

The Rwandan Rural Solar System Project
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Abstract: A numerical cost minimizing off-grid solar photovoltaic (PV) system model is designed and applied to a hypothetical load in rural Rwanda. The energy generated by the PV system is derived from measured high frequency solar irradiance data produced by the Rwandan Automated Weather Station (AWS) system. In particular, the AWS at Bakokwe, Rwanda is used in the cost minimizing algorithm. The raw AWS data is aggregated to an 8760 hour time series, so that the model results reflect a system that can operate reliably across the entire distribution of annual solar irradiance. The primary tradeoff in the model is between solar PV modules and batteries. The total system cost, including PV modules, batteries, solar charger and inverter at Bakokwe, Rwanda is \$3080.

Introduction: In 2014, the World Bank's estimate of the population in Rwanda with access to electricity was 20 percent and only 9 percent in the rural areas where the majority of the population lives.¹ The Rwanda Development Board estimated 40.5 percent of Rwandans had access rate to electricity by August 2017.² Of the 40.5 percent of the population with access, 29.5 percent had access through the central power grid and 11 percent had access through off-grid power sources. The Rwanda Development Board's goal of 100 percent access by 2023/24 includes a projection of 48 percent access from off-grid electricity. The ability for Rwandans to meet and the Rwanda Development Board to support these goals will depend on the efficiency of the investments made in off grid electricity supply. The Rwandan Meteorological Agency's investment in solar irradiance measurement instruments at 22 automated weather stations (AWS) is part of the government's information campaign to support investment in solar power generation capacity. The solar irradiance data measured and transmitted by the AWS needs to be processed and applied in order to support informed decisions about solar generation system siting and sizing. The following is an illustrative example of the processing and application of Rwandan AWS solar irradiance data for the optimal sizing of a reliable off-grid solar system.

Literature Review: The research on solar power generation in Rwanda spans several fields, but there is considerable room for additional research. Previous work includes estimates of solar irradiance in the atmospheric science literature, data validation for Rwandan AWS, and economic analysis of the willingness (or ability) for rural populations to pay for solar energy. There is also a literature on optimal design of micro-grid and hybrid off-grid systems.

Safari and Gasore estimate global solar radiation for different locations in Rwanda using sunshine based models. They calibrate the model using long term data on monthly mean daily sunshine hours and measured (weather station) daily global solar radiation data from Kigali

¹ The World Bank <https://data.worldbank.org/indicator/EG.ELC.ACCS.ZS?locations=RW> accessed 2/28/2018

² The Rwanda Development Board <http://www.rdb.rw/rdb/energy.html> accessed 2/28/2018.

Airport. The purpose of this research is to support solar photovoltaic system modeling. Safari and Gasore claim the model performs well because of its fit to the Kigali Airport data, but there is no out of sample forecast assessment, because there was no data direct measurements of solar radiation at the other locations for which they estimate radiation from observed sunshine hours.

Beyer and Habyarimana assess the quality of the Rwanda AWS irradiance data against data derived from satellite sources, including NASA SSE estimates of daily average insolation by month. The authors find large deviations between several AWS sites and NASA data. Beyer and Habyarimana also compare estimated generation (kWh) of PV systems based on satellite derived irradiance data against measured production of an installed system. They find average errors of approximately 25 percent for several locations in Eastern Africa and conclude that more localized data are preferred.

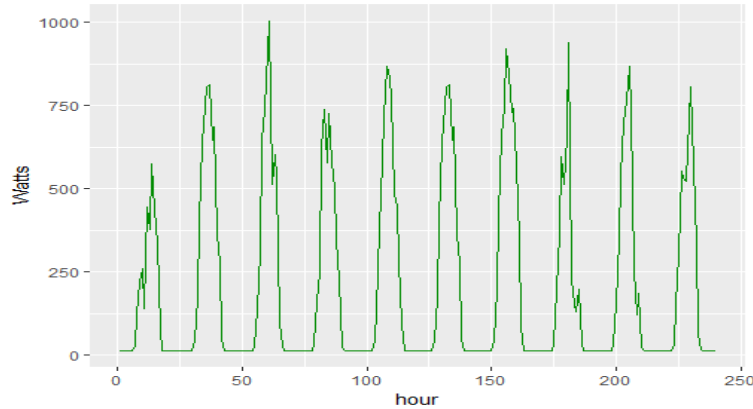
A number of studies optimize micro grid systems by combining solar, wind, hydroelectric and conventional combustion generators. For instance, Ashok develops an iterative numerical model to find the optimal system components for an isolated rural community in India. Ashok finds the combination of micro-hydro and wind generation to be the optimal system design in that application.

Grimm et al, estimate the willingness to pay for electricity in rural Rwanda, and find it is too low to cover the cost of providing access. The willingness to pay is a function of income, and therefore this result may only represent an inability to pay. Therefore, investment in electricity infrastructure to provide access to rural populations will require subsidies. Grimm et al suggest it is more rationale to promote off-grid solar than grid-based electrification because of its better cost-benefit performance.³

Data: The Rwandan Meteorological Agency and Rwandan Environmental Management Authority work together to maintain the country's AWS system and provide public access to the data those AWS produce. A map of the locations of the weather stations is included in the Appendix as Figure A1. The AWS measure temperature, barometric pressure, soil moisture content, humidity, wind speed and solar irradiance. A set of raw measurement data was collected for the Bakokwe AWS for the time period between January 1, 2017 and December 31, 2017. The dataset includes 101,060 observations of solar irradiance (W/m^2), approximately one measurement every five minutes. In order to simplify the application of the data which follows, the mean of the raw data was averaged by hour to produce an hourly time series, i.e. 8760 solar irradiance values. Given the high frequency of the data, the hourly average irradiance is a reasonable estimate of the integral of irradiance over the hour, so we can consider the hourly irradiance value to be an insolation value (Wh/m^2). The complete 8760 hourly measure of insolation at Bakokwe AWS is shown in the appendix as Figure A2. A 240 hour portion of the time series at Bakokwe is show here in Figure 1.

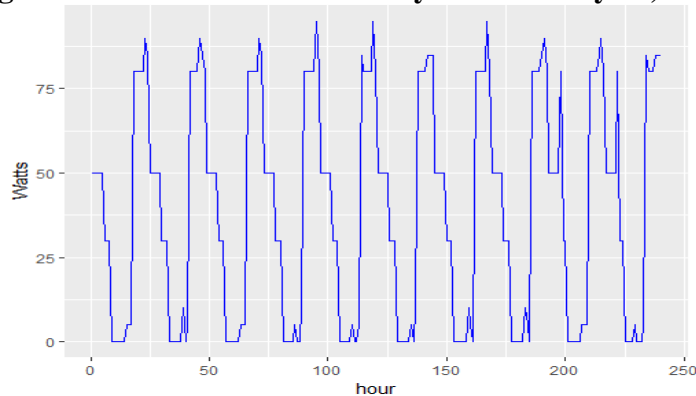
³ Grimm, M., Lenz, L., Peters, J., & Sievert, M. (2017). Demand for off-grid solar electricity—Experimental evidence from Rwanda.

Figure 1: Bakokwe Insolation Data – January 1 to January 10, 2017



In order to model the reliability and cost of an off-grid solar PV system, a system load is required. System load is difficult to estimate, given the population of interest's lack of access to electricity makes their current load zero. In order to illustrate the application of a simple cost minimizing algorithm for an off-grid solar system, we construct a hypothetical rural Rwandan hourly load profile. The profile imagines a single household with loads driven primarily by lighting demand with the addition of small accessory loads for radio or phone charging. The lighting loads are determined by the daily sunrise and sunset values.⁴ At sundown there is a 50W outdoor light switched on and this light stays on until the sun rises. There are also three 10W indoor lights on a single switch. These lights come on after sundown, switch off at midnight, and come on again for three hours in the morning. In addition, three 5W accessory loads are switched on for one hour twice a day with different probability in the morning, afternoon and evening. A 240 hour sample of the load data is shown below in Figure 2.

Figure 2: Bakokwe Load January 1 to January 10, 2017

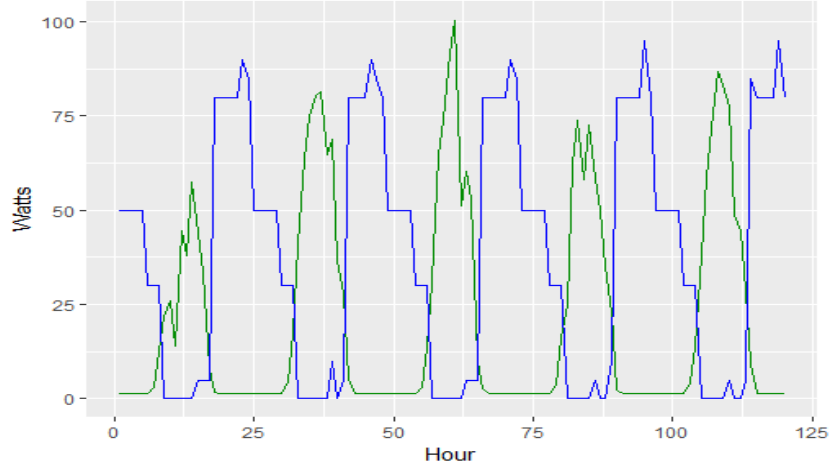


Off-grid Solar PV System Model: One challenge of an off-grid system is the lack of scale and diversity in the load and generation. Diversity of a large set of loads leads to a predictable smooth load normally characterized by some base load. The particular challenge of a solar PV off-grid system is the correlation between solar irradiance and lighting load, its negative and in the present case near one. Therefore, all energy derived from the solar irradiance must be stored

⁴ Sun up and sun down data available at <https://www.timeanddate.com/sun/rwanda/kigali>

and discharged in hours without irradiance to meet demand or loads. This characteristic of the solar PV system under current consideration is illustrated in Figure 3. The green line represents 10 percent of the irradiance value and blue represents the load. Ten percent of irradiance is used to yield comparable scales in the time series and is also a reasonable approximation of PV module efficiency.

Figure 3: Rural Rwanda Solar PV System



The off-grid solar PV system model will be used to compare the cost and reliability performance of different system configurations. System components such as the inverter and the solar charge controller will be considered fixed inputs in the model. The PV modules and battery components will be allowed to vary. The PV modules and batteries are complements, in part, and substitutes, in part. A detailed investigation of the solar irradiance data can help with the intuition here, but is omitted for space and clarity considerations. The two variable components are complements, because without PV modules no energy can be added to the batteries and without batteries no energy generated in the solar hours can be stored for use in the lighting load hours. However, higher relative levels of PV modules reduce the need for excess battery capacity because energy needed for the coming evening can be generated during the current day. On the other hand, higher relative battery capacity can facilitate storage from higher irradiance days for use on later lower irradiance days when not enough energy is generated from the PV modules.

The objective of the off-grid solar PV system model is to find the cost minimizing combination of solar PV modules and batteries necessary to meet a minimum state of charge (reliability) constraint. The objective function is a simple linear combination of the PV module and battery inputs.

$$Cost = \beta_1 pv.cap + \beta_2 bat.cap$$

Where $pv.cap$ is the discrete number of $200W/m^2$ PV modules and $bat.cap$ is the discrete number of 600Wh batteries. The parameters β_1 and β_2 are the per unit cost of the 200W PV module and 600Wh battery respectively. In this application $\beta_1 = \$0.4/W$ and $\beta_2 = \$0.2/Wh$.

The constraint on the optimization is a state of charge (or reliability) constraint. The reliability constraint in the current application is violated if the system battery pack is discharged below 55 percent of capacity in any period.

$$bat_t(pv.cap, bat.cap, solar_t, bat_{t-1}, load_t) < 0.55bat.cap$$

Where bat_t is the state of charge (production) function which takes $pv.cap$, $bat.cap$, $solar_t$, bat_{t-1} , and $load_t$ as arguments and returns the energy available in time period t , $solar_t$ is the times series of observed solar irradiance data, bat_{t-1} is the energy left in the battery from time period $t - 1$, and $load_t$ is the household load in period t . In practice the reliability constraint can be relaxed to allow some non-zero number of hours to violate the constraint. In instances where the number of violations is low, it may be reasonable to sacrifice some ‘reliability’ for the reduced cost. Formally, the cost minimization problem is as follows.

$$\min_{pv.cap, bat.cap} \beta_1 pv.cap + \beta_2 bat.cap$$

$$s.t. \quad 0.55bat.cap - bat(pv.cap, bat.cap, solar_t, bat_{t-1}, load_t) < 0 \quad \forall t \in \{1, 2, \dots, 8760\}$$

Cost Minimization Methodology: The cost minimization problem stated above may appear at first glance consistent with standard cost minimization from economic theory. The objective function is a linear (and therefore concave cost function) and production is determined, in part, by the productive inputs, our choice variables $pv.cap$ and $bat.cap$. There are several considerations which complicate the problem. The constraint applies in each period. The functional form of the production is not immediately obvious, and upon further consideration includes discontinuities associated with the $pv.cap$ and $bat.cap$. Production (battery charge) is a function of exogenous variables $solar_t$ and $load_t$ as well as a continuation value bat_{t-1} . In addition, the inputs are discrete rather than continuous. This makes an optimization problem difficult to solve with techniques that rely on calculus. Given these considerations, we employ a numerical method for solving the optimization problem. Specifically, we employ an exhaustive grid search over a reasonable range of combinations of the inputs. For the current application, the choice variables, $pv.cap$ and $bat.cap$, are allowed to range from 1 to 10. Therefore, the optimization program searches 100 different combinations of inputs. For each combination, the program calculates the cost of the input combination and counts the number of periods in which the constraint is violated.

The algorithm begins by setting $bat.cap = 1 \times bat.inc$ and $pv.cap = 1 \times pv.inc$, where $bat.inc = 600$ and $pv.inc = 200$. The cost is calculated and stored in a 10 by 10 cost matrix in position 1,1. The cost for the first combination is $\beta_1 pv.cap + \beta_2 bat.cap = 0.4 \times 1 \times 200 + 0.2 \times 1 \times 600 = 80 + 120 = 200$. The state of charge of the battery at time zero is set to the battery capacity, $bat_0 = bat.cap$. In each period (hour) the level of energy provided to battery is $pv_t = pv.size \times solar_t \times \eta$, where η is the efficiency of the solar PV system and $solar_t$ is the solar insolation in W/m^2 . The $pv.size$ is the number of 200W panels which are approximately $1 m^2$, and therefore the units on $pv.size \times solar_t$ is Wh. The level of energy extracted from the battery in each period is $load_t/\gamma$ where γ is the efficiency of the battery. The efficiency of the PV modules, η , and efficiency of the batteries, γ , are 0.1 and 0.8 respectively. In addition, a count variable $violation_t$ is set to zero in period zero, i.e. $violation_0 = 0$.

In each period $t \in \{1,2, \dots, 8760\}$, the following logic is applied.

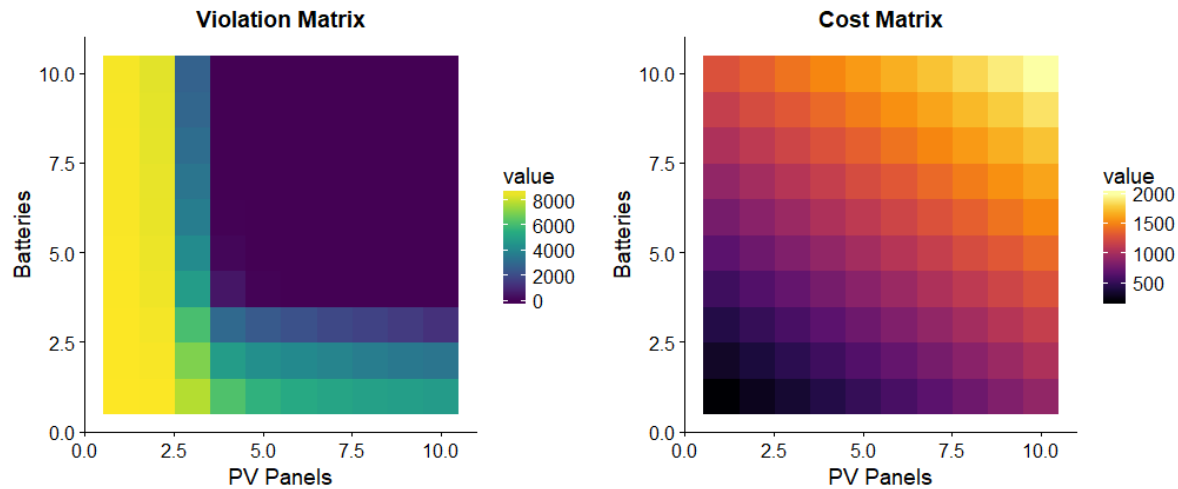
| | |
|------|--|
| | $x = bat_{t-1} + pv_t - load_t/\gamma$ |
| If | $x > bat.cap$ |
| Then | $bat_t = bat.cap$ |
| Else | $bat_t = x$ |
| If | $bat_t < bat.cap \times 0.55$ |
| Then | $violation_t = violation_{t-1} + 1$ |
| Else | $violation_t = violation_{t-1}$ |

In period 8760, the value of $violation_{8760}$ is returned and stored in a violation matrix in position 1,1. Once this is completed for the first combination, $bat.inc = 600$ and $pv.inc = 200$, an incremental change is made such that $bat.inc = 600$ and $pv.inc = 400$ and the cost and violation values are recalculated and stored in the respective matrices. The procedure is carried out for all 100 combinations in the set $\{(pv.cap, bat.cap), pv.cap = \{1,2, \dots, 10\}, bat.cap = \{1,2, \dots, 10\}\}$. The cost matrix is then searched for the minimum value subject to the requirement that the corresponding value in the violation matrix is zero.

Results

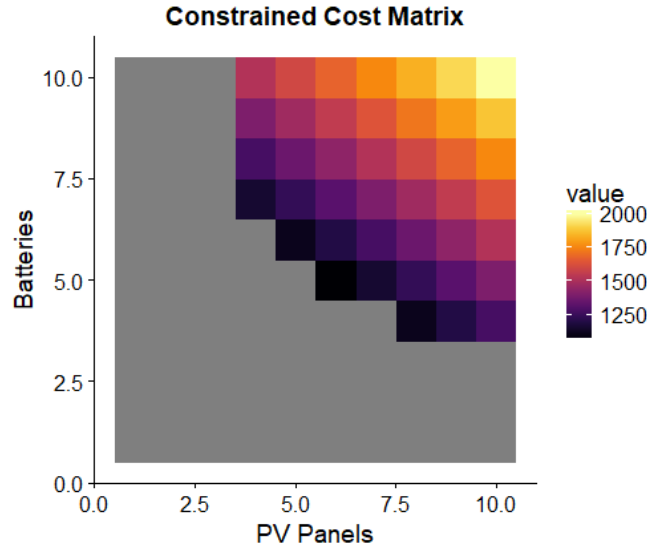
The results of the cost minimizing solar PV system reasonably sized exhaustive grid search are presented below. Figure 4 shows a visualization of the violation and cost matrices.

Figure 4: Violation and Cost Matrices



Where the darkest blue section of the violation matrix illustrates the combinations of $pv.cap$ and $bat.cap$ where the constraint is violated in zero hours. The cost matrix illustrates the linear nature of the objective of the cost function with a greater gradient in the battery dimension. Figure 5 reduces the cost matrix by excluding the combinations where the violation matrix is non-zero.

Figure 5: The Results Matrix



The minimum cost system which satisfies the reliability constraint is $pv.cap = 6m^2 \times 200W/m^2 = 1200W$ and $bat.cap = 5 \times 600Wh = 3000Wh$. The cost of that system is $\frac{\$0.4}{W} \times 1200W + \frac{\$0.2}{Wh} \times 3000Wh = \1080 . If the cost of a solar charger and inverter are included, the total cost is $\$1080 + \$400 + \$2000 = \3480 .

Conclusion and Future Work: The availability of surface measurements of solar irradiance in Rwanda provides a primary input for the optimization of solar generation. The isolated solar PV system model designed around the Bakokwe AWS measurements provides an illustration of one type of optimization, cost minimization of PV system at a fixed location. In addition to the Bakokwe AWS site we have collected data for six other sites. Once this data is cleaned, it can be applied to the same cost minimization procedure and the relative cost of providing reliable power from isolated solar PV systems across rural Rwanda can be discovered. This is a second type of optimization, optimal siting of solar generation. In addition, the difference between of Bakokwe and other AWS site and the NASA SSE irradiance data deserves further investigation. What are the causes of the differences? Are there other explanatory variables, like local topography, which drive the differences? Also, the availability of the a geographically disperse set of AWS provides the opportunity to interpolate between the points to estimate solar irradiance at any location within Rwanda.

Bibliography

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Appendix

Figure A1: Rwandan AWS

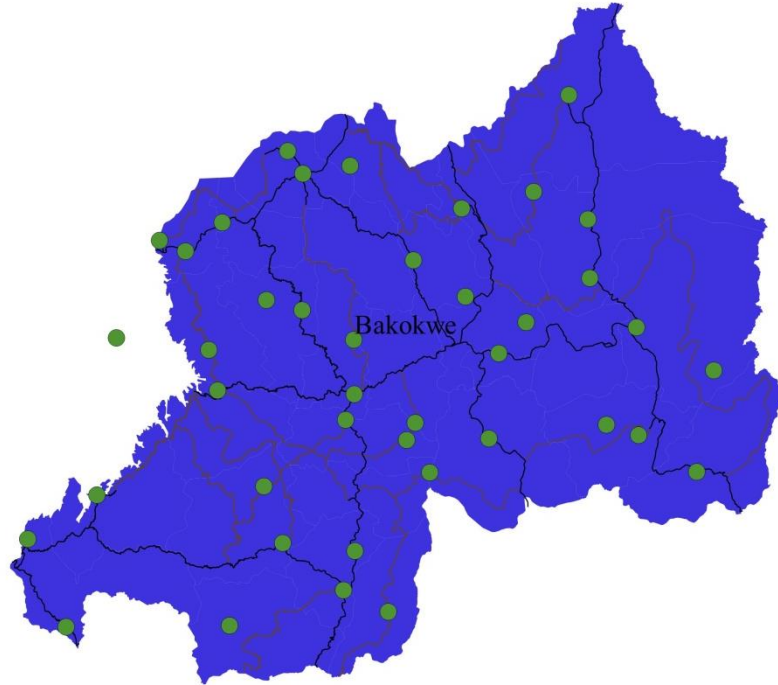
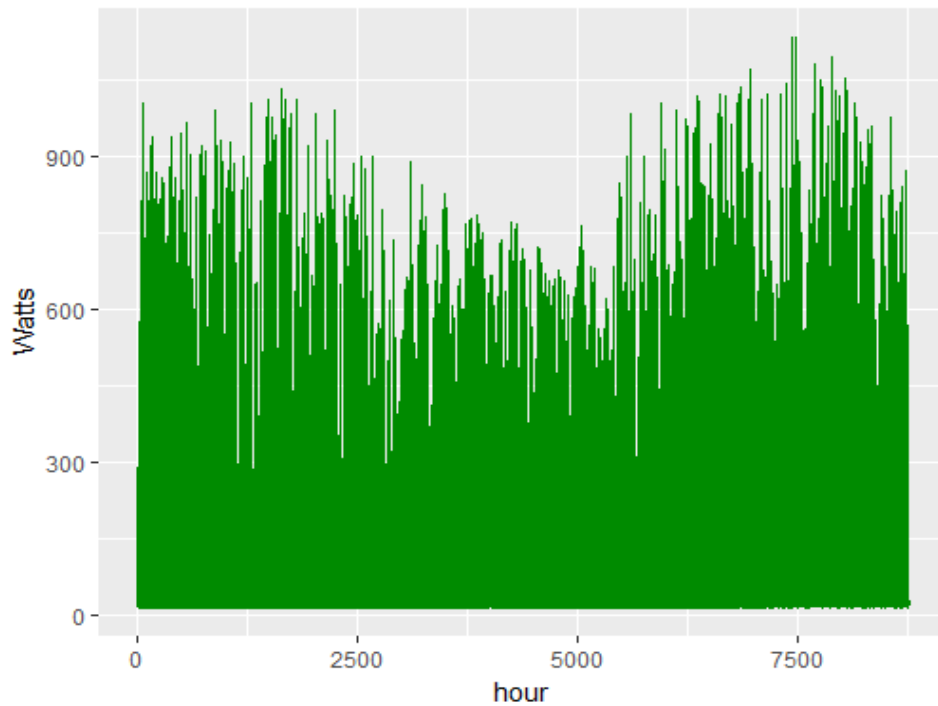


Figure A2: Bakokwe Insolation Data – January 1 to January 10, 2017

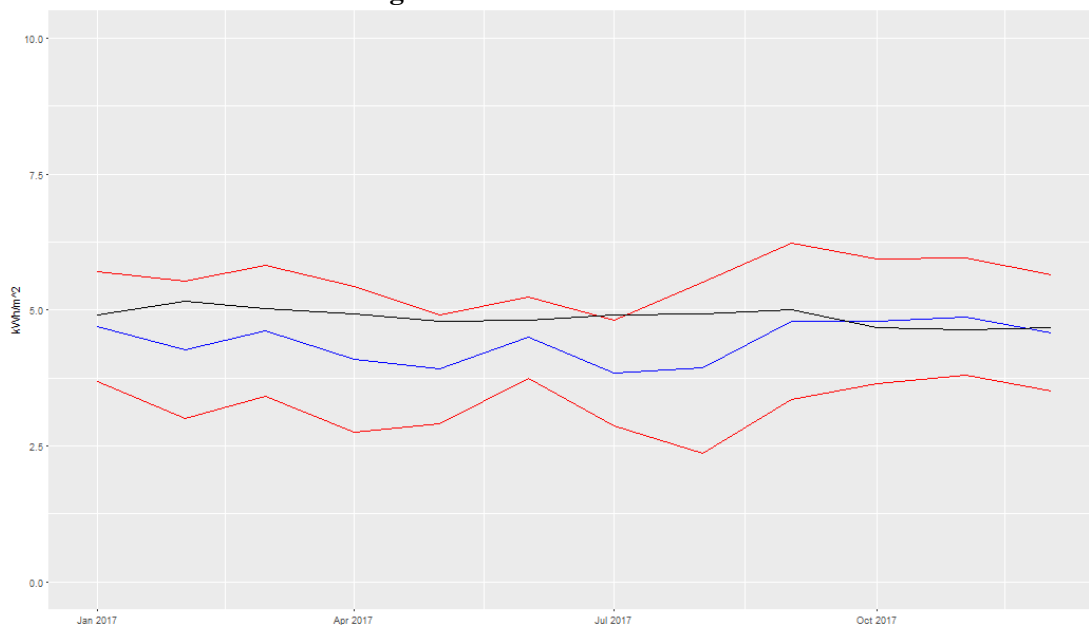


Note on validation of Bakokwe AWS: Average daily insolation by month was calculated in order to compare the data against other sources. Table 1 includes the daily average kWh per meter squared figures for AWS and NASA. These two series and AWS ± 1 standard deviation are shown graphically. Figure 3 shows this graphically. We feel pretty good about what this tells us about the validity of the data collected from the Bakokwe AWS.

Table A1: Verification with NASA⁵

| Month | Rwanda AWS | NASA | Difference |
|-------|------------|---------|------------|
| Jan | 4.694416 | 4.9 | -0.2056 |
| Feb | 4.271674 | 5.17 | -0.8983 |
| Mar | 4.616558 | 5.03 | -0.4134 |
| Apr | 4.101780 | 4.93 | -0.8282 |
| May | 3.915872 | 4.8 | -0.8841 |
| Jun | 4.495937 | 4.81 | -0.3141 |
| Jul | 3.848759 | 4.9 | -1.0512 |
| Aug | 3.930063 | 4.92 | -0.9899 |
| Sep | 4.791194 | 5 | -0.2088 |
| Oct | 4.793519 | 4.68 | 0.1135 |
| Nov | 4.876519 | 4.63 | 0.2465 |
| Dec | 4.576539 | 4.68 | -0.1035 |
| Mean | 4.409403 | 4.87083 | -0.46143 |

Figure A3: Verification with NASA



⁵ NASA Surface meteorology and Solar Energy - URL Parent: <https://eosweb.larc.nasa.gov/cgi-bin/sse/grid.cgi?email=luke.s@wildmail.com>
 URL: https://eosweb.larc.nasa.gov/cgi-bin/sse/grid.cgi?&num=210089&lat=-1.85&hgt=100&submit=Submit&veg=17&sitelev=&email=luke.s@wildmail.com&p=grid_id&step=2&lon=29.8