5) a) find we find the vectors :

$$V_1 = 1$$

$$V_2 = \cos(kt)$$ norm $\rightarrow$

$$V_3 = \sin(kt)$$

$$c_0 = \frac{\sqrt{2}}{2\sqrt{\pi}}$$

$$c_k = \sin(kt)/\sqrt{\pi}$$

$$\partial_k = \cos(kt)/\sqrt{\pi}$$

b) $$f_0 = (c_0 \cdot t^2) \cdot c_0 + \sum_{k=0}^{+\infty} (c_k \cdot t^2) \cdot c_k + \sum_{k=0}^{\infty} (\partial_k \cdot t^2) \cdot \partial_k$$

$$= \frac{\sqrt{2}}{3} \pi^{5/2} + \frac{4\sqrt{\pi}\,(-1)^k}{k^2} + 0$$

c) distance $$= \| f - f_0 \|^2 = \langle f, f \rangle^2 - 2 \cdot \langle f, f_0 \rangle$$

$$+ \langle f_0, f_0 \rangle^2$$

$$= \frac{6\pi^{10}}{25} - 2 \cdot 4\pi^{7/2} \frac{(\sqrt{2}\,\pi^2 k^2 + 12(-1)^k)}{8 k^2} + 4\pi^4 \frac{(\sqrt{2}\,\pi^5 k^2 + 12\,(-1)^k)^4}{81 k^8}$$