3)

Batch 1	Batch 2	Batch 3	Batch 4	Batch 5
23.46	23.59	23.51	23.28	23.29
23.48	23.46	23.64	23.40	23.46
23.56	23.42	23.46	23.37	23.37
23.39	23.49	23.52	23.46	23.32
23.40	23.50	23.49	23.39	23.38

a) Is this a random effects model? Why?

This is a random effects model. There is no deterministic decision made by the statistician in selecting the batches. If they were to take a bunch of batches from the night shift, this would be a fixed effects model where all results can only be translated to the results of night shifts. With them being randomly selected, this is a random effects model.

b) Is there a significant variation in calcium content from batch to batch? Use $\alpha = 0.05$.

Source	SS	df	MS	F	P-value
Treatment	0.14	4	0.035	17.5	2.5779×10^{-6}
Error	0.04	20	0.002		
Total	0.18	24			

$$SS_{Trt} = \frac{1}{n}\sum_{1}^{a} y_{i\cdot}^2 - \frac{(y_{\cdot\cdot})^2}{N}$$
$$= \frac{1}{5}(68{,}701) - \frac{343501.5}{25}$$
$$= 13740.2 - 13740.06$$
$$= 0.14$$

$$SS_{Tot} = \sum_{1}^{a}\sum_{1}^{n} y_{ij}^2 - \frac{(y_{\cdot\cdot})^2}{N}$$
$$= 13740.24 - \frac{343501.5}{25}$$
$$= 13{,}740.24 - 13740.06$$

With a p-value less than 0.05, there is evidence to reject the null hypothesis that all the means are the same. There is evidence to suggest that at least one mean is different.

c) Estimate the variance components for this model.

$$\hat{\sigma}^2 = MS_E = 0.002$$

$$\hat{\sigma}_\tau^2 = \frac{MS_{Trt} - MS_E}{n} = \frac{0.035 - 0.002}{5} = \frac{0.033}{5} = 0.0066$$