Chapter 3-7 Test Material

5 Pieces of a Statistical Test

1. Null Hypothesis
2. Alternative Hypothesis } Hypotheses
3. Test Statistic
4. Rejection Region
5. Conclusion/interpretation

Confidence Intervals for Z's: $\bar{y} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

Test for μ, when σ is known

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: \mu \le \mu_0$ $H_1: \mu > \mu_0$		$Z_0 \ge Z_\alpha$
Left	$H_0: \mu \ge \mu_0$ $H_1: \mu < \mu_0$	$Z_0 = \frac{\bar{y}-\mu_0}{\sigma/\sqrt{n}}$	$Z_0 \le Z_\alpha$
Two	$H_0: \mu = \mu_0$ $H_1: \mu \ne \mu_0$		$\lvert Z_0 \rvert \ge Z_{\alpha/2}$

$(100-\alpha)\%$ CI for μ, when σ is unknown

$\bar{y} \pm t_{\alpha/2, df} \frac{s}{\sqrt{n}}$

Test for μ, when σ is unknown

Tail(s)	Hypothesis	Test Statistic	Rejection Region
Right	$H_0: \mu \le \mu_0$ $H_1: \mu > \mu_0$		$t_0 \ge t_\alpha$
Left	$H_0: \mu \ge \mu_0$ $H_1: \mu < \mu_0$	$t_0 = \frac{\bar{y}-\mu_0}{s/\sqrt{n}}$	$t \le t_\alpha$
Two	$H_0: \mu = \mu_0$ $H_1: \mu \ne \mu_0$		$\lvert t_0 \rvert \ge t_{\alpha/2}$

Test for $\mu_1 - \mu_2$ assuming independent samples & equal variances

Tail(s)	Hypothesis	Test Statistic	Rejection Region
Right	$H_0: \mu_1 - \mu_2 \le D$ $H_1: \mu_1 - \mu_2 \ge D$	$df = n_1 + n_2 - 2$	$t_0 \ge t_{\alpha, df}$
Left	$H_0: \mu_1 - \mu_2 \ge D$ $H_1: \mu_1 - \mu_2 < D$	$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - D}{s_p \sqrt{1/n_1 + 1/n_2}}$ Pooled st. dev. $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} \leftarrow df$	$t_0 \le t_{\alpha, df}$ (negative)
Two	$H_0: \mu_1 - \mu_2 = D$ $H_1: \mu_1 - \mu_2 \ne D$	D_0 = hypothesized difference	$\lvert t_0 \rvert \ge t_{\alpha/2, df}$

$(100-\alpha)\%$ C.I. for $\mu_1 - \mu_2$ assuming independent samples & $\sigma_1 = \sigma_2$

$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2, df} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

Satterwaite's Approximation - Use when $\sigma_1^2 \neq \sigma_2^2$ $\quad df = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$

D_0 = hypothesized difference

Test for $\mu_1 - \mu_2$ assuming independent samples & unequal variances

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: \mu_1 - \mu_2 \leq D_0$ $H_1: \mu_1 - \mu_2 > D_0$	$(100-\alpha)\%$ CI for $\mu_1 - \mu_2$ assuming samples for $\sigma_1^2 \neq \sigma_2^2$ $CI = (\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2, df} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$t_0 \geq t_{\alpha/2, df}$
Left	$H_0: \mu_1 - \mu_2 \geq D_0$ $H_1: \mu_1 - \mu_2 < D_0$	$t_0 = \dfrac{(\bar{y}_1 - \bar{y}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	$t_0 \leq t_{\alpha, df}$ (neg)
Two	$H_0: \mu_1 - \mu_2 = D_0$ $H_1: \mu_1 - \mu_2 \neq D_0$		$\lvert t_0 \rvert \geq t_{\alpha/2, df}$

Wilcoxon Rank Sum Test

Assumptions:

1. Independent Samp.
2. Pop. distributions same

Test for Two Independent Medians ($n_1 \leq 10$, $n_2 \leq 10$)

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: M_1 - M_2 \leq 0$ $H_1: M_1 - M_2 > 0$	$T_0 = \sum$ ranks of Sample 1	$T_0 > T_{upper}$
Left	$H_0: M_1 - M_2 \geq 0$ $H_1: M_1 - M_2 < 0$		$T_0 < T_{lower}$
Two	$H_0: M_1 - M_2 = 0$ $H_1: M_1 - M_2 \neq 0$		$T_0 \geq T_U$ or $T_0 < T_L$

Sampling Distribution

$\mu_T = \dfrac{n_1(n_1 + n_2 + 1)}{2}$

$\sigma_T^2 = \dfrac{n_1 n_2}{12}(n_1 + n_2 + 1)$

Notes for →

- $T_0 = \sum$ Ranks for Sample 1
- μ_T & σ_T^2 as defined Previously

Test for Two Independent Medians ($n_2 > 10$, $n_1 > 10$)

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: M_1 - M_2 \leq 0$ $H_1: M_1 - M_2 > 0$	$Z_0 = \dfrac{T_0 - \mu_T}{\sigma_T}$. If ties	$Z_0 \geq Z_\alpha$
Left	$H_0: M_1 - M_2 \geq 0$ $H_1: M_2 - M_2 < 0$	$\sigma_T^2 = \dfrac{n_1 n_2}{12}\left[(n_1 + n_2 + 1) - \dfrac{\sum_{j=1}^{k} t_j(t_j^2 - 1)}{(n_1 + n_2)(n_1 + n_2 - 1)}\right]$	$Z_0 \leq -Z_\alpha$
Two	$H_0: M_1 - M_2 = 0$ $H_1: M_2 - M_2 \neq 0$		$\lvert Z_0 \rvert \geq Z_{\alpha/2}$

$(100-\alpha)\%$ around $\mu_d = \mu_1 - \mu_2 = \bar{d} \pm t_{\alpha/2, df} \dfrac{s_d}{\sqrt{n}}$,

Notes: $\bar{d}$ is mean of d_i's, s_d is std. dev of d_i's, n is # of obs. $df = n - 1$

Test for $\mu_1 - \mu_2$ for Paired Data → Assumptions:

1) Samp. dist of d_i's are normal
2) d_i's are independent

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: \mu_1 - \mu_2 \leq \mu_d$ $H_1: \mu_1 - \mu_2 > \mu_d$	$\mu_d = \mu_1 - \mu_2$	$t_0 \geq t_{\alpha, df}$
Left	$H_0: \mu_1 - \mu_2 \geq \mu_d$ $H_1: \mu_1 - \mu_2 < \mu_d$	$t_0 = \dfrac{\bar{d} - \mu_0}{s_d/\sqrt{n}}$	$t_0 \leq -t_{\alpha, df}$
Two	$H_0: \mu_1 - \mu_2 = \mu_d$ $H_1: \mu_1 - \mu_2 \neq \mu_d$		$\lvert t_0 \rvert \geq t_{\alpha/2, df}$

NonParametric of Paired Data T-test

Assumptions:

1) Population of d_i's be Symmetric

M = median of pop of diffs.

The Wilcoxon Signed Rank Test $n > 50$ (after deleting Zero Values)

Tail(s)	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: M \leq d_0$ $H_1: M > d_0$	$Z_0 = \dfrac{T_0 - \mu_T}{\sigma_T}$	$Z_0 \geq Z_\alpha$
Left	$H_0: M \geq d_0$ $H_1: M < d_0$		$Z_0 \leq Z_\alpha$
Two	$H_0: M = d_0$ $H_1: M \neq d_0$		$\lvert Z_0 \rvert \geq Z_{\alpha/2}$

- D_0 = hypothesized diffs
- T_- = sum of neg ranks
- T_+ = sum of pos. ranks
- T_α = Crit. val in Tbl 6
- $\mu_T = \dfrac{n(n+1)}{4}$

The Wilcoxon signed Rank Test $n \leq 50$ (after deleting Zero Values)

Tails	Hypotheses	Test Statistic	Rejection Region
Right	$H_0: M \leq D_0$ $H_1: M > D_0$	$T_0 = T_-$	$Z_0 \geq Z_\alpha$
Left	$H_0: M \geq D_0$ $H_1: M < D_0$	$T_0 = T_+$	$Z_0 \leq Z_\alpha$
Two	$H_0: M = D_0$ $H_1: M \neq D_0$	$T_0 = \min(T_-, T_+)$	$\lvert Z_0 \rvert \geq Z_{\alpha/2}$

σ_T^2 when we have ties $= \frac{1}{24}\left[n(n+1)(2n+1) - \frac{1}{2}\sum_j t_j(t_j - 1)(t_j + 1)\right]$ $\quad\Big|\quad$ σ_T^2 when ~~ties~~ No Ties $= \sqrt{\dfrac{n(n+1)(2n+1)}{24}}$