Supplement A: Descriptions of symbols

The description of each symbol appeared in Supplement B and Supplement C is list as below:

Table 1. Descriptions of symbols

Symbols	Descriptions
η	Free surface elevation
U_A, V_A	Vertical average velocity in x, y direction, respectively
U, V, W	Velocity in x, y, σ direction, respectively
D	Fluid column depth
f	The Coriolis parameter
g	The gravitational acceleration
ρ_0	Constant density
ρ	Situ density
T	Potential temperature
S	Salinity
R	Surface solar radiation incident
$q^2/2$	Turbulence kinetic energy
l	Turbulence length scale
$q^2l/2$	Production of turbulence kinetic energy and turbulence length scale
dti	Time step of baroclinic mode
dte	Time step of barotropic mode
dx	Grid increment in x direction
dy	Grid increment in y direction
A_M	Horizontal kinematic viscosity
A_H	Horizontal heat diffusivity
K_M	Vertical kinematic viscosity
K_H	Vertical mixing coefficient of heat and salinity
K_q	Vertical mixing coefficient of turbulence kinetic energy
tclim	Climatology of temperature
sclim	Climatology of salinity
rmean	Horizontal mean density field in z-coordinate

Supplement B: Continuous governing equations

The equations governing the baroclinic (internal) mode in GOMO are the three-dimention hydrostatic primitive equations.

$$\frac{\partial \eta}{\partial t}+\frac{\partial UD}{\partial x}+\frac{\partial VD}{\partial y}+\frac{\partial W}{\partial \sigma}=0 \tag{1}$$

$$\frac{\partial UD}{\partial t}+\frac{\partial U^2D}{\partial x}+\frac{\partial UVD}{\partial y}+\frac{\partial UW}{\partial \sigma}-fVD+gD\frac{\partial \eta}{\partial x}=\frac{\partial}{\partial \sigma}\left(\frac{K_M}{D}\frac{\partial U}{\partial \sigma}\right)+\frac{gD^2}{\rho_0}\frac{\partial}{\partial x}\int_\sigma^0 \rho' d\sigma'-\frac{gD}{\rho_0}\frac{\partial D}{\partial x}\int_\sigma^0 \sigma'\frac{\partial \rho'}{\partial \sigma'}d\sigma'+F_u \tag{2}$$

$$\frac{\partial VD}{\partial t}+\frac{\partial UVD}{\partial x}+\frac{\partial V^2D}{\partial y}+\frac{\partial VW}{\partial \sigma}+fUD+gD\frac{\partial \eta}{\partial y}=\frac{\partial}{\partial \sigma}\left(\frac{K_M}{D}\frac{\partial V}{\partial \sigma}\right)+\frac{gD^2}{\rho_0}\frac{\partial}{\partial y}\int_\sigma^0 \rho' d\sigma'-\frac{gD}{\rho_0}\frac{\partial D}{\partial y}\int_\sigma^0 \sigma'\frac{\partial \rho'}{\partial \sigma'}d\sigma'+F_v \tag{3}$$

$$\frac{\partial TD}{\partial t}+\frac{\partial TUD}{\partial x}+\frac{\partial TVD}{\partial y}+\frac{\partial TW}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left(K_H\frac{\partial T}{\partial \sigma}\right)+F_T+\frac{\partial R}{\partial \sigma} \tag{4}$$

$$\frac{\partial SD}{\partial t}+\frac{\partial SUD}{\partial x}+\frac{\partial SVD}{\partial y}+\frac{\partial SW}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left(K_H\frac{\partial S}{\partial \sigma}\right)+F_S \tag{5}$$

$$\rho=\rho(T,S,p) \tag{6}$$

$$\frac{\partial q^2D}{\partial t}+\frac{\partial Uq^2D}{\partial x}+\frac{\partial Vq^2D}{\partial y}+\frac{\partial Wq^2}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left(\frac{K_q}{D}\frac{\partial q^2}{\partial \sigma}\right)+\frac{2K_M}{D}\left[\left(\frac{\partial U}{\partial \sigma}\right)^2+\left(\frac{\partial V}{\partial \sigma}\right)^2\right]+\frac{2g}{\rho_0}K_H\frac{\partial \rho}{\partial \sigma}-\frac{2Dq^3}{B_1 l}+F_{q^2} \tag{7}$$

$$\frac{\partial q^2lD}{\partial t}+\frac{\partial Uq^2lD}{\partial x}+\frac{\partial Vq^2lD}{\partial y}+\frac{\partial Wq^2l}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left(\frac{K_q}{D}\frac{\partial q^2l}{\partial \sigma}\right)+E_1l\left\{\frac{K_M}{D}\left[\left(\frac{\partial U}{\partial \sigma}\right)^2+\left(\frac{\partial V}{\partial \sigma}\right)^2\right]+\frac{qE_3}{\rho_0}K_H\frac{\partial \rho}{\partial \sigma}\right\}-\frac{Dq^3}{B_1}\widetilde{W}+F_{q^2l} \tag{8}$$

Where F_u, F_v, F_{q^2}, and F_{q^2l} are horizontal kinematic viscosity terms of u, v, q^2, and q^2l, respectivly. F_T, F_S are horizontal diffusion terms of T, S, respectivly.

$$F_u=\frac{\partial}{\partial x}(2A_MD\frac{\partial U}{\partial x})+\frac{\partial}{\partial y}\left[A_MD(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x})\right] \tag{9}$$

$$F_v=\frac{\partial}{\partial y}(2A_MD\frac{\partial V}{\partial y})+\frac{\partial}{\partial x}\left[A_MD(\frac{\partial U}{\partial y}+\frac{\partial V}{\partial x})\right] \tag{10}$$

$$F_T=\frac{\partial}{\partial x}(A_HH\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(A_HH\frac{\partial T}{\partial y}) \tag{11}$$

$$F_S=\frac{\partial}{\partial x}(A_HH\frac{\partial S}{\partial x})+\frac{\partial}{\partial y}(A_HH\frac{\partial S}{\partial y}) \tag{12}$$

$$F_{q^2}=\frac{\partial}{\partial x}(A_MH\frac{\partial q^2}{\partial x})+\frac{\partial}{\partial y}(A_MH\frac{\partial q^2}{\partial y}) \tag{13}$$

$$F_{q^2l}=\frac{\partial}{\partial x}(A_MH\frac{\partial q^2l}{\partial x})+\frac{\partial}{\partial y}(A_MH\frac{\partial q^2l}{\partial y}) \tag{14}$$

The equations governing the barotropic (external) mode in GOMO are obtained by vertically integrating the baroclinic equations.

$$\frac{\partial \eta}{\partial t}+\frac{\partial U_AD}{\partial x}+\frac{\partial V_AD}{\partial y}=0 \tag{15}$$

$$\frac{\partial U_AD}{\partial t}+\frac{\partial (U_A)^2D}{\partial x}+\frac{\partial U_AV_AD}{\partial y}-fV_AD+gD\frac{\partial \eta}{\partial x}=\tilde{F}_{u_a}-wu(0)+wu(-1)-\frac{gD}{\rho_0}\int_{-1}^0\int_\sigma^0\left[D\frac{\partial \rho'}{\partial x}-\frac{\partial D}{\partial x}\sigma'\frac{\partial \rho'}{\partial \sigma}\right]d\sigma' d\sigma+G_{u_a} \tag{16}$$

$$\frac{\partial V_AD}{\partial t}+\frac{\partial U_AV_AD}{\partial y}+\frac{\partial (V_A)^2D}{\partial y}+fU_AD+gD\frac{\partial \eta}{\partial y}=\tilde{F}_{v_a}-wv(0)+wv(-1)-\frac{gD}{\rho_0}\int_{-1}^0\int_\sigma^0\left[D\frac{\partial \rho'}{\partial y}-\frac{\partial D}{\partial y}\sigma'\frac{\partial \rho'}{\partial \sigma}\right]d\sigma' d\sigma+G_{v_a} \tag{17}$$

Where $\tilde{F}_{u_a}$, $\tilde{F}_{v_a}$ are the horizontal kinematic viscosity terms of U_A, V_A, respectivly. G_{u_a}, G_{v_a} are the dispersion terms of U_A, V_A, respectivly. The subscript 'A' denotes vertical integration.

$$\tilde{F}_{u_a} = \frac{\partial}{\partial x}\left[2H(AA_M)\frac{\partial U_A}{\partial x}\right] + \frac{\partial}{\partial y}\left[H(AA_M)\left(\frac{\partial U_A}{\partial y} + \frac{\partial V_A}{\partial x}\right)\right] \tag{18}$$

$$\tilde{F}_{v_a} = \frac{\partial}{\partial y}\left[2H(AA_M)\frac{\partial V_A}{\partial y}\right] + \frac{\partial}{\partial x}\left[H(AA_M)\left(\frac{\partial U_A}{\partial y} + \frac{\partial V_A}{\partial x}\right)\right] \tag{19}$$

$$G_{u_a} = \frac{\partial^2 (U_A)^2 D}{\partial x^2} + \frac{\partial^2 U_A V_A D}{\partial x \partial y} - \tilde{F}_{u_a} - \frac{\partial^2 (U^2)_A D}{\partial x^2} - \frac{\partial^2 (UV)_A D}{\partial y^2} + (F_u)_A \tag{20}$$

$$G_{v_a} = \frac{\partial^2 U_A V_A D}{\partial x \partial y} + \frac{\partial^2 (V_A)^2 D}{\partial y^2} - \tilde{F}_{v_a} - \frac{\partial^2 (UV)_A D}{\partial x^2} - \frac{\partial^2 (V^2)_A D}{\partial y^2} + (F_v)_A \tag{21}$$

$$U_A = \int_{-1}^{0} U d\sigma \tag{22}$$

$$V_A = \int_{-1}^{0} V d\sigma \tag{23}$$

$$(U^2)_A = \int_{-1}^{0} U^2 d\sigma \tag{24}$$

$$(UV)_A = \int_{-1}^{0} UV d\sigma \tag{25}$$

$$(V^2)_A = \int_{-1}^{0} V^2 d\sigma \tag{26}$$

$$(F_u)_A = \int_{-1}^{0} F_u d\sigma \tag{27}$$

$$(F_v)_A = \int_{-1}^{0} F_v d\sigma \tag{28}$$

Supplement C: Discrete governing equations

The discrete governing equations of baroclinic (internal) mode expressed by operators are shown as below:

$$\frac{\eta^{t+1}-\eta^{t-1}}{2dti} + \delta^x_f(\overline{D}^x_b U) + \delta^y_f(\overline{D}^y_b V) + \delta^\sigma_f(W) = 0 \tag{29}$$

$$\frac{(\overline{D}^x_b U)^{t+1}-(\overline{D}^x_b U)^{t-1}}{2dti} + \delta^x_b\left[\overline{(\overline{D}^x_b U)}^x_f \overline{U}^x_f\right] + \delta^y_f\left[\overline{(\overline{D}^y_b V)}^x_b \overline{U}^y_b\right] + \delta^\sigma_f(\overline{W}^x_b \overline{U}^\sigma_b) - \overline{(\tilde{f}\overline{V}^y_f D)}^x_b - \overline{(f\overline{V}^y_f D)}^x_b + g\overline{D}^x_b \delta^x_b(\eta) = \delta^\sigma_b\left[\frac{\overline{K_M}^x_b}{(\overline{D}^x_b)^{t+1}}\delta^\sigma_f(U^{t+1})\right] + \frac{g(\overline{D}^x_b)^2}{\rho_0}\int_{\sigma_k}^0\left[\delta^x_b(\overline{\rho'}^\sigma_b) - \frac{\sigma\ \delta^x_b(D)}{\overline{D}^x_b}\delta^\sigma_b(\overline{\rho'}^x_b)\right]d\sigma + F_u \tag{30}$$

$$\frac{(\overline{D}^y_b V)^{t+1}-(\overline{D}^y_b V)^{t-1}}{2dti} + \delta^x_f\left[\overline{(\overline{D}^x_b U)}^y_b \overline{V}^x_b\right] + \delta^y_b\left[\overline{(\overline{D}^y_b V)}^y_f \overline{V}^y_f\right] + \delta^\sigma_f(\overline{W}^y_b \overline{V}^\sigma_b) + \overline{(\tilde{f}\overline{U}^x_f D)}^y_b + \overline{(f\overline{U}^x_f D)}^y_b + g\overline{D}^y_b \delta^y_b(\eta) = \delta^\sigma_b\left[\frac{\overline{K_M}^y_b}{(\overline{D}^y_b)^{t+1}}\delta^\sigma_f(V^{t+1})\right] + \frac{g(\overline{D}^y_b)^2}{\rho_0}\int_{\sigma_k}^0\left[\delta^y_b(\overline{\rho'}^\sigma_b) - \frac{\sigma\ \delta^y_b(D)}{\overline{D}^y_b}\delta^\sigma_b(\overline{\rho'}^y_b)\right]d\sigma + F_v \tag{31}$$

$$\frac{(TD)^{t+1}-(TD)^{t-1}}{2dti} + \delta^x_f(\overline{T}^x_b U\overline{D}^x_b) + \delta^y_f(\overline{T}^y_b V\overline{D}^y_b) + \delta^\sigma_f(\overline{T}^\sigma_b W) = \delta^\sigma_b\left[\frac{K_H}{D^{t+1}}\delta^\sigma_f(T^{t+1})\right] + F_T + \delta^\sigma_f R \tag{32}$$

$$\frac{(SD)^{t+1}-(SD)^{t-1}}{2dti} + \delta^x_f(\overline{S}^x_b U\overline{D}^x_b) + \delta^y_f(\overline{S}^y_b V\overline{D}^y_b) + \delta^\sigma_f(\overline{S}^\sigma_b W) = \delta^\sigma_b\left[\frac{K_H}{D^{t+1}}\delta^\sigma_f(S^{t+1})\right] + F_S \tag{33}$$

$$\rho = \rho(T,S,p) \tag{34}$$

$$\frac{(q^2D)^{t+1}-(q^2D)^{t-1}}{2dti} + \delta^x_f(\overline{U}^\sigma_b \overline{q^2}^x_b \overline{D}^x_b) + \delta^y_f(\overline{V}^\sigma_b \overline{q^2}^y_b \overline{D}^y_b) + \delta^\sigma_c(Wq^2) = \delta^\sigma_b\left[\frac{\overline{K_q}^\sigma_f}{D^{t+1}}\delta^\sigma_f(q^2)^{t+1}\right] + \frac{2K_M}{D}\left\{\left[\delta^\sigma_b(\overline{U}^x_f)\right]^2 + \left[\delta^\sigma_b(\overline{V}^y_f)\right]^2\right\} + \frac{2g}{\rho_0}K_H\delta^\sigma_b(\rho) - \frac{2Dq^3}{B_1 l} + F_{q^2} \tag{35}$$

$$\frac{(q^2lD)^{t+1}-(q^2lD)^{t-1}}{2dti} + \delta^x_f(\overline{U}^\sigma_b \overline{q^2 l}^x_b \overline{D}^x_b) + \delta^y_f(\overline{V}^\sigma_b \overline{q^2 l}^y_b \overline{D}^y_b) + \delta^\sigma_c(Wq^2l) = \delta^\sigma_b\left[\frac{\overline{K_q}^\sigma_f}{D^{t+1}}\delta^\sigma_f(q^2l)^{t+1}\right] + lE_1\frac{K_M}{D}\left\{\left[\delta^\sigma_b(\overline{U}^x_f)\right]^2 + \left[\delta^\sigma_b(\overline{V}^y_f)\right]^2\right\} + \frac{lE_1 g}{\rho_0}K_H\delta^\sigma_b(\rho) - \frac{Dq^3}{B_1}\left\{1 + E_2\left[\frac{l}{\kappa D}\left(\frac{-1}{\sigma} + \frac{1}{1+\sigma}\right)\right]^2\right\} + F_{q^2l} \tag{36}$$

Where F_u,F_v,F_{q^2}, and F_{q^2l} are horizontal kinematic viscosity terms of u, v, q^2, and q^2l, respectivly. F_T, F_S are horizontal diffusion terms of T, S, respectivly.

$$F_u = \delta^x_b\left[2A_M D\delta^x_f(U^{t-1})\right] + \delta^y_f\left\{\overline{(\overline{A_M}^x_b)}^y_b \overline{(\overline{D}^x_b)}^y_b\left[\delta^x_b(V)^{t-1} + \delta^y_b(U)^{t-1}\right]\right\} \tag{37}$$

$$F_v = \delta^y_b\left[2A_M D\delta^y_f(V^{t-1})\right] + \delta^x_f\left\{\overline{(\overline{A_M}^x_b)}^y_b \overline{(\overline{D}^x_b)}^y_b\left[\delta^x_b(V)^{t-1} + \delta^y_b(U)^{t-1}\right]\right\} \tag{38}$$

$$F_T = \delta^x_f\left[\overline{A_H}^x_b \overline{H}^x_b \delta^x_b(T^{t-1} - T_{CLIM})\right] + \delta^y_f\left[\overline{A_H}^y_b \overline{H}^y_b \delta^y_b(T^{t-1} - T_{CLIM})\right] \tag{39}$$

$$F_S = \delta^x_f\left[(\overline{A_H}^x_b \overline{H}^x_b \delta^x_b(S^{t-1} - S_{CLIM})\right] + \delta^y_f\left[\overline{A_H}^y_b \overline{H}^y_b \delta^y_b(S^{t-1} - S_{CLIM})\right] \tag{40}$$

$$F_{q^2} = \delta^x_f\left[\overline{(\overline{A_M}^x_b)}^\sigma_b \overline{H}^x_b \delta^x_b(q^2)^{t-1}\right] + \delta^y_f\left[\overline{\overline{A_M}^y_b}^\sigma_b \overline{H}^y_b \delta^y_b(q^2)^{t-1}\right] \tag{41}$$

$$F_{q^2l} = \delta^x_f\left[\overline{(\overline{A_M}^x_b)}^\sigma_b \overline{H}^x_b \delta^x_b(q^2l)^{t-1}\right] + \delta^y_f\left[\overline{\overline{A_M}^y_b}^\sigma_b \overline{H}^y_b \delta^y_b(q^2l)^{t-1}\right] \tag{42}$$

The discrete governing equations of barotropic (external) mode expressed by operators are shown as below:

$$\frac{\eta^{t+1}-\eta^{t-1}}{2dte} + \delta^x_f(\overline{D}^x_b\ U_A) + \delta^y_f(\overline{D}^y_b\ V_A) = 0 \tag{43}$$

$$\frac{(\overline{D}^x_b U_A)^{t+1}-(\overline{D}^x_b U_A)^{t-1}}{2dte} + \delta^x_b\left[\overline{(\overline{D}^x_b U_A)}^x_f \overline{(U_A)}^x_f\right] + \delta^y_f\left[\overline{(\overline{D}^y_b V_A)}^x_b \overline{(U_A)}^y_b\right] - \overline{\left[\tilde{f}_A \overline{(V_A)}^y_f D\right]}^x_b - \overline{\left[f\overline{(V_A)}^y_f D\right]}^x_b + g\overline{D}^x_b\delta^x_b(\eta) = \delta^x_b\left\{2(AA_M)D\delta^x_f\left[(U_A)^{t-1}\right]\right\} + \delta^y_f\left\{\overline{\left[\overline{(AA_M)}^x_b\right]}^y_b \overline{(\overline{D}^x_b)}^y_b\left[\delta^x_b(V_A) + \delta^y_b(U_A)\right]^{t-1}\right\} + \phi_x \tag{44}$$

$$\frac{(\overline{D}^y_b V_A)^{t+1}-(\overline{D}^y_b V_A)^{t-1}}{2dte} + \delta^x_f\left[\overline{(\overline{D}^x_b U_A)}^y_b \overline{(V_A)}^x_b\right] + \delta^y_b\left[\overline{(\overline{D}^y_b V_A)}^y_f \overline{(V_A)}^y_f\right] + \overline{\left[\tilde{f}_A \overline{(U_A)}^x_f D\right]}^y_b + \overline{\left[f\overline{(U_A)}^x_f D\right]}^y_b + g\overline{D}^y_b\delta^y_b(\eta) = \delta^y_b\left\{2(AA_M)D\delta^y_f\left[(V_A)^{t-1}\right]\right\} + \delta^x_f\left\{\overline{\left[\overline{(AA_M)}^x_b\right]}^y_b \overline{(\overline{D}^x_b)}^y_b\left[\delta^x_b(V_A) + \delta^y_b(U_A)\right]^{t-1}\right\} + \phi_y \tag{45}$$

where

$$\phi_x = -WU(0) + WU(-1) - \frac{g(\overline{D}_b^x)^2}{\rho_0} \int_{-1}^{0} \left\{ \left[\int_{\sigma}^{0} \delta_b^x \overline{(\rho'_{zz})}_b^{\sigma} d\sigma' \right] d\sigma \right\} + \frac{g\overline{D}^x \delta_b^x D}{\rho_0} \int_{-1}^{0} \left\{ \left[\int_{\sigma}^{0} \overline{\sigma}_b^{\sigma} \delta_b^{\sigma} (\overline{\rho'}_b^x) \right] d\sigma \right\} + G_x \tag{46}$$

$$\phi_y = -WV(0) + WV(-1) - \frac{g(\overline{D}_b^y)^2}{\rho_0} \int_{-1}^{0} \left\{ \left[\int_{\sigma}^{0} \delta_b^y \overline{(\rho'_{zz})}_b^{\sigma} d\sigma' \right] d\sigma \right\} + \frac{g\overline{D}^y \delta_b^y D}{\rho_0} \int_{-1}^{0} \left\{ \left[\int_{\sigma}^{0} \overline{\sigma}_b^{\sigma} \delta_b^{\sigma} (\overline{\rho'}_b^y) \right] d\sigma \right\} + G_y \tag{47}$$