$$\left(\frac{\partial^2}{\partial x^2} - \frac{v}{c^2}\cdot\frac{\partial}{\partial t}\cdot\frac{\partial}{\partial x} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi(x,t)=0$$

$u = x + at$, $v = x + bt$

$$\frac{\partial}{\partial x} = \frac{\partial u}{\partial x}\cdot\frac{\partial}{\partial u} + \frac{\partial v}{\partial x}\cdot\frac{\partial}{\partial v} = \frac{\partial}{\partial u} + \frac{\partial}{\partial v}$$

$$\frac{\partial}{\partial t} = \frac{\partial u}{\partial t}\cdot\frac{\partial}{\partial u} + \frac{\partial v}{\partial t}\cdot\frac{\partial}{\partial v} = a\cdot\frac{\partial}{\partial u} + b\frac{\partial}{\partial v}$$

$$\frac{\partial^2}{\partial x^2} = \left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right)^2 = \frac{\partial^2}{\partial u^2} + 2\cdot\frac{\partial}{\partial u}\cdot\frac{\partial}{\partial v} + \frac{\partial^2}{\partial v^2}$$

$$\frac{\partial^2}{\partial t\,\partial x} = \frac{\partial}{\partial t}\cdot\left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right) = \left(a\frac{\partial}{\partial u} + b\frac{\partial}{\partial v}\right)\left(\frac{\partial}{\partial u} + \frac{\partial}{\partial v}\right) = a\frac{\partial^2}{\partial u^2} + (a+b)\frac{\partial^2}{\partial u\partial v} + b^2\frac{\partial}{\partial v^2}$$

$$\frac{\partial^2}{\partial t^2} = \left(a\frac{\partial}{\partial u} + b\frac{\partial}{\partial v}\right)^2 = a^2\frac{\partial^2}{\partial u^2} + 2ab\cdot\frac{\partial^2}{\partial u\partial v} + b^2\frac{\partial^2}{\partial v^2}$$

$\boxed{\text{items die moeten verwijderd worden!}}$

dus:

$$\frac{\partial^2}{\partial u^2} + 2\cdot\frac{\partial}{\partial u}\cdot\frac{\partial}{\partial v} + \frac{\partial^2}{\partial v^2} - \left(\frac{v}{c^2}\left(a\frac{\partial^2}{\partial u^2} + (a+b)\frac{\partial^2}{\partial u\partial v} + b^2\frac{\partial}{\partial v^2}\right)\right)$$

$$-\frac{1}{c^2}\left(a^2\frac{\partial^2}{\partial u^2} + 2ab\cdot\frac{\partial^2}{\partial u\partial v} + \frac{b^2\partial^2}{\partial v^2}\right) = 0$$

$$\Rightarrow \frac{\partial^2}{\partial u^2}\cdot\left(1 - \frac{va}{c^2} - \frac{a^2}{c^2}\right) = 0 \qquad \left\{\begin{array}{l} c^2 - va - a^2 = 0 \\ c^2 - vb - b^2 = 0 \end{array}\right.$$

$$\Rightarrow \frac{\partial^2}{\partial v^2}\left(1 - \frac{b\cdot v}{c^2} - \frac{b^2}{c^2}\right) = 0$$

$$\Leftrightarrow \left\{\begin{array}{l} a^2 + va - c^2 = 0 \\ b^2 + vb - c^2 = 0 \\ \Leftrightarrow \left\{\begin{array}{l} a = \dfrac{-v \pm \sqrt{v^2+4c^2}}{2} \\ b = \dfrac{-v \pm \sqrt{v^2+4c^2}}{2} \end{array}\right. \end{array}\right.$$

∇

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