```
> restart: with(LinearAlgebra):
> # Define the function
> f := (x, y) -> x^3*exp(-(x^2 + y^2));
```

$$f := (x, y) \mapsto x^3 \cdot e^{-y^2 - x^2} \quad (1)$$

```
> df_dx := diff(f(x, y), x);
  df_dy := diff(f(x, y), y);
```

$$df\_dx := 3\,x^2\,e^{-x^2-y^2} - 2\,x^4\,e^{-x^2-y^2}$$

$$df\_dy := -2\,x^3\,y\,e^{-x^2-y^2} \quad (2)$$

```
> stationary_points := solve({df_dx = 0, df_dy = 0}, {x, y});
```

$$stationary\_points := \{x = 0, y = y\}, \{x = RootOf(2\,\_Z^2 - 3), y = 0\} \quad (3)$$

```
> d2f_dx2 := diff(f(x, y), x, x);
  d2f_dy2 := diff(f(x, y), y, y);
  d2f_dxdy := diff(f(x, y), x, y);
  Hessian := Matrix([[d2f_dx2, d2f_dxdy], [d2f_dxdy, d2f_dy2]]);
```

$$d2f\_dx2 := 6\,x\,e^{-x^2-y^2} - 14\,x^3\,e^{-x^2-y^2} + 4\,x^5\,e^{-x^2-y^2}$$

$$d2f\_dy2 := -2\,x^3\,e^{-x^2-y^2} + 4\,x^3\,y^2\,e^{-x^2-y^2}$$

$$d2f\_dxdy := -6\,x^2\,y\,e^{-x^2-y^2} + 4\,x^4\,y\,e^{-x^2-y^2}$$

$$Hessian := \quad (4)$$

$$\begin{bmatrix} 6\,x\,e^{-x^2-y^2} - 14\,x^3\,e^{-x^2-y^2} + 4\,x^5\,e^{-x^2-y^2} & -6\,x^2\,y\,e^{-x^2-y^2} + 4\,x^4\,y\,e^{-x^2-y^2} \\ -6\,x^2\,y\,e^{-x^2-y^2} + 4\,x^4\,y\,e^{-x^2-y^2} & -2\,x^3\,e^{-x^2-y^2} + 4\,x^3\,y^2\,e^{-x^2-y^2} \end{bmatrix}$$

```
>
> Hessian_at_points := subs(stationary_points, Hessian);
```

$$Hessian\_at\_points := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (5)$$

```
> eigenvalues := LinearAlgebra:-Eigenvalues(Hessian_at_points);
```

$$eigenvalues := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (6)$$

```
> # Analyse the behaviour along the y-axis if needed
> df_dx_at_x0 := subs(x = 0, df_dx);
  df_dy_at_x0 := subs(x = 0, df_dy);
```

$$df\_dx\_at\_x0 := 0$$

$$df\_dy\_at\_x0 := 0 \quad (7)$$

```
>
```