

we define: $s = i\omega$
 \swarrow Laplace \searrow Fourier

$$\rightarrow F\left(\frac{e^{-bt} \cdot \sin(at) \cdot \Theta(t)}{a}\right)(\omega)$$

$$= \mathcal{L}\left(\frac{e^{-bt} \cdot \sin(at)}{a}\right)(s)$$

$$= \frac{1}{a} \cdot \mathcal{L}\left((e^{-bt}) \sin(at)\right)(s)$$

$$= \frac{1}{a} \cdot \mathcal{L}\left(\sin(at)\right)(s+b) \quad \text{shift}$$

$$= \frac{1}{a} \cdot \frac{a}{a^2 + (s+b)^2} = \frac{1}{a^2 + (i\omega + b)^2}$$