1) golf vgl: $\partial_t^2 u(x,t) = c^2 \cdot \partial_x^2 u(x,t)$

$u(L,t) = h_0$

$u'(0,t) = 0$

$u(x,0) = f(x)$

$u'(x,0) = g(x)$

① Scheiding van variabelen:

$u(x,t) = X(x) \cdot T(t)$

$\Rightarrow T'' X = c^2 X'' T$

$\Leftrightarrow \frac{T''}{T} = c^2 \cdot \frac{X''}{X} \quad \rightarrow \begin{cases} T'' = T c^2 \sigma \\ X'' = X \sigma \end{cases}$

Homogene oplossing:

$\sigma > 0: \quad c_1 \cdot e^{\sqrt{\sigma} x} + c_2 e^{-\sqrt{\sigma} x}$

$u'(0,t) \rightarrow \sqrt{\sigma} \cdot (c_1 - c_2) = 0$

$u(L,t) \rightarrow c_1 \cdot e^{\sqrt{\sigma} L} + c_2 e^{-\sqrt{\sigma} \cdot L} = 0$

$\Leftrightarrow c_1 = c_2 = 0$! triviaal

$\sigma = 0:$

$c_1 x + c_2 = 0$

$u'(0,t) \rightarrow c_1 = 0$

$u(L,t) \rightarrow c_1 L + c_2 = 0$) triviaal

$\sigma < 0:$

$c_1 \cdot \cos(kx) + c_2 \cdot \sin(kx) = 0$

$u'(0,t) \Rightarrow c_2 \cdot k \cdot \cos(kL) = 0$

$\rightarrow k = \frac{\pi}{2L} \cdot (2m+1) \quad \rightarrow \sigma = -\left(\frac{\pi}{2L}(2m-1)\right)^2$

$\rightarrow X_m(x) = \cos\left(\frac{\pi}{2L}(2m+1) \cdot x\right)$

$T'' = T c^2 \cdot \sigma$

$\rightarrow T_m(t) = s_m \cdot \sin\left(\frac{\pi(2m-1) \cdot ct}{2L}\right) + t_m \cdot \cos\left(\frac{\pi(2m-1)ct}{2L}\right)$

$u_h(x,t) = \sum_{m=0}^{\infty} \left(s_m \cdot \sin\left(\frac{\pi(2m-1)ct}{2L}\right) + t_m \cdot \cos\left(\frac{\pi(2m-1)cT}{2L}\right)\right) \cdot \cos\left(\frac{\pi}{2L}(2m+1) \cdot x\right)$

Inhomogene oplossing:

$u_p(x) = c_1 x + c_2$

for: $u'(0,t) = 0$

$\rightarrow c_1 = 0$

for $u(L,t) = h_0$

$\rightarrow c_2 = h_0 \quad \rightarrow u_p(x) = h_0$

total solution:

$$\boxed{u(x,t) = h_0 + \sum_{m=0}^{\infty}\left(s_m \cos\left(\frac{\pi(2m-1)ct}{2L}\right) + t_m \cdot \sin(\ldots)\right) \cos\left(\frac{\pi}{2L}(2m+1)x\right)}$$

② Find coefficients

1) $u(x,0) = f(x) = h_0 + \sum_{m=0}^{\infty} s_m \cdot \cos\left(\frac{\pi}{2L} \cdot (2m+1) \cdot x\right)$

$\Leftrightarrow s_m = \frac{2}{L} \cdot \int_0^L \cos\left(\frac{\pi}{2L}(2m+1)x\right)(f(x) - h_0)\, dx$

2) $u'(x,0) = \sum_{m=0}^{\infty} t_m \cdot \frac{\pi}{2L}(2m-1)c \cdot \cos\left(\frac{\pi}{2L} \cdot (2m+1) \cdot x\right) = g(x)$

$t_m = \frac{4}{\pi(2m-1)c} \cdot \int_0^L g(x) \cdot \cos\left(\frac{\pi}{2L} \cdot (2m+1)x\right) \cdot dx$