$F = \nabla\varphi + \nabla \times A$

$F = \nabla\varphi$

$\Leftrightarrow \nabla F = \Delta\varphi$

$= 3x^2 + 3y^2 + 3z^2$

$\Rightarrow \nabla\varphi = x^3, y^3, z^3$

$\varphi = \frac{1}{4}\cdot(x^4 + y^4 + z^4)$

$\nabla \times A = F - \nabla\varphi$

$= (x^2(x+y-z) - x^3,\ y^2(y+z-x) - y^3,\ z^2(z+x-y) - z^3)$

$A = \dots$

→ check divergence! $\neq 0$!

$\nabla A' = \nabla A + \Delta\varphi = 0$

$\Leftrightarrow \Delta\psi = x^2 z + y^2 z$

$\Leftrightarrow \nabla\psi = \left\{\frac{x^3}{3}z,\ \frac{y^3}{3}z,\ \frac{1}{12}(x^4 + y^4)\right\}$

$\Leftrightarrow \psi = \frac{1}{12}z(x^4 + y^4)$

So $A = \left\{-\frac{y^2 z(-z+2x)}{2} + \frac{x^3}{3}z,\right.$

$\left.-\frac{x^2 z(-z+2y)}{2} + \frac{y^3}{3}z,\ \frac{1}{12}(x^4+y^4)\right\}$

→ check Divergence and it should be 0!