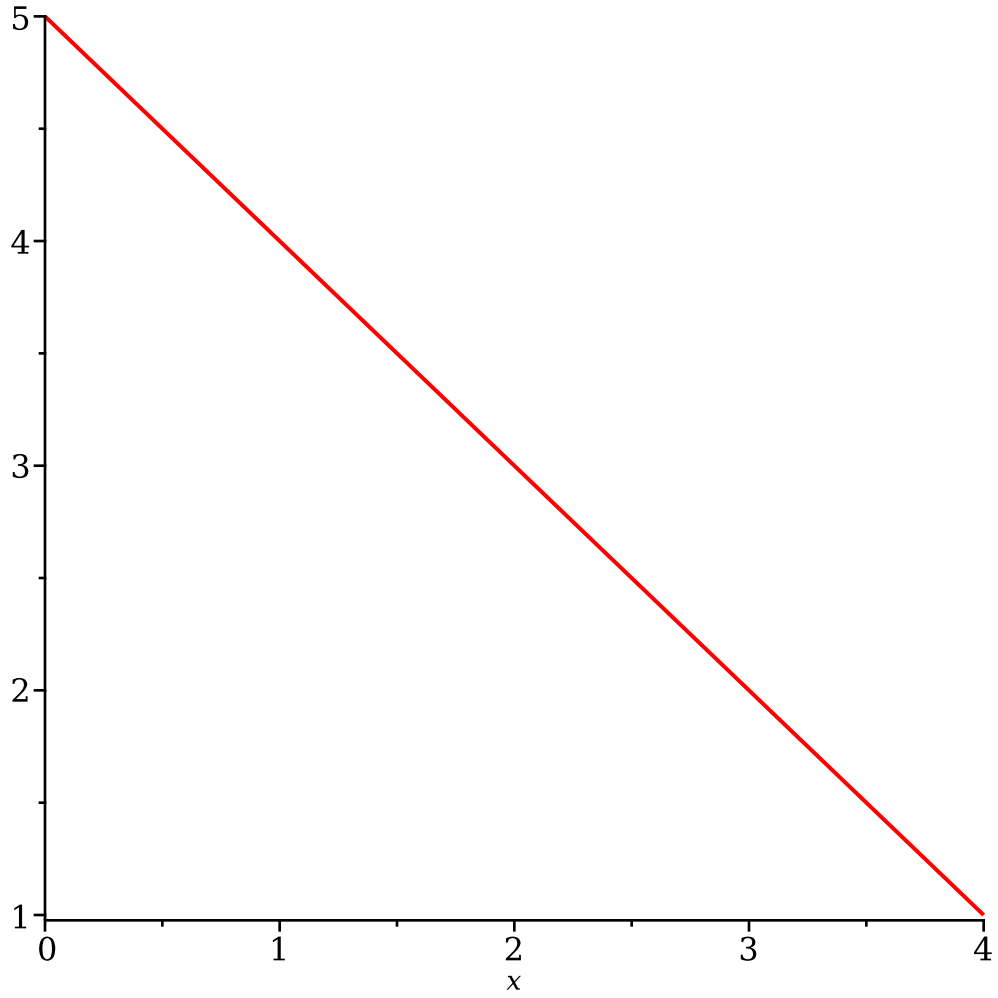
```
> h0 := 1;
  L  := 4;
  c  := 1;
```

$$h0 := 1$$
$$L := 4$$
$$c := 1 \quad (1.1)$$

```
> f := x-> -x+5;
  g := x-> x^2;
```

$$f := x \mapsto -x + 5$$
$$g := x \mapsto x^2 \quad (1.2)$$

```
> plot(f(x),x=0..L, color=red);
```



Particuliere oplossing

```
> ut := x-> h0;
```

$$ut := x \mapsto h0 \tag{1.3}$$

Homogene oplossing: Coefficienten

```
> s := n -> 2/L * int(cos((2*n+1)*Pi*x/(2*L))*(f(x)-ut(x)), x=0..L)
  ;
```

$$s := n \mapsto \frac{2 \cdot \left(\int_0^L \cos\left(\frac{(2 \cdot n + 1) \cdot \pi \cdot x}{2 \cdot L} \right) \cdot (f(x) - ut(x)) \, dx \right)}{L} \tag{1.4}$$

```
> t := n -> 4/((2*n+1)*Pi) * int(cos((2*n+1)*Pi*x/(2*L))*(g(x)-diff
  (ut(x),x)), x=0..L);
```

$$t := n \mapsto \frac{4 \cdot \left(\int_0^L \cos\left(\frac{(2 \cdot n + 1) \cdot \pi \cdot x}{2 \cdot L} \right) \cdot \left(g(x) - \frac{d}{dx} ut(x) \right) dx \right)}{(2 \cdot n + 1) \cdot \pi} \tag{1.5}$$

Homogene oplossing: Oplossing

```
> uh := (x,t,N) -> sum( (s(n)*cos(c*(2*n+1)*Pi*t/(2*L)) + t(n)*sin
```

```
(c*(2*n+1)*Pi*t/(2*L)))*cos((2*n+1)*Pi*x/(2*L)), n=0..N);
```

$$uh := (x, t, N) \to \sum_{n=0}^{N} \left(s(n) \cos\left(\frac{1}{2} \frac{c\,(2\,n+1)\,\pi\,t}{L} \right) + t(n) \sin\left(\frac{1}{2} \frac{c\,(2\,n+1)\,\pi\,t}{L} \right) \right) \cos\left(\frac{1}{2} \frac{(2\,n+1)\,\pi\,x}{L} \right) \quad (1.6)$$

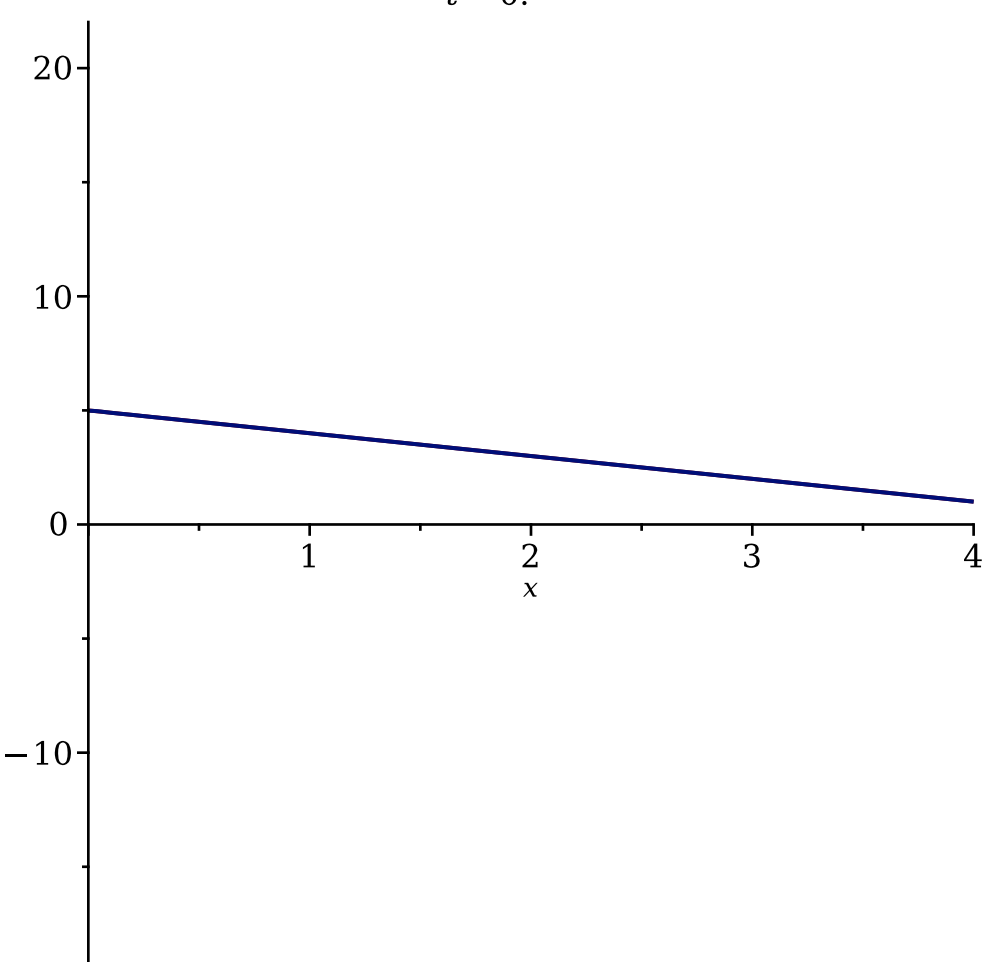
Oplossing

```
> u := (x,t,N) -> ut(x) + uh(x,t,N);
```

$$u := (x, t, N) \mapsto ut(x) + uh(x, t, N) \quad (1.7)$$

Visualisatie

```
> animate(plot, [{u(x,t,50),f(x)}, x=0..L], t=0..20, frames=100);
```




```
> plot({u(x,5*32,50),u(x,16,50), u(x,0,50)}, x=0..L);
```

Error, (in plot) expecting a real constant as range endpoint but received L

Oefening 3:

```
> restart:with(plots):
```

Gegevens

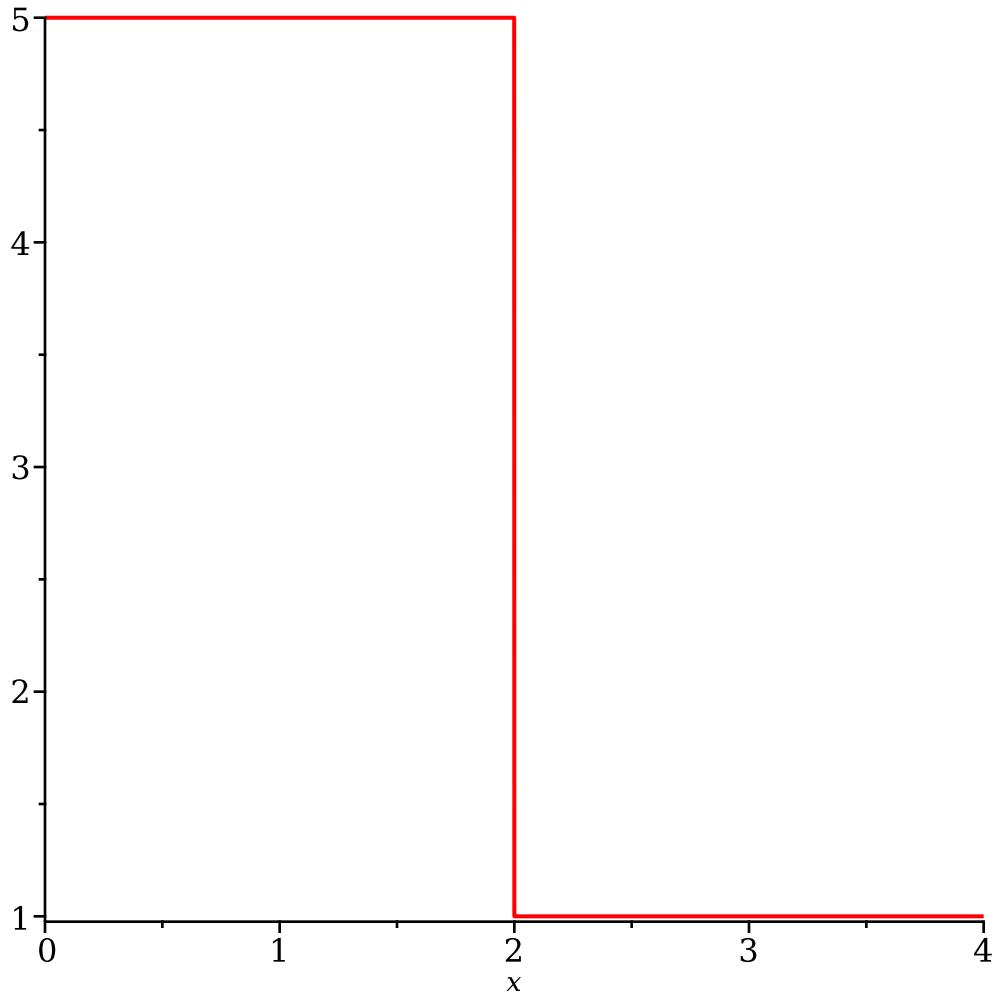
```
> h0 := 1;
  L  := 4;
  c  := 1;
```

$$h0 := 1$$
$$L := 4$$
$$c := 1 \qquad (2.1)$$

```
> f := x-> piecewise(x<2,5, x>2,1);
  g := x-> x^2;
```

$$f := x \mapsto \begin{cases} 5 & x < 2 \\ 1 & 2 < x \end{cases}$$
$$g := x \mapsto x^2 \qquad (2.2)$$

```
> plot(f(x),x=0..L, color=red);
```



Particuliere oplossing

```
> ut := x-> h0;
```

$$ut := x \mapsto h0 \qquad (2.3)$$

Homogene oplossing: Coefficienten

```
> s := n -> 2/L * int(cos((2*n+1)*Pi*x/(2*L))*(f(x)-ut(x)), x=0..L)
```

;

$$s := n \mapsto \frac{2 \cdot \left(\int_0^L \cos\left(\frac{(2 \cdot n + 1) \cdot \pi \cdot x}{2 \cdot L} \right) \cdot (f(x) - ut(x)) \, dx \right)}{L} \tag{2.4}$$

```
> t := n -> 4/((2*n+1)*Pi) * int(cos((2*n+1)*Pi*x/(2*L))*(g(x)-diff
  (ut(x),x)), x=0..L);
```

$$t := n \mapsto \frac{4 \cdot \left(\int_0^L \cos\left(\frac{(2 \cdot n + 1) \cdot \pi \cdot x}{2 \cdot L} \right) \cdot \left(g(x) - \frac{d}{dx} ut(x) \right) dx \right)}{(2 \cdot n + 1) \cdot \pi} \tag{2.5}$$

Homogene oplossing: Oplossing

```
> uh := (x,t,N) -> sum( (s(n)*cos(c*(2*n+1)*Pi*t/(2*L)) + t(n)*sin
  (c*(2*n+1)*Pi*t/(2*L)))*cos((2*n+1)*Pi*x/(2*L)), n=0..N);
```

$$uh := (x, t, N) \to \sum_{n=0}^{N} \left(s(n) \cos\left(\frac{1}{2} \frac{c\,(2\,n+1)\,\pi\,t}{L} \right) + t(n) \sin\left(\frac{1}{2} \frac{c\,(2\,n+1)\,\pi\,t}{L} \right) \right) \cos\left(\frac{1}{2} \frac{(2\,n+1)\,\pi\,x}{L} \right) \tag{2.6}$$

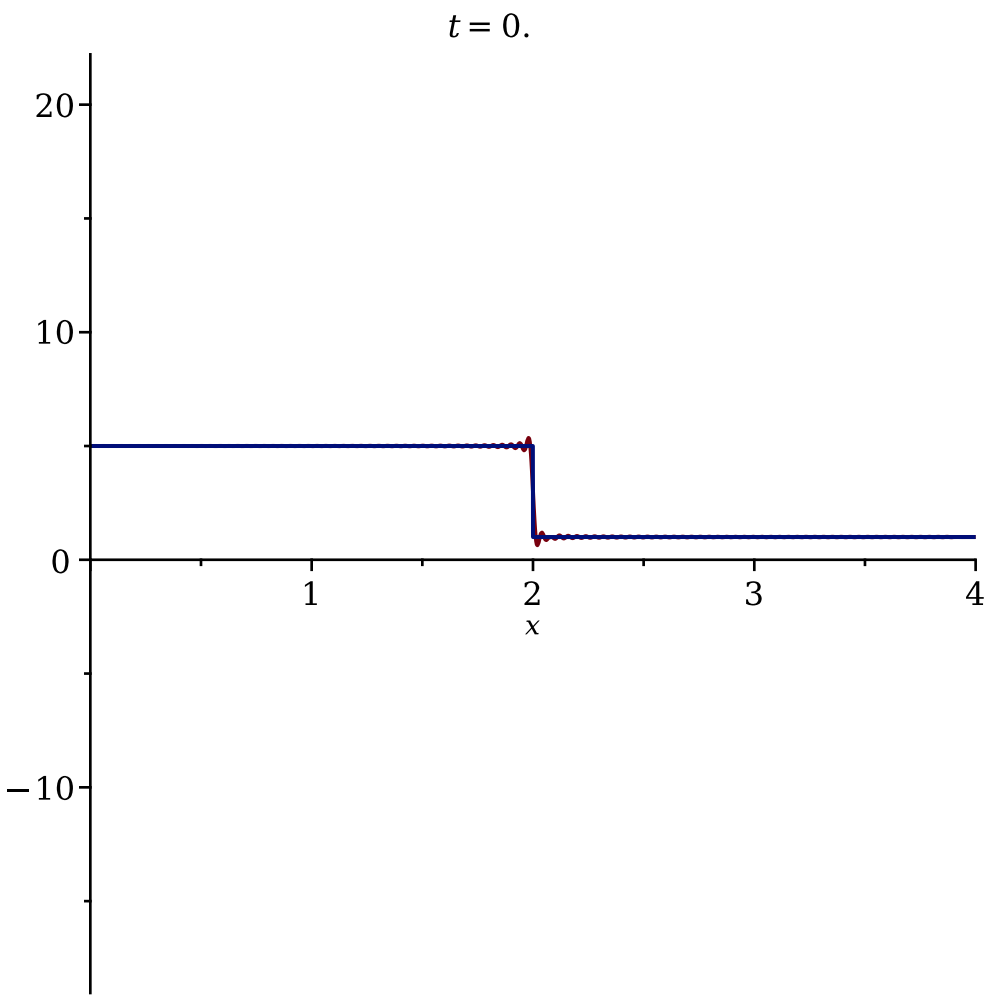
Oplossing

```
> u := (x,t,N) -> ut(x) + uh(x,t,N);
```

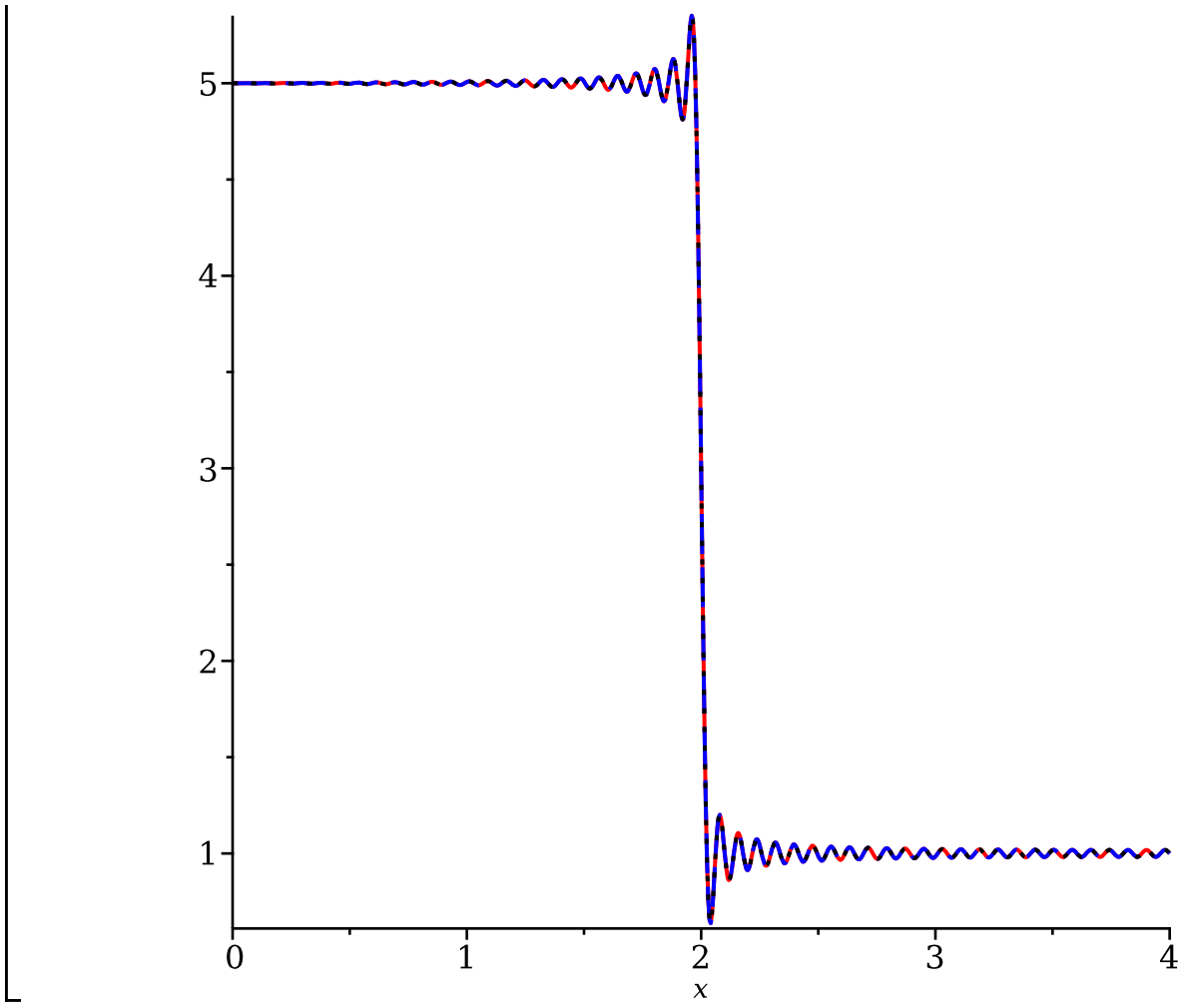
$$u := (x, t, N) \mapsto ut(x) + uh(x, t, N) \tag{2.7}$$

Visualisatie

```
> animate(plot, [{u(x,t,200),f(x)}, x=0..L], t=0..20, frames=100);
```




```
> plot0 := plot(u(x,0,100), x=0..L, color=red):
  plot1 := plot(u(x,16,100), x=0..L, color=blue, linestyle="dash"):
  plot2 := plot(u(x,5*16,100), x=0..L, color=black, linestyle=
  "dot"):
  display(plot0, plot1, plot2);
```


5
4
3
2
1
0
1
2
3
4
x