Q 4

FOUNDATIONS OF ARTIFICIAL INTELLIGENCE

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M.Tech. AI (Semester I)

Consider the problem of placing k knights on an n×n chessboard such that no two knights are attacking each other, where k is given and $k \leq n^2$.

Observation:

n = 1, max(k) =1

K

n = 2, max(k) = 4

K	K
K	K

n = 3, max(k) = 5

K		K
	K	
K		K

	K	
K	K	K
	K	

1. Choose a CSP formulation. In your formulation, what are the variables?

Answer

$K_{i,}$ one for each of k Knights, where

1 <= i <= k

Example - When k = 4, we will have the following possible variables:

K_1, K_2, K_3 and K_4

2. What are the possible values of each variable?

Answer

Each variable can take one of the possible values for tuple (g, h).

g can take values from 0 to (n - 1).

h can take values from 0 to (n – 1).

Example - When k = 4, each variable can take one of the following values:

(0,0)	(0,1)	(0,2)	(0,3)
(1,0)	(1,1)	(1,2)	(1,3)
(2,0)	(2,1)	(2,2)	(2,3)
(3,0)	(3,1)	(3,2)	(3,3)

3. **What sets of variables are constrained, and how?**

Answer

Every pair of knights (K_r, K_s),

where 1 <= r <= n, 1 <= s <= n and r != s,

is constrained, such that no two knights can be on the same square or on the squares separated by a knight's move.

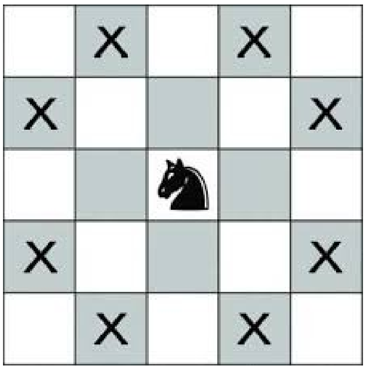

Image Source: HackerEarth

(0,0)	(0,1)	(0,2)	(0,3)	(0,4)
(1,0)	(1,1)	(1,2)	(1,3)	(1,4)
(2,0)	(2,1)	**(2,2)**	(2,3)	(2,4)
(3,0)	(3,1)	(3,2)	(3,3)	(3,4)
(4,0)	(4,1)	(4,2)	(4,3)	(4,4)

(0,0)	(i-2, j-1)	(0,2)	(i-2, j+1)	(0,4)
(i-1, j-2)	(1,1)	(1,2)	(1,3)	(i-1, j+2)
(2,0)	(2,1)	**(i, j)**	(2,3)	(2,4)
(i+1, j-2)	(3,1)	(3,2)	(3,3)	(i+1, j+2)
(4,0)	(i+2, j-1)	(4,2)	(i+2, j+1)	(4,4)

4. Now consider the problem of putting as many knights as possible on the board without any attacks. Explain how to solve this as a search problem.

Answer

Here, we can assume the number of available knights to be same as the number of squares on the chessboard.

Now, we need to find the maximum number of knights that we can place on the chessboard.

We can solve this problem using backtracking search.

- Place the knights one by one starting from the first row and first column and move forward to the first row and second column such that no two knights attack each other.
- When one row gets over, we move to the next row.
- Before placing a knight, we always check if the square is an attacking position of some other knight.
- If it is, we need to move forward and keep looking for another square.
- If it is not, we can place the knight.
- While following this procedure, we need to create a new board every time we place a knight.
- This is done because if we get one solution and we need other solutions, then we can backtrack to our old board with the old configuration of knights which can then be explored for other possible solutions.
- The process of backtracking is continued till we get all our possible solutions.
- Once we have identified all possible solutions, we can easily identify the maximum number of knights that we can place on the chessboard.