

Biometric Recognition

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Model Solutions

Solution 01. Let the gray scale image be I , and the intensity of at a pixel at location (x,y) be $I(x,y)$. We can follow the procedure below to convert values in the range of 0-7 to 3, 8-15 to 11 and so on.

For every pixle $I(x,y)$ of the image do step 1 to 4.

1. Let x = integer part of $I(x,y)/8$
2. Let $y = y * 8$
3. Let $z = y + 3$
4. Set $I(x,y) = z$

Solution 02.

- **Translate** by 45 columns to right and 16 rows up. $(100+45,66+16) = (145,82)$
- **Rotate** 30° anti clockwise.

$$\begin{aligned}x' &= x \times \cos 30 - y \times \sin 30 \\&= 145 \times 0.866 - 82 \times 0.5 \\&= 83 \\y' &= x \times \sin 30 + y \times \cos 30 \\&= 145 \times 0.5 - 82 \times 0.866 \\&= 144\end{aligned}$$

Solution 03. Laplacian of a function $f(x, y)$ of two variables is defined as $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. Because derivatives of any order are linear operations, the Laplacian is a linear operation. We know that

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \quad (1)$$

and

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \quad (2)$$

Therefore

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

The diagonal terms can be incorporated by adding two more terms one for each of the two diagonal directions. The form of each new term is the same as either equation-1 or equation-2 but coordinates are along the diagonal.

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Solution 04. Let $Match$ represents total matchings that pass the threshold Th then, based on correct match (CM) we can determine false non match (FNM) and false match (FM) as shown in Table 1.

- $FNM = 9 - CM$;
- $FM = Match - CM$;
- $FNMR = \frac{FNM \times 100}{9}$
- $FMR = \frac{FM \times 100}{72}$

Th	Match	CM	FNM	FM
28	81	9	0	72
29	79	8	1	71
30	76	8	1	68
31	74	7	2	67
32	71	6	3	65
33	66	6	3	60
34	62	6	3	56
35	60	6	3	54
36	54	5	4	49
37	51	5	4	46
38	46	5	4	41
39	37	3	6	34
40	27	2	7	25
41	21	2	7	19
42	16	1	8	15
43	9	1	8	8
44	2	1	8	1
45	1	0	0	1

Table 1: Match, CM, FNM, FM at Various Thresholds

We can see form Figure 1 that FNMR and FMR intersect near the threshold 38; so Equal Error Rate ($EER = 0.5(FMR + FNMR)$) is 50.69. When 38 is selected as operating threshold accuracy ($accuracy = 100 - EER$) is found to be 49.31.

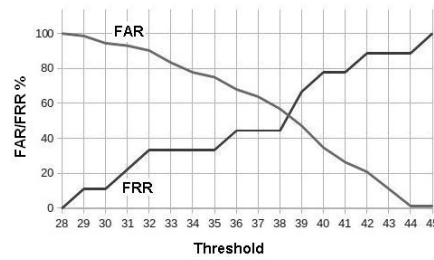


Figure 1: FMR, FNMR Curves

Solution 05.

- Dissimilarity score returned by face 0.18. Since the score lies between 0 and 1, we can convert this to similarity score by subtracting from one. Therefore, similarity score for face = $1 - 0.18 = 0.82$
- To scale this score in the range 1 to 100, we have to multiply the value by 100. so normalized similarity score for face = $0.82 \times 100 = 82$.
- Fingerprint similarity score 65 is added to this to get fused score = $82 + 65 = 147$
- Further, to normalize the fused score in range 1 to 100, we should divide this by 2. so final score is $147/2 = 73.5$

Solution 06. Consider the Table 2

At Thresh-old	Genuine Count	Imposters Count	False Re-ject (FR)	False Ac-cept (FA)	FRR % = FR/10	FAR % = FA/10	FAR-FRR	Accuracy = $\frac{100 - FAR + FRR }{2}$
64	0	1	0	10	0	100	100	50
70	0	1	0	9	0	90	90	55
71	0	1	0	8	0	80	80	60
74	0	1	0	7	0	70	70	65
76	0	1	0	6	0	60	60	70
81	1	1	0	5	0	50	50	75
82	0	0	1	4	10	40	30	75
83	0	1	1	4	10	40	30	75
84	0	1	1	3	10	30	20	80
85	0	0	1	2	10	20	10	85
87	1	1	1	2	10	20	10	85
88	1	0	2	1	20	10	10	85
89	0	1	3	1	30	10	20	80
90	1	0	3	0	30	0	30	85
91	1	0	4	0	40	0	40	80
92	1	0	5	0	50	0	50	75
94	1	0	6	0	60	0	60	70
95	1	0	7	0	70	0	70	65
96	1	0	8	0	80	0	80	60
98	1	0	9	0	90	0	90	55
99	0	0	10	0	100	0	100	50

Table 2: Analysis at various thresholds

1. Difference between FAR and FRR is minimum at threshold 85 and 88 so either of one can be selected as decision threshold, let 85 is selected.
2. Equal error rate (EER) $\frac{FAR + FRR}{2} = 15\%$ at a point where FRR = 10%, FAR = 20%.
3. Maximum Accuracy is 85% that is also at threshold 85 with FRR = 10%, FAR = 20%.

Solution 07.

1. **For Airport Access** Consider the product matrix is given in Table 3

Biometric Drawbacks	Finger	Face	Iris
Cooperation Needed	10×10	1×10	3×10
Social Stigma	10×3	3×3	3×3
Population Missing	1×3	1×3	1×3
Inconvenience	1×3	1×3	3×3
Acquisition time	1×10	1×10	3×10
Failure to Enroll	3×3	1×3	10×3
Failure to Image	3×10	3×10	3×10
#FA per 10K	1×10	3×10	1×10
#FR per 10K	3×10	10×10	1×10
Template Size	3×3	10×3	10×3
Installation Cost	1×3	1×3	3×3
Running cost	1×3	1×3	3×3
Cost per Match	3×3	1×3	1×3
	249	237	212

Table 3: Scores Against Drawbacks

Best system is Iris.

2. **For Credit Card:** Consider the product matrix is given in Table 4

Biometric Drawbacks	Finger	Face	Iris
Cooperation Needed	10×1	1×1	3×1
Social Stigma	10×3	3×3	3×3
Population Missing	1×3	1×3	1×3
Inconvenience	1×1	1×1	3×1
Acquisition time	1×3	1×3	3×3
Failure to Enroll	3×10	1×10	10×10
Failure to Image	3×10	3×10	3×10
#FA per 10K	1×10	3×10	1×10
#FR per 10K	3×3	10×3	1×3
Template Size	3×10	10×10	10×10
Installation Cost	1×10	1×10	3×10
Running cost	1×10	1×10	3×10
Cost per Match	3×3	1×3	1×3
	185	240	333

Table 4: Scores Against Drawbacks

Best system is Fingerprint.

3. **For Physical Access:** Consider the product matrix is given in Table 5

Biometric Drawbacks	Finger	Face	Iris
Cooperation Needed	10×1	1×1	3×1
Social Stigma	10×3	3×3	3×3
Population Missing	1×1	1×1	1×1
Inconvenience	1×3	1×3	3×3
Acquisition time	1×10	1×10	3×10
Failure to Enroll	3×3	1×3	10×3
Failure to Image	3×3	3×3	3×3
#FA per 10K	1×3	3×3	1×3
#FR per 10K	3×3	10×3	1×3
Template Size	3×1	10×1	10×1
Installation Cost	1×3	1×3	3×3
Running cost	1×3	1×3	3×3
Cost per Match	3×1	1×1	1×1
	96	92	126

Table 5: Scores Against Drawbacks

Best system is Face.

Solution 08. Combined parameters for multimodel biometric are shown in Table 6. Total is given by $FAR \times CFA + FRR \times CFR$.

Trait	FAR	FRR	CFA	CFR	Total
Face	3.0%	2.5%	5	2	20
Iris	0.5%	0.8%	3	8	7.9
Ear	1.0%	2.0%	4	6	16
Face + Iris	0.5%	0.8%	$0.75 \times 8 = 6$	$0.75 \times 10 = 7.5$	9
Face + Ear	1.0%	2.0%	$0.75 \times 9 = 6.75$	$0.75 \times 8 = 6$	18.75
Iris + Ear	0.5%	0.8%	$0.75 \times 7 = 5.25$	$0.75 \times 14 = 10.5$	11.025

Table 6: Combining traits.

Therefore, suitable biometric pair for selection is Iris alone.

Solution 09. Applying Haar wavelet transformation of second level on following image in Table 7.

$I_1 =$	12	4	23	12	22	18	12	16
	23	15	14	14	20	1	23	4
	19	5	24	15	21	2	11	3
	17	21	24	18	23	19	9	22
	14	23	19	9	3	10	15	23
	21	16	21	4	4	11	16	1
	23	13	17	7	15	13	12	18
	3	11	16	13	14	17	13	15

$I_2 =$	3	8	23	14	19	11	8	4
	6	4	1	24	18	9	12	15
	16	24	3	14	1	15	23	5
	10	34	7	18	5	23	19	21
	15	15	12	19	16	3	4	23
	13	13	23	20	12	4	15	16
	16	12	5	21	24	23	5	13
	6	1	6	22	13	14	14	11

Table 7: Images

Produces result as shown in Table 8 and 9.

Harr of $I_1 =$	65.0000	56.5000	-7.0000	6.5000	9.1924	9.1924	16.2635	3.5355
	57.5000	50.0000	4.5000	-6.5000	-0.7071	14.1421	-5.3033	-0.3536
	-6.5000	1.5000	2.5000	-3.5000	2.1213	-1.4142	0	7.0711
	6.0000	-8.5000	6.0000	-7.0000	-2.1213	4.9497	-4.5962	5.3033
	-5.3033	7.0711	-10.2530	6.3640	0	5.5000	-7.5000	-11.5000
	-6.0104	-12.7279	-3.8891	-0.7071	9.0000	1.5000	7.5000	10.5000
	1.0607	6.7175	-1.0607	-8.1317	-7.0000	-3.5000	0	-11.5000
	6.0104	-0.3536	9.5459	-1.7678	9.0000	3.5000	2.5000	-2.0000

Table 8: I_1 Haar Results

Harr of $I_2 =$	52.2500	52.0000	0.2500	-1.5000	-12.3744	-12.7279	-5.3033	6.0104
	54.7500	52.5000	-9.2500	2.0000	3.1820	-12.7279	7.4246	-8.8388
	-10.7500	-4.0000	-20.7500	10.5000	10.2530	2.8284	17.3241	-5.3033
	10.2500	-6.0000	0.2500	-13.5000	-3.1820	9.8995	7.4246	-5.3033
	4.5962	-4.2426	-3.8891	6.3640	-3.5000	16.0000	-0.5000	3.5000
	-4.2426	-8.4853	1.4142	0	8.0000	0	2.0000	10.0000
	-2.8284	-0.3536	5.6569	2.4749	0	-5.0000	2.5000	-9.0000
	6.7175	4.5962	8.1317	9.5459	-0.5000	0	1.0000	-5.5000

Table 9: I_2 Haar Results

Similarity between two images can be obtained by using bit level Hamming distance on feature vector. Feature vector of the above images is 2×2 sub matrix of second level haar wavelet transformation as below (drop fractional part to make values integer).

$$\text{Feature of } I_1 = \begin{bmatrix} 65 & 56 \\ 57 & 50 \end{bmatrix} = 65 \ 56 \ 57 \ 50 = 1000001 \ 0111000 \ 0111001 \ 0110010$$

$$\text{Feature of } I_2 = \begin{bmatrix} 52 & 52 \\ 54 & 52 \end{bmatrix} = 52 \ 52 \ 54 \ 52 = 0110100 \ 0110100 \ 0110110 \ 0110100$$

Hamming distance can be obtained by fraction of similar bits.

$$\begin{array}{rcl} \text{Feature of } I_1 & = & 1000001 \ 0111000 \ 0111001 \ 0110010 \\ \text{Feature of } I_2 & = & 0110100 \ 0110100 \ 0110110 \ 0110100 \\ \hline \text{Similar bits} & & 0001010 \ 1110011 \ 1110000 \ 1111001 \end{array}$$

$$\text{Similarity Score} = \frac{15}{28} = 0.5357$$

Solution 10.

Total matchings: 49

Total genuine matchings: 07

Total imposter matchings: 42

• **For Face:**

At Thresh- old	Genuine Count	Imposters Count	False Re- ject (FR)	False Ac- cept (FA)	FRR % = FR/7	FAR % = FA/42	FAR- FRR	Accuracy = $100 - \frac{ FAR+FRR }{2}$
0.0	0	0	0	42	0.00	100.00	100.00	50.00
0.1	0	5	0	42	0.00	100.00	100.00	50.00
0.2	0	3	0	37	0.00	88.10	88.10	55.95
0.3	0	6	0	34	0.00	80.95	80.95	59.52
0.4	1	11	0	28	0.00	66.67	66.67	66.67
0.5	0	7	1	17	14.29	40.48	26.19	72.62
0.6	1	5	1	10	14.29	23.81	9.52	80.95
0.7	1	1	2	5	28.57	11.90	16.67	79.76
0.8	1	3	3	4	42.86	9.52	33.33	73.81
0.9	3	1	4	1	57.14	2.38	54.76	70.24
1.0	0	0	7	0	100.00	0.00	100.00	50.00

Table 10: Face

Based on Table 10.

1. Difference between FAR and FRR is minimum at threshold 0.60 so appropriate threshold is 0.60.
2. Equal error rate ($EER = \frac{FAR+FRR}{2}$) = 19.05%, at a point where FRR = 14.29%, FAR = 23.81% Maximum Accuracy is 80.95% that is achieved and FRR = 14.29%, FAR = 23.81%.
3. Minimum Expected Error: probability that a matching score is importer $P_I = 42/49$, probability that a matching score is genuine $P_G = 7/49$. Minimum expected error is given by $E_{min} = FAR \times P_I + FRR \times P_G$ where FAR and FRR are given at point of EER. Therefore,

$$E_{min} = 23.81 \times \frac{42}{49} + 14.29 \times \frac{7}{49} = 22.45\%$$

- **For Ear:** dissimilarity score can be converted to similarity by subtracting the score by 1.0 to get following values shown in Table 11.

B	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.4	0.3	0.2	0.4	0.1	0.4	0.2
S_2	0.4	0.6	0.3	0.1	0.5	0.2	0.9
S_3	0.2	0.4	0.8	0.4	0.3	0.9	0.2
S_4	0.3	0.9	0.3	0.3	0.2	0.2	0.2
S_5	0.3	0.2	0.1	0.2	0.9	0.2	0.8
S_6	0.3	0.9	0.2	0.3	0.4	0.8	0.3
S_7	0.8	0.3	0.3	0.2	0.2	0.1	0.7

Table 11: Similarity Scores for Ear

At Thresh-old	Genuine Count	Imposters Count	False Re-ject (FR)	False Ac-cept (FA)	FRR % = FR/7	FAR % = FA/42	FAR-FRR	Accuracy = $100 - \frac{ FAR+FRR }{2}$
0.0	0	0	0	42	0.00	100.00	100.00	50.00
0.1	0	4	0	42	0.00	100.00	100.00	50.00
0.2	0	14	0	38	0.00	90.48	90.48	54.76
0.3	1	11	0	24	0.00	57.14	57.14	71.43
0.4	1	6	1	13	14.29	30.95	16.67	77.38
0.5	0	1	2	7	28.57	16.67	11.90	77.38
0.6	1	0	2	6	28.57	14.29	14.29	78.57
0.7	1	0	3	6	42.86	14.29	28.57	71.43
0.8	2	2	4	6	57.14	14.29	42.86	64.29
0.9	1	4	6	4	85.71	9.52	76.19	52.38
1.0	0	0	7	0	100.00	0.00	100.00	50.00

Table 12: Ear

Based on above Table 12

1. Appropriate threshold: difference between FAR and FRR is minimum at threshold 0.50 so appropriate threshold is 0.50.
2. Equal error rate (EER) $\frac{FAR+FRR}{2} = 22.62\%$ at a point where FRR = 28.57%, FAR = 16.67% Maximum Accuracy is 78.57% that is achieved and FRR = 28.57%, FAR = 14.29%.
3. Minimum Expected Error: probability that a matching score is importer $P_I = 42/49$, probability that a matching score is genuine $P_G = 7/47$. Minimum expected error is given by $E_{min} = FAR \times P_I + FRR \times P_G$ where FAR and FRR are given at point of EER. Therefore,

$$E_{min} = 28.57 \times \frac{42}{49} + 16.67 \times \frac{7}{49} = 26.87\%$$

- **For Iris:** Can be normalized by dividing the score by 100 to get following values as shown in Table 13.

B	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.9	0.5	0.5	0.2	0.1	0.1	0.3
S_2	0.1	0.8	0.2	0.3	0.8	0.4	0.4
S_3	0.3	0.1	0.5	0.4	0.4	0.4	0.8
S_4	0.1	0.3	0.3	0.8	0.2	0.3	0.7
S_5	0.8	0.2	0.1	0.2	0.4	0.1	0.2
S_6	0.3	0.9	0.2	0.1	0.2	0.9	0.3
S_7	0.8	0.4	0.2	0.4	0.2	0.4	0.9

Table 13: Normalized Scores for Ear

At Thresh-old	Genuine Count	Imposters Count	False Re-ject (FR)	False Ac-cept (FA)	FRR % = FR/7	FAR % = FA/42	FAR-FRR	Accuracy = $100 - \frac{ FAR+FRR }{2}$
0.0	0	0	0	42	0.00	100.00	100.00	50.00
0.1	0	8	0	42	0.00	100.00	100.00	50.00
0.2	0	10	0	34	0.00	80.95	80.95	59.52
0.3	0	8	0	24	0.00	57.14	57.14	71.43
0.4	1	8	0	16	0.00	38.10	38.10	80.95
0.5	1	2	1	8	14.29	19.05	4.76	83.33
0.6	0	0	2	6	28.57	14.29	14.29	78.57
0.7	0	1	2	6	28.57	14.29	14.29	78.57
0.8	2	4	2	5	28.57	11.90	16.67	79.76
0.9	3	1	4	1	57.14	2.38	54.76	70.24
1.0	0	0	7	0	100.00	0.00	100.00	50.00

Table 14: Iris

Based on Table 14.

1. Appropriate threshold: difference between FAR and FRR is minimum at threshold 0.50 so appropriate threshold is 0.50.
2. Equal error rate (EER) $\frac{FAR+FRR}{2} = 16.67\%$ at a point where FRR = 14.29%, FAR = 19.05%
Maximum Accuracy is 83.33% that is achieved and FRR = 14.29%, FAR = 19.05%.
3. Minimum Expected Error: probability that a matching score is importer $P_I = 42/49$, probability that a matching score is genuine $P_G = 7/47$. Minimum expected error is given by $E_{min} = FAR \times P_I + FRR \times P_G$ where FAR and FRR are given at point of EER. Therefore,

$$E_{min} = 14.29 \times \frac{42}{49} + 19.05 \times \frac{7}{49} = 14.97\%$$

- **Fused matching score:** with accuracy of Face, Ear and Iris are 95%, 97% and 99% Weight for individual traits t can be obtained using formula

$$W_t = \frac{Accuracy_t}{\sum Accuracy_t}$$

Therefore, $W_{Face} = 95/(95 + 97 + 99) = 0.326$, $W_{Ear} = 97/(95 + 97 + 99) = 0.333$, and $W_{Iris} = 99/(95 + 97 + 99) = 0.340$. Fused score for similarity can be obtained by using normalized similarity scores as

$$fuse_score = \sum_{t \in \{Face, Ear, Iris\}} W_t \times score_t$$

Table 15 shows fused scores.

B	S_1	S_2	S_3	S_4	S_5	S_6	S_7
S_1	0.7326	0.4329	0.4322	0.2664	0.0999	0.2976	0.4294
S_2	0.2976	0.7326	0.2983	0.2005	0.6993	0.3656	0.5987
S_3	0.4294	0.1998	0.7298	0.4322	0.4315	0.6313	0.4690
S_4	0.1665	0.5973	0.3649	0.6001	0.2324	0.3316	0.4024
S_5	0.4697	0.2650	0.1977	0.2650	0.6313	0.2310	0.4322
S_6	0.2345	0.8991	0.2650	0.2643	0.2990	0.8658	0.3649
S_7	0.5710	0.3337	0.2983	0.4308	0.3302	0.2345	0.6695

Table 15: Fused Scores

At Thresh-old	Genuine Count	Imposters Count	False Re-ject (FR)	False Ac-cept (FA)	FRR % = FR/7	FAR % = FA/42	FAR-FRR	Accuracy = $\frac{100}{ FAR+FRR } - \frac{1}{2}$
0.0999	0	1	0	42	0.00	100.00	100.00	50.00
0.1665	0	1	0	41	0.00	97.62	97.62	51.19
0.1977	0	1	0	40	0.00	95.24	95.24	52.38
0.1998	0	1	0	39	0.00	92.86	92.86	53.57
0.2005	0	1	0	38	0.00	90.48	90.48	54.76
0.2310	0	1	0	37	0.00	88.10	88.10	55.95
0.2324	0	1	0	36	0.00	85.71	85.71	57.14
0.2345	0	2	0	35	0.00	83.33	83.33	58.33
0.2643	0	1	0	33	0.00	78.57	78.57	60.71
0.2650	0	3	0	32	0.00	76.19	76.19	61.90
0.2664	0	1	0	29	0.00	69.05	69.05	65.48
0.2976	0	2	0	28	0.00	66.67	66.67	66.67
0.2983	0	2	0	26	0.00	61.90	61.90	69.05
0.2990	0	1	0	24	0.00	57.14	57.14	71.43
0.3302	0	1	0	23	0.00	54.76	54.76	72.62
0.3316	0	1	0	22	0.00	52.38	52.38	73.81
0.3337	0	1	0	21	0.00	50.00	50.00	75.00
0.3649	0	2	0	20	0.00	47.62	47.62	76.19
0.3656	0	1	0	18	0.00	42.86	42.86	78.57
0.4024	0	1	0	17	0.00	40.48	40.48	79.76
0.4294	0	2	0	16	0.00	38.10	38.10	80.95
0.4308	0	1	0	14	0.00	33.33	33.33	83.33
0.4315	0	1	0	13	0.00	30.95	30.95	84.52
0.4322	0	3	0	12	0.00	28.57	28.57	85.71
0.4329	0	1	0	9	0.00	21.43	21.43	89.29
0.4690	0	1	0	8	0.00	19.05	19.05	90.48
0.4697	0	1	0	7	0.00	16.67	16.67	91.67
0.5710	0	1	0	6	0.00	14.29	14.29	92.86
0.5973	0	1	0	5	0.00	11.90	11.90	94.05
0.5987	0	1	0	4	0.00	9.52	9.52	95.24
0.6001	1	0	0	3	0.00	7.14	7.14	96.43
0.6313	1	1	1	3	14.29	7.14	7.14	89.29
0.6695	1	0	2	2	28.57	4.76	23.81	83.33
0.6993	0	1	3	2	42.86	4.76	38.10	76.19
0.7298	1	0	3	1	42.86	2.38	40.48	77.38
0.7326	2	0	4	1	57.14	2.38	54.76	70.24
0.8658	1	0	6	1	85.71	2.38	83.33	55.95
0.8991	0	1	7	1	100.00	2.38	97.62	48.81

Table 16: Fused score at various threshold.

Based on Table 16

1. Appropriate threshold: difference between FAR and FRR is minimum at threshold 0.6001 so appropriate threshold is 0.6001.
2. Equal error rate (EER) $\frac{FAR+FRR}{2} = 3.57\%$ at a point where FRR = 0.00%, FAR = 7.14%
Maximum Accuracy is 96.43% that is achieved and FRR = 0.00%, FAR = 7.14%.

3. To determine optimum value of k so that the system provides 100% CRR for top k best matches in case of identification, consider fused scores in sorted order as given in Table 17.

S_1	0.7326 (S_1)	0.4329 (S_2)	0.4322 (S_3)	0.4294 (S_7)	0.2976 (S_6)	0.2664 (S_4)	0.0999 (S_5)
S_2	0.7326 (S_2)	0.6993 (S_5)	0.5987 (S_7)	0.3656 (S_6)	0.2983 (S_3)	0.2976 (S_1)	0.2005 (S_4)
S_3	0.7298 (S_3)	0.6313 (S_6)	0.4690 (S_7)	0.4322 (S_3)	0.4315 (S_5)	0.4294 (S_1)	0.1998 (S_2)
S_4	0.6001 (S_4)	0.5973 (S_2)	0.4024 (S_7)	0.3649 (S_3)	0.3316 (S_6)	0.2324 (S_5)	0.1665 (S_1)
S_5	0.6313 (S_5)	0.4697 (S_1)	0.4322 (S_7)	0.2650 (S_2)	0.2650 (S_4)	0.2310 (S_6)	0.1977 (S_3)
S_6	0.8991 (S_2)	0.8658 (S_6)	0.3649 (S_7)	0.2990 (S_5)	0.2650 (S_3)	0.2643 (S_4)	0.2345 (S_1)
S_7	0.6695 (S_7)	0.5710 (S_1)	0.4308 (S_4)	0.3337 (S_2)	0.3302 (S_5)	0.2983 (S_3)	0.2345 (S_6)

Table 17: Sorted Fused Scores

To get 100% CRR, one should consider $k = 2$ top best matches.