

```

> restart;

# Parameters
k := 4:
u0 := 2:
v0 := 1:

# Time range
t_range := 0..5:

# Underdamped case ( $\gamma^2 < 4k$ )
gamma1 := 2:
omega := sqrt(4*k - gamma1^2)/2:
u_underdamped := u0*exp(-gamma1*t/2)*cos(omega*t) +
    (v0 + (gamma1/2)*u0)*exp(-gamma1*t/2)*sin(omega*t)
    /omega:

# Critically damped case ( $\gamma^2 = 4k$ )
gamma2 := 2*sqrt(k):
u_critical := (u0 + (v0 + gamma2*u0/2)*t)*exp(-gamma2*t/2):

# Overdamped case ( $\gamma^2 > 4k$ )
gamma3 := 5:
r1 := (-gamma3 - sqrt(gamma3^2 - 4*k))/2:
r2 := (-gamma3 + sqrt(gamma3^2 - 4*k))/2:
u_overdamped := ((u0*r2 - v0)/(r2 - r1))*exp(r1*t) +
    ((v0 - u0*r1)/(r2 - r1))*exp(r2*t):

# Plot the three cases
plots[display](
    [plot(u_underdamped, t = t_range, color = blue, thickness = 2, legend
        = "Underdamped"),
    plot(u_critical, t = t_range, color = green, thickness = 2, legend
        = "Critically Damped"),
    plot(u_overdamped, t = t_range, color = red, thickness = 2, legend
        = "Overdamped")],
    title = "Damped Oscillator Solutions",
    labels = ["Time (t)", "Displacement (u(t))"],
    gridlines = true
);

```

