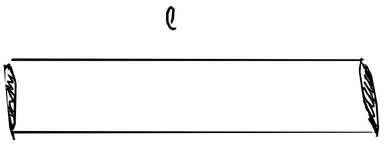
$u(x,0) = f(x)$

$= 3\cos^2 \frac{\pi x}{2l} \quad , 0 < x < l$



i) warmtevgl: $\frac{\partial u}{\partial t} = \alpha^2 \cdot \frac{\partial^2 u}{\partial x^2}$

isolated rand:

$\frac{\partial u}{\partial x}\Big|_{x=0} = \frac{\partial u}{\partial x}\Big|_{x=l} = 0$

$\frac{d}{dt} \cdot \int_0^l u(x,t) \cdot dx$

$= \int_0^l \frac{\partial u(x,t)}{\partial t} \cdot dx = \int_0^l \alpha^2 \cdot \underbrace{\frac{\partial^2 u}{\partial x^2} \cdot dx}_{=0} = 0$

ii) we need to reach equilibrium

$\int_0^l T_c \cdot dx = T_c l$ -> moet gelijk zijn aan $\underbrace{\int_0^l f(x) \cdot dx}_{\text{via maple: } \frac{3}{2} l}$

so: $T_c \cdot l = \frac{3}{2} l \Leftrightarrow \boxed{T_c = \frac{3}{2}}$

iii) 1) Separation of variables:

$u(x,t) = X(x) \cdot T(t)$

$\Rightarrow$ 1) $\frac{dT}{dt} + \lambda T = 0$

1) $\frac{d^2 X}{dx^2} + \frac{\lambda}{\alpha^2} X = 0$ met $k^2 = \frac{\lambda}{\alpha^2}$

$\Rightarrow \frac{d^2 X}{dx^2} + k^2 X = 0 \to X(x) = A\cos(kx) + B\sin(kx)$

if $x = 0$: $\frac{dX}{dx} = -Ak \cdot \sin(kx) + Bk \cdot \cos(kx)$

$= Bk = 0 \Rightarrow \boxed{B = 0}$

if $x = l$: $\frac{dX}{dx} = -Ak\sin(kl)$

omdat $\frac{dX}{dx}\Big|_{x=l} = 0 \Rightarrow \sin(kl) = 0$

$\Leftrightarrow kl = n \cdot \pi \Leftrightarrow \boxed{k = \frac{n\pi}{l}}$

$X(x) = A \cdot \cos\left(\frac{n\pi x}{l}\right)$

Now for: $\frac{dT}{dt} + \lambda T = 0$ (eerste orde diff vgl)

$\Leftrightarrow T(t) = C \cdot e^{-\lambda t}$ met $\lambda = a^2 k^2 = a^2 \cdot \frac{n^2 \pi^2}{l^2}$

$\Rightarrow T(t) = c \cdot e^{-\frac{a^2 \cdot n^2 \cdot \pi^2}{l^2} \cdot t}$

algemene oplossing is dus:

$u(x,t) = \sum_{n=0}^{\infty} a_n \cdot e^{-\frac{a^2 n^2 \cdot \pi^2 \cdot t}{l^2}} \cdot \cos\left(\frac{n\pi x}{l}\right)$

$= a_0 + \sum_{n=1}^{\infty} a_n$ "

$= \frac{2}{l} \cdot \int_0^l f(x) \cdot \cos\left(\frac{n\pi x}{l}\right) dx$, met $f(x) = 3\cos^2\left(\frac{\pi x}{2l}\right)$

$= \frac{6\sin(n\pi) \cdot \cos\left(\frac{n\pi}{l}\right)^2}{\pi n}$ en $a_0 =$

$\frac{1}{l} \int_0^l f(x) \cdot dx = \frac{3}{2}$